Cooperative Beamforming in Ad-Hoc Networks with Sublinear Transmission Power

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***Abstract*—The efficiency of routing algorithms in ad-hoc networks is measured by delay, throughput, and energy. Here, we focus on routing algorithms optimizing energy consumption while providing small routing delay. For this, we exploit the sender beamforming gain in the line-of-sight path-loss model, where multiple nodes (each with a single antenna) cooperate for beamforming and the routing algorithms provide distributed self-synchronization. While direct point-to-point communication over distance d in the line-of-sight model needs transmission power $\Theta\left(d^2\right)$, and multi-hop power needs power $\Theta\left(d\right)$ and delay $\Theta\left(d\right)$, we can reduce the power to $\Theta(\sqrt{d})$ or $\Theta\left(\log d\right)$ depending on the geometry. We present three algorithms with different trade-offs. The first algorithm is designed for grid nodes in the plane and has a point-to-point delay of $\Theta\left(\log d\right)$ and overall power consumption of $\Theta(\sqrt{d})$. The second algorithm for the same geometry decreases the delay to $\Theta\left(\frac{1}{\varepsilon}\log\log d\right)$ with power $\Theta((\sqrt{d})^{1+\varepsilon})$ for $\varepsilon > 0$. The third algorithm requires a three-dimensional grid network and achieves a delay of $\Theta\left(\log d\right)$ and reduces the energy needed by all nodes to $\Theta\left(\log d\right)$.**

***Index Terms*—Ad-hoc networks, wireless communication, beamforming, MIMO, line-of-sight, energy consumption**

I. Introduction

In mobile networks most of the energy at the nodes is consumed by radio transmission. Compared to centralized network structures ad-hoc networks allow a reduction of this factor, since shorter distances can reduce the transmission energy. While in practice this factor is limited, since there is some base power consumption at the electronic devices, in theory this effect favors long paths with very small hops over shorter paths with long hops [3]. Furthermore, tradeoffs between energy, delay, and network throughput can be observed. While this geometric based network model models connections as point-to-point connections, it turns out that in reality radio communication is better modeled by an analog continuous model, where effects like superposition and the impact of a channel matrix reflecting the environment are considered. While this model is much more involved, it allows the design of modern communication schemes like MIMO (multiple input multiple output), which is the basis of the last recent wireless communication standards.

So, the network algorithm community has gained interest in exploring the possibility offered by MIMO communications. Most of this research is concerned with the astonishing improvements concerning network throughput (e.g. [10]) and increased communication range. Here, we will turn our focus towards possible energy savings, where we start with theoretical considerations towards beamforming in ad-hoc networks, i.e. when individual radio nodes cooperate in order to form a super-positioned signal with increased strength towards a specified direction.

Beamforming can increase the transmission range or can be used to reduce transmission power. This is reached by combining the radio power of multiple antennas forming so called beams in contrast to the emission of an omni-directional radiation pattern with equal power in all directions. The setup of such a radio beam can be energy consuming [9]. This is especially the case if multiple antennas are not connected to the same device but multiple devices collaborate with their single antenna for collaborative beamforming. The nodes have to exchange the data which should be transmitted via beamforming from all devices and they have to synchronize, e.g in an open-loop, in order to coordinate phase-synchronization at the receiving target device within the beam.

On-the-fly phase-synchronization is one key future of our unicast algorithm for ad-hoc networks in [7]. The basic idea of that unicast algorithm is to transmit a message from the source to the destination in a multi-hop strategy and let multiple nodes in each hop collaborate for sender beamforming to increase the transmission range. We show that $\Theta\left(\log\log d\right)$ routing hops are sufficient for a point-to-point communication with distance d, although the transmission power of each node is restricted such that it can only reach the next neighbors without the help of other nodes. In this paper, we optimize these algorithms for energy efficiency. In particular, we present three algorithms, from which two algorithms are designed for nodes placed in the plane (see Section IV,V) and the third one for nodes placed in three-dimensional space providing different trade-offs between routing speed and energy consumption (see Section VI).

The presentation of the three algorithm in Sections IV-VI is preceded by a survey of related work in Section II, which also contains a brief description of the underlying unicast algorithm of [7]. This is followed by the model for wireless communication in Section III which we use in the analysis of the algorithms. We give a final conclusion in Section VII.

II. Related Work

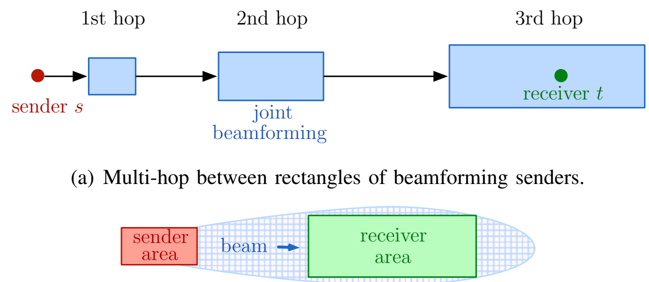
In [9] the challenges of low-power in wireless sensor networks are addressed. Sensor networks are a special case where mobile nodes are assumed to function over years without external power supply. The authors analyze in detail the energy consumption for communication consisting of software and hardware. They consider the power consumption of wireless hardware for the start-up from sleep mode and the transmission power, where the start-up energy is significantly high for short packets.

Jayaweera compares in [8] the energy consumption of a 2×1 MISO system, i.e. two antennas for sender beamforming and one receiver antenna to a single antennas communication (SISO). They observe that a variable data rate of M-QAM for different transmission distances can considerably improve the the performance of the system and thus reduce the energy consumption. He also analyses cooperative beamforming, where the data is distributed between nodes in a local area which then perform cooperative beamforming. For two beamforming sender, he shows that the energy consumption is halved.

