Particle Swarm Optimization in Stationary and Dynamic Environments

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Abstract

Inspired by social behavior of bird flocking or fish schooling, Eberhart and Kennedy first developed the particle swarm optimization (PSO) algorithm in 1995. PSO, as a branch of evolutionary computation, has been successfully applied in many research and application areas in the past several years, e.g., global optimization, artificial neural network training, and fuzzy system control, etc.. Especially, for global optimization, PSO has shown its superior advantages and effectiveness.

Although PSO is an effective tool for global optimization problems, it shows weakness while solving complex problems (e.g., shifted, rotated, and compositional problems) or dynamic problems (e.g., the moving peak problem and the DF1 function). This is especially true for the original PSO algorithm.

In order to improve the performance of PSO to solve complex problems, we present a novel algorithm, called self-learning PSO (SLPSO). In SLPSO, each particle has four different learning strategies to deal with different situations in the search space. The cooperation of the four learning strategies is implemented by an adaptive framework at the individual level, which can enable each particle to choose the optimal learning strategy according to the properties of its own local fitness landscape. This flexible learning mechanism is able to automatically balance the behavior of exploration and exploitation for each particle in the entire search space during the whole running process.

Another major contribution of this work is to adapt PSO to dynamic environments, we propose an idea that applies hierarchical clustering techniques to generate multiple populations. This idea is the first attempt to solve some open issues when using multiple population methods in dynamic environments, such as, how to define the size of search region of a sub-population, how many individuals are needed in each sub-population, and how many sub-populations are needed, etc.. Experimental study has shown that this idea is effective to locate and track multiple peaks in dynamic environments.

Declaration

The content of this submission was undertaken in the Department of Computer Science, University of Leicester and supervised by Dr. Shengxiang Yang during the period of registration. I hereby declare that the materials of this submission have not previously been published for a degree or diploma at any other university or institute. All the materials submitted for assessment are from my own research, except the reference work in any format by other authors, which are properly acknowledged in the content.

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3. S. Yang and C. Li. A clustering particle swarm optimizer for locating and tracking multiple optima in dynamic environments. *IEEE Transactions on Evolutionary Computation*, published online first: 26 August 2010. IEEE Press.
4. C. Li and S. Yang. A clustering particle swarm optimizer for dynamic optimization. Proceedings of the *2009 IEEE Congress on Evolutionary Computation*, pp. 439-446, 2009. IEEE Press.
5. C. Li and S. Yang. An adaptive learning particle swarm optimizer for function optimization. Proceedings of the *2009 IEEE Congress on Evolutionary Computation*, pp. 381-388, 2009. IEEE Press.
6. C. Li and S. Yang. A generalized approach to construct benchmark problems for dynamic optimization. Proceedings of the *7th Int. Conf. on Simulated Evolution and Learning*, pp. 391-400, 2008. Springer.
7. C. Li and S. Yang. An island based hybrid evolutionary algorithm for optimization. Proceedings of the *7th Int. Conf. on Simulated Evolution and Learning*, pp. 180-189, 2008. Springer.
8. C. Li, S. Yang and I. A. Korejo. An adaptive mutation operator for particle swarm optimization. Proceedings of the *2008 UK Workshop on Computational Intelligence*, pp. 165-170, 2008.

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Chapter 1

Introduction

Optimization, in a sense, has existed since humankind was born, which is one of the oldest scientific issues. From cutting edge technologies to our daily life, we are always tackling this problem to attempt to get maximum profit with minimum cost. Global optimization is a branch of applied mathematics and numerical analysis that deals with the optimization of a function or a set of functions to find the best possible solutions from a solution set. Analytically deterministic methods can be used to solve simple traditional global optimization problems, however, it becomes impossible or impractical to apply deterministic methods to solve global optimization problems that are not differentiable, not continuous, implicit, or have too many local optima. An implicit function here is a function where the dependent variable is not expressed explicitly by the independent variable. For example, the output y of a function f can be explicitly represented by the given input x: $y=f(x)$. By contrast, the output y of an implicit function can only be obtained from the input x by solving an equation of the form : $R(x, y)=0$.

In addition, most real-world problems are not well-defined due to the limitation of our knowledge or they are changing with time. Therefore, it is impossible to achieve the exact global optimal solutions and algorithms are required to adapt to the environments if the environments are dynamic. Another kind of global

optimization problems have a huge number of local optima, particularly in high dimensional search space. This property sometimes makes deterministic methods impractical to enumerate all local optima within bearable time. And even worse, deterministic methods are not able to enumerate all possible local optima due to high complexity of the search space. As a result, non-deterministic methods are needed to obtain the best possible solutions that may be a bit inferior to the global optimum within bearable time.

Evolutionary algorithms (EAs) are inspired by natural evolution of survival of the fittest. EAs are population based, stochastic, and heuristic optimization methods. The properties of parallel computation and self-adaptation enable EAs to be an ideal tool for solving optimization problems, especially for complicated problems that are considered to be impractical to be solved by traditional methods. In addition, other advantages using EAs are no or little information needed, no requirement of a differentiable or continuous objective function, and ease of implementation, etc.. These characteristics of EAs lead to a huge number of algorithms proposed over the past few decades. There are four major classes of EAs: genetic algorithm (GA) [38], evolution strategies (ES) [93], evolutionary programming (EP) [31], and genetic programming (GP) [54].

Inspired by the social behavior of organisms, particle swarm optimization (PSO, 1995) [26, 51], ant colony optimization (ACO, 1991) [22], and artificial bee colony (ABC, 2005) [46] techniques have attracted more and more researchers recently. These research approaches are swarm intelligence (SI) based methods, which are an important branch of artificial intelligence.

Besides the above branches, some other research methods (e.g., differential evolution (DE, 1995) [104], estimation of distribution algorithm (EDA, 1999) [57, 139], and gene expression programming (GEP, 2001) [30]) have also become hot research topics.

1.1 Challenges for EAs in the Continuous Space

Although EAs are ideal methods to solve complex optimization problems, there are some challenges when they are applied in real-world applications, such as, how to balance an algorithm's search behavior between exploration and exploitation, which is also called the exploration and exploitation dilemma. Furthermore, some important features of EAs in stationary environments will prevent the population of an EA from exploring promising areas in dynamic environments. For example, convergence is an important property of EAs in stationary environments, but this property turns out to be a disaster for the performance of EAs in dynamic environments. This is because an EA can not search any more in a new environment if the population of the EA converges in the current environment. Therefore, a traditional EA can not locate and track new global optima in dynamic environments. The following sections will further discuss these challenges for EAs in both stationary and dynamic environments.

1.1.1 Challenges in Stationary Environments

The major challenge for EAs is how to avoid being trapped in local optima. Generally speaking, EAs will converge to some location(s) in the fitness landscape in time. An algorithm is called converged if no new candidate solutions can be produced or the algorithm keeps on searching in a quite small subset in the search space. However, an algorithm is called prematurely converged if it has converged to a local optimum and there are better locations existing in the fitness landscape than the area being currently searched. This issue is particularly challenging if there are a huge number of local optima when the number of dimensions of the objective function is high. The challenge lies in that it is impossible to determine whether the best solution known so far is the global optimum or

not. In other words, it is not clear when to stop the search process, how to explore new promising areas, or how to avoid repeated search when an algorithm attempts to explore un-examined areas, etc. As a result, a lot of effort has been made to attempt to solve these issues in different research areas, for example, GA [82, 95, 96, 113, 140], DE [24, 29, 91], PSO [6, 73, 125], and EAs with techniques from other research fields, e.g., orthogonal design [58], latin squares [60], and taguchi methods [112], etc.

The major consequence of premature convergence is the loss of diversity. Losing diversity means that the whole population enters a status where all individuals are similar. As a result, it is hard for an algorithm to make progress any more. According to the theory of self-organization [80], if a system is going to be in an equilibrium, the evolution process will be stagnated. Generally speaking, diversity loss can be relieved by maintaining diversity or increasing diversity. Behind the phenomenon of diversity loss, there exists the real challenging issue: how to balance exploitation and exploration for EAs. In the context of evolutionary computation, the term of exploitation means refining individuals by a small change of the current candidate solutions while exploration means finding new areas in the search space. In other words, exploitation is considered to intensify a population while exploration is to diversify a population. Therefore, generally, people believe that exploration and exploitation are two opposite forces for guiding the search behavior of individuals.

Actually, from the algorithm point of view, almost all EAs are subject to this issue. If an algorithm prefers exploitation, it normally has a fast convergence speed, but it may be easily trapped in local optima. On the other hand, if an algorithm is in favor of exploration, it may never improve candidate solutions well enough to find the global optima or it may take too long to find a better location by chance. How to trade-off between exploitation and exploration is one

of the major topics in this thesis.

From the problem point of view, what kind of difficulties for EAs is determined by the problem to be solved. In other words, problems with different properties of fitness landscapes bring about different challenges for EAs. For example, if there is enough gradient information of the global optimum available for algorithms, the global optimum can be easily found. However, if there are some deceptive regions in the fitness landscape, algorithms may be easily trapped in those deceptive areas and it is hard for them to reach the global optimum. More fitness landscapes with different properties will be discussed in Section 3.1 in Chapter 3.

1.1.2 Challenges in Dynamic Environments

Although most research in EAs has focused on static optimization problems over the last decades, in recent years, investigating EAs for dynamic optimization problems (DOPs) has attracted a growing interest from the EA community because EAs are intrinsically inspired by natural or biological evolution, which is always subject to an ever-changing environment, and, hence, EAs, with proper enhancements, have a potential to be good optimizers for DOPs. To solve DOPs, an optimization algorithm is required to not only find the global optimal solution under a specific environment, but also track the trajectory of the changing optima over dynamic environments. As a result, convergence is dangerous for algorithms to track the changing optima because no progress can be made in the current environment once the population has converged in the previous environment. Therefore, the biggest challenge for EAs in dynamic environments is how to increase or maintain diversity in changing environments [10].

Over the years, several approaches have been developed into traditional EAs to address DOPs [12, 44, 126], including diversity increasing and maintaining schemes [21, 34, 131], memory schemes [13, 130, 133], multi-population schemes

[11, 132], adaptive schemes [78, 127, 128], multi-objective optimization methods [18], and problem change detecting approaches [94]. By using multi-population schemes, several PSO algorithms have been recently proposed to address DOPs [10, 40, 41, 66, 84, 117].

The multi-population method has been shown an effective approach to enhancing the diversity of an algorithm for DOPs, with the aim of maintaining multiple populations on different peaks. The traditional method of using the multi-population method to find optima for multi-modal functions divides the whole search space into local sub-spaces, each of which may cover one or a small number of local optima, and then separately searches within these sub-spaces. Here, there are several key, usually difficult, issues to be addressed, e.g., how to guide individuals to move toward different promising sub-regions, how to define the area of each sub-region, how to determine the number of sub-populations needed, and how to generate sub-populations.

It is difficult to guide individuals to move toward different promising sub-regions in the search space. It requires algorithms to be able to find relatively better local optima rather than very bad ones. It also requires algorithms to be able to distribute individuals to as many different promising local areas as possible. Because of the complexity of a fitness landscape, it is impossible to predict or estimate the exact shape of sub-regions where local optima are located, particularly in high dimensional problems with many local optima. Due to this difficulty, an algorithm is required to detect the shape of local optima by itself. The optimal number of sub-populations is determined by the property of the fitness landscape. Intuitively, the more number of local optima in the fitness landscape, the more number of sub-populations is needed. Generating sub-populations is also difficult as they should be distributed in different promising sub-areas.

In order to address the key issues in global optimization as well as how to

effectively use the multi-population method in dynamic environments, this thesis mainly introduces some novel ideas for both global optimization problems and DOPs, respectively.

1.2 Motivation

In order to alleviate the problems that EAs suffer, as explained above, several new ideas are introduced in this section.

1.2.1 Intelligence at the Individual Level

For most EAs, evolutionary progress is achieved by exchanging information among individuals. The way of exchanging information is the same for all individuals for most EAs. Although the whole population is able to move toward promising areas in the fitness landscape, the search process is at the population level. It is hard for a single individual to make its own step according to the current local fitness landscape where it is. In other words, a single individual can not independently search or make its own decision based on the property of its local fitness landscape. Generally speaking, individual level intelligence is needed for a particular individual to deal with different situations. For example, different problems may have different properties due to different shapes of the fitness landscapes. In order to effectively solve problems with different properties, individuals may need different learning strategies to deal with different situations. This may also be true even for a specific problem because the shape of local fitness landscape in different sub-regions of a particular problem may be quite different, such as composition benchmark functions. Therefore, in order to make individuals intelligent enough to deal with different situations independently, we need to implement several learning strategies for each individual.

From another point of view, actually, the aim of introducing individual level of intelligence is to try to balance the search behavior between exploration and exploitation. If individuals are able to independently search in the search space, they may be distributed in quite different sub-areas of the whole fitness landscape and the search progress is achieved mainly based on the information learned from their local fitness landscape rather than from the whole fitness landscape. In this mode, an individual mainly uses information obtained from its local fitness landscape to help its search. Therefore, this working mechanism will alleviate the premature convergence problem and increase the chance of finding the global optimum.

1.2.2 External Memory to Record Explored Areas

Generally speaking, for most EAs, individuals do not possess the capability of memorizing their previous search trajectory. Population distribution of next generation is made by a certain evolutionary model, e.g., crossover, mutation, and selection operations in GAs or by a specific update model, say PSO, DE, and EP, etc. But this evolutionary progress is only made based on the current population. That is, no previous search information is used to guide future search. Intuitively, we will have two advantages if we can properly use previous trajectory information: 1) first, the previous trajectory information would help to accelerate the exploitation process; 2) second, we would avoid redundant search. For example, if we can properly extract useful information of the evolutionary trajectory of the population, we will be able to predict the future movement direction of the current population. This strategy would be very effective especially when individuals are in the exploitation status around a local peak in the fitness landscape. If an individual can memorize its previously explored areas in the fitness landscape, then this knowledge can be used for its future search by avoiding those explored ar-

eas. This idea is able to encourage individuals to explore more promising areas in non-explored areas in the fitness landscape. Therefore, this kind of memory techniques would accelerate the evolutionary progress and increase the global search capability.

1.2.3 Independent Self-Restart Strategy

Convergence is one of the most important features of EAs. However, the question is that we do not know whether a population converges to the global optimum and it is even hard to know whether a population converges or not. So far, none of EAs can guarantee to always find the global optimum, especially for some complex multi-modal problems. Unfortunately, we can not let EAs run forever. Normally, EAs are given a specific stop criterion to stop their running. When a population converges, it does not contribute to the search any more. At this moment, if the stop criterion is not satisfied, how to effectively use the remaining computational resources is quite important. Normally, restart strategies can be used to activate stagnant individuals. However, there is a potential problem when we restart a set of individuals at the same time. The problem is that different individuals may converge at different number of iterations. In addition, sometimes, it is hard to judge whether the whole population converges or not. For example, all individuals have converged but are distributed in different sub-areas in the fitness landscape. If this case happens, it is difficult to judge whether a population converges or not. Although some methods can be used to check population convergence, they will bring new challenges when applying those methods. Considering the individual level of intelligence, if we can monitor the evolutionary status for each single individual during the run time, then each individual will be able to automatically restart when it converges.

1.2.4 Tracking Multiple Optima in Dynamic Environments

Different from the aim in stationary environments, the aim in dynamic environments, generally speaking, is to locate and track multiple optima rather than only the global optimum. In dynamic environments, we do not know which local optimum in the current environment will become the global optimum in the next or future environment. However, we do know the relatively "good" local optima have a larger probability to be the new global optimum than those local optima with bad fitness if the environmental change is continuous and mild. Experimental research [10, 11, 40, 41, 66, 84, 117, 132] has shown that multiple population methods are effective to locate and track multiple optima in dynamic environments.

However, as discussed above, the key question of using multiple population methods for DOPs is how to effectively generate sub-populations. The most common approach to generating multiple populations is to randomly generate a series of sub-populations across the whole search space. This method is simple and easy to implement. However, the biggest issue is the overlapping problem among randomly generated sub-populations. In order to remove the overlapping problem, we need to consider how to distribute a number of sub-populations in different sub-areas in the fitness landscape.

1.2.5 Ideas Implemented into PSO Algorithms

In PSO, a population of particles "fly" through the search space. Each particle follows the previous best position found by its neighbor particles and the previous best position found by itself. In the past decade, PSO has been actively studied and applied to many academic and real world problems with promising results due to its properties of simplicity and effectiveness.

PSO, on the one hand, is an effective optimization tool. It has some advantages

for solving problems: 1) it is easy to describe; 2) it is easy to implement; 3) it has a fast convergence speed; 4) it is robust to solve different problems by tuning parameters and the population topology. However, on the other hand, there are some disadvantages of PSO: 1) there is no mechanism to avoid premature convergence; 2) the application areas are relatively few.

So far, most PSO algorithms globally use a single learning pattern for all particles. This means all particles in a swarm use a same learning strategy. The monotonic learning pattern may cause the lack of intelligence for a particular particle, which makes it unable to deal with different complex situations.

To bring particles more intelligence to deal with different situations, we can start from the two basic models in PSO. There are two main models in PSO, called *gbest* (global best) and *lbest* (local best), respectively. The two models differ in the way of defining the neighborhood for each particle. In the *gbest* model, the neighborhood of a particle consists of the particles in the whole swarm, which share information between each other. On the contrary, in the *lbest* model, the neighborhood of a particle is defined by several fixed particles. The two models give different performance in different problems. Kennedy and Eberhart [49] and Poli et al. [90] pointed out that the *gbest* model has a faster convergence speed but also has a higher probability of getting stuck in local optima than the *lbest* model. On the contrary, the *lbest* model is less vulnerable to the attraction of local optima but has a slower convergence speed than the *gbest* model.

Therefore, in order to achieve a good performance of PSO in terms of the trade-off between exploration and exploitation in static environments, a PSO algorithm needs to balance its search between the *lbest* and *gbest* models. However, this is not an easy task. If we let each particle simultaneously learn from both its *pbest* position and the *gbest* position to update itself, the algorithm may suffer from the disadvantages of both models. One solution might be to implement

the cognition component and the social component separately. Considering this idea, each particle can focus on exploitation by learning from its individual *pbest* position or focus on convergence by learning from the *gbest* particle. The idea enables particles located in different regions in the fitness landscape to carry out local search or global search, or vice versa, for a particular particle in different evolutionary stages. Therefore, the probability of avoiding being trapped in the basins of attraction of local optima may be increased.

To generate a number of sub-populations without overlapping when applying multiple population methods for solving DOPs, we may first randomly generate an initial population with a large number of individuals and then split it into a series of sub-populations, each of which has a small number of individuals. To split a population, a simple method is to evenly randomly assign individuals into a certain number of sub-populations. It sounds an easy task. However, it is actually very hard to obtain proper division. The difficulties lie in two aspects: first, the optimal number of sub-populations is unknown; second, deciding which individuals should be assigned into one group is also a challenge. The first question is quite problem-dependent. Problems with different properties of fitness landscape need different optimal number of sub-populations to solve. It is also true even for a same problem with different number of dimensions. For example, with multi-modal problems, the number of peaks normally will exponentially increase when the number of dimensions increases. Intuitively, the larger the number of local optima in the fitness landscape, the larger the number of sub-populations that are needed. The second issue concerns the distribution of individuals in the fitness landscape. Individuals around a same local optimum should be classified into one group and individuals far away from each other should be clustered into different groups. By taking into account these two considerations, the hierarchical clustering methods would be the best option.

1.3 Aims and Objectives

The major aim of this thesis is to develop effective approaches for global optimization problems in both static and dynamic environments. The ideas will be implemented based on the PSO algorithm. Therefore, to achieve this main aim, the following objectives are established:

1. To study PSO's working mechanism to understand the search behavior of PSO.
2. To improve the performance of PSO by using some new ideas proposed in this thesis.
3. To establish a PSO algorithm for global optimization problems.
4. To work out an effective approach to handling dynamism in dynamic environments.
5. To establish a PSO algorithm for DOPs.

1.4 Methodology

To improve the performance of the basic PSO algorithm, several different approaches are proposed to avoid some disadvantages in this thesis, e.g., increasing diversity by mutation methods, multi-population methods, hybrid algorithms, modification of learning strategies, and adaptive techniques, etc. In this thesis, we propose two novel algorithms (self-learning PSO and clustering PSO) using adaptive learning mechanisms and multi-population techniques, respectively.

It is very hard to give an accurate analysis of a real PSO algorithm's performance (e.g., convergence and diversity) due to its stochastic movement. Even if we cannot give an accurate behavior analysis of a PSO algorithm, we can predict some general search behavior based on its working mechanism. If given some

assumptions, the convergence behavior of the original PSO can be proved by mathematical methods. For example, the convergence behavior is given in the thesis by assuming that the personal best position and the global best position do not change during the search process.

Another important method to test an algorithm's performance is to perform experimental study on benchmark problems. As we know, each benchmark problem is proposed to test certain properties of an algorithm. Theoretically, if we perform a complete test on all benchmark problems, we can conclude the algorithm's performance. However, it is impractical to test all benchmarks. Normally, to test an algorithm's performance, it requires to choose benchmarks that have different properties. In this thesis, all proposed PSOs are tested on a certain number of benchmarks.

In addition, comparing an algorithm's performance with other algorithms under the same comparison condition is also an useful method. Since, usually, all published algorithms (especially state-of-the-art algorithms) have been examined by many researchers, they normally have some distinguished performance on some test benchmarks. Therefore, an algorithm's performance can be shown by comparing with state-of-the-art algorithms. All algorithms proposed in the thesis are compared experimentally with other state-of-the-art algorithms to further analyze their performance.

1.5 Contributions

In this thesis, we present the following approaches to achieving the aforementioned objectives when using PSO to solve global optimization problems.

For static global optimization problems, we propose.

1. A set of learning strategies for particles.

2. An adaptive learning framework at the individual level.
3. A self-restart mechanism for particles which are in the convergence status.
4. A method of extracting useful information from improved particles.
5. An external memory for avoiding explored areas in the search space.

For DOPs, we propose.

1. A self-adaptive method to create multiple populations.
2. Approaches for removing over-lapping and over-crowding during the search progress.
3. A restart mechanism to deal with dynamism.

1.6 Outline

The rest of the thesis is organized as follows.

Chapter 2 gives an introduction of global optimization algorithms and PSO methods, including the original PSO, the working mechanism, trajectory analysis based on a simple model, and some improved PSOs. The main research on PSO can be classified into four categories: population topology, diversity maintaining, hybridization with auxiliary operations, and adaptive PSO. The corresponding research work that has been done for each topic is outlined in this chapter. Our proposed algorithms, an adaptive mutation PSO [65] and an island based hybrid evolutionary algorithm [67], are also described in detail.

Static and dynamic optimization benchmark problems used in this thesis are introduced in Chapter 3 and Chapter 4, respectively. The difficulties to solve global optimization problems and DOPs are also discussed in the corresponding chapter. For static optimization problems, we choose 45 test problems, including traditional functions, traditional functions with noise, shifted functions, ro-

tated shifted functions, and complex hybrid composition functions. For dynamic benchmarks, three different popular problems are presented, they are the moving peaks benchmark (MPB) [13], the DF1 generator [79], and the general dynamic benchmark generator (GDBG) [64, 68]. The problems introduced in these two chapters are used to test the performance of the proposed algorithms for static and dynamic problems, respectively.

The technical detail of the self-learning particle swarm optimizer (SLPSO) is introduced in Chapter 5. The major components of SLPSO include a set of four learning strategies, the adaptive learning framework, the information update method for the global best particle, the self-restart scheme, and the generalized parameter tuning method.

Chapter 6 describes the clustering particle swarm optimizer (CPSO) for DOPs. Several common issues, e.g., how to guide particles to move toward different promising sub-regions, how to define the area of each sub-region, how to determine the number of sub-swarms needed, and how to generate sub-swarms, are considered and solved by a single linkage hierarchical clustering method.

Experimental studies of SLPSO and CPSO are present in Chapter 7 and Chapter 8, respectively. The experiments include the effectiveness study of their components, the analysis of algorithm configuration, and comparison with other peer algorithms on the corresponding benchmarks introduced in Chapter 3 and Chapter 4, respectively.

Finally, Chapter 9 concludes the thesis and gives some discussion on the future work as well as the expectation of real intelligent algorithms that are able to self-learn and self-evolve in evolutionary computation.

Chapter 2

Particle Swarm Optimization

This chapter presents an overview of global optimization algorithms, description of the original PSO algorithm, and the literature review of PSO in both stationary and dynamic environments. Two of our PSO based algorithms are also described at the end of this chapter.

2.1 Global Optimization Algorithms

Global optimization is a fast growing area. It plays an important role in computer science, artificial intelligence and related research areas and the research is important as its applications are related to engineering, computational chemistry, finance, and many other fields.