De Freitas et al. [1] use MIMO techniques in wireless sensor networks to reduce the energy consumption. They differentiate between energy consumption for sending and receiving in a transmission and state in which cases multi-hop routing outperforms single-hop direct communication. They propose to use either cooperative MIMO between clusters of sensors or cooperative beamforming for receiving (SIMO) or sending (MISO). They present simulations where they compare single-hop, multi-hop, and communication with MIMO techniques and conclude that MIMO techniques are advantageous over multi-hop when data is sensitive to delay and cooperative MIMO techniques are more energy efficient for more than four hops.

The authors of [2] propose a solution for energy-efficient communication over long distances by using collaborative beamforming. Their cross layer approach coordinates several nodes on the MAC layer for cooperative beamforming on the physical layer. In a two phases protocol, they first spread the information in a local area followed by a second phase where the receivers repeat the received analog signal with adjusted phase and amplification for cooperative beamforming. Synchronization is reached by a central solution, where a selected node plays the role of a cluster head, which synchronizes the beamforming senders in the local area. Using the known positions of the collaborating nodes relatively to the cluster head, the nodes can set up collaborative beamforming in an open-loop approach. This approach also allows to send several messages at the same time using the same cooperative beamforming nodes to different directions. Rayleigh fading is used their and sender beamforming is optimized in an open-loop approach to a single destination. Our approach differs here, since we assume that the signal propagation in a certain area is homogenous according the line-of-sight model, i.e. reception delay corresponds to the distance to the sending cluster and we assume that we can synchronize using the reception times and delays.

The algorithms, presented here, advance our techniques presented in [7] and its corresponding technical report [6]. There, we present two unicast algorithms with focus on the transmission delay $\Theta\left(\log\log d\right)$ for distance d between sender and receiver. The sum of all nodes' transmission power is $\Theta\left(d\right)$. Both algorithms are based on multi-hop communication where in each each hop the information is transmitted from a rectangular area of beamforming senders to a rectangular area of multiple receivers, see Figure 1. The number of nodes



(a) Multi-hop between rectangles of beamforming senders.

(b) Beamforming from sender to receiver rectangle

Fig. 1. Scheme of the $\mathcal{O}(\log\log n)$-Unicast algorithm [6].

in the rectangular area of successive hops grows in each round, increasing the beamforming gain. In turn, the higher beamforming gain enables higher reception range and results in a speed-up resulting in only $\Theta\left(\log\log n\right)$ hops. Cooperative sender beamforming, i.e. when distributed nodes send phase-synchronized the same signal, requires distributed synchronization. Multiple receivers in a rectangular area can deduce their phase synchronization from the reception time alone. The key argument is that the direction of the propagation of receiving a message in the previous round and resending in the current round is nearly the same. If all receivers send the information in the next hop with the same delay, then the synchronization allows the desired beamforming gain. In the

Algorithm 1 Unicast of [6]

```
procedure RECEIVE(receiver r, message m, time t)
    if ISINRECTANGLE(round (t), r) then        ▷ active nodes
        WAIT(ψ (round (t) , r))                 ▷ phase correction
        SEND(m)              ▷ coordinated beamforming sending
```

first algorithm, the receiver nodes have to correct phases by a small delay $\psi\left(i, \mathbf{r}\right)$ which can be computed from the the receiver position $\mathbf{r}$ and the round number i. This correction is necessary when the node is located apart from the direct line between source and sink. In the second algorithm, we avoid this phase correction and each node simply synchronizes itself using the reception time. This only results in a constant increase of the running time.

III. The Physical Model

The algorithms of this work are based on the following model for signal transmission over a communication channel[1]. For analyzing sender beamforming, a channel includes m

[1]We use the same model in [7] extended by considering the effect of polarization for the three-dimensional case.

transmitters which simultaneously emit a signal, i.e. the i-th sender emits signal x_i. Following [11], the signal output y at a receiver depends on the signal inputs $x_1, \ldots, x_m$ of the senders as

$$y = \sum_{i=1}^{m} h_i \cdot x_i \ . \tag{1}$$

We assume that all nodes emit the same input signal $x = x_i$ with the same transmission power but with a time shift in order to phase-synchronize the signals at the target with output y which enables beamforming gain. We assume that data is modulated on a single carrier with frequency f_c respectively wavelength λ, e.g. via QAM (quadrature amplitude modulation).

Realistic channel models cover many effects, e.g. multi-path propagation, diffraction, changing environment, node movement, etc.. In this work, we focus on the line-of-sight model which solely describes the signal transmission on the direct path from transmitter to receiver. Thus, the baseband channel gain h_i for the i-th sender node is

$$h_i \quad = \quad \frac{1}{\|\mathbf{u_i}, \mathbf{v}\|} \cdot e^{-\frac{j2\pi}{\lambda} \cdot \|\mathbf{u_i}, \mathbf{v}\|} \ . \tag{2}$$

This involves an attenuation factor $\|\mathbf{u_i}, \mathbf{v}\|^{-1}$ due to path-loss for distance $\|\mathbf{u_i}, \mathbf{v}\|$ between the i-th transmitter position u_i and a receiver position v. Since the power is proportional to the square of the signal strength this corresponds to the standard energy path-loss model for line-of-sight and the far-field assumption with $\|\mathbf{u_i}, \mathbf{v}\| > 2\lambda$ where the energy decreases proportional to $\|\mathbf{u_i}, \mathbf{v}\|^{-2}$. The signal propagation from a sender to a receiver with speed of light c leads to signal delay $\|\mathbf{u_i}, \mathbf{v}\| / c$. For a given carrier frequency $\lambda = f/c$, this results in a phase shift $\gamma = \left(\frac{2\pi}{\lambda} \cdot \|\mathbf{u_i}, \mathbf{v}\|\right)$. Hereby, $j = \sqrt{-1}$ denotes the imaginary unit in the complex phase shift $e^{-j\gamma}$.