Global optimization problems are difficult to solve. There are a wide variety of techniques dealing with these problems. Generally speaking, global optimization algorithms can be divided into two basic classes: deterministic and probabilistic algorithms [122]. Figure 2.1 describes the rough classification of global optimization algorithms. Global optimization is the process of finding the best possible candidate solution to a given problem within a reasonable time limit. Deterministic algorithms are often used when there is a clear relation between the possible

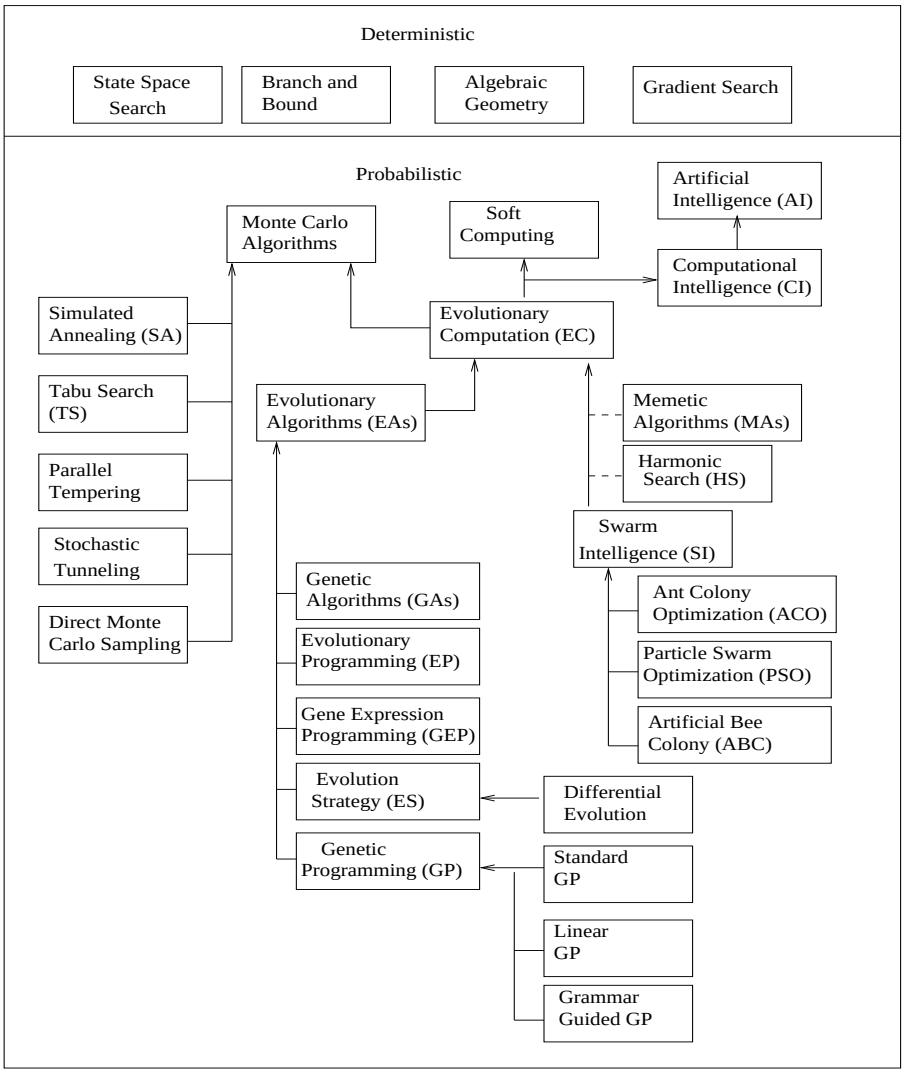


Figure 2.1: Classification of global optimization algorithms [122]

solutions and the objective function. Then the search space can be effectively explored. If the relation is not so obvious or too complicated, or the number of dimensions is too high, it will be very hard for deterministic algorithms to find the global optima. In other words, there is not enough or no gradient information of the objective function for traditional methods to find the global optima in the search space. Actually, it seems a search in a black box for traditional methods if the fitness landscape is very complex. Trying them would be possible an ex-

haustive enumeration of the search space or the search just in a local search space instead of the complete fitness landscape.

In the case where no gradient information of the objective function exists, probabilistic algorithms come into play. Stochastic approaches can deal with this kind of problems much easier and more effective than deterministic algorithms. The big advantage of probabilistic algorithms is that they are simple and easy to implement and they are robust in the situation where the objective function is dynamic or noised. Monte Carlo methods are a class of algorithms dealing with random calculation and most stochastic algorithms are Monte Carlo based approaches.

Heuristics is an important topic involved in many probabilistic algorithms. It is the process of gathering the current information by an algorithm to help it to decide how to generate the next candidate solution or which solutions should be processed next. It normally uses statistical information obtained from samples in the search space or some abstract models from natural phenomenons or physical processes. Well-known algorithms are simulated annealing (SA) and EAs. Simulated annealing, for example, decides which solution to be processed next according to the Boltzmann probability factor of atom configurations of solidifying metal melts. EAs, inspired by natural selection and survival of the fittest in the biology world, use some mechanisms copied from nature, e.g., reproduction, mutation, recombination, and selection, to evolve candidate solutions to promising areas in the search space.

Tabu Search (TS) uses memory structures to enhance performance of local search methods. In TS, each point obtained by the algorithm in the memory must be not visited again so that the algorithm is less likely to get stuck in local optima. Memetic Algorithms (MAs) are one kind of Evolutionary Computation (EC), which are population based methods with individual learning or local search

operation.

Genetic Algorithms (GAs) are population based stochastic approaches. GAs simulate the process of natural evolution where progress is made by mechanisms of mutation, crossover, and selection. Different from GAs, a typical EP algorithm does not contain crossover operation. ES is also a heuristic approach based on the idea of adaptation and evolution. Like EP, mutation and selection are the major search operators in ES. The differences between EP and ES lie in that there is no typical recombination mechanisms in EP. In addition, EP typically uses stochastic selection via a tournament while ES uses deterministic selection where the worst individual is removed normally from the population. GEP, a new evolutionary algorithm, evolves computer programs. In GEP, the computer programs are encoded in linear representation (genotype) and then translated into expression trees (phenotype). All the main genetic operators can be applied in GEP, including mutation, crossover, and recombination.

Swarm Intelligence (SI) is another important optimization methods of evolutionary computation. SI is the property of a system whereby the collective behaviors of (unsophisticated) agents interacting locally with their environment cause coherent functional global patterns to emerge. It takes inspiration from collective behaviors, such as ant colonies, bird flocking, animal herding, bacterial growth, and fish schooling. In the following sections in this chapter, one of SI methods, particle swarm optimization (PSO), is going to be discussed in detail.

2.2 The Original PSO

In PSO, a swarm of particles "fly" through the search space. Each particle keeps track of its coordinates in the problem space which are associated with the best solution (fitness) it has achieved so far. This value is called *pbest*. Another "best" value that is tracked by a particle is the best value, obtained so far by any particle

in the neighbors of the particle. This location is called *lbest*. When a particle takes all the population as its topological neighbors, the best value is a global best and is called *gbest*.

The PSO concept consists of, at each time step, changing the velocity of (accelerating) each particle toward its *pbest* and *lbest* locations (local version of PSO). Acceleration is weighted by a random term, with separate random numbers being generated for acceleration toward *pbest* and *lbest* locations. PSO was first introduced in 1995. It is a very efficient stochastic optimization tool for optimization problems. Recently, more and more researchers have been attracted by this promising research area. It is demonstrated that PSO gets better results in a faster, cheaper way compared with other methods.

Another reason that PSO is attractive is that there are few parameters to adjust. One version, with slight variations, works well in a wide variety of applications. PSO has been used for approaches that can be used across a wide range of applications, as well as for specific applications focused on a specific requirement.

Ever since PSO was first introduced, several major versions of PSO algorithms have been developed [90]. The following version modified by Shi and Eberhart [100] will be used in this thesis. Each particle i is represented by a position vector $\vec{x}_i$ and a velocity vector $\vec{v}_i$, which are updated as follows:

$$v'^{d}_{i} = \omega v^{d}_{i} + \eta_1 r_1 (x^{d}_{pbest_i} - x^{d}_{i}) + \eta_2 r_2 (x^{d}_{gbest} - x^{d}_{i}) \tag{2.1}$$

$$x'^{d}_{i} = x^{d}_{i} + v'^{d}_{i}, \tag{2.2}$$

where x'^{d}_{i} and x^{d}_{i} represent the current and previous positions in the d-th dimension of particle i respectively; v'_i and v_i are the current and previous velocity of particle i respectively; $\vec{x}_{pbest_i}$ and $\vec{x}_{gbest}$ are the best position found by particle i so far and the best position found by the whole swarm so far respectively; $\omega \in (0, 1)$ is an inertia

Algorithm 2.1 Particle Swarm Optimizer

```
Generate the initial swarm by randomly generating the position and velocity for each particle;
Evaluate the fitness of each particle;
repeat
    for each particle i do
        Update particle i according to Eqs. (2.1) and (2.2);
        if f(x_i) < f(x_pbest_i) then
            x_pbest_i := x_i;
            if f(x_i) < f(x_gbest) then
                x_gbest := x_i;
            end if
        end if
    end for
until the stop criterion is satisfied
```

weight, which determines how much the previous velocity is preserved; η_1 and η_2 are the acceleration constants, and r_1 and r_2 are random numbers generated in the interval $[0.0, 1.0]$ uniformly. The framework of the original PSO algorithm is shown in Algorithm 2.1.

Figure 2.2 illustrates the trajectory analysis of a particle in the fitness landscape. In PSO, each particle shares the information with its neighbors. The second and the third components on the right of Eq. (2.1) are called cognition and social components in PSO, respectively. The updating formula Eqs. (2.1) and (2.2) show that PSO combines the cognition component of each particle with the social component of particles in a group. The social component suggests that individuals ignore their own experience and adjust their behavior according to the previous best particle in the neighborhood of the group. On the other hand, the cognition component treats individuals as isolated beings and adjusts their behavior only according to their own experience.

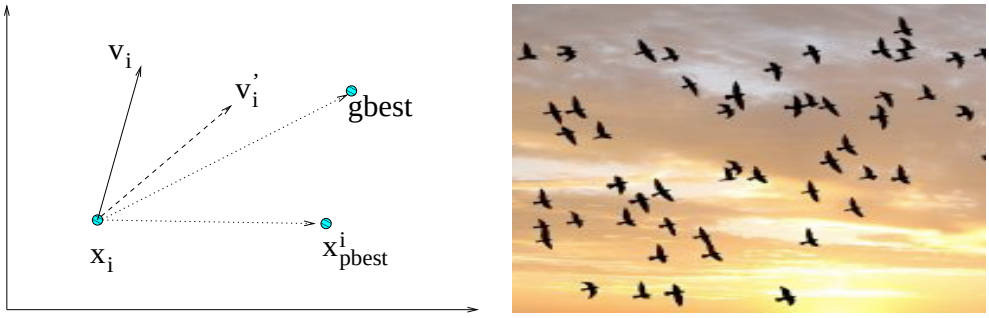


Figure 2.2: Particle trajectory analysis in PSO

2.3 Trajectory Analysis of the Standard PSO [19]

From the mathematic theoretical analysis by Clerc and Kennedy [19], the trajectory of a particle $\vec{x_i}$ in PSO converges to a weighted mean of $\vec{p_i}$ and $\vec{p_g}$. For simplicity, it should be noticed that we use $\vec{p_i}$ and $\vec{p_g}$ to represent a particle's personal best position and the global best position rather than $\vec{x}_{pbest_i}$ and $\vec{x}_{gbest}$ defined in Eq. (2.1) in this section, respectively. During the search progress, a particle will "fly" to its personal best so far position and the global best so far position. According to the update equation Eq. (2.1), the personal best position of the particle will gradually move closer to the global best position. Therefore, all the particles will eventually converge to the global best position. This information-sharing mechanism gives PSO a very fast speed of convergence. Meanwhile, because of this mechanism, PSO cannot guarantee to find the global optima. In fact, particles usually converge to a local optimum.

Once the whole swarm of particles are trapped into a local optimum, where $\vec{p_i}$ can be assumed to be the same as $\vec{p_g}$, all the particles converge to $\vec{p_g}$. Under this condition, the velocity update equation simply becomes:

$$\vec{v_i'} = \omega \vec{v_i} \tag{2.3}$$

When the iteration becomes infinite, the velocity $\vec{v_i}$ of a particle will be close to 0

due to $\omega \in (0, 1)$. After that, the position of particle $\vec{x_i}$ will not change. Therefore, PSO has no capability of jumping out of the local optimum. This is the reason that PSO often fails in finding the global optima.

In order to have a deep analysis of PSO's work mechanism, the trajectory of particle i in one dimension is given in the following mathematical formulas on the assumption that $pbest_i$ and $gbest$ keep constant over some generations. The following equations can be obtained by Eqs. (2.1) and (2.2):

$$x_i(t+1) = x_i(t) + v_i(t+1) \tag{2.4a}$$

$$v_i(t+1) = \omega v_i(t) + c_1 r_1(p_i - x_i(t)) + c_2 r_2(p_g - x_i(t)) \tag{2.4b}$$

$$v_i(t) \leq V_{max} \tag{2.4c}$$

Set $\varphi_1 = c_1 r_1$, $\varphi_2 = c_2 r_2$,$\varphi = \varphi_1 + \varphi_2$, the update equations become:

$$x_i(t+1) = \omega v_i(t) + (1-\varphi)x_i(t) + \varphi_1 p_i(t) + \varphi_2 p_g(t) \tag{2.5a}$$

$$v_i(t+1) = \omega v_i(t) - \varphi x_i(t) + \varphi_1 p_i(t) + \varphi_2 p_g(t) \tag{2.5b}$$

By substituting Eq. (2.5a) into Eq. (2.5b), the following non-homogeneous recurrence relation is obtained:

$$x_i(t+1) = (1 + \omega - \varphi)x_i(t) - \omega x_i(t-1) + \varphi_1 p_i(t) + \varphi_2 p_g(t) \tag{2.6}$$

The characteristic equation corresponding to the recurrence relation is:

$$x^2 + (1 + \omega - \varphi)x + \omega = 0 \tag{2.7}$$

Given the initial condition: $x_i(0) = x_0$, $x_i(1) = x_1$, and assuming that $p_i(t)$ and $p_g(t)$ keep constant over t, the explicit closed form of the recurrence relation is then

given by:

$$x_i(t) = k_1 + k_2\alpha^t + k_3\beta^t \tag{2.8}$$

where,

$$k_1 = \frac{\varphi_1 p_i(t)+\varphi_2 p_g(t)}{\varphi},\ \gamma = \sqrt{(1+\omega-\varphi)^2-4\omega}$$
$$\alpha = \frac{1+\omega-\varphi+\gamma}{2},\ \beta = \frac{1+\omega-\varphi-\gamma}{2}$$
$$x_2 = (1+\omega-\varphi)x_1 - \omega x_0 + \varphi_1 p_i(t) + \varphi_2 p_g(t)$$
$$k_2 = \frac{\beta(x_0-x_1)-x_1+x2}{\gamma(\alpha-1)},\ k_3 = \frac{\alpha(x_1-x_0)+x_1-x2}{\gamma(\beta-1)}$$

For simplicity, we assume that φ_1 and φ_2 are constant. When the following limitation converges, the position sequence of particle i also converges:

$$\lim_{t\to\infty} x_i(t) = \lim_{t\to\infty}(k_1 + k_2\alpha^t + k_3\beta^t) \tag{2.9}$$

Then, the following result is obtained:

1). if $\max(\|\alpha\|, \|\beta\|) > 1$, $x_i(t)$ diverges

2). if $\max(\|\alpha\|, \|\beta\|) < 1$, $x_i(t)$ converges

When $x_i(t)$ converges, $\lim_{t\to\infty} x_i(t)$ becomes:

$$\lim_{t\to\infty} x_i(t) = k_1 = \frac{\varphi_1 p_i(t) + \varphi_2 p_g(t)}{\varphi} \tag{2.10}$$

If φ_1 and φ_2 are not constants, but are distributed within a range, e.g., uniformly distributed numbers, then the expectation of φ_1 and φ_2 are:

$$E(\varphi_1) = c_1\int_0^1 \frac{x}{1-0}dx = \frac{c_1}{2} \quad E(\varphi_2) = c_2\int_0^1 \frac{x}{1-0}dx = \frac{c_2}{2}$$

Finally, the limitation becomes:

$$\lim_{t\to\infty} x_i(t) = \frac{c_1 p_i(t) + c_2 p_g(t)}{c_1+c_2} = \frac{c_1}{c_1+c_2}p_i(t) + (1-\frac{c_1}{c_1+c_2})p_g(t) \tag{2.11}$$

From Eq. (2.11), we have that particle i converges to a weighted mean of $\vec{p}_i$ and $\vec{p}_g$.

2.4 PSO in Static Environments

Due to its simplicity and effectiveness, PSO has become popular and many improved versions have been reported in the literature since it was first introduced. Most research on performance improvement can be classified into four categories: population topology [43, 76], maintaining diversity [6, 73, 125], hybridization with auxiliary search operators [1, 77, 89], and adaptive PSO [20, 39, 138]. They are briefly reviewed below.

2.4.1 Population Topology

Population topology has a significant effect on the performance of PSO since it determines the way particles communicate or share information with each other. Population topologies can be divided into static and dynamic topologies. For static topologies, communication structures of circles, wheels, stars, and randomly-assigned edges were tested in [53]. The test [53] has shown that algorithms' performance is different on different problems depending on the topology used. Then, Kennedy and Mendes [50] have tested a large number of aspects of social-network topology on five test functions. After that, a fully informed PSO (FIPS) was introduced in [76] by Mendes. Mendes gave a comprehensive test on the effect of population topology in [75]. In FIPS, a particle uses a stochastic average of *pbest*s from all of its neighbors instead of using its own *pbest* position and the *gbest* position. A combination of *gbest* and *lbest* models was implemented by Parsopoulos and Vrahatis [86]. In order to give a standard form for PSO, Bratton and Kennedy proposed a standard version of PSO (SPSO) in [14]. In SPSO [14], a local ring population topology is used and the experimental results have shown

that the *lbest* model is more reliable than the *gbest* model on many test problems.

For dynamic topologies, Suganthan [107] suggested a dynamically adjusted neighbour model, where the search begins with a *lbest* model and gradually increases the neighbourhood size to become a *gbest* model. Hu and Eberhart [39] applied a dynamic topology where some closest particles in the objective space are chosen in every iteration. Liang and Suganthan [43] developed a comprehensive learning PSO (CLPSO) for multi-modal problems. In CLPSO, a particle uses different particles' historical best information to update its velocity, and for each different dimension a particle can potentially learn from a different exemplar. It has been reported that the topology of CLPSO can prevent premature convergence by increasing diversity on multi-modal functions, either with or without correlation between variables.

2.4.2 PSO with Diversity Control

Ratnaweera et al. [92] stated that the lack of population diversity in PSO algorithms is a factor that makes algorithms prematurely converged to local optima. Several approaches of diversity control have been introduced in order to avoid the whole swarm converging to a single optimum. A PSO algorithm with self-organized criticality was implemented in [73]. To help PSO attain more diversity in [73], a "critical value" is created when two particles are too close to each other. In [6], diversity control was implemented by preventing too many particles to get crowded in one sub-region of the search space. Negative entropy was added into PSO in [125] to discourage premature convergence.

Other researchers have attempted to use multi-swarm methods to maintain the diversity of PSO. In [17], a niching PSO (NichePSO) was proposed by incorporating a cognitive only PSO model with the guaranteed convergence PSO (GCPSO) algorithm [114]. Parrott and Li developed a speciation based PSO

(SPSO) [61, 83], which dynamically adjusts the number and size of swarms by constructing an ordered list of particles, ranked according to their fitness, with spatially close particles joining a particular species. The atomic swarm approach has been adapted to track multiple optima simultaneously with multiple swarms in dynamic environments by Blackwell and Branke [9, 10].

2.4.3 Hybrid PSO

Hybridization of EAs is becoming more popular. The hybrid EAs extend the scale of problems that one particular EA can solve by combining ideas from other EAs. Among the hybrid EAs, hybrid PSO is an attractive topic. One of the first hybridized PSO was developed by Angeline [1] where a selection scheme is introduced. Miranda and Fonseca [77] applied the idea of evolution strategies in PSO. In [120], fast evolutionary programming (FEP) [136] was modified by replacing Cauchy mutation with a version of PSO velocity. Hybrid PSO based on GP was proposed in [88, 89]. A cooperative PSO (CPSO-H) was proposed in [115], which employs the idea of splitting the search space into smaller solution vectors and combines it with PSO of the *gbest* model. CPSO-H [115] has shown great improvement over the original PSO algorithm on multi-modal problems. An island based model was produced by combining FEP [136], estimation of distribution algorithm (EDA) [57, 139] with PSO of the *gbest* model.

2.4.4 PSO with Adaptation

Besides the above three active research topics, adaptation is another promising research trend in PSO. An adaptive scheme that decreases ω linearly with iteration was introduced by Shi and Eberhart in [100], and then they also proposed a fuzzy adaptive ω scheme for PSO in [98]. Ratnaweera et al. [92] developed a self-organizing hierarchical PSO with time-varying acceleration coefficients, where a

larger η_1 and a smaller η_2 were set at the beginning and gradually reversed during the search progress. Clerc [20] presented an adaptive version of the constriction factor, population size, and number of neighborhood for PSO. Recently, an adaptive PSO version (APSO)[138] with adaptive ω, η_1 and η_2 was proposed by Zhan et al.. In APSO, four evolutionary states including "exploitation", "exploration", "convergence", and "jumping out" were defined. Coincidentally, the four learning operators in ALPSO [134] play the same roles as the four evolutionary states defined in APSO [138] but the way of implementation is different. While the four learning operators in ALPSO is updated by an operator adaptation scheme [134], the evolutionary states in APSO were estimated by evaluating the population distribution and particle fitness. In each evolutionary state, APSO gives one corresponding equation to adjust the value of η_1 and η_2. Accordingly, the value of ω was tuned using a sigmoid mapping of evolutionary factor f in each evolutionary state. It was reported that the APSO algorithm substantially enhanced the performance of PSO in terms of the convergence speed, global optimality, solution accuracy, and algorithm reliability on 12 test problems.

In order to bring more intelligence at the individual level, an adaptive learning PSO (ALPSO) that utilizes four learning strategies was introduced in [70]. In ALPSO, each particle has four learning sources: its historical best position (*pbest*), the *pbest* position of its nearest particle ($pbest_{nearest}$), the global best position (*gbest*), and a random position nearby (*prand*). The four learning strategies have different properties that play different roles during the search progress. Learning from *pbest* and learning from $pbest_{nearest}$, which are used as local search strategies, are to focus on exploitation and exploration, respectively. Learning from *gbest* and learning from *prand* are used as global search strategies although they have different roles: the former helps ALPSO to control convergence while the latter helps ALPSO to avoid being trapped in the basins of attraction of local optima.

Based on our previous work in [70], an updated version of ALPSO (ALPSO-II) was proposed in [71]. Compared with ALPSO, some new functions were introduced in ALPSO-II, including two updated learning strategies, particle status monitoring mechanism, and controlling the number of particles that learn from the global best position. These new features greatly enhance the performance of ALPSO. The experimental results in [71] also show that ALPSO-II outperforms ALPSO in terms of the convergence speed and solution accuracy.

In the two versions of ALPSO, using an adaptive technique proposed in [65], four learning operators that are based on the four corresponding learning strategies cooperate with each other to search in the entire search space. Each particle can self-adjust its learning strategies for global search or local search. Therefore, the four learning operators cooperate to enable particles to balance between exploitation and exploration during the whole search progress. This adaptive learning framework gives each particle the flexibility so that it can choose the best learning strategy to self-adjust its behavior according to the property of the local fitness landscape. In other words, this flexibility gives each particle the individual level of intelligence to deal with the fitness landscape with different properties.

2.5 PSO in Dynamic Environments

Many researchers have considered multi-populations as a means of enhancing the diversity of EAs to address DOPs. Kennedy [48] proposed a PSO algorithm that uses a *k*-means clustering algorithm to identify the centers of different clusters of particles in the population, and then uses these cluster centers to substitute the personal best or neighborhood best positions. In order to allow cluster centers to be stabilized, the *k*-means algorithm iterates three times. The limitation of this clustering approach lies in that the number of clusters must be predefined.

Branke et al. proposed a self organizing scouts (SOS) [11] algorithm that has

been shown to give promising results on DOPs with many peaks. In SOS, the population is composed of a parent population that searches through the entire search space and child populations that track local optima. The parent population is regularly analyzed to check the condition for creating child populations, which are split off from the parent population. Although the total number of individuals is constant since no new individuals are introduced, the size of each child population is adjusted regularly.

Brits et al. [16] proposed a *nbest* PSO algorithm which is in particular designed for locating multiple solutions to a system of equations. The *nbest* PSO algorithm defines the "neighborhood" of a particle as the closest particles in the population. The neighborhood best for each particle is defined to be the average of the positions of these closest particles. In [17], a niching PSO (NichePSO) algorithm was proposed by incorporating a cognitive only PSO model and the guaranteed convergence PSO (GCPSO) algorithm [114]. NichePSO maintains a main swarm that can create a sub-swarm once a niche is identified. The main swarm is trained by the cognition only model [52]. If a particle's fitness shows a little change over a small number of generations, then a new sub-swarm is created with the particle and its closest neighbors. NichePSO uses some rules to decide the absorption of particles into a sub-swarm and the merging operation between two sub-swarms, which mainly depends on the radius of the involved sub-swarms.

Parrott and Li developed a speciation based PSO (SPSO) [61, 83] algorithm, which dynamically adjusts the number and size of swarms by constructing an ordered list of particles, ranked according to their fitness, with spatially close particles joining a particular species. At each generation, SPSO aims to identify multiple species seeds within a swarm. Once a species seed has been identified, all the particles within its radius are assigned to that same species. Parrott and Li also proposed an improved version of SPSO with a mechanism to remove

redundant duplicate particles in species in [84]. In [4], Bird and Li developed an adaptive niching PSO (ANPSO) algorithm which adaptively determines the radius of a species by using the population statistics. Recently, Bird and Li introduced another improved version of SPSO using a least square regression (rSPSO) in [5].

The atomic swarm approach has been adapted to track multiple optima simultaneously with multiple swarms in dynamic environments by Blackwell and Branke [9, 10]. In their approach, a charged swarm is used for maintaining the diversity of the swarm, and an exclusion principle ensures that no more than one swarm surround a single peak. In [10], anti-convergence is introduced to detect new peaks by sharing information among all sub-swarms. This strategy was experimentally shown to be efficient for the moving peak problem (MPB) [13].

To specify the number of clusters within the k-means PSO algorithm, Passaro and Starita [87] used the optimization of a criterion function in a probabilistic mixture-model framework. In this framework, the particles are assumed to be generated by a mix of several probabilistic distributions. Each different cluster corresponds to a different distribution. Then, finding the optimal number k is equivalent to fitting the model with the observed data while optimizing some criteria. The performance of their algorithm was reported better than SPSO [61] and ANPSO [4] for static problems.

A collaborative evolutionary swarm optimization (CESO) was proposed in [74]. In CESO, two swarms, which use the crowding differential evolution (CDE) [111] and PSO model, respectively, cooperate with each other by a collaborative mechanism. The swarm using CDE is responsible for preserving diversity while the PSO swarm is used for tracking the global optimum. The competitive results were reported in [74].

Inspired by the SOS algorithm [11], a fast multi-swarm optimization (FMSO)

algorithm was proposed in [66] to locate and track multiple optima in dynamic environments. In FMSO, a parent swarm is used as a basic swarm to detect the most promising area when the environment changes, and a group of child swarms are used to search the local optimum in their own sub-spaces. Each child swarm has a search radius, and there is no overlap among all child swarms by repelling them from each other. If the distance between two child swarms is less then their radius, then the whole swarm of the worse one is removed. This guarantees that no more than one child swarm covers a single peak.

In order to address the key issues relevant to the multi-swarm method, e.g., how to guide particles to move toward different promising sub-regions, how to define the area of each sub-region, how to determine the number of sub-swarms needed, and how to generate sub-swarms, a clustering PSO (CPSO) algorithm has recently been proposed for DOPs in [69]. In CPSO, each particle learns from its own historical best position and the historical best position of its nearest neighbor other than the global best position as in the basic PSO algorithm. The velocity update equation for training a particle i is as follows:

$$v'^{d}_{i} = \omega v^{d}_{i} + \eta_1 r^{d}_{i}(x^{d}_{pbest_i} - x^{d}_{i}) + \eta_2 \cdot r^{d}_{i} \cdot (x^{d}_{pbest_{i\_n}} - x^{d}_{i}), \tag{2.12}$$

where $\vec{x}_{pbest_{i\_n}}$ is the personal best position of the particle that is nearest to particle i. This learning strategy enables particles in CPSO adaptively detect sub-regions by themselves and assign them to different neighborhoods. Using a hierarchical clustering method, the whole swarm in CPSO can be divided into sub-swarms that cover different local regions. In order to accelerate the local search, a learning strategy for the global best particle was also introduced in CPSO. CPSO has shown some promising results according to the preliminary experimental study in [69].

In [134], there are some simplifications compared with the original CPSO. First, the training process is removed in [134]. Second, in the original CPSO

[69], the hierarchical clustering method involves two phases of clustering: rough clustering and refining clustering. In [134], the hierarchical clustering method is simplified into only one phase. The experimental results in [134] show the efficiency of the clustering particle swarm optimizer for locating and tracking multiple optima in dynamic environments in comparison with other PSO models based on the multi-swarm method.

2.6 Adaptive Mutation PSO

In order to improve the performance of PSO, the addition of a mutation operator to PSO should enhance its global search capacity. There are mainly two types of mutation operators: one type is based on particle position [28, 37, 55, 56, 103] and the other type is based on particle velocity [63, 92, 118, 119]. The former methods are by far the most common techniques found in the literature, while, the work of the latter one is very few. In this section, the adaptive mutation PSO [65] is introduced.

Different mutation operators can be used to help PSO jump out of local optima. However, a mutation operator may be more effective than other ones on a certain type of problems and may be worse on another type of problems. In fact, it is the same even for a specific problem at different stage of the optimization process. That is, the best mutation results can not be achieved by a single mutation operator, instead several mutation operators may have to be applied at different stages to obtain the best performance. This section introduces a mutation operator that can adaptively select the most suitable mutation operator for different problems. Before presenting the adaptive mutation operator, three mutation operators designed for the global best particle are described as follows.

2.6.1 Three Mutation Operators

A. Cauchy mutation operator

$$\vec{v}'_g = \vec{v}_g exp(\delta) \tag{2.13}$$

$$\vec{x}'_g = \vec{x}_g + \vec{v}'_g \delta_g \tag{2.14}$$

where $\vec{x}_g$ and $\vec{v}_g$ represent the position and velocity of the global best particle. δ and δ_g denote Cauchy random numbers with the scale parameter of 1.

B. Gaussian mutation operator

$$\vec{v}'_g = \vec{v}_g exp(N) \tag{2.15}$$

$$\vec{x}'_g = \vec{x}_g + \vec{v}'_g N_g \tag{2.16}$$

where $\vec{x}_g$ and $\vec{v}_g$ represent the position and velocity of global best particle. N and N_g are Gaussian distribution numbers with the mean 0 and the variance 1.

C. Levy mutation operator

$$\vec{v}'_g = \vec{v}_g exp(L(\alpha)) \tag{2.17}$$

$$\vec{x}'_g = \vec{x}_g + \vec{v}'_g L_g(\alpha), \tag{2.18}$$

where $L(\alpha)$ and $L_g(\alpha)$ are random numbers generated from the Levy distribution with a parameter α, which is set to 1.3.

2.6.2 The Adaptive Mutation Operator

In this section, the adaptive mutation operator, which uses the three mutation operators described above according to their selection ratios, is introduced. All mutation operators have an equal initial selection ratio with 1/3. Each mutation

operator is applied according to its selection ratio and its offspring fitness is evaluated. The mutation operators that result in higher fitness values of offspring have their selection ratios increased. The mutation operators that result in lower fitness values of offspring have their selection ratios decreased. Gradually, the most suitable mutation operator will be chosen automatically and controls all the mutation behavior in the whole swarm. Without loss of generality, we discuss the minimization optimization problems.

First, some definitions are given below: The progress value $prog_i(t)$ of operator i at generation t is defined as follows:

$$prog_i(t) = \sum_{j=1}^{M_i(t)} f(p_j^i(t)) - \min\left(f(p_j^i(t)), f(c_j^i(t))\right), \tag{2.19}$$

where $p_j^i(t)$ and $c_j^i(t)$ denote a parent and its child produced by mutation operator i at generation t and $M_i(t)$ is the number of particles that select mutation operator i to mutate at iteration t.

The reward value $reward_i(t)$ of operator i at generation t is defined as follows:

$$reward_i(t) = exp(\frac{prog_i(t)}{\sum_{j=1}^{N} prog_j(t)}\alpha + \frac{s_i}{M_i(t)}(1-\alpha)) + c_i p_i(t) - 1 \tag{2.20}$$

where s_i is the number of particles whose children have a better fitness than themselves after being mutated by mutation operator i, $p_i(t)$ is the selection ratio of mutation operator i at generation t, α is a random weight between $(0, 1)$, N is the number of mutation operators, and c_i is a penalty factor for mutation operator i, which is defined as follows:

$$c_i = \begin{cases} 0.9, & \text{if } s_i = 0 \text{ and } p_i(t) = \max_{j=1}^{N}\left(p_j(t)\right) \\ 1, & \text{otherwise} \end{cases} \tag{2.21}$$

If the current best operator has no contribution and it is also the best operator in the previous generation, then the selection ratio of the current best operator will decrease.

With the above definitions, the selection ratio of mutation operator i is updated according to the following equation:

$$p_i(t+1) = \frac{reward_i(t)}{\sum_{j=1}^{N} reward_j(t)}(1 - N * \gamma) + \gamma, \tag{2.22}$$

where γ is the minimum selection ratio for each mutation operator, which was set to 0.01 for all the experiments in [65]. This selection ratio update equation considers four factors: the progress value, the ratio of successful mutations, previous selection ratio, and the minimum selection ratio. Another important parameter for the adaptive mutation operator is the frequency of updating the selection ratios of mutation operators. That is, the selection ratio of each mutation operator can be updated at a fixed frequency, e.g., every U_f generations, instead of every generation.

It should be noticed that the choice of the random weight α in Eq.(2.20) and the penalty factor of value of 0.9 in Eq.(2.21) was developed empirically from experiments results. They may not be the optimal choice for all problems. From experimental results, we found that the value of the penalty factor in Eq.(2.21) should be large enough to avoid improper severe punishment to the best learning operator due to temporal bad performance.

The framework of the PSO algorithm with adaptive mutation is described in Algorithm 2.2. It was reported that the algorithms with mutation operator perform better than the original PSO on most problems used in [65]. And the adaptive mutation operator presents at least the second best result among all the mutation algorithms on all test problems. By introducing mutation, PSO greatly improves its global search capability. Although different mutation operators give

Algorithm 2.2 Adaptive Mutation PSO

1: Generate the initial swarm by randomly generating the position and velocity for each particle;
2: Evaluate the fitness of each particle;
3: **repeat**
4: **for** each particle i **do**
5: Update particle i according to Eqs. (2.1) and (2.2);
6: **if** $f(\vec{x}_i) < f(\vec{x}_{pbest_i})$ **then**
7: $\vec{x}_{pbest_i} := \vec{x}_i$;
8: **if** $f(\vec{x}_i) < f(\vec{x}_{gbest})$ **then**
9: $\vec{x}_{gbest} := \vec{x}_i$;
10: **end if**
11: **end if**
12: **end for**
13: Mutate $\vec{x}_{gbest}$ by one of the three mutation operators with the probability of its selection ratio for T times.
14: Compare the best one $\vec{x}^*_{gbest}$ of the mutants with $\vec{x}_{gbest}$ and select the better one as the new global best particle.
15: update the selection ratio for each mutation operator according to Eq. (2.22) every U_f generations.
16: **until** the stop criterion is satisfied

different performance on different test problems, the adaptive mutation operator shows a balanced performance on all test problems. That is, the adaptive mutation is more robust than any single mutation operator investigated in [65].

2.7 Island Based Hybrid Evolutionary Algorithm

As reported in the literature, several heuristical techniques have been used to improve the general efficiency of EAs. Zmuda et al. [141] introduced a hybrid evolutionary learning scheme for synthesizing multi-class pattern recognition systems. Wang [116] developed a hybrid approach to improve the performance of EAs for a simulation optimization problem. A hybrid evolutionary PSO algorithm was proposed by Shi et al. [99]. The hybrid approach executes two systems simultaneously and selects P individuals from each system for exchanging after

the designated N iterations. The individuals with a larger fitness have more opportunities of being selected. A hybrid technique that combines GA and PSO, called genetic swarm optimization (GSO), was proposed by Grimaldi et al. [35] for solving an electromagnetic optimization problem. Li and Wang et al. [63, 117] proposed a hybrid PSO using Cauchy mutation to reduce the probability of being trapped in local optima for PSO.

In this section, an island based hybrid evolutionary algorithm (IHEA) [67] is proposed based on PSO, FEP [135, 136] and EDA [57, 139]. An island model using different evolutionary strategies is designed for improving the optimization performance of the component algorithms.

The main idea of IHEA is that migration of individuals among different islands can increase the diversity of each island, so it can reduce the probability of premature convergence. In IHEA, there are three sub-populations residing in three different islands, which use PSO, FEP, and EDA algorithms, respectively, to search global optima in the whole shared search space. Since different islands use different evolutionary techniques, they probably follow different search directions in the whole shared search space. That is, they explore different areas in the whole search space. However, they are not independent to search. They exchange their own updated information periodically between each other by migration of promising individuals. The information sharing mechanism is helpful to search unexplored space where probably the global optima are located.

PSO in IHEA can be regarded as a fast local search operator for exploitation, EDA is used for exploring new promising area in the whole search space. FEP can be taken as a mutation operator because of its long jump capability. The cooperative search among the three islands is helpful for them to explore new promising areas. It greatly reduces the probability of premature convergence. Hence, the global search capability is improved. The main framework of IHEA is

Algorithm 2.3 Island Based Hybrid Evolutionary Algorithm

1: Initialize the three population, set $k = 1$;
2: Evaluate the fitness of each particle;
3: **repeat**
4: Use PSO, EDA and FEP to optimize each population respectively;
5: Migrate the best individual among the three population to the other two population;
6: $k = k + 1$;
7: **until** the stop criterion is satisfied

shown in Algorithm 2.3.

Another big issue in IHEA is the population resource allocation among different islands. As we know, different problems may have totally different landscapes, such as the number of local or global optima, the difference among local optima, the location of local or global optima, and so on. Different problems may need different algorithms to solve it. Hence, we should allocate different population resources on different islands. For example, we can allocate the most population resources to the island that is more effective than other algorithms to solve the problem.

For the EDA algorithm used in [67], the truncation selection method was used as usual in the literature. A major issue in EDA is how to build a probability distribution model. The Gaussian model with diagonal covariance matrix (GM/DCM) [57] is used in IHEA.

IHEA was compared with SPSO, FEP, and EDA on eight test problems in [67]. It was reported that all the results obtained by IHEA are better than that of the other three algorithms. The information sharing mechanism is developed among the three islands in [67] and greatly reduces the probability of being trapped in a local optimum for IHEA. The global search capability of IHEA is better than the three component algorithms.

2.8 Summary

This chapter introduced some fundamental knowledge of PSO, including the origin of PSO, the basic PSO algorithm, and the trajectory analysis of a particle. This chapter also reviewed the brief literature of PSO in both stationary and dynamic spaces. Two of our proposed PSO algorithms, including the AMPSO and the IHEA, are introduced at the end of this chapter.

Chapter 3

Global Optimization Problems

The most common form of global optimization is the minimization of one real-valued function f in the parameter-space $\vec{x} \in \mathbb{R}$. That is, to find an optimum solution $\vec{x^*}$, which satisfies the following equation for any $\vec{x} \in \mathbb{R}$:

$$f(\vec{x^*}) \leq f(\vec{x}), \vec{x} \in \mathbb{R} \tag{3.1}$$

In this thesis, the global optimization problems to be solved are problems of single objective and without constraints in the decision space except the constraint of the search domain. Global optimization test functions have been becoming more and more complex, from simple unimodal function to rotated shifted multi-modal function to hybrid composition benchmark proposed recently [106]. Finding the global optima of a function has become much more challenging and has been practically impossible for many problems so far.

3.1 Introduction

Global optimization is about finding the best possible solution(s) for a given problem. For a single objective function, global optimum is either its maximum

value or its minimum value depending on what you are looking for. For example, in design model (normally described by a mathematical model with several variables and parameters), we will maximize the performance of the model in terms of a certain aspects, such as strength. On the other hand, we will minimize the usage of materials to save cost. In this thesis, without loss of generality for all global optimization problems, we are about to find their minimum function values. Therefore, we just discuss the minimization problems in the following chapters.

For a given minimization problem, a global optimum is an optimum of the entire domain $\mathbb{A}$ which yields the lowest value while a local optimum is an optimum of a subset of $\mathbb{X}$ where its value is the lowest in its neighbors. The formal definition of a local optimum $\vec{x}_l^*$ can be described as:

$$\forall \vec{x} \, \exists \epsilon > 0 : f(\vec{x}_l^*) \leq f(\vec{x}) \, \forall \vec{x} \in \mathbb{X}, \|\vec{x} - \vec{x}_l^*\| < \epsilon \tag{3.2}$$

Generally speaking, the difficulty for algorithms to find the global optimum is how to prevent individuals from being trapped in the basins of attractions of local optima. Generally speaking, an objective function is difficult if it is not differentiable, not continuous, or implicit, or has too many local optima. The difficulty can be roughly explained from Figure 3.1, which shows some different kinds of fitness landscapes.

Most objective functions are multi-modal problems, which have many local optima in the fitness landscape. This unsteady or fluctuating fitness landscapes makes the objective function complicated and difficult to optimize because optimizers do not know the right direction during the process. For example, if the fitness landscape is too rugged or irregular, as in Figure 3.1-d, it is hard for optimizers to learn useful information to guide the search. Sometimes, the fitness landscape shows deceptiveness, e.g., Figure 3.1-e, which will mislead optimizers

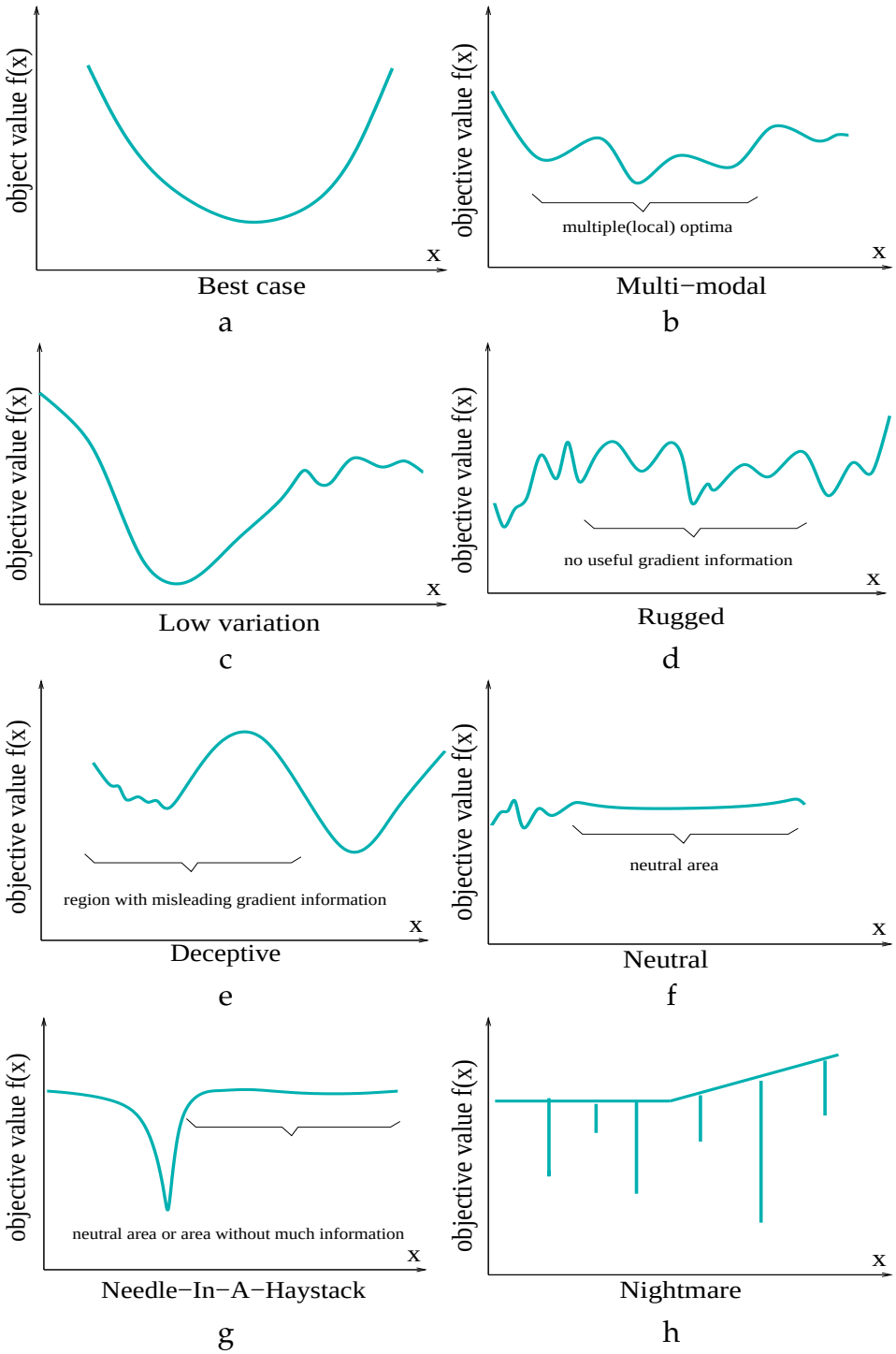


Figure 3.1: Different properties of fitness landscapes

away from the global optimum. It is also challenging if the global optimum is situated on a plateau of the fitness landscape, as illustrated in Figure 3.1-f and Fig-

ure 3.1-g, because an optimizer can not find any gradient information to guide the search. In addition, many other aspects also make the fitness landscape difficult to optimize, e.g., noise, dynamism, rotation, shift, and dimensional linkage, etc.

In the following section, we introduce some global optimization benchmarks, including un-rotated, shifted, rotated shifted as well as composition functions, used in this thesis.

3.2 Test Functions

In this section, we chose 45 test functions, including traditional functions, traditional functions with noise, shifted functions, and rotated shifted functions, which are widely used in the literature [43, 76, 136, 138] as well as the complex hybrid composition functions proposed recently in [42, 106], to test an algorithm's performance. The details of these test functions are given in Table 3.1, Table 3.2, and Table 3.3. The 45 functions are divided into six groups in terms of their properties: traditional problems (f_1-f_{12}) , noisy problems (f_{19}-f_{22}), shifted problems (f_{15}-f_{18}, f_{31}-f_{32}, and f_{36}-f_{38}), rotated problems (f_{23}-f_{26}), rotated shifted problems (f_{27}-f_{30}, f_{33}-f_{35}, and f_{39}), and hybrid composition functions (f_{13}-f_{14} and f_{40}-f_{45}). Table 3.4 shows the parameter settings for some particular functions (f_3, f_{12}-f_{14}, f_{23}-f_{30}). The detailed parameter setting for functions f_{31}-f_{45} can be found in [106] with the corresponding functions.

3.2.1 Traditional Test Problems

To test the performance of the proposed algorithms on standard test problems, we selected 12 test functions. Five of them are unimodal problems, which are considered easy to solve. They are the Sphere function (f_1), Rosenbrock function (f_8), Schwefel_2_22 (f_9), Schwefel_1_2 (f_{10}), and Schwefel_2_21 (f_{11}). The others in

Table 3.1: The test functions, where f_{min} is the minimum value of a function and $S \in R_n$

Name	Test Function	S	f_{min}
Sphere	$f_1(\vec{x}) = \sum_{i=1}^{n} x_i^2$	$[-100, 100]$	0
Rastrigin	$f_2(\vec{x}) = \sum_{i=1}^{n} (x_i^2 - 10\cos(2\pi x_i) + 10)$	[-5.12, 5.12]	0
Noncont_Rastrigin	$f_3(\vec{x}) = \sum_{i=1}^{n} (y_i^2 - 10\cos(2\pi y_i) + 10)$	[-5.12, 5.12]	0
Weierstrass	$f_4(\vec{x}) = \sum_{i=1}^{n} (\sum_{k=0}^{k_{max}} [a^k \cos(2\pi b^k (x_i + 0.5))]) - n \sum_{k=0}^{k_{max}} [a^k \cos(\pi b^k)]$, $a = 0.5, b = 3, k_{max} = 20$	[-0.5,0.5]	0
Griewank	$f_5(\vec{x}) = \frac{1}{4000} \sum_{i=1}^{n} (x_i - 100)^2 - \prod_{i=1}^{n} \cos(\frac{x_i - 100}{\sqrt{i}}) + 1$	[-600, 600]	0
Schwefel	$f_6(\vec{x}) = 418.9829 \cdot n + \sum_{i=1}^{n} -x_i \sin(\sqrt{\lvert x_i \rvert})$	[-500, 500]	0
Ackley	$f_7(\vec{x}) = -20\exp(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}) - \exp(\frac{1}{n}\sum_{i=1}^{n} \cos(2\pi x_i)) + 20 + e$	[-32, 32]	0
Rosenbrock	$f_8(\vec{x}) = \sum_{i=1}^{n} 100(x_{i+1}^2 - x_i)^2 + (x_i - 1)^2)$	[-2.048, 2.048]	0
Schwefel_2_22	$f_9(\vec{x}) = \sum_{i=1}^{n} \lvert x_i \rvert + \prod_{i=1}^{n} \lvert x_i \rvert$	[-10, 10]	0
Schwefel_1_2	$f_{10}(\vec{x}) = \sum_{i=1}^{n} (\sum_{j=1}^{i} x_j)^2$	[-100, 100]	0
Schwefel_2_21	$f_{11}(\vec{x}) = \max_{i=1}^{n} \lvert x_i \rvert$	[-100, 100]	0
Penalized_1	$f_{12}(\vec{x}) = \frac{\pi}{30}\{10\sin^2(\pi y_1) + \sum_{i=1}^{n-1} (y_i - 1)^2 \cdot [1 + 10\sin^2(\pi y_{i+1})] + (y_n - 1)^2\} + \sum_{i=1}^{n} u(x_i, 5, 100, 4), y_i = 1 + (x_i + 1)/4$	[-50, 50]	0
H_Com	$f_{13}(\vec{x})$ =Hybrid Composition function (CF4) in [42]	[-5, 5]	0
RH_Com	$f_{14}(\vec{x})$ =Hybrid Composition function (CF4) with rotation in [42]	[-5, 5]	0

Table 3.2: Test functions of f_{15} to f_{30}, where "O" represents the original problems, "N","S", "R", and "RS" represent the modified problems by adding noise, shifting, rotating, and combination of shifting and rotating, respectively.