To compensate the phase shift in the channel, the sender nodes in our model can delay the signal resulting in a phase shift ϕ_i. For maximum beamforming gain, the phase shift of the channel γ_i is compensated by the delay of the i-th sender such that $e^{-j\gamma_i} \cdot e^{j\phi_i} = 0$ for all $1 \leq i \leq m$ which is the case for $\gamma_i = \phi_i$.

So, the line-of-sight model with stationary nodes is the most powerful model to enable beamforming gain, since the beamforming only depends on the geometry of the nodes and the setup of delays at the senders in the non-fading channel and may be considered as upper bound what is possible with beamforming gain.

We will limit our algorithms to phase shifts between the m senders less than one wave, i.e. $|\gamma_i - \gamma_k| \leq 2\pi$ for senders i and k. Thus, intersymbol interference is not expected and correlation at the receiver over a symbol frame will be proportional to the analysis of an infinite long carrier $e^{j2\pi f \cdot t}$. Thus, it suffices to analyze the input-output equation

$$h = \sum_{i=1}^{m} h_i \cdot e^{j\phi_i} \ . \tag{3}$$

In [4] we show that the width of the main beam is determined by the sender geometry and the wavelength. We also estimate position and strengths of the side beams for randomized positions of the senders.

As usual, interfering signals and analog errors are modeled as additive white Gaussian noise $w \sim \mathcal{N}\left(0, \sigma^2\right)$ with variance σ^2. The received signal power at the receiver is modeled by $P = |y|^2$. We model that a signal can be received if the signal to noise ratio is larger than a threshold τ, i.e. $\text{SNR} = \frac{P}{N} \geq \tau$, where N is the energy of the noise. Since we are only interested in the asymptotic behavior, we choose $\tau = 1$.

We do not restrict the individual transmission power of each node, but try to minimize the sum of all nodes' transmission power during the unicast of a message. All nodes are placed in a unit distance grid in two or three dimensions.

The goal of the following unicast algorithms is to save the overall transmission energy. There are three approaches. Approach one and two are an invariant of the original unicast but with smaller growth of the rectangles. This increases the transmission delay but decreases the transmission energy from $\Theta(d)$ in the original unicast down to $\Theta(\sqrt{d})$. The first approach will have logarithmic transmission delay whereby the second algorithm has double-logarithmic delay. The third approach is an extension of the algorithm in three-dimensions where the beamforming senders are in a cuboid with the dimensions $w \times h \times b$. Here, logarithmic energy for a unicast operation is possible, i.e. $\Theta(\log d)$ for distance d.

IV. Unicast I with Delay $\Theta(\log d)$ and Transmission Energy $\Theta(\sqrt{d})$

The routing algorithm Unicast I consists of $\log_b n$ subsequent multi-hop steps (see Fig. 1(a)) for some $b > 1$. The i-th multi-hop step is performed with sender beamforming from an array of coordinated senders to an array of coordinated receivers (see Figure 2). The topology of each array performing

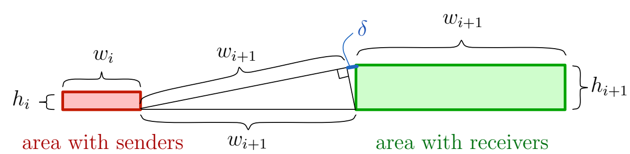


Fig. 2. Unicast step with sender beamforming from a $(w_i \times h_i)$ sender array to a $(w_{i+1} \times h_{i+1})$ receiver array in distance w_{i+1}

sender beamforming is a rectangle. The dimensions $(w_i \times h_i)$ of each array of the i-th hop with width w_i, height h_i, and transmission power p_i of each node are chosen as stated in Lemma 1.

Lemma 1 **[6]** *Sender nodes in a rectangular area $w_i \times h_i$ can reach with cooperative sender beamforming and transmission power p_i per node all nodes in rectangle in distance w_{i+1} and dimensions $w_{i+1} \times h_{i+1}$ if the following inequalities are* *satisfied.*

$$
\begin{aligned}
h_{i+1}^2 &\leq \frac{1}{4}\lambda \cdot w_{i+1} \ , && (4)\\
w_{i+1} &\leq \frac{1}{3\sqrt{2}} h_i \cdot w_i \cdot \sqrt{p_i} \ , && (5)\\
w_{i+1} &\geq q \cdot w_i \ , && (6)\\
h_i &\leq w_i \ , && (7)\\
p_i &\geq 1 \ . && (8)
\end{aligned}
$$

Besides a successful transmission to all nodes in the $w_{i+1} \times h_{i+1}$ rectangular area we also like to synchronize the phases of these nodes for the successive next hop of sender beamforming. Setting the ratio of width w_{i+1} and height h_{i+1} according to Equation (4) makes sure that all nodes have a phase-synchronization error less than $\frac{\pi}{4}$ for beam-formed sending to the target, i.e. if all nodes repeat the received message after a fixed delay then the phase shift in the super-positioned signal is less than $\frac{\pi}{4}$ at the target position. Equation (5) ensures that the reception range $3w_{i+1}$ of the beamforming senders is large enough to reach all nodes in the receiver array. The proof of Equations (4-5) can be found in [6], Lemma 1 and 2.