	O	N	S	R	RS		O	N	S	R	RS
Sphere	f_1	f_{19}	f_{18}	f_{23}	f_{27}	Schwefel	f_6	f_{20}	f_{15}	f_{25}	f_{28}
Rastrigin	f_2	f_{22}	f_{17}	f_{24}	f_{30}	Ackley	f_7	f_{21}	f_{16}	f_{26}	f_{29}

this group are multi-modal problems where the number of local optima increases exponentially with the number of dimensions. These problems appear to be difficult to solve for many optimization algorithms. The Rastrigin function (f_2) has a large number of local optima, which may easily trap algorithms into a local optimum. Hence, to solve this problem, algorithms should maintain a large diversity. The Noncontinuous Rastrigin function (f_3) is constructed based on the Rastrigin function and has the same number of local optima as the continuous Rastrigin function. The Weierstrass function (f_4) is continuous but differentiable only on a set of points. The Griewank function (f_5) has linkages among variables, which make it difficult to find the global optimum but the difficulty decreases

Table 3.3: Test functions of f_{31} to f_{45} chosen from [106]

f	Name&function number in [106]	S	f_{min}
f_{31}	S_Sphere_CEC05(F_1)	[-100,100]	-450
f_{32}	S_Rastrigin_CEC05(F_9)	[-5,5]	-330
f_{33}	RS_Rastrigin_CEC05(F_{10})	[-5,5]	-330
f_{34}	RS_Weierstrass_CEC05(F_{11})	[-0.5,0.5]	90
f_{35}	RS_Ackley_Bound_CEC05(F_8)	[-32,32]	-140
f_{36}	S_Rosenbrock_CEC05(F_6)	[-100,100]	390
f_{37}	S_Schwefel_1_2_CEC05(F_2)	[-100,100]	-450
f_{38}	S_Schwefel_1_2_Noisy_CEC05(F_4)	[-100,100]	-450
f_{39}	RS_Elliptic_CEC05(F_3)	[-100,100]	-450
f_{40}	Com_CEC05(F_{15})	[-5,5]	120
f_{41}	H_Com_CEC05(F_{16})	[-5,5]	120
f_{42}	H_Com_Noisy_CEC05(F_{17})	[-5,5]	120
f_{43}	RH_Com_CEC05(F_{18})	[-5,5]	10
f_{44}	RH_Com_NarrowBasin_CEC05(F_{19})	[-5,5]	10
f_{45}	RH_Com_Bound_CEC05(F_{20})	[-5,5]	10

when the number of dimensions increases [123]. The Ackley function (f_7) has one narrow global optimum basin and many minor local optima. But it can be easily solved as its local optima are shallow. The Schwefel function (f_6) is difficult to solve due to its deep local optima being far from the global optimum. The Penalized function 1 (f_{12}) is easy to solve as its local optima are shallow.

Table 3.4: Parameters settings for f_3, f_{12}, f_{13}, f_{14}, the rotated and rotated shifted functions

f	Parameter values
f_3	$y_i = \begin{cases} x_i, & \lvert x_i\rvert < 1/2, \\ round(2x_i), & \lvert x_i\rvert \geq 1/2 \end{cases}$
f_{12}	$u(x,a,k,m) = \begin{cases} k(x-a)^m, & x > a, \\ 0, & -a \leq x \leq a, \\ k(-x-a)^m, & x < -a. \end{cases}$
f_{13}	m=10, $\mathbf{M}_{1-10}(f_{13})$=identity matrix, $c_{1-10}(f_{14})$=2
f_{14}	g_{1-2} = Sphere function, g_{3-4}=Rastrigin function, g_{5-6}=Weierstrass function
	g_{7-8}=Griewank function,g_{9-10}=Ackley function
	$bias_k = 100(k-1), k = 1, 2, \ldots, 10$
f_{29}	c=100
f_{23}–f_{28},f_{30}	c=2

3.2.2 Noisy Test Problems

In order to test an algorithm's performance in noisy environments, the functions f_{19} -f_{22} are modified from four traditional test functions (f_1, f_6, f_2, and f_7) by adding

noise in each dimension by:

$$f(\vec{x}) = g(\vec{x} - 0.01 \cdot \vec{o}_{rand})$$

where $\vec{o}_{rand}$ is a vector of uniformly distributed random numbers within $(0, 1)$ generated for each fitness evaluation.

3.2.3 Shifted Test Problems

In some problems, the global optimum has the same parameter value in all dimensions (e.g., the Rosenbrock function (f_8) has a global optimum of [1,1,...,1]). In some other problems, the global optimum is always located in the origin (e.g., the Rastrigin function (f_2) has a global optimum at the origin [0,0,...0]). Due to the simple properties of the global optimum in these types of test problems, some researchers may develop algorithms that specifically exploit these properties. For example, if the global optimal values of one dimension is found by the algorithm, the values of all remaining dimensions can be easily obtained by copying the found value to these dimensions. To avoid this bias, the shifted problems f_{15}-f_{18} are extended from existing standard benchmark problems by shifting the global optimum to a random position within the search range. The shifted functions f_{31}-f_{32} and f_{36}-f_{38} were chosen from [106]. The functions with prefix of "S_" are shifted problems in Table 3.2 and Table 3.3. The global optima of the problems in this group were shifted by:

$$f(\vec{x}) = g(\vec{x} - \vec{o}_{rand})$$

where the new global optimum $\vec{o}_{new} = \vec{o}_{rand} + \vec{o}_{old}$, where $\vec{o}_{rand}$, $\vec{o}_{old}$, and $\vec{o}_{new}$ are the random position, the original global optimum, and the new global optimum, respectively. $\vec{o}_{rand}$ is initialized once at the beginning of the run and keeps the same for all the fitness evaluations during the whole run.

3.2.4 Rotated Shifted Test Problems

Some multi-modal functions in the first group are separable problems. This type of problems have a special property: they can be solved by using D one-dimensional search methods, like the ones used in some co-evolutionary algorithms. To avoid this bias, we rotated and shifted some functions in the first group. The functions with prefix of "RS_" are rotated shifted problems in Table 3.2 and Table 3.3. We have eight rotated and shifted problems in this group, which are rotated and shifted by the method used in [106]:

$$f(\vec{x}) = g((\vec{x} - \vec{o}_{rand}) * \mathbf{M})$$

where **M** is a linear transformation matrix generated using:

$$\mathbf{M} = \mathbf{P} * \mathbf{N} * \mathbf{Q}$$

where **P** and **Q** are orthogonal matrices obtained by classical Gram-Schmidt method, **N** is a diagonal matrix where the main diagonal entries are produced by:

$$n_{ii} = c^{\frac{u_i - min(u)}{max(u) - min(u)}}, u = U(1, D)$$

where D is the number of dimensions, $U(1, D)$ is a random number uniformly distributed in $[1, D]$; c=$Cond(\mathbf{M})$ is the condition number of the matrix **M**, and c is a constant which is set to different values as listed in Table 3.4 for the four functions in this group.

3.2.5 Hybrid Composition Test Problems

In order to further test an algorithm's performance on complex problems, eight hybrid composition functions were chosen. The functions f_{13} and f_{14} were defined

in [42] and the other six (f_{40}-f_{45}) listed in Table 3.3 were defined in [106]. Hybrid composition functions are constructed using some basic benchmark functions to create even more challenging problems with a randomly located global optimum and several randomly located deep local optima.

The composition function can be described as:

$$f(\vec{x}) = \sum_{i=1}^{m} (w_i \cdot (g'_i((\vec{x} - \vec{o}_{rand_i})/\lambda_i * \mathbf{M}_i) + bias_i)) \tag{3.3}$$

where $f(\vec{x})$ is the composition function; $g_i(\vec{x})$ is i-th basic function used to construct the composition function; m is the number of basic functions; $\mathbf{M}_i$ is the linear transformation matrix for each $g_i(\vec{x})$; $\vec{o}_{old_i} + \vec{o}_{rand_i}$ is the optimum of the changed $g_i(\vec{x})$ caused by shifting the landscape. $\vec{o}_{old_i}$ and $\vec{o}_{rand_i}$ are the optima of the original $g_i(\vec{x})$ without any change and a random position generated for the changed $g_i(\vec{x})$, respectively. The value of $\vec{o}_{old_i}$ is 0 for all the basic functions used in this group. The weight value w_i for each $g_i(\vec{x})$ is calculated as:

$$w_i = exp(-sqrt(\frac{\sum_{k=1}^{n} (x_k - o_i^k + o_{old_i}^k)^2}{2n\sigma_i^2}))$$

$$w_i = \begin{cases} w_i & \text{if } w_i = max(w_i) \\ w_i \cdot (1 - max(w_i)^{10}) & \text{if } w_i \neq max(w_i) \end{cases}$$

$$w_i = w_i / \sum_{i=1}^{m} w_i$$

where σ_i is the converge range factor of $g_i(\vec{x})$, whose default value is 1.0; λ_i is the stretch factor for each $g_i(\vec{x})$, which is defined as:

$$\lambda_i = \sigma_i \cdot \frac{X_{max} - X_{min}}{x_{max}^i - x_{min}^i}$$

where $[X_{max}, X_{min}]^n$ is the search range of $f(x)$ and $[x^i_{max}, x^i_{min}]^n$ is the search range of $g_i(\vec{x})$.

In Eq. (3.3), $g'_i(\vec{x}) = C \cdot g_i(\vec{x})/|g^i_{max}|$, where C is a predefined constant, which is set to 2000, and g^i_{max} is the estimated maximum value of $g_i(\vec{x})$, which is estimated as:

$$g^i_{max} = g_i(x_{max} \cdot M_i)$$

The details of functions f_{13} and f_{14} are given in Table 3.4. The only difference between functions f_{13} and f_{14} is that f_{14} is rotated while f_{13} is not. This makes f_{14} more complex than f_{13}. Although in [106] the authors proposed eleven hybrid composition functions, we chose only six typical problems (f_{40}-f_{45}) for testing because the other functions also have similar properties and difficulty level as the ones we selected.

It should be noticed that there are three pairs of functions in Table 3.2 and Table 3.3 where each pair of functions have similar property. They are (f_{17}, f_{32}), (f_{18},f_{31}), and (f_{30},f_{33}). For each pair, the differences are that the former chosen from [106] have bias value but no in the later and the locations of the global optima are also different. However, for convenient comparison, we still take the two functions of each pair as different functions.

3.3 Performance Metrics

We test all algorithms over a certain number of independent runs on a specific problem and then calculate the statistical results in different ways. In order to show an algorithm's performance in terms of different aspects, we use several performance metrics, such as, the mean value, variance, and success rate. To compare two different algorithms' performance on a particular problem, we perform a two-tailed T-test operation. To compare overall performances of algorithms on

all the 45 static problems, we calculate the winning ratio and equal ratio, which are defined in the following section.

To evaluate an algorithm's performance on static problems, we record the mean value of the difference between the best result found by the algorithm and the global optimum value over a certain number of runs (N) for each problem:

$$mean = \frac{1}{N}\sum_{k=1}^{N}(f(\vec{x_k}) - f(\vec{x^*}))$$

where $\vec{x_k}$ and $\vec{x^*}$ represent the best solution found by the algorithm in the kth run and the global optimum, respectively.

To compare two algorithms' performance on the statistical level, the two-tailed t-test with 58 degrees of freedom at a 0.05 level of significance was conducted between two algorithms with N=30. The performance difference is significant between two algorithms if the absolute value of the t-test result is greater than 2.0.

To compare two algorithms' overall performance on the 45 problems, we define a winning ratio of algorithm a against algorithm b ($w_r(a,b)$) together with an equal ratio ($e_r(a,b)$) as follows:

$$w_r(a,b) = \sum_{f=1}^{F} B_f(a,b) / \sum_{f=1}^{F} B_f(b,a) \tag{3.4}$$

$$e_r(a,b) = \sum_{f=1}^{F} (1 - B_f(a,b) - B_f(b,a))/F \tag{3.5}$$

where F=45 and if the performance of algorithm a is significantly better than algorithm b on function f in terms of the t-test result, $B_f(a,b)$ returns 1; otherwise $B_f(a,b)$ returns 0. From the two equations, we can see that the larger the winning ratio and the smaller the equal ratio, the bigger the overall performance difference between algorithm a and algorithm b.

Table 3.5: Accuracy level of the 45 problems

Accuracy level	Function
1.0e-6	$f_1, f_7, f_9, f_{11}, f_{12}, f_{16}, f_{18}, f_{19}, f_{21}, f_{23}$ $f_{26}, f_{27}, f_{29}, f_{31}, f_{35}, f_{39}$
0.01	$f_2, f_3, f_4, f_5, f_6, f_8, f_{10}, f_{15}, f_{17}, f_{20}, f_{22}, f_{24}$ $f_{25}, f_{28}, f_{30}, f_{32}, f_{33}, f_{34}, f_{36}, f_{37}, f_{38}$
0.1	$f_{13}, f_{14}, f_{40}, f_{41}, f_{42}, f_{43}, f_{44}, f_{45}$

Another performance metric is the success rate, which is to calculate the rate of the number of successful runs over the total number of runs. A successful run means an algorithm achieves the fixed accuracy level within a fixed number of evaluations for a particular problem. The accuracy level for each test problem is give in Table 3.5.

3.4 Summary

This chapter briefly discussed the difficulties for algorithms to solve global optimization problems. Some examples of how to modify traditional problems to make them harder to solve, e.g., rotating and shifting the fitness landscape, adding noise, and the composition method, are introduced. To evaluate and compare performances of different algorithms investigated in this thesis, some performance metrics are also defined in this chapter.

Chapter 4

Dynamic Optimization Problems

In recent years, there has been a growing interest in studying EAs for DOPs due to its importance in real world applications since many real world optimization problems are DOPs. Over the years, a number of dynamic problem generators have been used in the literature to create dynamic test environments to compare the performance of EAs. Generally speaking, they can be roughly divided into two types.

For the first type, the environment is just switched between several stationary problems or several states of a problem. For example, many researchers tested their approaches on the dynamic knapsack problem where the weight capacity of the knapsack changes over time, usually oscillating between two or more fixed values [59, 81]. Cobb and Grefenstette [21] used a dynamic environment that oscillates between two different fitness landscapes. For this type of generators, the environmental dynamics is mainly characterized by the speed of change measured in EA generations.

The second type of DOP generators construct dynamic environments by re-shaping a predefined fitness landscape. For example, Branke [13] proposed the MPB problem, which consists of a number of peaks in a multi-dimensional landscape, where the height, width, and position of each peak can be altered a little

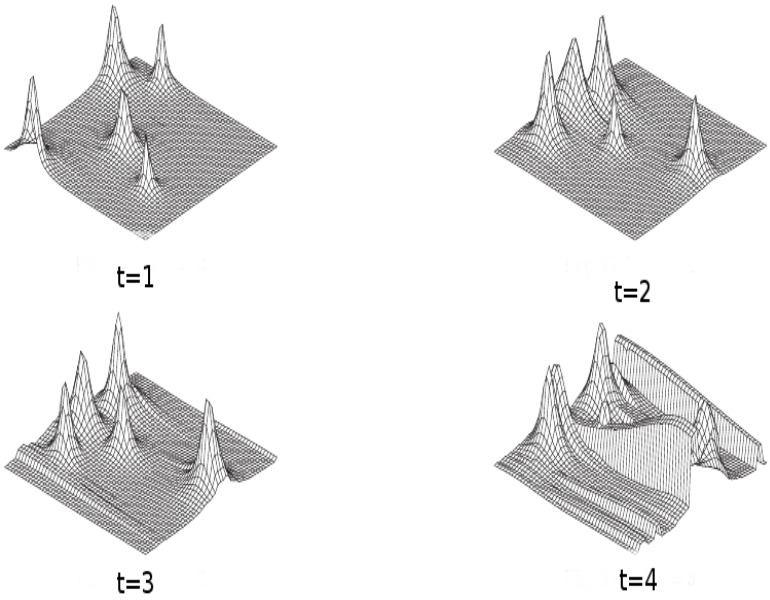


Figure 4.1: Dynamism in the fitness landscape

every time the environment changes. This function is capable of generating a given number of peaks in a given number of dimensions that vary both spatially (position and shape of a peak) and in terms of fitness. Morrison and De Jong [79] also defined a dynamic problem generator that is similar to the MPB problem. To evaluate an algorithm's performance under different kinds of dynamisms, Li and Yang proposed the GDBG benchmark [64, 68], which has seven different environmental change types.

To understand how the fitness landscape changes over time, Figure 4.1 shows four episodes in different time steps of the MPB problem [13]. It can be seen from the figure, some elements are changing from time step $t = 1$ to time step $t = 4$. The height, width, shape, location, and the number of peaks are changing over time.

Change types are also variable. For example, in the GDBG [64, 68] system, it has small step change, large step change, random change, chaotic change, recurrent change, recurrent change with noise, and dimensional change. Therefore, all

these dynamic characteristics require algorithms to quickly response and adapt to the new environments, and make DOPs more complex and harder to optimize than stationary problems.

In this chapter, three popular dynamic benchmarks are introduced to test an algorithm's performance in dynamic environments. They are the MPB problem [13], the DF1 function generator [79], and the GDBG benchmark [64, 68].

4.1 The MPB Problem

The MPB problem proposed by Branke [13] has been widely used as dynamic benchmark problems in the literature. Within the MPB problem, the optima can be varied by three features, e.g., the location, height, and width of peaks. For the D-dimensional landscape, the problem is defined as follows:

$$F(\vec{x}, t) = \max_{i=1,\ldots,p} \frac{H_i(t)}{1 + W_i(t) \sum_{j=1}^{D} (x_j(t) - X_{ij}(t))^2} \tag{4.1}$$

where $W_i(t)$ and $H_i(t)$ are the height and width of peak i at time t, respectively, and $X_{ij}(t)$ is the j-th element of the location of peak i at time t. The p independently specified peaks are blended together by the *max* function. The position of each peak is shifted in a random direction by a vector $\vec{v_i}$ of a distance s (s is also called the shift length, which determines the severity of the problem dynamics), and the move of a single peak can be described as follows:

$$\vec{v_i}(t) = \frac{s}{\left|\vec{r} + \vec{v_i}(t-1)\right|}((1-\lambda)\vec{r} + \lambda\vec{v_i}(t-1)) \tag{4.2}$$

where the shift vector $\vec{v_i}(t)$ is a linear combination of a random vector $\vec{r}$ and the previous shift vector $\vec{v_i}(t-1)$ and is normalized to the shift length s. The correlated parameter λ is set to 0, which implies that the peak movements are uncorrelated.

Table 4.1: Default settings for the MPB problem

Parameter	Value
p (number of peaks)	10
change frequency	5000
height severity	7.0
width severity	1.0
peak shape	cone
basic function	no
shift length s	1.0
number of dimensions D	5
correlation coefficient λ	0
S	[0, 100]
H	[30.0, 70.0]
W	[1, 12]
I	50.0

More formally, a change of a single peak can be described as follows:

$$H_i(t) = H_i(t-1) + height\_severity * \sigma \tag{4.3}$$

$$W_i(t) = W_i(t-1) + width\_severity * \sigma \tag{4.4}$$

$$\vec{X}_i(t) = \vec{X}_i(t)(t-1) + \vec{v}_i(t) \tag{4.5}$$

where σ is a normally distributed random number with mean zero and variation one.

The default settings and definition of the benchmark used in this thesis can be found in Table 4.1. In Table 4.1, the change frequency is the number of evaluations between two successive changes where the term "evaluation" means that an individual is created and its fitness is evaluated, S denotes the range of allele values, and I denotes the initial height for all peaks. The dynamism of changes is described as follows. The height of peaks is shifted randomly in the range $H = [30, 70]$ and the width of peaks is shifted randomly in the range $W = [1, 12]$.

4.2 The DF1 Generator

The dynamic problem generator DF1, proposed by Morrison and De Jong [79], is a kind of moving peaks benchmark generators. Within the DF1 generator, the base landscape in the D-dimensional real space is defined as:

$$f(\vec{x}) = \max_{i=1,\ldots,p} \left[H_i - R_i \times \sqrt{\sum_{j=1}^{D} (x_j - X_{ij})^2} \right] \tag{4.6}$$

where $\vec{x} = (x_1, \cdots, x_D)$ is a point in the landscape, p specifies the number of peaks (or cones), and each peak i is independently specified by its height H_i, its slope R_i, and its center $X_i = (X_{i1}, \cdots, X_{iD})$. The fitness at a point on the surface is assigned the maximum height of all optima at that point; the optima with the greatest height at a point is said to be visible at that point.

Just like the MPB problem [13], DF1 creates dynamic problems by changing the features, i.e., the location, height, and slope of each peak independently. The dynamics are controlled by the Logistics function given by:

$$\Delta_t = A \cdot \Delta_{t-1} \cdot (1 - \Delta_{t-1}) \tag{4.7}$$

where A is a constant value in the range [1.0, 4.0] and Δ_t is used as the step size of changing a particular parameter (i.e., the location, height, or slope) of peaks at iteration t after scaled by a scale factor s in order to reduce step sizes that may be larger than intended for each step. The Logistics function allows a wide range of dynamic performance by a simple change of the value of A, from simple constant step sizes, to step sizes that alternate between two values, to step sizes that rotate through several values, to completely chaotic step sizes. The default parameter settings are shown in Table 4.2. More details on DF1 can be found in [79].

Table 4.2: Parameter Settings for the DF1 Function

Parameter	value
number of dimensions D	2
A	1.2,3.3,3.99
number of peaks p	3
change frequency U	6000
population size	60
H	[1.0,5.0]
W	[1.0,5.0]
S	[-1.0,1.0]
scale factor s	0.4

4.3 The GDBG System

The GDBG system proposed by Li and Yang [64] defines a DOP as below:

$$F = \min f(x, \phi, t) \tag{4.8}$$

where f is the cost function, x is a feasible solution in the solution set, t is the real-world time, and ϕ is the system control parameter(s). The GDBG system constructs dynamic environments by changing the system control parameter(s). It can be described as follows:

$$\phi(t+1) = \phi(t) \oplus \Delta\phi \tag{4.9}$$

where $\Delta\phi$ is a deviation from the current system control parameters. Then, the new environment at the next moment $t+1$ is obtained as follows:

$$f(x, \phi, t+1) = f(x, \phi(t) \oplus \Delta\phi, t) \tag{4.10}$$

The GDBG system can be instantiated to construct DOPs in different space domains, including binary, real, and combinatorial spaces. In this thesis, we use

the real space instance to construct real-encoded DOPs as described in [68]. For convenience, without special note, the term of "GDBG" benchmark refers to the instances in real space in the GDBG system [64] in this thesis. In total, there are seven basic test functions defined and each test function has seven change types, which are small step change, large step change, random change, chaotic change, recurrent change, recurrent change with noise, and dimensional change. The seven change types are defined as follows:

T1 (small step): $\Delta\phi = \alpha \cdot \|\phi\| \cdot r \cdot \phi_{severity}$

T2 (large step): $\Delta\phi = \|\phi\| \cdot (\alpha \cdot sign(r) + (\alpha_{max} - \alpha) \cdot r) \cdot \phi_{severity}$

T3 (random): $\Delta\phi = N(0,1) \cdot \phi_{severity}$

T4 (chaotic): $\phi(t+1) = A \cdot \phi(t) \cdot (1 - \phi(t)/\|\phi\|)$

T5 (recurrent): $\phi(t+1) = \phi_{min} + \|\phi\|(\sin(\frac{2\pi}{P}t + \varphi) + 1)/2$

T6 (recurrent with noise): $\phi(t+1) = \phi_{min} + \|\phi\|(\sin(\frac{2\pi}{P}t + \varphi) + 1)/2 + N(0,1) \cdot noise_{severity}$

T7 (dimensional change): $D(t+1) = D(t) + flag \cdot \Delta D$

where $\|\phi\|$ is the range of ϕ, $\phi_{severity} \in (0,1)$ is the change severity of ϕ, ϕ_{min} is the minimum value of ϕ, $noise_{severity} \in (0,1)$ is the noise severity in the recurrent with noise change. $\alpha \in (0,1)$ and $\alpha_{max} \in (0,1)$ are constant values, which are set to 0.04 and 0.1 in the GDBG system, respectively. The Logistics function is used in the chaotic change type, where A is a positive constant in the range $(1.0, 4.0)$. If ϕ is a vector, the initial values of the items in ϕ should be different within $\|\phi\|$ in the chaotic change. P is the period of the recurrent change and the recurrent change with noise, φ is the initial phase, r is a random number in $(-1, 1)$, $sign(x)$ returns 1 when x is greater than 0, -1 when x is less than 0, or 0 otherwise. $N(0,1)$ denotes a normally distributed one-dimensional random number with mean zero

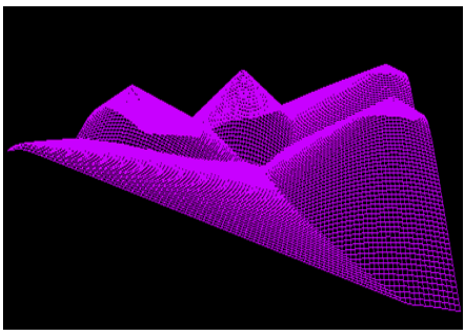
Table 4.3: Default settings for the GDBG benchmark

Parameter	Value
number of dimensions D	fixed: 10; changing: [5-15]
search range	$x \in [-5, 5]^D$
number of functions or peaks p	10
change frequency U	$10,000 \times D$ fitness evaluations
number of changes K	$K = 60$
period P	12
severity of noisy	$noisy_{severity} = 0.8$
chaotic constant A	3.67
step severity	$\alpha = 0.04$
maximum of α	$\alpha_{max} = 0.1$
height range	$h \in [10, 100]$
initial height	$initial\_height = 50$
severity of height change	$\phi\_h_{severity} = 5.0$
sampling frequency s_f	100 fitness evalutions

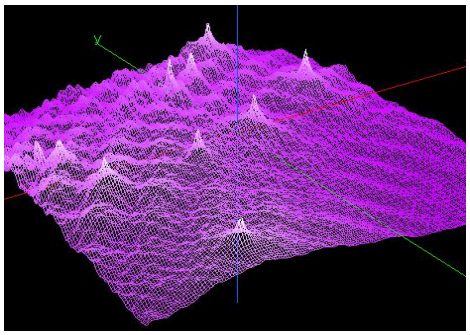
and standard deviation one. ΔD is a predefined constant, which is set to 1 by default. If $D(t) = D_{max}$, $flag = -1$; if $D(t) = D_{min}$, $flag = 1$. D_{max} and D_{min} are the maximum and minimum number of dimensions, respectively.

The seven basic test functions defined in [68] are the rotation peak function with 10 peaks (F_1 with $p = 10$), the rotation peak function with 50 peaks (F_1 with $p = 50$), the composition of Sphere's functions (F_2), the composition of Rastrigin's functions (F_3), the composition of Griewank's functions (F_4), the composition of Ackley's functions (F_5), and the hybrid composition function (F_6). The parameters of the seven problems are set the same as in [68]. The general parameter settings are given in Table 4.3 and the detailed settings of F_1 to F_6 can be found from [68].