In Equation (6), factor q is the base of the exponential speed-up, i.e. progress $\Theta\left(q^i\right)$ in round i. The power is limited by a constant in Equation (8), i.e. a node can reach all next neighbors.

Theorem 1 *The rectangular area $w_i \times h_i$ in round i with power p_i per node is*

$$
\begin{aligned}
w_i &= q^i \cdot w_0 \ ,\\
h_i &= h_0 \cdot (\sqrt{q})^i \ \textit{with } h_0 = \sqrt{\frac{w_0 \cdot \lambda}{4}} \ ,\\
p_i &\geq \frac{72 \cdot q^2}{w_0 \cdot \lambda} \cdot q^{-i} \ .
\end{aligned}
$$

The communication of Unicast I of distance d and $\log_q d$ hops needs an overall transmission power of $\Theta\left(\sqrt{d/\lambda}\right)$.

Proof: The width w_i can be derived from (6) with start rectangle width w_0. The height of the rectangles is derived from rearranging (6).

$$
h_i = \sqrt{\frac{1}{4}\lambda w_i} = h_0 \cdot (\sqrt{q})^i \ \text{with } h_0 = \sqrt{\frac{w_0 \cdot \lambda}{4}}
$$

The reception range in round i is

$$
w_{i+1} \leq \frac{h_0 w_0}{3\sqrt{2}} \cdot q^{\frac{3}{2}i} \cdot \sqrt{p_i} = \frac{w_0^{\frac{3}{2}}\sqrt{\lambda}}{3\sqrt{8}} \cdot q^{\frac{3}{2}i} \cdot \sqrt{p_i} \ .
$$

Inserting inequality (6) gives for the power p_i of a sender node in round i:

$$
\begin{aligned}
q \cdot \underbrace{q^i \cdot w_0}_{w_i} &\leq \frac{w_0^{\frac{3}{2}}\sqrt{\lambda}}{3\sqrt{8}} \cdot q^{\frac{3}{2}i} \cdot \sqrt{p_i}\\
\sqrt{p_i} &\geq \frac{q \cdot 3\sqrt{8}}{\sqrt{w_0 \cdot \lambda}} \cdot q^{-i/2}\\
p_i &\geq \frac{72 \cdot q^2}{w_0 \cdot \lambda} \cdot q^{-i} \ .
\end{aligned}
$$

The power of all nodes involved in a unicast operation is then

$$
\begin{aligned}
P_{\text{tx,I}} &= \sum_{i=0}^{\lceil \log_q d \rceil - 1} h_i \cdot w_i \cdot p_i\\
&= \frac{36 \cdot q^2 \cdot \sqrt{w_0}}{\sqrt{\lambda}} \cdot \sum_{i=0}^{\lceil \log_q d \rceil - 1} (\sqrt{q})^i\\
&= \frac{36 \cdot q^2 \cdot \sqrt{w_0}}{\sqrt{\lambda}} \cdot \frac{\sqrt{d} - 1}{\sqrt{q} - 1}\\
&= \frac{36 \cdot q^2 \cdot \sqrt{w_0}}{\sqrt{\lambda} \cdot (\sqrt{q} - 1)} \cdot \left(\sqrt{d} - q^{-1/2}\right)\\
&= \Theta\left(\sqrt{d/\lambda}\right) \ .
\end{aligned}
$$

■

So, the energy consumption to transmit a message over distance d is $\Theta\left(\sqrt{d/\lambda}\right)$ and the transmission delay is $\Theta\left(\log_b d\right)$.

Corollary 1 *In an ad-hoc network with n nodes placed in a $\sqrt{n} \times \sqrt{n}$ grid, the energy consumption of Unicast I is $\Theta\left(\sqrt[4]{n}/\sqrt{\lambda}\right)$ with transmission delay $\Theta\left(\log_b n\right)$.*

The number of nodes N_1, involved in the execution of Unicast I, is bounded by the following.

$$
\begin{aligned}
N_1 &= \sum_{i=0}^{\lceil \log_q d \rceil - 1} h_i \cdot w_i\\
&= \frac{\sqrt{\lambda} \cdot w_0^{3/2}}{2} \sum_{i=0}^{\lceil \log_q d \rceil - 1} \left(q^{3/2}\right)^i\\
&= \frac{\sqrt{\lambda} \cdot w_0^{3/2}}{2} \cdot \frac{\left(q^{3/2}\right)^{\log_q d} - 1}{q^{3/2} - 1}\\
&= \frac{\sqrt{\lambda} \cdot w_0^{3/2}}{2} \cdot \frac{d^{3/2} - 1}{q^{3/2} - 1}\\
&= \Theta\left(\sqrt{\lambda} \cdot d^{3/2}\right)
\end{aligned}
$$

We neglect energy consumption for standby, but possibly have to consider the energy consumption by p_{rx} for signal-processing at each receiver node. This energy consumption of all nodes involved in the operation is then

$$
P_{\text{rx,I}} = N_1 \cdot p_{\text{rx}} = \Theta\left(\sqrt{\lambda} \cdot d^{3/2}\right) \ .
$$

There are different constant factors involved for the transmission power and the receiving power, and only if the reception power can be neglected compared to the transmission power, this unicast algorithm makes energy-wise sense.

V. Unicast II with Delay $\Theta(\log\log d)$ and Transmission Energy $\Theta(\sqrt{d})$

In Unicast I, we slowed down the transmission delay of the algorithm of [6] from $\Theta(\log\log d)$ to $\Theta(\log d)$ in order to reduce the total energy consumption of the unicast. In the following algorithm Unicast II, we combine small transmission delay $\Theta\left(\log_{1+\varepsilon}\log_{w_0} d\right)$ and small transmission power $\Theta(\sqrt{d})$. Yet, note that for a small ε it holds

$$\log_{1+\varepsilon} x = \frac{\log x}{\log(1+\varepsilon)} \geq \frac{\log x}{\varepsilon} .$$

So, for small $\varepsilon \to 0$, $\frac{1}{\varepsilon}$ will be the dominating factor which slows the delay down.

Lemma 2 *A $w_i \times h_i$ rectangle of beamforming senders can reach all senders in an exponentially larger rectangle $w_{i+1} \times h_{i+1}$, i.e. the areas have sizes $A_{i+1} \geq A_{i+1}^{\alpha}$ for $\alpha > 1$, if it holds for the rectangles*

$$h_{i+1}^2 \leq \frac{1}{4}\lambda \cdot w_{i+1} \quad (9)$$

$$w_{i+1} \leq \frac{1}{3\sqrt{2}} h_i \cdot w_i \cdot \sqrt{p_i} \quad (10)$$

$$w_{i+1} \geq w_i^{\alpha} \quad (11)$$

$$h_i \leq w_i \quad (12)$$

$$p_i \geq 1 \quad (13)$$

The rectangular area of nodes has dimensions

$$\begin{aligned} w_i &= (w_0)^{\alpha^i} \text{ and} \\ h_i &\leq \frac{\sqrt{\lambda}}{2} \cdot (\sqrt{w_0})^{\alpha^i} . \end{aligned}$$

Proof: The recursion of the i-th height of the rectangular area can be solved as follows.

$$\begin{aligned} h_{i+1}^2 &\leq \frac{1}{4}\lambda \cdot w_{i+1} = \frac{\lambda}{4} \cdot (w_0)^{\alpha^{i+1}} \\ h_i &\leq \frac{\sqrt{\lambda}}{2} \cdot (\sqrt{w_0})^{\alpha^i} \end{aligned}$$

■

This Lemma helps us to prove the following claim.

Theorem 2 *The delay of a point-to-point communication of distance d with Unicast II is $\Theta\left(\log_\alpha \log_{w_0} d\right)$. The corresponding transmission power is $\Theta\left(d^{\frac{1}{2}+2\varepsilon}\right)$. The energy consumption for signal processing of all nodes in a communication is $\Theta\left(\sqrt{\lambda} \cdot d^{3/2}\right)$.*

Proof: To reach a rectangle of size at least d we need k rounds.

$$\begin{aligned} d &= \left\lfloor (w_0)^{\alpha^k} \right\rfloor \\ k &= \left\lfloor \log_\alpha \log_{w_0} d \right\rfloor \end{aligned}$$

The power p_i of each sender in the i-th rectangle is the following.

$$\begin{aligned} \sqrt{p_i} &\geq 3\sqrt{2} \cdot \frac{w_{i+1}}{h_i \cdot w_i} \\ &= 3\sqrt{2} \cdot \frac{(w_0)^{\alpha^{i+1}}}{\frac{\sqrt{\lambda}}{2} \cdot \left(\sqrt{w_0}\right)^{\alpha^i} \cdot (w_0)^{\alpha^i}} \\ &= \frac{6\sqrt{2}}{\sqrt{\lambda}} \cdot (w_0)^{\alpha^{i+1} - \frac{\alpha^i}{2} - \alpha^i} \\ &= \frac{6\sqrt{2}}{\sqrt{\lambda}} \cdot \left(w_0^{\alpha - \frac{3}{2}}\right)^{\alpha^i} , \\ p_i &\geq \frac{72}{\lambda} \cdot \left(w_0^{2\alpha-3}\right)^{\alpha^i} . \end{aligned}$$

The energy consumption in order to reach distance d is:

$$\begin{aligned} P_{\text{tx,II}} &= \sum_{i=0}^{\lfloor \log_\alpha \log_{w_0} d \rfloor} w_i \cdot h_i \cdot p_i \\ &\leq \sum_{i=0}^{\lfloor \log_\alpha \log_{w_0} d \rfloor} \underbrace{(w_0)^{\alpha^i}}_{w_i} \cdot \underbrace{\frac{\sqrt{\lambda}}{2} \cdot \left(\sqrt{w_0}\right)^{\alpha^i}}_{h_i} \cdot \underbrace{\frac{72}{\lambda} \cdot \left(w_0^{2\alpha-3}\right)^{\alpha^i}}_{p_i} \\ &= 36\sqrt{\lambda} \sum_{i=0}^{\lfloor \log_\alpha \log_{w_0} d \rfloor} (w_0)^{\left(\frac{3}{2}+2\alpha-3\right)\cdot\alpha^i} \\ &= 36\sqrt{\lambda} \sum_{i=0}^{\lfloor \log_\alpha \log_{w_0} d \rfloor} \left(w_0^{\left(2\alpha - \frac{3}{2}\right)}\right)^{\alpha^i} \end{aligned}$$

To achieve a double-exponential growth we need $\alpha > 1$ and we choose $\alpha = 1 + \varepsilon$ and $\varepsilon > 0$. The energy consumption is then

$$P_{\text{tx,II}} \leq 36\sqrt{\lambda} \sum_{i=0}^{\lfloor \log_{1+\varepsilon} \log_{w_0} d \rfloor} \left(w_0^{\left(\frac{1}{2}+2\varepsilon\right)}\right)^{(1+\varepsilon)^i} .$$