Generally speaking, the GDBG benchmark is much harder to locate and track the global optima than the MPB and DF1 problems. Due to the huge number of local optima and the higher number of dimensions of the GDBG benchmarks, the complexity of some test functions (e.g., F_2 - F_6) is much higher than the MPB and DF1 problems. For example, the visual comparison of the complexity of a



One episode of the MPB problem



One episode of the GDBG bechmark

Figure 4.2: Comparison of the MPB problem and the GDBG benchmark

fitness landscape created by the MPB problem and a fitness landscape created by the GDBG benchmark can be found from Figure 4.2. Figure 4.2 clearly shows that the fitness landscape of the GDBG benchmark is much rougher than that of the MPB problem, and the number of local optima of the GDBG benchmark is also far more than that of the MPB problem.

4.4 Performance Metrics

For different dynamic generators, different performance metrics are defined to evaluate an algorithm's performance. The details of these performance metrics are described in the following sections. It should be noticed that the problems used in dynamic environments are maximization problems that are different from the global optimization problems used in this thesis.

4.4.1 Performance Metrics for the MPB Problem

The performance measure used for the MPB problem is the offline error, which is defined as follows:

$$\mu = \frac{1}{K}\sum_{k=1}^{K}(h_k - f_k), \tag{4.11}$$

where f_k is the best solution obtained by an algorithm just before the k-th environmental change, h_k is the optimum value of the k-th environment, μ is the average of all differences between h_k and f_k over the environmental changes, and K is the total number of environments.

4.4.2 Performance Metrics for the DF1 Function

Two performance measure methods, as used in [84], are described for the DF1 problem. They are the global offline error (e_g) and the average offline error (e_{avg}). The global offline error is defined as follows:

$$e_k = 1 - f_{gk}/h_{gk} \tag{4.12}$$

$$e_g = \frac{1}{K}\sum_{k=1}^{K} e_k \tag{4.13}$$

where f_{gk} and h_{gk} are the best particle's fitness and the height of the global optimum just before the k-th environmental change, and K is the total number of environments.

In order to measure the ability of an algorithm to track multiple optima in addition to the global optimum, the average of the best "local" errors since the last environmental change over all visible peaks is calculated as follows:

$$e_{avg_k} = \frac{1}{|p_k|}\sum_{i=1}^{|p_k|} e_{ik} \tag{4.14}$$

where e_{ik} is relative error of the best particle's fitness on a visible peak i to the height h_{ik} of peak i at the last generation of environment k, p_k denotes those peaks that are visible at the last generation of environment k, and $|p_k|$ is the number of

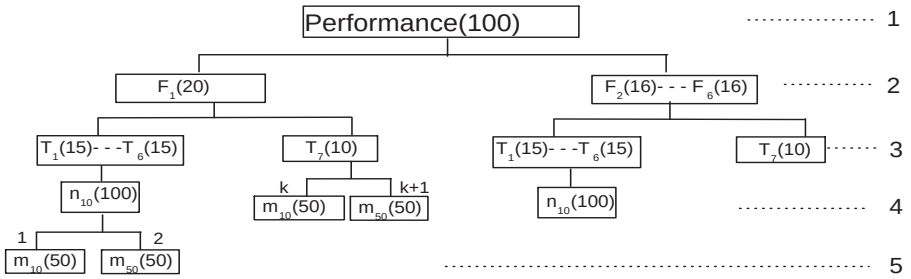


Figure 4.3: Overall performance marking measurement

such peaks. So, the average offline error e_{avg} is calculated as:

$$e_{avg} = \frac{1}{K} \sum_{k=1}^{K} e_{avg_k} \tag{4.15}$$

When calculating e_{avg_k}, a particle is considered to be on an optimum if the optimum is visible at the location of the particle. If no particle is on an optimum, the local error for the optimum is the difference between the optimum's height and the fitness of the closest particle within the radius of the optimum's apex. If there is no particle on a peak, that peak is not included in the calculation of the average local error.

4.4.3 Performance Metrics for the GDBG Benchmark

There are 49 test cases in total constructed from the seven test problems in the GDBG benchmark. For an algorithm on each test case, the offline error (e_{off}) and its standard variance (STD) are recorded, which are defined as in [68] as follows:

$$e_{off} = \frac{1}{R * K} \sum_{r=1}^{R} \sum_{k=1}^{K} e_{r,k}^{last} \tag{4.16}$$

$$STD = \sqrt{\frac{\sum_{r=1}^{R}\sum_{k=1}^{K}(e_{r,k}^{last} - e_{off})^2}{R * K - 1}} \tag{4.17}$$

where R and K are the total number of runs and the number of environmental changes for each run, respectively, and $e_{r,k}^{last} = |f(\vec{x}_{best}(r,k)) - f(\vec{x^*}(r,k))|$, where $\vec{x^*}(r,k)$ is the global optimum of the k-th environment and $\vec{x}_{best}(r,k)$ is the position of the best particle of the last generation of the k-th environment during the r-th run.

The overall performance of an algorithm on all 49 test cases is calculated using the method proposed in [68], which is illustrated in Figure 4.3. In the figure, F_1-F_6 denote the six functions defined in the GDBG benchmark in [68], T_1-T_7 represent the seven change types defined in Section 4.3, n_{10} means that the number of dimensions is ten, and m_{10} and m_{50} denote that the number of peaks is 10 and 50, respectively.

Each test case i is assigned a weight w_i and the sum of weights of all the test cases is 1.0. The mark obtained by an algorithm on test case $i \in \{1, \ldots, 49\}$ is calculated by:

$$mark_i = \frac{w_i}{R * K}\sum_{r=1}^{R}\sum_{k=1}^{K}\left(r_{rk}^{last} / \left(1 + \frac{1}{S}\sum_{s=1}^{S}(1 - r_{rk}^{s})\right)\right) \tag{4.18}$$

where r_{rk}^{last} is the relative ratio of the best particle fitness of the last generation to the global optimum of the k-th environment, r_{rk}^{s} is the relative ratio of the best particle's fitness to the global optimum at the s-th sampling during the k-th environment (*initial population should be sampled*), and $S = U/s\_f$ is the total number of samples for each environment. The relative ratio r_{rk}^{s} is defined by

$$r_{rk}^{s} = \begin{cases} \frac{f(\vec{x}_{best}(r,k,s))}{f(\vec{x^*}(r,k))}, & f = F_1 \\ \frac{f(\vec{x^*}(r,k))}{f(\vec{x}_{best}(r,k,s))}, & f \in \{F_2, F_3, F_4, F_5, F_6\} \end{cases} \tag{4.19}$$

where $\vec{x}_{best}(r,k,s)$ is the position of the best particle up to the s-th sample in the

k-th environment during the r-th run.

The overall performance of an algorithm on all the test cases is then calculated as follows:

$$performance = 100 \times \sum_{i=1}^{49} mark_i \tag{4.20}$$

4.5 Summary

This chapter introduced some basic methods of how to build the dynamic environments to test the performance of EAs. Three popular benchmark generators, including the MPB problem, the DF1 function, and the GDBG benchmark, are described as well as their corresponding performance metrics.

Chapter 5

Self-learning Particle Swarm Optimizer

So far, most PSO algorithms globally use a single learning pattern for all particles, which means that all particles in a swarm use a same learning strategy. The monotonic learning pattern may cause the lack of intelligence for a particular particle, which makes it unable to deal with different complex situations. Actually, due to different difficulties of different fitness landscapes, we need to develop an intelligent system with different characteristics rather than a single learning pattern to deal with different complex situations.

In order to bring more intelligence to the individual level, this chapter will introduce a novel algorithm, called self-learning PSO (SLPSO). It will systematically describe the working mechanism of SLPSO in the following aspects. First, a set of four learning strategies are introduced, which have different properties to help particles deal with fitness landscapes with different properties. Second, an adaptive learning framework is presented to enable each particle to adaptively choose the best learning strategy according to its local fitness landscape. Third, in order to extract useful information from improved particles, an information up-

date method for the global best particle is described. Fourth, a monitoring method is developed, which is able to accurately identify whether a general particle has converged or not. Fifth, to increase diversity, a restart scheme is implemented to create new swarms that are composed of restarted particles that are eliminated from old swarms. Finally, a generalized parameter tuning method is introduced for general problems. Also, multiple population methods are used to encourage particles to explore unseen areas in the fitness landscape. How to systemically organize these components together and make them work efficiently, will be discussed in detail in this chapter.

5.1 General Considerations

In this section, the open issues, which are challenges for EAs introduced in Section 1.1 in Chapter 1, will be further discussed at a deep level in PSO, e.g., how to tradeoff the performance between the *gbest* model and the *lbest* model, how to avoid premature convergence, and what kind of intelligence a general particle may need.

5.1.1 Tradeoff between the *gbest* and *lbest* Models

It is generally believed that the *gbest* model biases more toward exploitation, while the *lbest* model focuses more on exploration. Although there are many improved versions of PSO, the question of how to balance the performance of the *gbest* and *lbest* models is still an important open issue, especially for multi-modal problems.

In the *gbest* model, all particles' social behavior is strictly constrained by learning information from the *gbest* particle. The *gbest* particle is more attractive than particles' personal *pbest* positions because of the higher fitness of the global best particle. Therefore, particles are easily attracted by the *gbest* particle and quickly

converge to the region around the *gbest* particle even though that region does not contain the global optimum. In the *gbest* model, once the whole swarm converges to a local optimum, all particles will lose the capability of jumping out of the basin of attraction of that local optimum since their velocity has reduced to zero.

In the *lbest* model, particles do not learn from the *gbest* particle of the whole swarm but from the best particle of their neighborhood. Because of that, particles that do not belong to the neighborhood of the *gbest* particle will not communicate with the *gbest* particle. As a result, in the *lbest* model, particles are not easily trapped in the area around the *gbest* particle. However, the trade-off is that the convergence speed may be slower compared with the *gbest* model.

5.1.2 Premature Convergence

Another important issue is how to avoid premature convergence for EAs. Convergence is one of the key performance metrics of EAs. Generally speaking, if a swarm of particles converges to a point in the search space, it means that the evolutionary process comes to an end. All individuals become inactive since they have lost the search capability. According to the theory of self-organization [80], if the system is going to be in an equilibrium, the evolution process will be stagnated. However, converging to a global optimum is not guaranteed for EAs. So, we need to maintain the social diversity of swarm to keep on the evolutionary process.

5.1.3 Individual Level of Intelligence

In order to achieve a good performance, a PSO algorithm needs to balance its search between the *lbest* and *gbest* models. However, this is not an easy task. If we let each particle simultaneously learn from both *pbest* and *gbest* to update itself, the algorithm may suffer from the disadvantages of both models. One solution might

be to implement the cognition component and the social component separately. In this way, each particle can focus on exploitation by learning from its individual *pbest* position or focus on convergence by learning from the *gbest* particle. The idea enables particles of different locations in the fitness landscape to carry out local search or global search, or vice versa, in different evolutionary stages. Different particles can play different roles (exploitation or convergence) during the search progress, and even the same particle can play different roles during the search progress. Thanks to this flexibility, the probability of avoiding PSO from being trapped in the basins of attraction of local optima may be increased. However, there is a difficulty in implementing this idea: the appropriate moment for a particle to learn from *gbest* or *pbest* is very hard to know.

To implement the idea above that allows particles to carry out different types of search (local or global) at different evolutionary stages, we need an adaptive method to automatically choose one type of search strategies. The method is adaptive, i.e., it enables particles to adaptively switch from learning from global information to learning from local information, and vice versa. Thanks to this adaptive property, we can overcome the aforementioned difficulty of identifying the appropriate learning stage for each particle. That is, which learning strategy and when to apply that strategy are determined by the property of the local fitness landscape where a particle is. The adaptive method will be further described later in Section 5.3.

5.1.4 Maintaining Diversity by Intelligence

Maintaining diversity is an important method to prevent a population from being trapped in local optima. There are several methods to maintain population diversity, e.g., maintaining diversity through the whole run, multi-population methods, restart methods, etc. The restart method is simple and easy to imple-

ment. However, to effectively use this technique, several open issues should be addressed. For example, how to restart particles, and when to perform the restart operation.

Maintaining the social diversity of a swarm can be achieved by refreshing the inactive particles (converged particles). That is, we restart a particle if it converges to a point in the search space. How to check the converged status of a particle and how to introduce fresh particles will also be described in the following sections.

How to avoid explored areas in the fitness landscape is a challenging question. Actually, maintaining diversity is a kind of idea that attempts to encourage particles to explore new areas. However, little or no attempt has been made to prevent particles from searching the areas that have been explored. To attempt to achieve this goal, first we need to give an answer to the question of which regions should be memorized by a particular particle. Second, we need to determine how to prevent particles from searching the areas covered by the memorized regions. These two issues will be discussed in the following sections in this chapter.

5.2 Learning Strategies in SLPSO

Inspired by the idea of division of labor, we can assign to particles different roles, e.g., converging to the global best particle, exploitation of its personal best position, exploring new promising areas, or jumping out of local optima. Which role a particular particle should play is determined by the local fitness landscape where it is. To assign to a particular particle individual level of intelligence, we need to consider the possible situations of the local fitness landscape.

Take Figure 5.1 as an example, which shows the fitness landscape of a composition function with ten components, labeled by 1, 2, 3 . . . 10, respectively, in two dimensions. The best learning strategy for particles in the sub-regions covered by components 1, 2, 3, 4, and 10 may be to learn from their own personal best

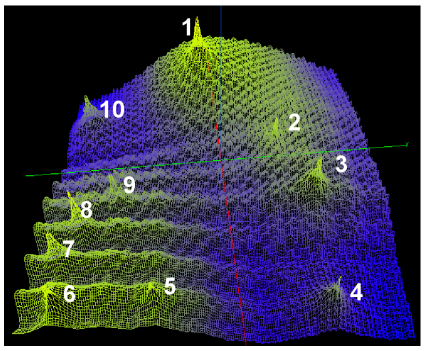


Figure 5.1: The fitness landscape of a composition function with ten components in two dimensions

positions since these components are relatively far away from each other and the shape of them is similar to the unimodal Sphere function. While, for particles in the sub-areas covered by components 5, 6, 7, 8, and 9, the best choice might be to learn from their neighbor, because these components are close to each other and they sit on several near ridges in the fitness landscape. The third case is when a particle converges to a local optimum, the jumping out strategy might help it escape from that local optimum. Another common case is when a particle stands on a slope, the best strategy may be to learn from its own *pbest* position or the particles with better fitness on the same slope. From Figure 5.1, we can conclude that which learning strategy a particle should use depends on its local fitness landscape. Therefore, we need to develop different learning strategies for particles in different situations.

Based on the above idea, we define four learning strategies and corresponding learning operators used in SLPSO. In SLPSO, the learning information for each particle comes from four sources: the archive of the *gbest* particle (*abest*), its individual *pbest* position, the *pbest* position of a random particle ($pbest_{rand}$) whose *pbest* is better than its own *pbest*, and a random position *prand* nearby. The four learning strategies play the roles of convergence, exploitation, exploration, and jumping out of the basins of attraction of local optima, respectively.

The four learning strategies enable each particle to independently deal with different situations. For each particle k, the learning equations corresponding to the four learning operators respectively are given as follows:
Operator a: learning from its *pbest* position

$$exploitation : v_k^d = \omega v_k^d + \eta \cdot r_k^d \cdot (pbest_k^d - x_k^d) \tag{5.1}$$

Operator b: learning from a random position nearby

$$jumping\ out : x_k^d = x_k^d + v_{avg}^d \cdot N(0,1) \tag{5.2}$$

Operator c: learning from the *pbest* of a random particle

$$exploration : v_k^d = \omega v_k^d + \eta \cdot r_k^d \cdot (pbest_{rand}^d - x_k^d) \tag{5.3}$$

Operator d: learning from the *abest* position

$$convergence : v_k^d = \omega v_k^d + \eta \cdot r_k^d \cdot (abest^d - x_k^d) \tag{5.4}$$

where $pbest_{rand}$ is the *pbest* of a random particle, which is better than $pbest_k$; the jumping step v_{avg}^d is the average velocity of all particles in the d-th dimension, which is calculated by $v_{avg}^d = \sum_{k=1}^{N} |v_k^d|/N$, where N is the population size; $N(0,1)$ is a random number generated from the normal distribution with mean 0 and variance 1; the *abest* position is used to store the best position found by SLPSO so far.

It should be noted that different from the standard PSO (SPSO), in SLPSO a bias selection scheme is added into the operator of learning from the $pbest_{rand}$ position. That is, a particle only learns from a $pbest_{rand}$ position that is better than its own historical best position *pbest*. Due to this scheme, more resources are given

Algorithm 5.1 *Update(operator i, particle k, fes)*

if $i = a$ **then**
  Update the velocity and position of particle k using operator a and Eq. (2.2);
else if $i = b$ **then**
  Update the position of particle k using operator b;
else if $i = c$ **then**
  Choose a random particle j that is not particle k;
  if $f(\vec{x}_{pbest_j}) < f(\vec{x}_{pbest_k})$ **then**
    Update the velocity and position of particle k using operator c and Eq. (2.2);
  else
    Update the velocity and position of particle j using operator c and Eq. (2.2);
  end if
else
  Update the velocity and position of particle k using operator d and Eq. (2.2);
end if
fes++;

where fes is the current number of fitness evaluations.

to the badly performing particles to improve the whole swarm. The procedure is described in Algorithm 5.1.

It should also be noticed that the *abest* position in Eq. (5.4) is different from the *gbest* particle of the whole swarm. Although it is the same position as the *gbest* particle in the initial population, it will be updated by Algorithm 5.2 (it will be explained later in Section 5.4) and gets better than the *gbest* particle. Different from the ALPSO algorithm in [70], all particles in SLPSO, including the *gbest* particle, learn from the *abest* position.

The four learning objectives of a particle in SLPSO can be visualized in Figure 5.2, where "$pself$" is a particle's current position, "*xrand*" is generated within a circle area with a radius of the mutation step (v_{avg}), *pbest_rand* is the *pbest* position of a random particle, and *abest* is the archive of the *gbest* particle. The position and velocity update framework in SLPSO is shown in Algorithm 5.1.

The first learning operator - learning from a particle's own *pbest* position, allows the particle to fully search in the area around its historical best position. Such kind of exploitation helps a particle to find a better position much easier

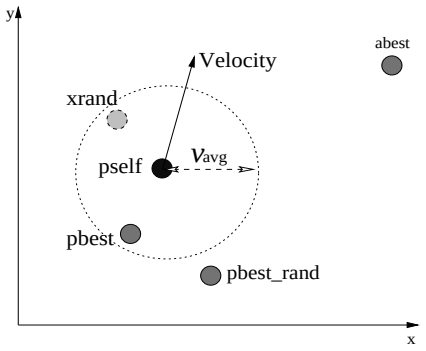


Figure 5.2: The four Learning objectives of a particle in SLPSO

than it would do by searching in a region that is far away from its *pbest* position.

The second operator - learning from a random position nearby, is useful in the case that a particle that already has converged to a local optimum but there is a more promising region nearby which has not been explored by any particle. The operator b will give particles a probability to jump to that promising region.

The third operator - communicating with a random neighbor, enables particles to explore the non-searched areas with a higher probability than learning from its nearest neighbor used in [70]. From the learning equation of Eq. (5.3), learning from different particles can actually alter the velocity as the distance is different from different random particles in two successive iterations. Therefore, this learning strategy is able to maintain the social diversity of a swarm in a certain level.

The fourth learning operator - learning from the *abest* position, focuses on increasing the convergence speed, which is important for global optimization algorithms. Like the *gbest* model, the information of the *abest* position is also shared by all other particles of the whole swarm in SLPSO. Learning from the *abest* position allows all other particles to quickly converge to the location of *abest*.

In SLPSO, the four learning options enable each particle to move to a promising position with a higher probability than the original PSO. The choice of which

learning option is the most suitable for a particular particle would depend on the local fitness landscape where it is. However, it is assumed that we cannot know how the fitness landscape looks like even though we have *a priori* knowledge of the fitness landscape. Instead, each particle should detect the shape of the local fitness landscape where it is currently in by itself. To implement the idea of adaptively detecting the shape of the local fitness landscape, we use the method proposed in [65] with some simplification, which enables a particle to choose the most suitable operator automatically. The simplified method is described in the following section.

5.3 The Adaptive Learning Mechanism

As analyzed above, the best learning strategy for a particular particle is determined by its local fitness landscape, so the optimal strategy for that particle may change with the change of its position during the evolutionary process. In this section, we will achieve two objectives: one is to provide a solution to how to choose the optimal learning strategy, and the other is to adapt this learning mechanism to the environmental change for a particular particle during the evolutionary process.

5.3.1 Ideas of Operator Adaptation

The four learning operators in SLPSO represent four different population topologies, which in turn allow particles to have four different communication structures. Each population structure determines a particle's neighborhood and the way it communicates with the neighborhood. By adaptively adjusting the population topology, we can adaptively adjust the way particles interact with each other and hence can make the PSO algorithm perform better in different situa-

tions. However, currently there is very little research on adaptation of population topologies. The most common adaptation methods in PSO are addition or removal of particles [20] and parameters tuning [92, 98, 100, 138].

The task of selecting operators from a set of alternatives has been comprehensively studied in EAs [27, 33, 102] and many techniques have been proposed to solve this problem [23, 101, 109, 110]. Inspired by the idea of probability matching [109], we introduce an adaptive framework using the aforementioned four learning operators, each of which is assigned with a selection ratio. This adaptive framework is an extension of the work we have done in [65]. Different from the work in [65], which only implemented the adaptive scheme at a population level, we will extend the adaptive scheme to the individual level and make it simpler than the version in [70]. The adaptive framework is based on the assumption that the most successful operator used in recent past iterations may be also successful in the future several iterations. The selection ratio of each operator is equally initialized to 1/4 for each particle and is adaptively updated according to its relative performance.

For each particle, one of the four learning operators is selected according to their selection ratios. The operator that results in a higher relative performance, which is evaluated by a combination of the offspring fitness, current success ratio, and previous selection ratio, will have its selection ratio increased. Gradually, the most suitable operator will be chosen automatically and control the leaning behavior of each particle in different local fitness landscapes during the evolutionary progress.

5.3.2 Selection Ratio Update

The update method of the selection ratios of the four operators is also modified compared with [70]. The operators' selection ratios for a particle are updated only

if it does not improve for U_f successive iterations, where U_f is called the updating frequency. During the updating period for each particle, the progress value and the reward value of operator i are calculated as follows.

The progress value $p_i^k(t)$ of operator i for particle k at iteration t is defined as:

$$p_i^k(t) = \begin{cases} \mid f(\vec{x}_k(t)) - f(\vec{x}_k(t-1)) \mid, & \text{if operator } i \text{ is chosen} \\ & \text{by } \vec{x}_k(t) \text{ and } \vec{x}_k(t) \text{ is better than } \vec{x}_k(t-1) \\ 0, & \text{otherwise} \end{cases} \tag{5.5}$$

The reward value $r_i^k(t)$ has three components, which are the normalized progress value, the success rate, and the previous selection ratio. It is defined by:

$$r_i^k(t) = \frac{p_i^k(t)}{\sum_{j=1}^{M} p_j^k(t)}\alpha + \frac{g_i^k}{G_i^k}(1-\alpha) + c_i^k s_i^k(t) \tag{5.6}$$

where g_i^k is the counter that records the number of successful learning times of particle k, in which its child is fitter than particle k by applying operator i since the last selection ratio update; G_i^k is the total number of iterations where operator i is selected by particle k since the last selection ratio update; $\frac{g_i^k}{G_i^k}$ is the success ratio of operator i for particle k; α is a random weight between 0.0 and 1.0; M is the number of operators; c_i^k is a penalty factor for operator i of particle k, which is defined as follows:

$$c_i^k = \begin{cases} 0.9, & \text{if } g_i^k = 0 \text{ and } s_i^k(t) = \max_{j=1}^{M}(s_j^k(t)) \\ 1, & \text{otherwise} \end{cases} \tag{5.7}$$

and $s_i^k(t)$ is the selection ratio of operator i for particle k at the current iteration.

Different from the reward calculation in Eq.(2.20) in [65], we removed the exponential operation in Eq.(5.6) for simplicity. By removing the exponential operation, here, the three components in Eq.(5.6) have roughly equal importance to calculate the reward.

By introducing the penalty factor in Eq. (5.6), the optimal learning strategy is able to adapt to the environmental change for a particular particle during the evolution process. As we know, when a particle moves to a new location in the search space, the local fitness landscape around may also change. If the property of the new local fitness landscape is quite different from the property of the previous one where it moves from, it means the old optimal learning strategy may not work any more in the new local fitness landscape. Therefore, it needs to re-select a new optimal learning strategy. This can be achieved by punishing the old optimal learning strategy like in Eq. (5.6).

Based on the above definitions, the selection ratio of operator i for particle k in the next iteration $t+1$ is updated according to the following equation:

$$s_i^k(t+1) = \frac{r_i^k(t)}{\sum_{j=1}^{M} r_j^k(t)}(1 - M * \gamma) + \gamma \tag{5.8}$$

where γ is the minimum selection ratio for each operator, which is set to 0.01.

5.3.3 Working Mechanism

According to the above definitions, we know that there is usually one operator that has the highest selection ratio with each particle. This operator must be the most successful one compared with the other operators at the current moment. However, when a particle converges or moves to a new local sub-region whose property is different from the previous one, this most successful operator may no longer bring any benefit to the particle. When this case happens, according to the punishing mechanism in Eq. (5.7), the selection ratio of that operator will decrease, while the selection ratios of the other operators will increase. So, a new most suitable operator will be adaptively selected based on its relatively better performance and the outdated operator will lose its domination automatically.