Note that

$$\begin{aligned} \sum_{i=0}^{k} w^{q^i} &= w^{q^k} \cdot \sum_{i=0}^{k} w^{q^i - q^k} \\ &= w^{q^k} \cdot \left(1 + \sum_{i=0}^{k-1} w^{q^i - q^k}\right) \\ &\leq w^{q^k} \cdot \left(1 + \sum_{i=0}^{k-1} w^{q^{k-1} - q^k}\right) \quad \text{for } q > 1 \\ &= w^{q^k} \cdot \left(1 + \frac{k}{w^{q^k(1-1/q)}}\right) \\ &= w^{q^k}\left(1 + o(1)\right) . \end{aligned}$$

Thus, the energy consumption for sending is

$$P_{\text{tx,II}} \leq 36\sqrt{\lambda} \cdot (1 + o(1)) \cdot d^{\frac{1}{2}+2\varepsilon} .$$

The number of active nodes in Unicast II is in the order of Unicast I.

$$\begin{aligned} N_2 &= \sum_{i=0}^{\lfloor \log_{1+\varepsilon} \log_{w_0} d \rfloor} w_i \cdot h_i \\ &\le \sum_{i=0}^{\lfloor \log_{1+\varepsilon} \log_{w_0} d \rfloor} \underbrace{(w_0)^{(1+\varepsilon)^i}}_{w_i} \cdot \underbrace{\frac{\sqrt{\lambda}}{2} \cdot \left(\sqrt{w_0}\right)^{(1+\varepsilon)^i}}_{h_i} \\ &= \frac{\sqrt{\lambda}}{2} \sum_{i=0}^{\lfloor \log_{1+\varepsilon} \log_{w_0} d \rfloor} \left(w_0^{3/2}\right)^{(1+\varepsilon)^i} \\ &= \frac{\sqrt{\lambda}}{2} \cdot (1 + o(1)) \cdot d^{3/2} \end{aligned}$$

and so is the constant power for signal processing with

$$P_{\text{rx,II}} = N_2 \cdot p_{\text{rx}} = \Theta\left(\sqrt{\lambda} \cdot d^{3/2}\right) .$$

■

VI. Unicast III with Delay $\Theta(\log d)$ and Transmission Energy $\Theta(\log d)$

There is a trade-off between the amount of nodes for sender beamforming and energy consumption. Beamforming can focus the sender energy onto beams and increase reception range which can reduce transmission power. However, when the expansion of beamforming senders is much bigger than the distance to the receiver, it gets unattractive to use the sender nodes being furthest away, because they have a high path loss to the receiver. For nodes placed on a one-dimensional line [5], we could show linear energy consumption $\Theta(d)$ for transmission distance d. The expansion of m beamforming sender is $\Theta(m)$ in this case. In the plane, we could decrease the expansion of m beamforming sender to a rectangle with a diameter $\Theta\left(m^{2/3}\right)$. This could decrease the transmission power to $\Theta\left(\sqrt{d}\right)$.

The spatial expansion of coordinated beamforming senders can further increased compared to the one-dimensional line and two-dimensional rectangle when selecting beamforming senders in a three-dimensional cuboid (see Figure 4).

Lemma 3 *If a single sender s sends a signal to a $w \times h \times b$ cuboidal cell in a distance of at least w (see Figure 3), then all nodes in this cell are phase-synchronized for sender beamforming towards the target with a phase error less than α if $h^2 \le \frac{\alpha}{2\pi}\lambda w$.*

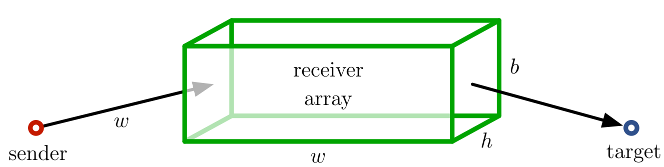


Fig. 3. Nodes in the $w \times h \times b$ cuboid receive the message from the sender in distance w and are phase-synchronized for sender beamforming to the target

Proof: Let x denote the signal of the sender s and y the signal at r. Then,

$$y = \frac{x}{\|\mathbf{s}, \mathbf{r}\|} \cdot e^{-\frac{j2\pi}{\lambda} \cdot \|\mathbf{s}, \mathbf{r}\|} .$$

Thus, the phase shift is described by $-\arg(\frac{y}{x}) = \frac{2\pi}{\lambda} \|\mathbf{s}, \mathbf{r}\|$. The difference of phase shifts is therefore

$$\begin{aligned} \delta &= \frac{2\pi}{\lambda} \|\mathbf{s}, \mathbf{r}\| - \frac{2\pi x}{\lambda} \\ &= \frac{2\pi}{\lambda} \left(\sqrt{r_x^2 + r_y^2 + r_z^2} - r_x\right) \\ &\le \frac{2\pi}{\lambda} r_x \left(\sqrt{1 + \left(\frac{\sqrt{2} r_y}{r_x}\right)^2} - 1\right) . \end{aligned}$$

This phase difference is maximized for $r_y = h$, $r_z = b$ and $r_x = w$ and by applying the relation $\sqrt{1+x^2} - 1 \le \frac{x^2}{2}$ for all $x \ge 0$ (see [6]) we get

$$\delta \le \frac{\pi}{\lambda} \frac{2r_y^2}{r_x} = \frac{\pi}{\lambda} \frac{2h^2}{w} .$$

From $h^2 \le \frac{\alpha}{2\pi}\lambda w$ it follows that $\delta \le \alpha$. ■

We choose dimensions of the cuboidal cells such that the error phase-synchronization is $\alpha \le \pi/4$, i.e. less than a wavelength. For sender beamforming, the nodes simply resend the message after a fixed time offset after receiving the message. The synchronization error α can be fixed with techniques presented in [6], which do not affect the energy consumption asymptotically.

In our model, the antennas of all nodes are aligned along the z-axis, i.e. a transmission between nodes with the same z-coordinate won't be affected by polarization. The polarization effect occurs when sending from a cuboid of beamforming senders to a cuboid of receivers. Assuming the distance between both cuboids in w_{i+1} and the height of the receiver cuboid is b_{i+1}, the elevation (angle) is at most

$$\theta = \arctan\left(\frac{b_{i+1}}{w_{i+1}} + \frac{\pi}{2}\right) .$$

Thus, the attenuation factor a_{pol} of polarization at sender and receiver is at most

$$a_{\text{pol}} = \cos\left(\tan^{-1}\left(\frac{b_{i+1}}{w_{i+1}}\right)\right) = \frac{1}{\sqrt{1 + \frac{b_{i+1}^2}{w_{i+1}^2}}} .$$

Furthermore, one has to consider the directional antenna behavior. If we assume a dipole antenna along the z-axis this results in an extra factor of $\cos(\theta)$, which results in the same calculation and an extra factor of a_{pol}.

$$a_{\text{pol}}^2 = \frac{1}{1 + \frac{b_{i+1}^2}{w_{i+1}^2}} \ge \frac{1}{1 + \frac{\lambda}{4w_{i+1}}} \ge \frac{4}{5}$$

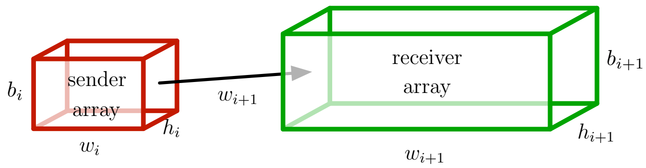


Fig. 4. Transmission from a $w_i \times h_i \times b_i$ cuboid of coordinated beamforming senders to receivers in a $w_{i+1} \times h_{i+1} \times b_{i+1}$ cuboid

Lemma 4 *A $w_i \times h_i \times b_i$-cuboidal cell of beamforming senders with transmission power p_i can reach any node in a $w_{i+1} \times h_{i+1} \times b_{i+1}$-cuboidal cell at distance w_{i+1} if*

$$\begin{aligned} h_{i+1} &\geq h_i \ , & (14)\\ b_{i+1} &\geq b_i \ , & (15)\\ w_{i+1} &\geq w_i \ , & (16)\\ w_{i+1} &\leq \frac{1}{10\sqrt{5}}\sqrt{p_i} w_i h_i b_i \ , & (17)\\ h_{i+1} &\leq w_{i+1} \ , & (18)\\ b_{i+1} &\leq w_{i+1} \ , & (19)\\ h_{i+1}^2 &\leq \frac{1}{4}\lambda w_{i+1} \ , & (20)\\ b_{i+1}^2 &\leq \frac{1}{4}\lambda w_{i+1} \ . & (21) \end{aligned}$$

Proof: For a sender antenna and a receiving antenna with relative offset (w, h, b) we have $w \in [w_{i+1}, w_i + w_{i+1}], b \in [0, b_{i+1}], h \in [0, h_{i+1}]$. The signal phase shift with respect to w is therefore described by

$$\sqrt{w^2+h^2+b^2} - w = w\left(\sqrt{1+\frac{h^2+b^2}{w^2}} - 1\right) \leq \frac{1}{2}\frac{h^2+b^2}{w}$$

using $\sqrt{1+\frac{h^2+b^2}{w^2}} \leq \frac{1}{2}\frac{h^2+b^2}{w^2}$. The last term ist bounded by

$$\begin{aligned} \frac{1}{2}\frac{(h_{i+1})^2+(b_{i+1})^2}{w_{i+1}} &\leq \frac{1}{2}\frac{(h_{i+1})^2+(b_{i+1})^2}{w_{i+1}} \\ &\leq \frac{1}{4}\frac{\lambda w_{i+1}}{w_{i+1}} \leq \frac{1}{4}\lambda \ . \end{aligned}$$

For the amplitude note that

$$\begin{aligned} \sqrt{w^2+h^2+b^2} &\leq \sqrt{(3w_{i+1})^2+(h_{i+1})^2+(b_{i+1})^2} \\ &\leq \sqrt{5} w_{i+1} \ . \end{aligned}$$

The number of senders is given by $w_i h_i b_i$ and the attenuation is given by at most

$$\frac{1}{\sqrt{5}w_{i+1}} \cdot \frac{1}{4} \cdot \frac{4}{5}\cos \pi/2 = \frac{1}{5\sqrt{5}w_{i+1}}$$

Since $\frac{w_i h_i b_i \sqrt{p_i}}{5\sqrt{5}} \geq 2w_{i+1} \geq w_i + w_{i+1}$ all receivers can be reached, if the original amplitude of each sender is $\sqrt{p_i}$. ■

Again we increase the size of the cuboids in each round according to these equations. The power will be decreased as well. The following recursion gives a valid choice for appropriate constants c_1 depending on λ.

$$\begin{aligned} w_{i+1} &:= 2 \cdot w_i & (22)\\ h_i &= \sqrt{w_i \lambda/4} & (23)\\ b_i &= \sqrt{w_i \lambda/4} & (24)\\ p_i &= \frac{c_1^2}{w_i^2} & (25) \end{aligned}$$

Theorem 3 *For $\lambda \leq 1$, the delay for the three-dimensional beamforming algorithm is $\Theta(\log d)$ for distance d between source and target. The energy consumption for transmission is $\Theta(\frac{1}{\lambda}\log d)$ and for total signal processing energy of $\Theta\left(d^2\right)$.*

Proof: Note that

$$w_i = (w_0)2^i \ .$$

Note that $w_{i+1} \geq w_i$ and also $h_{i+1} \geq h_i$ and $b_{i+1} \geq b_i$.