This is how the adaptive mechanism works. Based on the analysis of the adaptive working mechanism, we can see that SLPSO is able to choose the optimal learning strategy and is also able to adapt to the environmental change for each independent particle. This kind of individual level of intelligence extends the scale of problems that can be effectively solved by EAs.

In addition, it should be noticed that the selection ratios of the four operators are updated at the same time and not updated every iteration. Instead of counting the number of successive iterations for U_f as in [70], we just record the number of successive unsuccessful learning times (m_k, see Algorithm 5.6 in Section 5.8). If the value of the counter m_k for particle k does not reach the maximal value of U_f and particle k is improved by any operator, the value of m_k will be reset to 0. This method reduces the risk of punishing the best operator due to its temporally bad performance in a short period. After the selection ratios of the four learning operators are updated, all the parameters of each operator, including the progress value, the reward value, and the success ratio, is reset to 0, except the selection ratio. The optimal value of U_f depends on specific problems. If the value of U_f is too large, the algorithm may suffer from learning outdated information. On the other hand, if the value of U_f is too small, the algorithm may not have enough knowledge to estimate the optimal selection ratio properly. How to set this parameter for a general problem will be explained later in Section 5.7.

5.4 Information Update for the *abest* Position

In most PSO algorithms, the *gbest* particle is updated only when a better position than the current *gbest* particle is found. Once it is updated, the information of all dimensions of the *gbest* particle is replaced with that of the better position. This updating mechanism has one disadvantage: promising information may not always be preserved. For example, even though a particle has promising informa-

tion in a particular dimension, that information would still be lost if the particle has a lower fitness due to unpromising information from other dimensions. This problem, called "two step forward, one step back" in [115], has two opposite aspects to be considered regarding particles getting better or worse. Take the separable Sphere function as an example. For particles getting improvement, one aspect is that improvement of some dimensions on the gene level (the values of corresponding dimensions move closer to the origin) brings the improvement on the whole individual level. The other opposite aspect is that some dimensions get worse (the values of corresponding dimensions move away from the origin) although the whole individual gets better. Vice versa for the case of particles getting worse.

Whether a particle get better or worse depends on whether the benefit from the improvement dimensions is larger or less than the deterioration from the worsen dimensions. This is also a common phenomenon in most evolutionary algorithms. However, it is very difficult to find out which dimensions get better or worse. This is because, firstly there may be inter-relationships among dimensions and secondly the change of the objective function values may be not linearly related to the change of the decision space.

Although it is difficult to find out which dimensions get better, we can monitor the improved particles to attempt to extract improvement information from certain dimensions. If a particle gets better over time, there may be the case that the particle has some useful information in some certain dimensions even though it has a relatively low fitness value. In that case, other particles should learn that useful information. In SLPSO, the *abest* position learns the useful information from the dimensions of particles which show improvement over time. Once the promising information is extracted from the improved dimensions of a particle, the information of corresponding dimensions of the *abest* position is updated.

However, it is difficult to effectively implement this idea. For example, in order to check the information of which dimension of an improved particle is useful for the *abest* position, we have to check all the dimensions of that improved particle. That is because it is very difficult to know the information of which dimension or what combination of dimensions of the improved particle is useful for the *abest* position.

Although we do not know which dimensions or what combination of dimensions is useful for the *abest* position, we can assign a learning probability (P_l) of each dimension for the *abest* position to learn from the improved particle. Introducing the learning probability has two advantages: firstly the algorithm will save a lot of computational resources and secondly the algorithm can reduce the probability of learning potentially un-useful information for the *abest* position even if it also reduce the probability of learning useful information. That is, the algorithm can utilize the saved computational resources to further explore the search space. Intuitively, the benefit from the use of the saved computational resources may compensate the loss of reducing the probability of learning useful information. This is because, for many cases, the information of an improved particle may be not useful for the *abest* position as they distribute in different sub-regions of the search space. This means that assigning a learning probability may not affect too much the performance of SLPSO. Keep in mind, although the *abest* position gets better after a successful learning from an improved particle, some dimensions of the *abest* position may get worse. If this case happens, the *abest* position will learn potential bad information from improved particles, and the performance of SLPSO will be seriously affected. The evidence can be seen from the experimental results in the section of parameter sensitivity analysis in Chapter 7. Algorithm 5.2 describes the update framework of the *abest* position.

Algorithm 5.2 *UpdateAbest(particle k, fes)*

```
1: for each dimension d of abest do
2:    if rand() < P_l^k then
3:       x⃗_t_abest := x⃗_abest;
4:       x⃗_t_abest[d] := x⃗_k[d];
5:       Evaluate x⃗_t_abest;
6:       fes++;
7:       if f(x⃗_t_abest) < f(x⃗_abest) then
8:          x⃗_abest[d] := x⃗_t_abest[d];
9:       end if
10:   end if
11: end for
```

where P_l^k is the learning probability for the *abest* position to learn from particle k.

5.5 Monitoring Particles' Status

In this section, we will answer some open issues regarding the premature convergence problem, e.g., how to identify a converged particle, when to perform re-initialization operation, and which particles need to be restarted.

5.5.1 Population Re-initialization

Generally speaking, re-initialization is a common method to increase population diversity in EAs. However, there are some open issues while using this method, e.g., how to protect the re-initialized individuals from being eliminated since they usually have very bad fitness. When to perform re-initialization is also an open issue. Although the former issue does not exist in PSO since there is no selection operation, we still have the later problem.

There are several ways to check when to perform re-initialization. The first method is to check the population diversity by a given definition. If the population diversity is less than a threshold, we perform re-initialization for some random individuals or the whole population. The second is to monitor the global best individual. If it does not improve for a certain number of iterations, re-initialization can be launched. For PSO algorithms, we can monitor a particle's velocity. If the

velocity's magnitude of a particle reaches a threshold value, we can re-initialize that particle. Whichever method we use, we have to define a threshold value to perform this operation. However, it is very difficult to get an optimal threshold value for a particular problem. In addition, the threshold values for different problems probably are different because they are problem dependent.

The common problem of the above approaches is that they cannot examine whether an individual is in the stage of evolution or in the stage of convergence. If we can judge particles' status (evolution or convergence), we will know when to perform re-initialization. Consequently, the above problems of using re-initialization methods will also be avoided. In order to monitor a particle's status, we introduce a method to check whether a particle is in the convergence status or not.

5.5.2 Monitoring Particles' Status

In SLPSO, there is a mechanism of monitoring the performance of the four learning operators. The approach is to monitor the selection ratios of the four learning operators. Once a particle converges to a local optimum, none of the four operators can help it jump out of that local optimum. Due to the punishment mechanism, their selection ratios will go back to the initial stage where they would have similar equal values of 1/4. Hence, we can use this information to examine whether a particle is converged or not. By using this approach, we can easily avoid the above problems discussed regarding re-initialization.

To achieve this goal, in addition to calculate the common selection ratios defined in Section 5.3, we need to create a monitoring selection ratio for each learning operator. In SLPSO, every operation related to the update of the monitoring selection ratios is the same as described in Section 5.3 to update the common selection ratios except calculating the progress value $p_i'^k(t)$ of operator i for particle k at

iteration t, which is defined as:

$$p_i'^k(t) = \begin{cases} \mid f(\vec{x}_k(t)) - f(\vec{x}_k^{pbest}) \mid, & \text{if operator } i \text{ is chosen by} \\ & \vec{x}_k \text{ and } \vec{x}_k \text{ is better than } \vec{x}_k^{pbest} \\ 0, & \text{otherwise,} \end{cases} \tag{5.9}$$

To distinguish the definitions related to updating monitoring and common selection ratios in SLPSO, we put a prime symbol after each definition defined in Section 5.3, e.g., $p_i'^k(t)$ and $p_i^k(t)$ represent the monitoring progress value and common progress value of operator i for particle k at iteration t, respectively. And the four learning operators referred to by the two different terms are called common and monitoring operators in the following contents in this thesis.

The reason of introducing the monitoring selection ratios is that the common selection ratios may not be able to monitor a particle's status as a small fluctuation of the $pself$'s fitness value will cause a large disturbance of the values of the common selection ratios and this disturbance may affect SLPSO to correctly judge a particle's evolutionary status. However, the disturbance will not happen for the monitoring operators since the *pbest*'s fitness value of a particle never increases before re-initialization taking place during the evolution process. The corresponding evidence can be seen in the experimental study in Section 7.2.1 in Chapter 7.

In SLPSO, the common selection ratios and the monitoring selection ratios are updated at the same time and once they are updated, all the component parameters are reset to the initial states: progress values, reward values, success ratios are set to 0. The re-initialization of a particle is performed only if the variance of its monitoring selection ratios is less than a constant value of 0.05.

5.6 Controlling the Number of Particles That Learn from the *abest* Position

Learning from the *abest* position is used to accelerate the population convergence. It has the similar property as the PSO with *gbest* model. As a result, for the particles close to the *abest* position, the attraction to the *abest* position is too strong to give opportunity to the other three learning operators. In order to make full use of the adaptive learning mechanism, we need to control the number of particles that learn to the *abest* position. There are two reasons to do so.

First, the optimal number of particles that learn from the *abest* position is determined by the property of the problem to be solved. For example, to effectively solve the unimodal function, e.g., the Sphere function, we need to allow all particles to learn from the *abest* position so that all particles can quickly converge to the global optimum. On the contrary, for some multi-modal problems, we should allow most particles to do local search rather than to learn from the *abest* position so that they will have a higher probability to find the global optimum. The evidence can be seen from the experimental results in the section of parameter sensitivity analysis in Section 7.2.3 of Chapter 7.

Second, it is not fair for the other three operators to compete with the convergence operator, because all the other three operators contribute to the convergence operator. In other words, when a particle gets improvement through whichever of the other three operators, the convergence operator also gets benefit through direct or indirect ways: the direct way is that the improved particle becomes the new *abest* position, so that the *abest* position can be directly updated; the indirect way is that to extract useful information from improved particles. If the *abest* position succeeds, it will also indirectly get benefit.

Based on the above analysis, we need to control the number of particles that use

the convergence operator. However, it is hard to know which particles are suitable to use the convergence operator. To solve this problem, we randomly select a certain number of particles to use the convergence operator every iteration. To implement this idea, we need to update some information of the particles that are switched to use or not to use the convergence operator in two successive iterations. The information needed to be updated includes progress values, reward values, success ratios, and selection ratios.

If the switch happens, there are two cases for updating the related information. The first case is that particles use the convergence operator in previous iteration but do not in current iteration, and the second case is opposite to the first one. In the first case, for both common and monitoring operators, we need to remove the learning source of the *abest* position, and then normalize the selection ratios of the other three operators according to their current values and the other information of the three operators is kept the same. For the second case, we need to process the common and monitoring operators through different means. For the common operators, all the related information is reset to the initial states: progress values, reward values, and success ratios are set to 0 and selection ratios are set to 1/4 for the four common operators. While for the monitoring operators, the related information of the convergence operator is set to the initial states and the information of the other three operators keeps the same as the previous iteration except the selection ratios are re-normalized based on their current values. The update description can be seen in Algorithm 5.3.

5.7 Parameters Tuning in SLPSO

There are three key parameters in SLPSO: the update frequency (U_f), the learning probability (P_l), and the number of particles that learn to the *abest* position (M). The values of the three parameters significantly affect the performance of SLPSO

Algorithm 5.3 *UpdateLearningOpt(particle k)*

if CF_k!=true & PF_k=true **then**
  sum:=$\sum_{j=1}^{3} s_j^k$; sum':=$\sum_{j=1}^{3} s_j'^k$;
  for j=1 to 3 **do**
    s_j^k:=s_j^k/sum; $s_j'^k$:=$s_j'^k/sum'$;
  end for
  s_4^k:=0;$s_4'^k$:=0;
end if
if CF_k=true & PF_k!=true **then**
  for j=1 to 4 **do**
    p_j^k:=0;g_j^k:=0; G_j^k:=0; s_j^k:=1/4
  end for
  sum':=$\sum_{j=1}^{3} s_j'^k$;
  for j=1 to 3 **do**
    $s_j'^k = s_j'^k/sum'$*(1-1/4);
  end for
  $p_4'^k$:=0;$g_4'^k$:=0; $G_4'^k$:=0; $s_4'^k$:=1/4;
end if

where CF_k and PF_k are used to record whether particle k uses the convergence operator or not at current and previous iteration, respectively.

in most problems tested in this thesis. To achieve the best performance for SLPSO, different optimal values of U_f, P_l, and M are needed in different problems. This can be seen from the experimental results regarding the parameter sensitivity analysis in Section 7.2.3 in Chapter 7. For example, a small value of P_l helps SLPSO to achieve the best performance in the Sphere function while the same value negatively affects the performance of SLPSO in the Rastrigin problem. In order to achieve the best performance in different problems without manually tuning the parameters, this section suggests an approach to adjusting the values of the three parameters for a general problem.

5.7.1 Setting the Update Frequency

First, we present a general solution to set up the values of the update frequency (U_f) for SLPSO in all problems. Based on our previous work in [70], the optimal values of U_f mainly distribute from 1 and 10 in most problems tested in [70]. This

information is very useful for SLPSO to set up U_f for a particular problem even if we cannot know the optimal value of U_f for that problem. In order to use this information, we assign to each particle a different value of U_f instead of using a same value of U_f for all particles. The value of U_f for particle k is defined by the following empirical equation.

$$U_f^k = \max(10 * exp(-(1.6 \cdot k/N)^4), 1) \tag{5.10}$$

where N is the population size and U_f^k is the update frequency of particle k. By using this scheme, the values of U_f of all particles distribute from 1 to 10. As a result, it is possible that some particles might be able to achieve the optimal value of U_f for different problems.

5.7.2 Setting the Learning Probability

For the learning probability (P_l) parameter, we use the same method as used for setting up the update frequency, which is described as follows:

$$P_l^k = \max(1 - exp(-(1.6 \cdot k/N)^4), 0.05) \tag{5.11}$$

where N is the population size. Figure 5.3 describes the initial learning probability of each particle in a swarm with the population size of 10. From Figure 5.3, we can see that the learning probabilities of all particles distribute between 0.05 and 1.0.

In order to reduce the risk of using improper values of U_f and P_l for a particular particle, we generate a permutation of particles' index numbers at every iteration and then update the values of U_f and P_l for each particle.

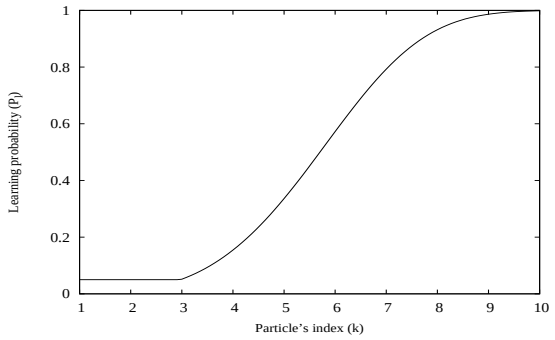


Figure 5.3: Initial learning probability of each particle in a swarm of 10 particles

5.7.3 Setting the Number of Particles Using the Convergence Operator

To set up how many particles should use the convergence operator, we use the following equation:

$$M(fes) = N \cdot (1 - exp(-100(fes/TFes)^3)) \tag{5.12}$$

where $TFes$ is the total number of fitness evaluations. Figure 5.4 shows the functional relation between M and fes in a swarm of 10 particles. From Figure 5.4, we can see that all particles initially do not use the convergence operator in order to focus on local search. However, to accelerate the convergence, the number of particles that use the convergence operator will gradually increase to the maximum value of 10 when the current fitness evaluations reaches 40 percentages of the total fitness evaluations.

In SLPSO, the inertia weight ω linearly decreases from 0.9 to 0.4 by the following equation:

$$\omega(fes) = 0.9 - 0.5 * fes/TFes \tag{5.13}$$

The parameter tuning equations used in SLPSO were developed empirically from our experimental study based on the problems selected in this thesis. Al-

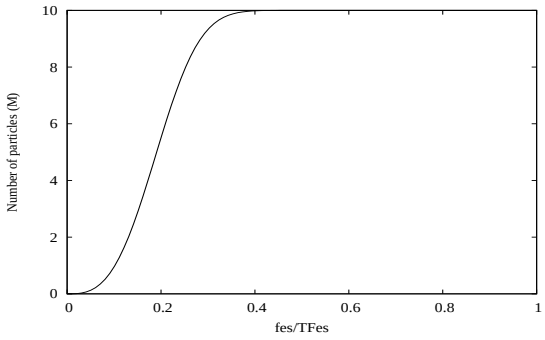


Figure 5.4: The number of particles that use the convergence operator at different iteration in a swarm of 10 particles

Algorithm 5.4 *UpdatePar()*

1: Create a permutation of index number;
2: Update U_f for each particle by Eq. (5.10);
3: Create another permutation of index number;
4: Update P_l for each particle by Eq. (5.11);
5: Calculate the number of particles using the convergence operator by Eq. (5.12);
6: Update related information of the four learning operators for each particle by Algorithm 5.3;
7: Calculate inertia weight ω by Eq. (5.13);

though these equations and the constants used in them, e.g., 1.6 in Eq.(5.10) and Eq.(5.11), may be not the optimal ones to set the values for the parameters of SLPSO, they were not randomly chosen but a result of a careful analysis. The analysis of all these equations is not discussed as it is not the main task of this thesis. Here, we just provide some ideas of how to tune the parameters for SLPSO for general problems, and users can design their own methods to set the parameters for SLPSO. Although the parameter tuning methods were developed based on the problems used in this thesis, they can also be used for new problems. These methods work well and the evidence can be seen from the comparison of SLPSO that uses these methods to set the parameters with other algorithms in Chapter 7. The parameter update operation at each iteration in SLPSO is described in Algorithm 5.4.

Algorithm 5.5 *Repel(particle k)*

1: **for** each visited position $\vec{visited}[i]_k$ **do**
2: d:=$dis(\vec{x}_k, \vec{visited}[i]_k)$
3: **if** $d < |v_k|$ **then**
4: **for** each dimension j **do**
5: $v_k^j += r_k^j \cdot \eta \cdot exp(-7 \cdot (d/|v_k|)^3) \cdot (x_k^j - visited[i]_k^j)$;
6: **end for**
7: **end if**
8: **end for**

where $r_k^j \in (0, 1)$ is a uniformly distributed random number

5.8 Framework of SLPSO

Besides the components described in the above sections, we use an external memory for restarted particles to avoid repeated search and multiple swarm methods to encourage particles to move toward un-explored areas in the search space.

5.8.1 External Memory

In SLPSO, due to the restart mechanism, re-initialized particles may be attracted to the positions they have visited. To encourage particles to explore new promising areas, we use an external memory for each particle to record the best position(s) found just before it restarts, and use the information to avoid the repeated search in the future. To implement this idea, we create a list (*visited*) to store the best position(s) found before each restart for each particle. However, the problem is how to use this information for a particular particle. This is because we can not directly use this information without considering the distance between the new re-started position and the visited positions. For example, if the current restart position is far away from its visited positions, we should not discourage it to move toward the visited positions since there may be promising areas on the way. To solve this problem, we assign to each particle a visual range with its own velocity. In other words, if a visited position is within the visual range

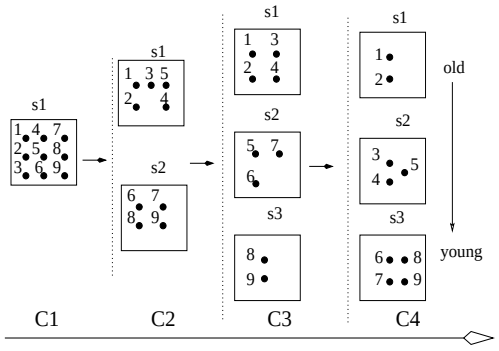


Figure 5.5: Flow chart of creating multiple swarms in SLPSO

of a particle by comparing with its current velocity, the particle should repel that visited position; otherwise, the particle should ignore that visited position. The implementation can be seen in Algorithm 5.5. It should be noticed that this procedure is performed after the velocity update and before the position update for each particle in Algorithm 5.1.

5.8.2 Multiple Swarms

Due to the restart mechanism, SLPSO is able to maintain the diversity. However, there is a problem of how to make full use of restarted particles. Because of the strong attraction to the *abest* position, restarted particles may be easily attracted by the *abest* position due to their bad fitness. In order to avoid the attraction to the *abest* position for restarted particles, we create new swarms using restarted particles.

Figure 5.5 shows the flow chart of how the restarted particles move from old swarms to young swarms or to a new swarm. In Figure 5.5, SLPSO starts with a main swarm $s1$ with nine particles at the initial stage $C1$. Particles 6, 7, 8, and 9 restart at stage $C2$, so they are split off from swarm $s1$ and form a new swarm $s2$. It should be noticed that a new swarm is created only from the youngest swarm, which is labeled with the largest number in each stage, such as swarm $s2$ at stage

$C2$, swarm $s3$ at stage $C3$, etc. And a restarted particle always moves from its old swarm to the next young swarm or a new swarm.

Together with the above components, the implementation of the SLPSO algorithm is summarized in Algorithm 5.6.

5.8.3 *Vmax* and Out of Search Range Handling in SLPSO

In SLPSO, we use the *Vmax* parameter to constrain the maximum velocity for each particle, and the value of *Vmax* is set to half of the search domain for a general problem. There are many methods of handling a particle that moves out of the search range in the literature, e.g., using boundary values, re-initialization, and not evaluating the particle. In SLPSO, we use a different method to constrain a particle within the search bounds: for each dimension, before updating its position, we first remember its position value ($x^d(t-1)$) of the previous iteration, then calculate a temporal value (x_t) by Algorithm 5.1, and finally, update its current position ($x^d(t)$) as follows:

$$x^d(t) = \begin{cases} R(X^d_{min}, x^d(t-1)) & \text{if } x_t < X^d_{min} \\ R(x^d(t-1), X^d_{max}) & \text{else if } x_t > X^d_{max} \\ x_t, & \text{else} \end{cases} \tag{5.14}$$

where $R(a, b)$ returns a uniformly distributed number within (a, b) and $[X^d_{min}, X^d_{max}]$ is the search range of the d-th dimension of the given problem.

5.8.4 Convergence and Diversity in SLPSO

The population convergence speed is one of key performance metrics for EAs and how to make the population quickly converge is an important issue. Another well-known open issue in EAs is how to maintain the population diversity. A common hypothesis is that high diversity is important to avoid premature convergence

Algorithm 5.6 The SLPSO Algorithm

Generate initial swarm and set up parameters for each particle;
Set fes :=0, iteration counter for initial swarm t:=0;
while $fes < T\_Fes$ **do**
  for each swarm $S[q]$ **do**
    for each particle k in swarm $S[q]$ **do**
      Select one learning operator i using the roulette wheel selection rule;
      $Update(i, k, fes)$;
      G_i^k++; $G_i'^k$++;
      if $f(\vec{x}_k(t)) < f(\vec{x}_k(t-1)$ **then**
        g_i^k++, and set $m_k := 0$;
        p_i^k+:=$f(\vec{x}_k(t-1)) - f(\vec{x}_k(t))$;
        Perform $UpdateAbest(k, fes)$ for the *abest* position;
      else
        $m_k := m_k + 1$;
      end if
      if $f(\vec{x}_k(t)) < f(\vec{x}_{pbest_k})$ **then**
        $g_i'^k$++; $p_i'^k$+:=$f(\vec{x}_{pbest_k}) - f(\vec{x}_k)$; $\vec{x}_{pbest_k} := \vec{x}_k$;
        if $f(\vec{x}_k) < f(\vec{x}_{abest})$ **then**
          $\vec{x}_{abest} := \vec{x}_k$;
        end if
      end if
      if $m_k \geq U_f^k$ **then**
        Update the common and monitoring selection ratios according to Eq. (5.8);
        for Each operator j **do**
          p_j^k:=0;g_j^k:=0; G_j^k:=0; $p_j'^k$:=0;$g_j'^k$:=0; $G_j'^k$:=0;
        end for
      end if
      if 0 <Var($\vec{s'}_k$) <= 0.05 **then**
        Re-initialize particle k; $S[q+1]$+:=$\vec{x}_k$;
      end if
    end for
    $UpdatePar()$;
    t++;
  end for
end while

where T_Fes is the total fitness evaluations for a run and Var($\vec{s'}_k$) is the variance of the four monitoring selection ratios for particle k.

and to escape from local optima. In this section, we will discuss how SLPSO achieves population convergence and maintains population diversity.

In SLPSO, the convergence operator plays an important role for the population convergence. This operator will guide all particles to converge to the *abest* position.

Since the *abest* position shares its information with all particles by the convergence operator and it is the best position found by all particles so far, all particles will eventually converge to the *abest* position in SLPSO.

From the working mechanism of SLPSO analyzed in the above sections, we can see that SLPSO has two ways to maintain the population diversity: one is non-deterministic and the other is deterministic. The non-deterministic way refers to the jumping-out operator. The jumping-out operator is used to help particles escape from local optima. Normally, it may help particles explore the search space in the beginning stage because it has an equal opportunity as the other three learning operators to be chosen. Then, if the fitness landscape is not suitable for the jumping-out operator to solve, its selection ratio will drastically drop to a very low level, which is rare to be selected. However, when particles are about to converge, the jumping-out operator will be revived due to the punishing mechanism and might help particles escape from local optima. If it works, the velocity of a particle can be regained to a certain level of the average velocity of all particles, which can be seen in Eq. (5.2). This way is non-deterministic, because it is not guaranteed to happen as the jumping-out operator might not work due to the improper jumping step.

The second way to maintain the population diversity is through the restart mechanism. As analyzed above, if none of the four learning operators can help a particle jump out of the local optimum where it converges, restart will be the last option to help it escape from that local optimum. After the restart, it will become an totally active particle again. This way is deterministic because it is guaranteed to happen for any particle. We take an extreme case where all particles converge to the global optimum as an example. As we know, none of the four learning operators works in this case and their selection ratios will absolutely go back to the initial state. Therefore, the restart mechanism still works.

5.8.5 Complexity of SLPSO

Compared with the basic PSO algorithm, SLPSO needs to perform extra computation on updating the selection ratios, the *abest* position, and the three parameters. For the update of selection ratios and the parameters, we do not need to re-evaluate particles, and the time complexity is O(N) (N is the population size) for each iteration. For the *abest* position update, although we need to re-evaluate particles, the re-evaluation happens only when particles get improvement, not every iteration. In addition, for each dimension of the *abest* position, the update is performed with a certain probability. According to the above component complexity analysis, we can see that the time complexity of SLPSO is not very high in comparison with the basic PSO algorithm.

5.9 Summary

In this chapter, we have introduced a novel SLPSO algorithm for global optimization problems. In SLPSO, each particle has a set of learning strategies, which are created by learning from four objectives: its own historical best position, a random neighbor, the global best position so far, and a random position nearby, respectively. The four learning objectives have different properties, which guide the particle to converge to the current global best position, exploit a local optimum, explore new promising areas, and jump out of a local optimum, respectively. In order to enable particles to automatically choose the appropriate learning objective at the appropriate moment during the search process, an adaptive selection mechanism is implemented by an adaptive framework at the individual level. For each particle, one of the four learning operators is selected according to their selection ratios. The selection ratio of each operator for each particle is equally initialized to 1/4 and is updated according to its relative performance.

To increase diversity, we introduce another mechanism to check particles' evolutionary status by monitoring the performance of the four monitoring operators. This mechanism enables SLPSO to automatically regain diversity by restarting those particles that have converged to local optima. Also, there are some other heuristic rules applied in SLPSO, e.g., extracting useful information from improved particles for the *abest* position, using an external memory to encourage each particle to explore new promising sub-regions, and introducing multiple-swarm methods to avoid the attraction to the exploited *abest* positions.

In order to make SLPSO effectively work on a general problem, several techniques are introduced. First, a bias selection is given to those badly performing particles to improve the whole swarm's performance. Second, to reduce the risk of punishing the current best operator for a particle because of the temporal bad performance of that operator in a short period, the selection ratios of the four learning operators are updated only if a particle is not improved for U_f successive iterations. Third, a learning probability P_l is introduced to solve the time-consuming problem and to reduce the probability of learning potential bad information when the *abest* position learns from an improved particle. Fourth, for a fair competition among the four learning operators, the scheme of controlling the number of particles that learn from the *abest* position is introduced. Finally, the parameter tuning methods for the three parameters (U_f, P_l, and M) are developed to set up SLPSO for solving a general problem.

Chapter 6

Clustering Particle Swarm Optimizer

In the real world, many optimization problems are dynamic. This requires an optimization algorithm not only to find the global optimal solution under a specific environment but also to track the trajectory of the changing optima with time over dynamic environments. To address this requirement, this chapter investigates a clustering particle swarm optimizer (CPSO) for DOPs. This algorithm employs a hierarchical clustering method to locate and track multiple peaks. A fast local search method is also introduced to search optimal solutions in a promising sub-region found by the clustering method.

6.1 Difficulties for PSO in Dynamic Environments

Similar to other EAs in many aspects, PSO has been shown to perform well for many static problems [85]. However, it is difficult for the basic PSO algorithm to optimize DOPs. The difficulty lies in two aspects: one is the outdated memory due to the changing environments and the other is the diversity loss due to the convergence. Of these two aspects, the diversity loss is by far more serious [10]. It has been demonstrated that the time taken for a partially converged swarm to re-diversify, find a shifted peak, and then re-converge is quite deleterious to the

performance of PSO [8].

In the basic PSO algorithm, the diversity loss is mainly due to the strong attraction of the global best particle, which results in that all the particles quickly converge to local or global optimum where the global best particle locates. This feature is beneficial for many stationary optimization problems. However, for DOPs, this feature is not good for PSO to track the changing optima. For DOPs, it is important to guide particles to search in different promising regions to obtain promising local optima as many as possible because these promising local optima may become the global best in the next new environment. Hence, local best particles are needed to guide the search in local regions in the search space. However, the question becomes how to determine which particles should be suitable as the neighborhood best and how to assign particles in different neighborhoods to move toward different sub-regions.

Several PSO algorithms have been recently proposed to address DOPs [10, 40, 41, 84, 117], of which using multi-swarms seems a good technique. The multi-swarm method can be used to enhance the diversity of the swarm, with the aim of maintaining multiple swarms on different peaks. The traditional method of using the multi-swarm method to find optima for multi-modal functions divides the whole search space into local sub-spaces, each of which may cover one or a small number of local optima, and then separately search within these sub-spaces. Here, there are several key, usually difficult, issues aforementioned in Chapter 1, e.g., how to guide particles to move toward different promising sub-regions, how to define the area of each sub-region, how to determine the number of sub-swarms needed, and how to generate sub-swarms. These issues will be further discussed in the following section.

6.2 General Considerations for Multi-swarms

In order to address the convergence problem of PSO for DOPs, the multi-swarm method can be used to maintain multiple swarms on different peaks, which are referred to as the optima in this thesis. For example, for the MPB problem [13], the highest peak in the fitness landscape is the global optimum and the other peaks with a lower height are local optima. Hence, for the multi-swarm method to work, the whole search space can be divided into several sub-regions. Each sub-region may contain one or more than one peak. Each sub-swarm covers one sub-region and exploits it. As mentioned above, when applying the multi-swarm method to achieve this purpose, there are several key issues to be considered.

The first issue concerns how to guide particles to move toward different promising sub-regions. This issue is important since if particles can not move to different sub-regions, PSO can not locate and track multiple optima. This requires that an algorithm should have a good global search capability to explore promising sub-regions. In [17], the cognitive only PSO model, which was tested by Kennedy [52], was used to train particles. Since there is no information sharing among particles in the cognitive only model, each particle just searches around its personal best position. This may cause the stagnation problem if there are deceptive sub-regions in the search space. Hence, in order to guide particles toward different promising sub-regions, particles should cooperate with the other nearby particles.

The second issue concerns how to define the area for each sub-region in the search space. The area of a sub-region determines how many peaks it may contain. If the area of a sub-region is too small, there is a potential problem that small isolated sub-swarms may search on a same local optimum. In this case, the number of individuals of each sub-swarm may be not enough to make any progress. However, if a sub-region is too large, there may be more than one peaks

within the sub-region covered by a sub-swarm. The best situation is that each sub-region just contains one peak. However, to achieve this goal is very hard due to the complexity of the search space, especially for real world problems. Traditionally, the search area of each sub-region is predefined by users according to preliminary experiments [83] or a formulated estimation [10], and the size of the search area is the same for all sub-regions. Obviously, it is not true that all the peaks have exactly the same shape or width in the whole search space. It is very hard to know the shape of a sub-region. Hence, how to define the search area of a sub-region is a very hard problem. Ideally, particles within the neighborhood of a peak should be able to calculate the sub-area by themselves.

How many sub-swarms are needed is the third issue to consider. From the experimental results in [10], the optimal number of sub-swarms is equal to the total number of peaks in the whole search space. The more peaks in the search space, the more sub-swarms we probably need. If too many sub-swarms distribute in the fitness landscape, the limited computation resources may be wasted. On the contrary, if there are too small number of sub-swarms, the PSO algorithm can not efficiently track different local optima. Again, the problem is that the number of peaks in the search space is usually unknown in advance, especially for real world problems. Although the number of peaks is given for some benchmark problems, we should assume that it is unknown to us.

Finally, how to generate sub-swarms is also an open issue. Generally speaking, sub-swarms are simply obtained by separating the main swarm according to some mechanism. In [48], a k-means clustering method was used to generate clusters. The limitation of the k-means method is that the number of clusters must be predefined. In the speciation based PSO [83], a new species is produced around a species seed. That is, all the particles within a radius r_s distance to a species seed are classified into a species corresponding to that species seed. Hence, a

new species is created by a given radius r_s around its seed. The number of sub-swarms is simply predetermined in mCPSO [10], although exclusion and anti-convergence strategies were used. Exclusion prevents sub-swarms from covering a single peak by an exclusion radius r_{excl}, and anti-convergence allows new peaks to be detected, which was implemented by defining a convergence radius r_{conv}. However, the serious disadvantage of SPSO and mCPSO is that those radius parameters must be given. In order to generate sub-swarms as accurate as possible, that is, only all particles on a same peak form a sub-swarm, the analysis of population distribution should be done before creating sub-swarms by some statistical methods.

If particles close to a peak can detect the peak by themselves, then they can classify themselves into a same cluster, and the search area can also be automatically defined when the new cluster is formed. This thinking motivated the proposal of CPSO in [69, 134]. In the following section, CPSO in its simplified version is described in detail to show how it overcomes the above problems when using the multi-swarm method.

6.3 Framework of the Clustering PSO

To address the above considerations for multi-swarm methods, a clustering method is introduced in CPSO. The clustering method can enable CPSO to assign particles to different promising sub-regions, adaptively adjust the number of sub-swarms needed, and automatically calculate the search region for each sub-swarm.

CPSO starts from an initial swarm, named the cradle swarm. Then, sub-swarms are created by a hierarchical clustering method. When sub-swarms are created, local search is launched on them in order to exploit potential peaks covered by these sub-swarms respectively. Finally, overlapping, convergence, and

Algorithm 6.1 The CPSO Algorithm

```
1: Create an empty convergence list clst to record the best particles of converged
   sub-swarms;
2: Create an empty list slst to record sub-swarms;
3: Set the fitness evaluation counter evals := 0;
4: Generate an initial cradle swarm C randomly;
5: Clustering(C, slst);
6: while the stop criterion is not satisfied do
7:    for each sub-swarm slst[i] do
8:       LocalSearch(slst[i], evals);
9:    end for
10:   CheckSubswarms(C, slst, clst);
11:   if |C| > 0 then
12:      LocalSearch(C, evals);
13:   end if
14:   if DetectChange(C, slst, clst, evals) then
15:      Clustering(C, slst);
16:   end if
17: end while
```

overcrowding checks are performed on the sub-swarms before the next iteration starts. If an environmental change is detected, a new cradle swarm will be randomly re-generated with the reservation of the positions located by all survived sub-swarms in the previous environment.

The framework of CPSO for DOPs is given in Algorithm 6.1. In the following sections, the major components of CPSO, including the clustering method, local search operator, sub-swarm checks, and detecting environmental changes, are described in detail, respectively.

6.4 Single Linkage Hierarchical Clustering

Some researchers have used the k-means clustering method to generate sub-swarms, the problem of the k-means method is that we do not know the optimum value of k for the current population. In addition, the optimum value of k is problem dependant. Setting k to a too large or a too small value will cause the

Algorithm 6.2 *Clustering*$(C, slst)$

```
1: Create a temporary cluster list G of size |C|;
2: for each particle i in C do
3:    G[i]:=C[i]; {i.e., each particle forms one cluster in G}
4: end for
5: Calculate the distance between all clusters (i.e., particles) in G and construct
   a distance matrix M of size |G| × |G|;
6: while TRUE do
7:    if FindNearestPair(G, r, s) = FALSE then
8:       Break;
9:    end if
10:   r := r + s; {i.e., merge clusters r and s into r}
11:   Delete the cluster s from G;
12:   Re-calculate all distances in M which have been affected by the merge of r
      and s;
13:   if each cluster in G has more than one particle then
14:      Break;
15:   end if
16: end while
17: slst := G;
18: Empty C;
```

problem of an improper number of sub-swarms, as discussed above. Traditionally, sub-swarms are created by directly using a number of swarms or simply splitting off from a main swarm. There is little or no analysis of distribution of individuals in the search space. Different from traditional clustering methods for multi-swarm based PSO algorithms, CPSO uses a single linkage hierarchical clustering method [45], as shown in Algorithm 6.2, to create sub-swarms.

In the clustering method, the distance $d(i, j)$ between two particles i and j in the D-dimensional space is defined as the Euclidean distance between them as follows:

$$d(i, j) = \sqrt{\sum_{d=1}^{D} (x_i^d - x_j^d)^2} \tag{6.1}$$

The distance of two clusters r and s in G, which is an element in distance matrix M in Algorithm 6.2 and is denoted as $M(r, s)$, is defined as the distance of the two

Algorithm 6.3 *FindNearestPair*(G, r, s)

1: $found := FALSE$;
2: $min\_dist := \sqrt{\sum_{i=1}^{D} (U_i - L_i)^2}$, where U_i and L_i are the upper and lower bounds of the i-th dimension of the search space;
3: **for** $i := 0$ to $|G|$ **do**
4: **for** $j := i + 1$ to $|G|$ **do**
5: **if** $(|G[i]| + |G[j]| > max\_subsize)$ **then**
6: continue;
7: **end if**
8: **if** $(min\_dist > M(G[i], G[j]))$ **then**
9: $min\_dist := M(G[i], G[j])$;
10: $r := G[i]$;
11: $s := G[j]$;
12: $found := TRUE$;
13: **end if**
14: **end for**
15: **end for**
16: Return $found$;

closest particles i and j that belong to clusters r and s respectively. $M(r, s)$ can be formulated as:

$$M(r, s) = \min_{i \in r, j \in s} d(i, j) \tag{6.2}$$

Given a cradle swarm C, the clustering method works as follows. It first creates a list G of clusters with each cluster only containing one particle in C. Then, in each iteration, it uses Algorithm 6.3 to find a pair of clusters r and s such that they are the closest among those pairs of clusters, of which the total number of particles in the two clusters is not greater than *max_subsize* (*max_subsize* is a prefixed maximum sub-swarm size), and, if successful, combines r and s into one cluster. This iteration continues until all clusters in G contain more than one particles. The value of *max_subsize* directly determines how many clusters can be obtained by the hierarchical clustering method. Definitely, it also determines the number of sub-swarms.

From the above description, it can be seen that using the above clustering

method, sub-swarms will be automatically created depending on the distribution of initial particles in the fitness landscape. The number of sub-swarms and the size of each sub-swarm are also automatically determined by the fitness landscape and the unique parameter *max_subsize*.

In this thesis, we have removed the training process used in the original CPSO [70]. In [70], the aim of training the initial swarm is to guide particles to move toward different promising sub-regions. After the training process, the clustering operation will be conducted to generate sub-swarms. From the experimental results, we found that training for the initial swarm is not necessary. There are no overlapping search areas among the sub-swarms that are produced from the initial swarm using the clustering method. Exploitation will then be carried out immediately in the own local search area of each sub-swarm and the sub-swarms will gradually move toward the local optima that are close to them, respectively. Finally, all sub-swarms will distribute in different sub-regions where local optima are located in the fitness landscape. So, the same objective as the training process used in the original CPSO [70] can be achieved without training in the simplified version. The test results regarding CPSO with and without the training process will be shown later in the experimental study section in Section 8.2.3 in chapter 8. The advantage of removing the training phase in the original CPSO lies in that more computational resources can be distributed to sub-swarms to perform local search. The refining clustering operation in the original CPSO is also removed in the simplified version because the number of sub-swarms can be controlled by the value of *max_subsize*. Setting a proper value for *max_subsize* can help CPSO allow one sub-swarm to cover a single peak. So, the refining clustering phase is also redundant. For example, if we set *max_subsize* to an extreme value (e.g., *max_subsize* = 1), then each sub-swarm contains only one particle and can just cover a single peak.

Algorithm 6.4 *LocalSearch*($S, evals$)

```
1: for each particle i ∈ S do
2:    Update particle i according to Eqs. (2.1) and (2.2);
3:    evals:=++evals%U;
4:    if particle i is better than pbest_i then
5:       Update pbest_i;
6:       LearnGBest(particle i, gbest, evals);
7:       if particle i is better than gbest then
8:          Update gbest;
9:       end if
10:   end if
11: end for
```

It should be noted that it is very difficult for algorithms to track all peaks in the fitness landscape, especially when we use a limited population resources to solve a problem with a large number of peaks in the search space. However, we can just track the peaks that have relatively higher heights compared with the other peaks in the fitness landscape since these peaks have a higher probability of becoming the highest peak in the next environment. In CPSO, if one sub-swarm covers more than one peaks in a local sub-region, particles would focus on the search on the relatively higher peaks in that local sub-region.

6.5 Local Search Strategy

When a sub-swarm is created using the above clustering method, it will undergo the local search process in order to exploit the sub-region covered by the sub-swarm. The framework of the local search process is described in Algorithm 6.4. In the local search process, in order for a sub-swarm to locate a local peak quickly, the PSO with the *gbest* model is used. That is, each particle in a sub-swarm also learns from the global best position *gbest* found by the sub-swarm.

In order to speed up the convergence process of sub-swarms, a linear decreasing scheme is also used in CPSO to adjust the inertia weight ω in Eq. (2.1) as

Algorithm 6.5 *LearnGBest*(*particle i*, *gbest*, *evals*)

1: **for** each dimension d of *gbest* **do**
2: $\vec{x}_{t\_gbest} := \vec{x}_{gbest}$ {$\vec{x}_{t\_gbest}$ is a temporary particle};
3: $x_{t\_gbest}[d] := x_i[d]$;
4: **if** $\vec{x}_{t\_gbest}$ is better than $\vec{x}_{gbest}$ **then**
5: $x_{gbest}[d] := x_{t\_gbest}[d]$;
6: **end if**
7: *evals*:=++*evals*%U;
8: **end for**

follows:

$$\omega = \omega_{max} - \frac{(\omega_{max} - \omega_{min}) \times c\_itr}{r\_itr} \tag{6.3}$$

where ω_{max} and ω_{min} are respectively the maximum and minimum value of ω, c_itr is the iteration counter for a sub-swarm, which starts from 0 when a sub-swarm is newly created, and r_itr is the remaining iterations before the next change when a sub-swarm is created, i.e., $r\_itr = (U - evals)/pop\_size$, where pop_size is the total number of particles in all sub-swarms and the cradle swarm.

In order to extract useful information for the *gbest* particle in each sub-swarm, the same idea as used to update the *abest* position in SLPSO with the learning probability of 1 is also used in CPSO, which is shown in Algorithm 6.5.

6.6 Check the Status of Sub-swarms

After the local search operation for all sub-swarms, they are checked regarding overlapping, convergence, and overcrowding. The checking of sub-swarms is as shown in Algorithm 6.6.

Traditionally, the overlapping check between two sub-swarms is carried out using their search radius. The search radius of a sub-swarm s can be calculated as follows.

$$radius(s) = \frac{1}{|s|} \sum_{i \in s} d(i, s_{center}), \tag{6.4}$$

Algorithm 6.6 *CheckSubswarms*($C, slst, clst$)

```
1: for each pair of sub-swarms (r, s) in slst do
2:    if r_overlap(r, s) > R_overlap then
3:       Merge r and s into r;
4:       Remove s from slst;
5:    end if
6: end for
7: for each sub-swarms r ∈ slst do
8:    if |r| > max_subsize then
9:       Remove worst (|r| − max_subsize) particles from r;
10:   end if
11: end for
12: for each sub-swarms s ∈ slst do
13:   if radius(s) < R_conv then
14:      Add gbest into clst;
15:      Remove s from slst;
16:   end if
17: end for
18: if |C| = 0 && |slst| = 0 then
19:   Add max_subsize random particles into C;
20: end if
```

where s_{center} is the center position of sub-swarm s and $|s|$ is the size of s. If any particle in a sub-swarm is within the search radius of another sub-swarm, then the overlapping search is assumed to occur. If the distance of the best particles of two sub-swarms is less than the sum of their search radius, then they are combined or one of them is removed. The above checking mechanism assumes that each sub-swarm just covers one peak. However, it is not true for real PSO algorithms. If a sub-swarm in a sub-region covers more than one peak, other sub-swarms that are within its search area should not be removed or combined together with this sub-swarm.

In order to reduce the risk of losing peaks that are being tracked, we adopt the following overlapping check scheme in CPSO. If two sub-swarms r and s are within each other's search area, an overlapping ratio between them, denoted $r_{overlap}(r, s)$, is calculated as follows. We first calculate the percentage of particles in

r which are within the search area of s and the percentage of particles in s which are within the search area of r, and then set $r_{overlap}(r, s)$ to the smaller one of the two percentages. The two sub-swarms r and s are combined only when $r_{overlap}(r, s)$ is greater than a threshold value $R_{operlap}$, which is set to 0.7.

In order to avoid too many particles searching on a single peak and hence save computing resources, an overcrowding check is performed on each sub-swarm in CPSO after the above overlapping check. If the number of particles in a sub-swarm is greater than *max_subsize*, then the particles with the worst personal best positions are removed one by one until the size of the sub-swarm is equal to *max_subsize*.

For DOPs, the best solutions found in the current environment may be useful for tracking the movements of peaks in the next environment. Hence, in CPSO, after the crowding check, the convergence check is carried out to see whether a sub-swarm has converged. A sub-swarm convergence list *clst* is used to record the best positions found by those converged sub-swarms in the current environment. If the radius of a sub-swarm is less than a small threshold value R_{conv}, which is set to 0.0001, the sub-swarm is regarded to be converged on a peak. If a sub-swarm is converged, its *gbest* is added into *clst* in order to be used in the next environment. Correspondingly, the converged sub-swarm is removed from the sub-swarm list: *slst*.

The removal of converged sub-swarm and combining two overlapping sub-swarms may result in the consequence of no particle surviving. If this happens, the algorithm will run forever. Therefore, if all sub-swarms are converged, *max_subsize* random particles will be generated into the current cradle swarm C to deal with the special situation.

6.7 Detecting Environmental Changes

Usually, for an algorithm to address DOPs efficiently, it is important to detect the environmental changes [94]. To detect the environmental changes, we may use the deterioration of the population performance or the time-averaged best performance as an indicator [12]. The fitness landscape change will affect all particles based on our experimental test on the MPB problem. Based on this fact, we can figure out several simple efficient methods to detect the environmental changes. Before updating *pbest* of each particle, we may re-evaluate its *pbest* position (e.g., the method used in [84]). If the fitness changes, it means that a change of the fitness landscape occurs. Another simple approach is to set several monitoring particles in the search space. The monitoring particles will be re-evaluated every iteration. If the environment changes, it will be detected by these monitoring particles using the above detecting method.

In CPSO, we use the global best particle over all sub-swarms as the monitoring particle to detect the environmental changes. Before updating the global best particle, we re-evaluate its fitness at each iteration. If its fitness changes, it indicates that an environmental change occurs. Once an environmental change is detected, CPSO takes the following actions, as shown in Algorithm 6.7; otherwise, if it fails to detect the change, the previous population will be used in the new environment. In order to adapt to the new environment quickly, we take the following actions. First, the best position *gbest* of each sub-swarm in *slst* before the change is saved into *clst*. Then, all particles are re-initialized to create a new cradle swarm and the particles stored in *clst* are added to the new cradle swarm by replacing the worst particles in it. Finally, *slst* and *clst* are re-set to be empty. Again, the clustering method will be performed to generate the new sub-swarm list.

Algorithm 6.7 *DetectChange*($C, slst, clst, evals$)

```
1: Re-evaluate the global best particle over all sub-swarms;
2: evals :=++evals%U;
3: if The fitness of the re-evaluated position changes then
4:    Save the gbest of each sub-swarm in slst into clst;
5:    Remove all sub-swarms in slst;
6:    Generate a new cradle swarm C;
7:    Add the particles in clst into C;
8:    Empty clst;
9:    Return TRUE;
10: else
11:    Return FALSE;
12: end if
```

6.8 Complexity Analysis

The major components of CPSO are the clustering operation, local search, and status checking of each sub-swarm. The clustering operation is performed only once at the very beginning when an environmental change is detected. From Algorithm 6.2, it can be seen that the time complexity of the clustering operation is $O(N^3)$, where N is the population size of the cradle swarm. We first compute all distances among each pair of particles in $O(N^2)$. For each iteration of merging two clusters r and s from the cluster list G, we find the nearest pair of clusters of G in $O(|G|^2)$ (if we use dynamic programming method, it would be reduced to $O(|G|log(|G|))$), then update the distance matrix M in $O(|G|)$. The number of clusters in G will decrease by 1 every iteration until the stop criterion is met. Finally, we perform the clustering operation in $O(N^3)$.

In the local search operation (Algorithm 6.4) for each sub-swarm, except performing the *gbest* learning on improved particles, there is no big difference from the basic PSO algorithm. In addition, the time complexity will reduce as performing overlapping and overcrowding check because of decreasing number of total particles.

The time complexity of the sub-swarm status check depends on how many

sub-swarms are produced by the clustering operation. However, it also will decrease as performing overlapping and convergence check for sub-swarms. In total, according to the above component complexity analysis, the extra computing time needed for CPSO is not so high in comparison with the basic PSO algorithm.

6.9 Comparison between CPSO and PSO with the *lbest* Model

In the PSO with the *lbest* model (PSO_{lbest}), if we define the neighborhood of a particle as its nearest *max_subsize* particles, and assign the best one of its neighborhood as its social component in Eqs. (2.1), it seems that we will get a similar search behavior as CPSO. This is because the clustering method in CPSO will assign the particles that are close to each other into a sub-swarm, which is the same as the neighborhood defined in PSO_{lbest}. However, CPSO has several major advantages in comparison with PSO_{lbest}.

First, CPSO can track multiple optima in dynamic environments. In CPSO, if more than one sub-swarms cover a same peak, they will finally be combined with each other into one sub-swarm by the overlapping check function. Hence, the positions found by the converged sub-swarms in *clst* are distributed on different peaks. Once an environmental change occurs, the elements in *clst* will be added into the new cradle swarm. When a peak moves not too far away from the previous location, the previous peak position locates on the slope of the new current peak. As we know, if this case happens, the previous peak location does help the search of the current peak in the new environment. However, PSO_{lbest} can not recognize such kind of peak locations even they are found by PSO_{lbest}. It is impossible to directly check which particles in the whole swarm are from different peaks since different peaks have quite different heights. Therefore, PSO_{lbest} can

not track multiple optima in dynamic environments.