Now

$$w_{i+1} = 2w_i = \frac{8}{c_1\lambda} w_i h_i b_i \sqrt{p_i} \leq \frac{1}{10\sqrt{5}}\sqrt{p_i} w_i h_i b_i \ ,$$

if $\frac{8}{c_1\lambda} \leq \frac{1}{10\sqrt{5}}$. So, we choose $c_1 = \frac{80\sqrt{5}}{\lambda}$.

We have $h_i \leq w_i$ and $b_i \leq w_i$, if $w_i \geq \frac{16}{\lambda^2}$. We have $h_i = \sqrt{w_i\lambda/4} \leq w_i$ if $w_i \geq \lambda/4$.

The delay is $\Theta(\log d)$ for distance d and the total energy for one round is given by

$$h_i b_i w_i p_i = \frac{c_1^2 \lambda w_i^2}{4w_i^2} = \frac{4,000}{\lambda} \ .$$

Summing over all $\log d$ rounds gives the total transmission energy of $\Theta(\frac{1}{\lambda}\log d)$.

To estimate the costs for signal-processing we again count the number of nodes, which is for round i

$$m_i = h_i \cdot w_i \cdot b_i = w_i^2 \cdot \frac{\lambda}{4}$$

These nodes have a reception range of

$$\sqrt{p_i} \cdot m_i = \frac{c_1}{w_i} \cdot \frac{w_i^2\lambda}{4} = 80\sqrt{5} \cdot w_i$$

The distance d from source to target is covered in round r, whereby for each round i we have to take into account the length of the rectangle w_i twice, for the rectangle and the transmission distance to the rectangle.

$$\begin{aligned} d &\leq w_0 + \sum_{i=1}^{r} 2w_0 \cdot 2^i = w_0\left(4 \cdot 2^r - 3\right) \\ r &\geq \log\left(\frac{d}{4w_0} + \frac{3}{4}\right) = \Theta\left(\log d\right) \end{aligned}$$

Summing over all r rounds gives a total number of nodes of

$$\sum_{i=0}^{r} w_i^2 \cdot \frac{\lambda}{8} = \frac{4}{3w_0^2}d^2 + \frac{2}{w_0}d + 5w_0^2 = \Theta\left(d^2\right) \ .$$

■

For a network diameter of $\sqrt[3]{n}$ the number of nodes involved in a unicast operation is $\mathcal{O}\left(n^{2/3}\right)$ which is asymptotically smaller than for the plane because of the smaller expected distance between source and target.

Corollary 2 *For $\lambda \leq 1$ and a three-dimensional network with diameter $\Theta\left(\sqrt[3]{n}\right)$, the delay for the three-dimensional beamforming algorithm is $\mathcal{O}(\log n)$. The energy consumption for transmission is $\mathcal{O}(\frac{1}{\lambda}\log n)$ and for signal processing $\mathcal{O}\left(n^{2/3}\right)$.*

With antennas aligned along the z-axis, the preceding algorithm is intended for routing in the x-y plane. If the the z-coordinate of source and target also differs, we can route in an inclined plane, which reduces the reception range of each node by a constant factor according to polarization. If the elevation (angle) is too large to reach the target in an inclined plane directly, we can gain height in a staircase like procedure. This only increases the routing speed and energy consumption by a constant factor and does not change the asymptotic results.

VII. Conclusion

We demonstrate unicast algorithms for the two-dimensional plane and three-dimensional space, for which a total sublinear transmission power $\Theta(\sqrt{d})$, resp. $\Theta(\log d)$, for distance d from source to target is possible in principle. We achieve this by combining a multi-hop routing scheme with cooperative beamforming. In each hop, selected senders cooperate for beamforming in order to increase the transmission distance, while in each round each sender reduces its transmission power. This algorithm is designed for ad-hoc networks with small traffic, sparse energy supply, and small transmission delay, e.g. for wireless sensor networks.

The three unicast algorithms (derived from [7]) are designed for either nodes placed in the plane or nodes in three-dimensional space. All algorithms need $\Theta\left(\log d\right)$ hops for a transmission distance of $\Theta\left(d\right)$ between source and target, which is the equivalent to the order of the transmission delay. The transmission power is $\Theta(\sqrt{d})$. In particular, if we choose devices for sender beamforming in a cuboidal cell in three-dimensional space, we can show transmission energy of $\Theta(\log d)$ along with a transmission delay of only $\Theta\left(\log d\right)$.

The overhead costs in terms of energy consumption for signal processing at each receiving node and for routing algorithms cannot be neglected. So, we see here a trade-off between energy consumption for sending and receiving and it will be part of future work to find the optimal solution for this tradeoff. However, if nodes are in reception mode anyway and energy consumption for that has not to be considered, these algorithms can be a real energy reduction.

While our model is restricted to the line-of-sight model implying a quadratic path loss, further work will concentrate on the energy savings for other path loss models. Likewise in [7] it turns out that the unicast algorithm presented here, use on-the-fly synchronization, i.e. each node simply needs to repeat the original message after some fixed time offset.

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