Second, CPSO can control overcrowding in a single peak. There are two aspects regarding the overcrowding over a single peak in CPSO. One is that more than one sub-swarms cover a single peak, and the other is that too many particles exist within one sub-swarm on a peak. The first problem can be solved by the overlapping check as analyzed above. Hence, those sub-swarms that are inferior to the best sub-swarm on a peak will be automatically removed. The second problem is solved by the overcrowding check in Algorithm 6.6. However, PSO_{lbest} can not solve the overcrowding problem since it can not check which particles locate on which peaks.

Third, CPSO has a higher probability of covering more local optima than PSO_{lbest} can do. In CPSO, since there is no communication among sub-swarms, each sub-swarm just searches its local area, and finally will converge to a local optimum if it survives till the next change takes place. Hence, every peak will be found if it is covered by a sub-swarm. However, in PSO_{lbest}, the *gbest* with a relatively better fitness of one particle's neighborhood may belong to the neighborhood of different particles. That is, particles from different peaks may share the same *gbest*. So, particles from the peaks with lower heights will be attracted by the *gbest* from the peak with a higher height. Finally, they will converge on that peak with a higher height and lose the track of the peaks with relatively lower heights.

Finally, CPSO can partially adaptively adjust the number of particles and sub-swarms needed to achieve the best performance based on its work mechanism. However, the number of particles in PSO_{lbest} is fixed during the whole run.

6.10 Summary

This chapter introduced a clustering PSO for DOPs to locate and track multiple peaks using the multiple swarm method. The single linkage hierarchical clustering method used in CPSO, which is the major contribution of the algorithm, solves several key issues when using the multiple swarm method, e.g., how to guide particles to move toward different promising sub-regions, how to define the area of each sub-region, how to determine the number of sub-swarms needed, and how to generate sub-swarms.

In addition, to improve the effectiveness of CPSO, several techniques are implemented in CPSO to check the status regarding overlapping, convergence, and overcrowding.

Chapter 7

Experimental Study of SLPSO

It is very hard to give an accurate analysis of a real PSO algorithm's performance (e.g., convergence and diversity) due to its stochastic movement. One strategy adopted to analyze a PSO algorithm's performance is to choose a set of functions that have different properties and levels of complexity, and compare the best solutions achieved by the algorithm with the global optima of the selected test functions. Another method to examine an algorithm's performance is to compare the algorithm with some state-of-the-art algorithms.

In this chapter, the experimental study of the SLPSO algorithm is performed based on the static problems (f_1-f_{45}) introduced in Chapter 3. In order to examine the ideas used in SLPSO and provide some evidence, the experimental study, including the analysis of the search behavior, the adaptive learning mechanism, the parameter sensitivity analysis, the learning strategy for the *abest* position, and the process of selection ratios of the four learning operators, are performed. In addition, some state-of-the-art PSO algorithms are chosen, including CPSO-H_k [115], FIPS [76], CLSPO [43], and APSO [138] as well as the standard PSO (SPSO) [14], to further test the performance of SLPSO.

Table 7.1: Configuration of Involved PSO Algorithms

Algorithm	Year	Population Topology	Parameter Settings
SPSO[14]	2007	Local ring	$\omega = 0.721348$, $\eta_1 = \eta_2 = 1.19315$
CPSO-H$_k$[115]	2004	Cooperative multi-swarm approach	ω: [0.4, 0.9], $\eta_1 = \eta_2 = 1.49$ k=6
FIPS[76]	2004	Local URing	$\chi = 0.7298$, $\sum c_i = 4.1$
CLPSO[43]	2006	Comprehensive learning	ω: [0.4, 0.9], $\eta = 2.0$
APSO[138]	2009	Global star	ω: [0.4, 0.9], $\eta_1 + \eta_2$: [3.0, 4.0] with adaptive tuning

7.1 Experimental Setup

Experiments were conducted to compare SLPSO with the five PSO algorithms on the 45 test problems in 10, 30, and 50 dimensions, respectively. The configuration of each peer algorithm, which is exactly the same as it appeared in the original paper, is given in Table 7.1.

Below is a brief description of the peer PSO algorithms taken from the literature to compare with SLPSO. The first algorithm is the cooperative PSO (CPSO-H$_k$) [115], which combines a cooperative PSO model with the original PSO. For this algorithm, we also use the same value of "*split factor*" $k = 6$ as it was used in the original paper [115]. The second algorithm is the fully informed PSO (FIPS) [76] with a U-ring topology that achieved the highest success rate. The third algorithm is the comprehensive learning PSO (CLSPO) [43], which was proposed for solving multi-modal problems and shows a good performance in comparison with eight other PSO algorithms. The fourth peer algorithm is the adaptive PSO (APSO) [138], which adaptively tunes the values of η_1, η_2, and ω based on the population distribution in the fitness landscape to achieve different purposes, such as exploration, exploitation, jumping out, and convergence. It was reported that APSO substantially enhanced the performance of PSO in terms of the convergence speed, the global optimality, the solution accuracy, and the algorithm reliability.

The fifth peer algorithm is the standard PSO (SPSO) proposed in [14]. We chose this algorithm to investigate how SLPSO performs compared to the standard version of PSO. For SLPSO, η_1 and η_2 were set to 1.496. V_{max} was set to half of the search domain for each test function, which can be seen from Table 3.1 and Table 3.3, and the default parameter settings suggested in Section 5.7 were used for all problems if there is no special note.

To fairly compare SLPSO with the other five algorithms, all algorithms were run independently 30 times on the 45 test problems. The initial population and the stop criterion are the same for all algorithms for each run. The maximal number of fitness evaluations (T_Fes) was used as the stop criterion for all algorithms on each test function. The swarm size and T_Fes were set to (10, 50000), (20, 100000), and (30, 300000) for dimensions 10, 30, and 50, respectively. The parameter settings of the other five algorithms are based on their optimal configurations suggested by the corresponding papers.

7.2 Working Mechanism of SLPSO

In this section, the experimental study, including the search behavior, the self-learning mechanism, parameter sensitivity analysis, the learning strategy for the *abest* position, and the common selection ratios of the four learning operators, is performed to examine the performance of SLPSO.

7.2.1 Analyzing the Search Behavior of SLPSO

This section provides evidence to support our assumption of SLPSO's working mechanism on the unimodal Sphere function (f_1) and multi-modal Schwefel function (f_6) in two dimensions. For simplicity, we only used five particles on both test functions over a single run of 10000 total fitness evaluations and all five particles

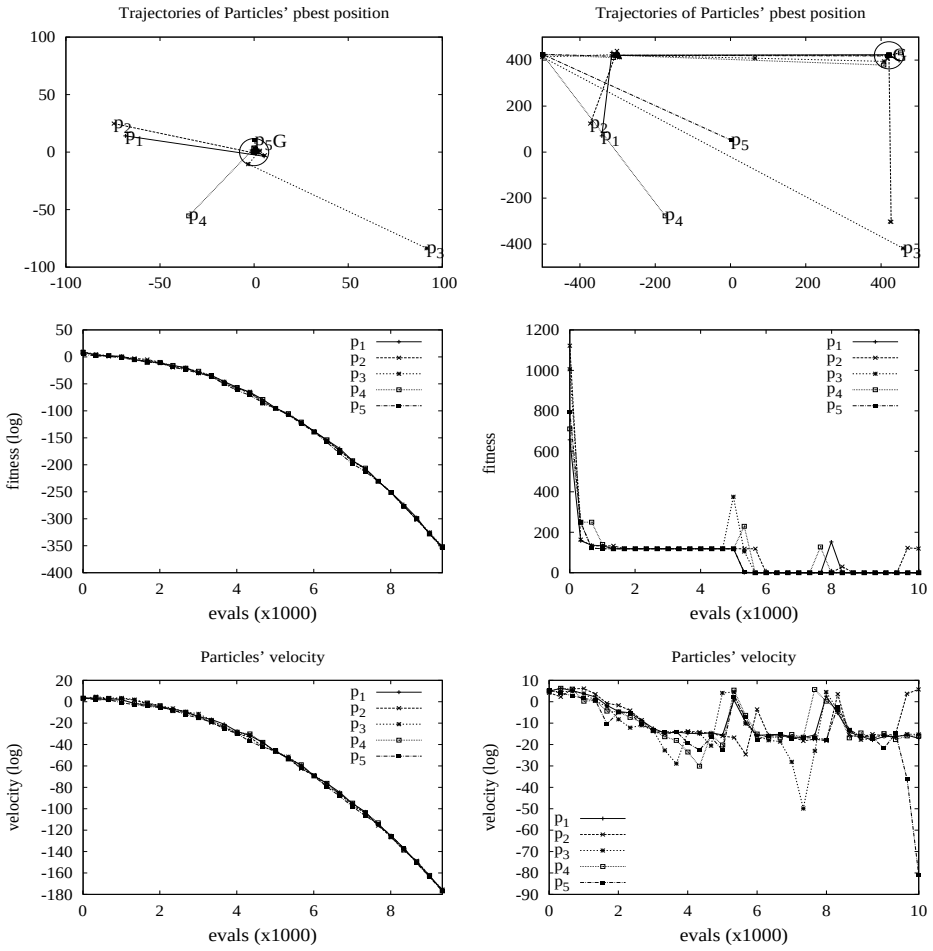


Figure 7.1: *pbest* trajectory, fitness process, and velocity of each particle on the Sphere function (left) and Schwefel function (right) in two dimensions

use the four learning operators from the beginning to the end of the evolution. In other words, different from the original SLPSO algorithm, there is no control of using the convergence operator for all particles.

Figure 7.1 shows the *pbest* trajectory, fitness process, and velocity process of the five particles on the Sphere and Schwefel functions, where the circle points labeled with "G" of the top two graphs are the locations of the global optima of the two functions. The five particles start the search at the locations of "P_i", $i = 1, 2, \ldots, 5$. The graphs on the left hand are the results on the Sphere function and the graphs on the right hand are the results on the Schwefel function. Figure 7.2 and Figure 7.3

show the common and monitoring selection ratios of the four operators of each particle on the two functions, respectively. Figure 7.4 shows an overview of the variance of the selection ratios of the common and monitoring operators on the two test functions.

Comparing the results of the Sphere and Schwefel functions in Figure 7.1, it can be easily seen that restart happened on the Schwefel function but not on the Sphere function. It should be noticed that we cannot clearly see the restart moment from the fitness process graphs on the Schwefel function where the fitness of the restarted particles shown in Figure 7.1 is much smaller than their initial fitness values. The reason lies in that the corresponding data were omitted when drawing the graphs. However, it is obvious to see from the velocity process graph where the velocity of the restarted particles is similar to the initial velocity. Although all particles succeeded to find the global optima of the two functions, the process was quite different. For the Schwefel function, it can be seen from the fitness process graph that all the five particles were trapped into local optima after 2000 fes, and the first restart took place for some particles at about 5000 fes and helped all particles to eventually reach the global optimum. However, because of no local optima in the Sphere function, all the particles easily found the global optimum without any restart. The comparison results show that SLPSO is intelligent to detect the fitness landscape with different properties. Thanks to the monitoring mechanism, it is capable of effectively utilizing the restart mechanism to maintain population diversity. However, to further analyze the search behavior, we need to observe the changes of selection ratios of the four learning operators for each particle.

From the selection ratios graphs of both functions in Figure 7.2 and Figure 7.3, it can be seen that the algorithm is indeed able to switch from one learning operator to another when necessary at the individual level to focus the individual on doing

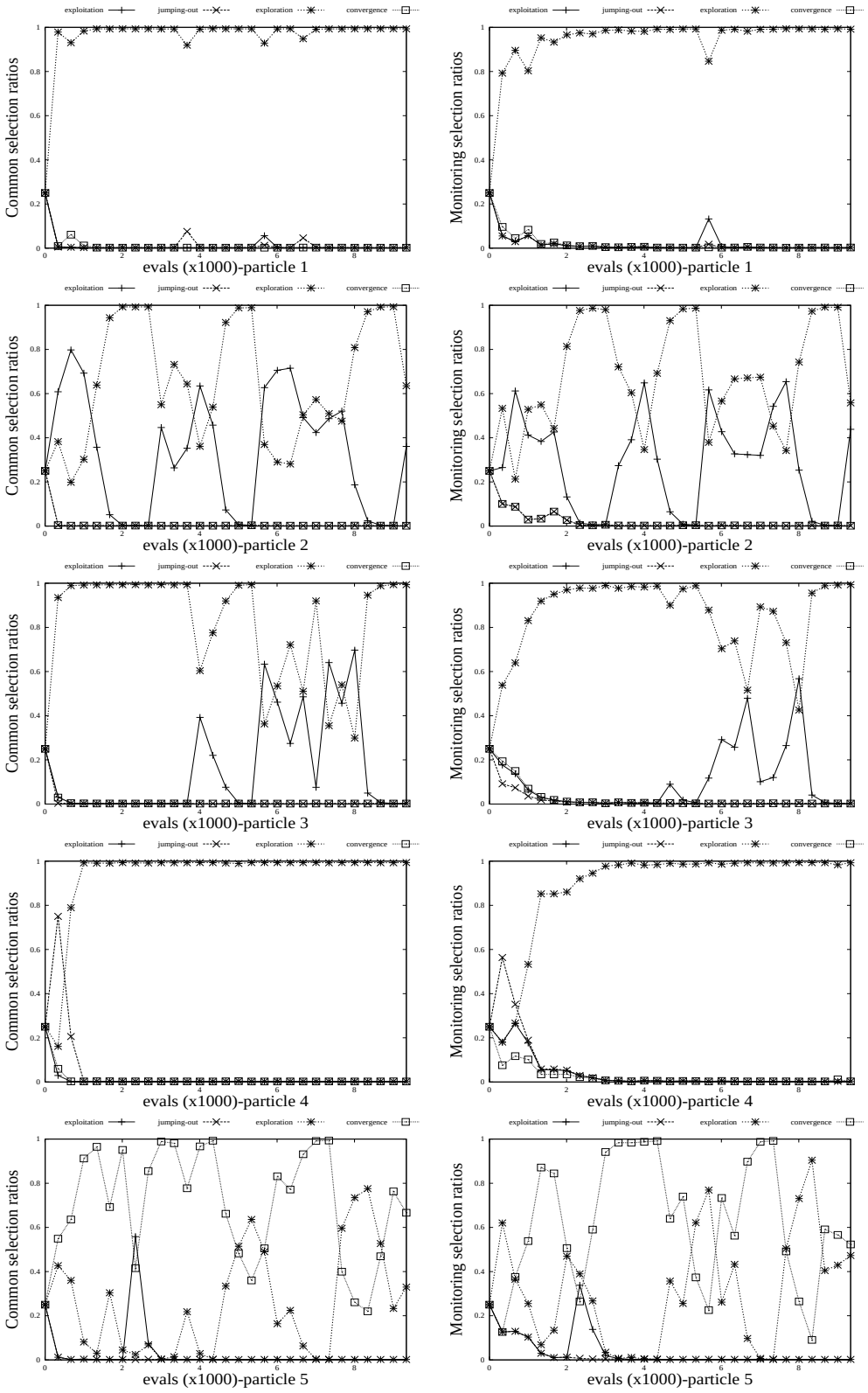


Figure 7.2: Selection ratios of the common and monitoring operators on the Sphere function (f_1) in two dimensions

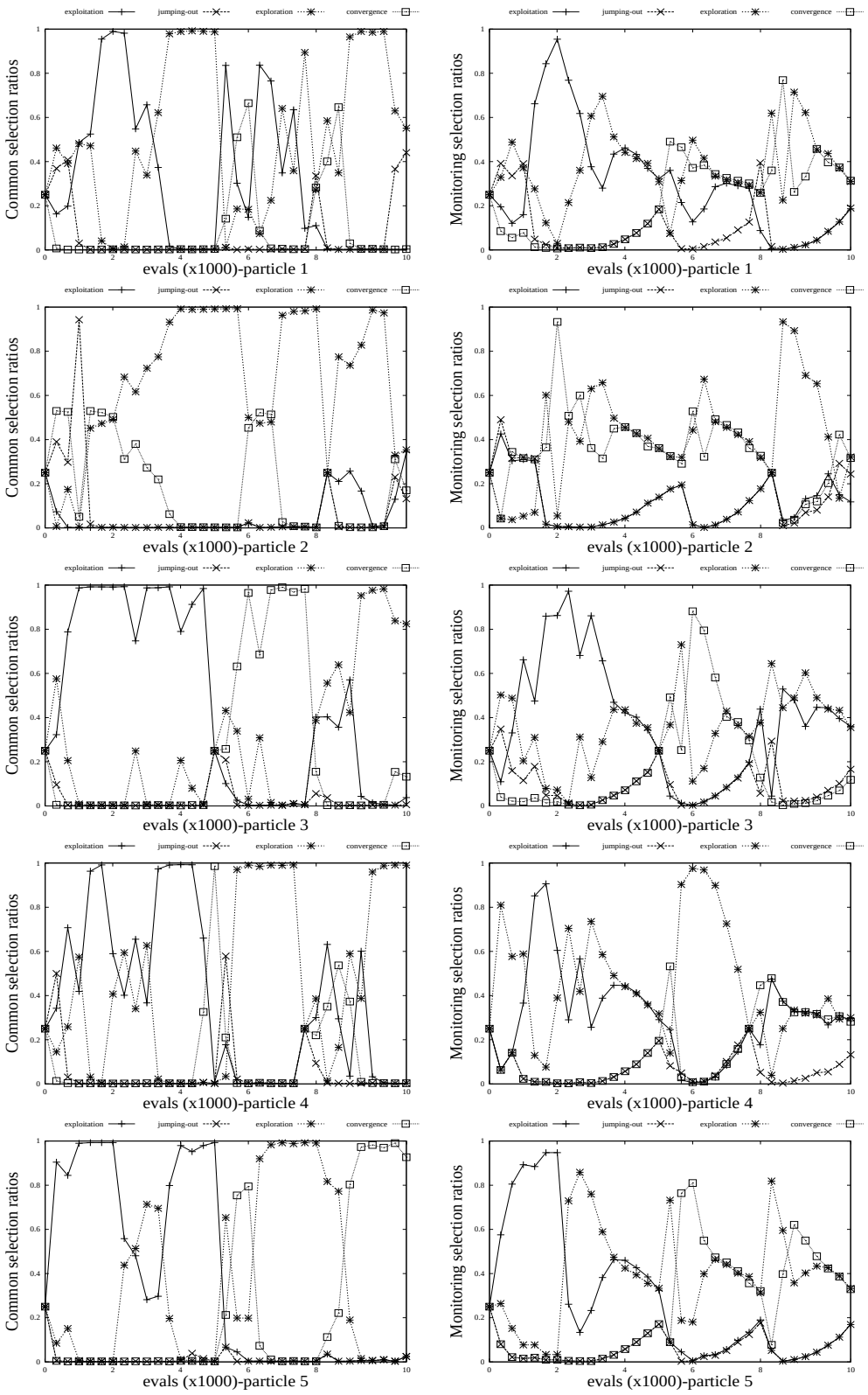


Figure 7.3: Selection ratios of the common and monitoring operators on the Schwefel function (f_6) in two dimensions

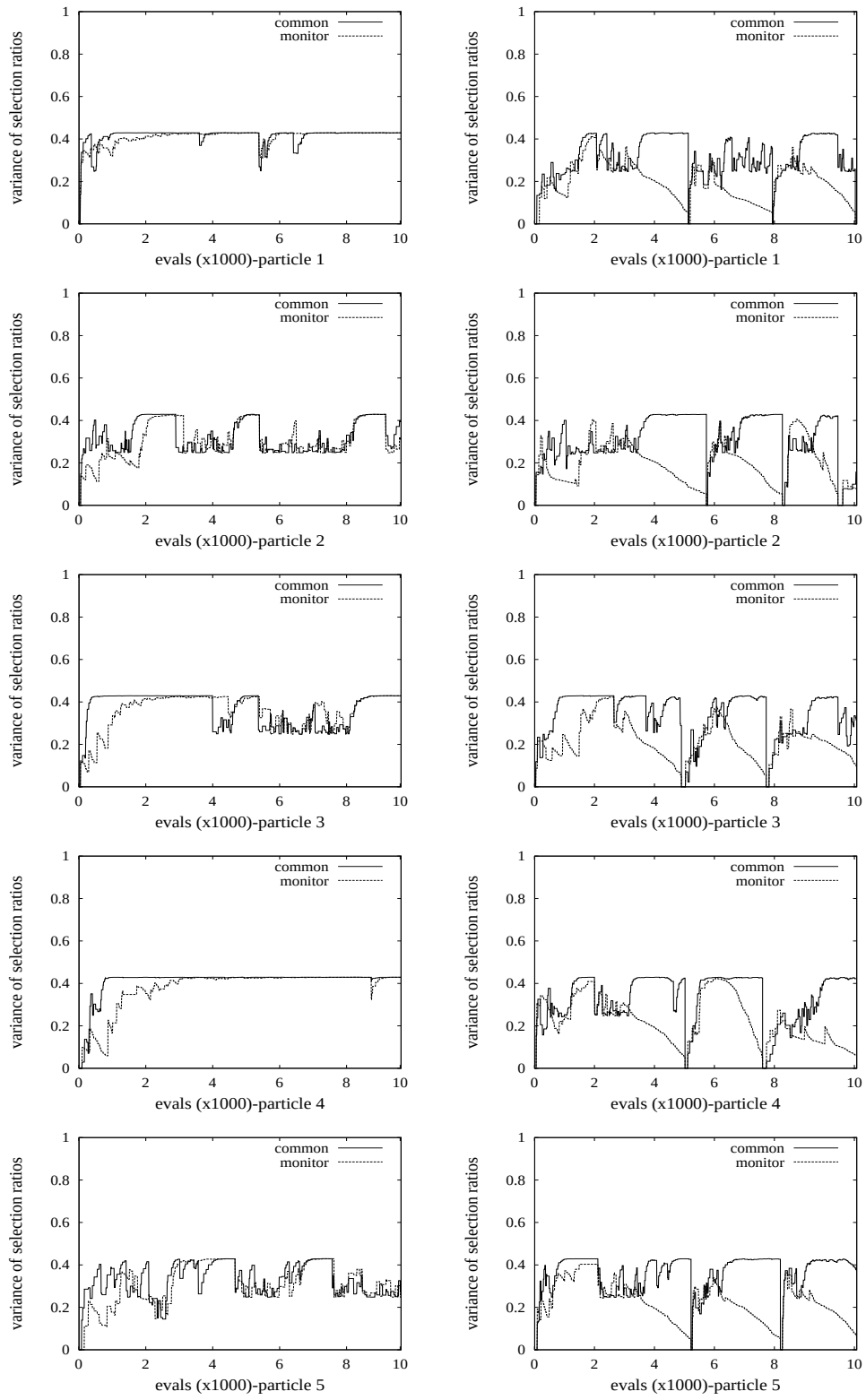


Figure 7.4: Process of variance of common and monitoring selection ratios of the four learning operators on the Sphere function (left) and the Schwefel function (right) in two dimensions

one of the following task: exploration, exploitation, convergence, or jumping out. In both Sphere and Schwefel problems, each particle was able to choose a new best learning objective when it could not get benefit from the old, outdated learning objective.

For particles in different sub-regions in the fitness landscape, the learning strategies with the largest selection ratios are different. In other words, SLPSO is capable to adapt to different environments by using different learning strategies. Take the Sphere function as an example. To reach the position of the global optimum, particles 1 and 4 used the exploration operator from the beginning to the end, while particles 2 and 3 switched between the exploration and exploitation operators several times during the whole evolutionary process. However, for particle 5, the most frequently used strategy was the convergence operator.

By comparing the common and monitoring selection ratios of the four learning operators on the Schwefel function, we can clearly see a particle's convergence status through the monitoring selection ratios but not through the common selection ratios. Take particle 1 as an example. The convergence status was shown twice when it converged in the monitoring selection ratios graph where the values of the four learning operators went back to the initial state. However, we cannot observe this trend in the corresponding common selection ratio graph. This validates our assumption that the common selection ratios cannot monitor a particle's status. On the contrary, for the graphs of the Sphere function in Figure 7.4, we cannot see the similar observations as on the Schwefel function as no re-initialization happened for the five particles and the process of the common and monitoring selection ratios has similar trend for all the five particles during the whole evolutionary process.

It should be noticed that the common selection ratios did show the initial state where they have equal values during the evolutionary process in Figure 7.3 and the

variance of the common selection ratios becomes 0 in Figure 7.4. This is because the common selection ratios of the four learning operators were reset to initial states when particles were re-initialized. This can be clearly seen from Figure 7.4. The results on the Schwefel function in Figure 7.4 obviously show that the variance of the monitoring selection ratios linearly decreases when particles tend to be convergence status till the restart happened. However, although particles are in the convergence status, the variance of common selection ratios is still in a very large level. For the Sphere function, we cannot observe the similar phenomenon with the monitoring selection ratios as for the Schwefel function.

By observing the search behavior of SLPSO on unimodal (Sphere) and multimodal (Schwefel) problems, we can conclude that the adaptive learning mechanism does enable SLPSO to deal with different situations at the individual level. In other words, each single particle is able to learn the knowledge of its surrounding local area and also is able to check its own evolutionary status so that it can make right decisions for some hard situations, e.g., which learning strategy should be used, how to check the convergence status, and when to restart if being trapped into local optimum, to adapt to the changing environments. By considering these issues, each particle can automatically change its search behavior according to the surrounding environments and its evolutionary status.

7.2.2 Self-Learning Mechanism Test

In order to further investigate the level of effectiveness that the adaptive learning mechanism can bring to SLPSO, we carried out experiments on SLPSO with the adaptive learning mechanism and on Non-SLPSO without the adaptive learning mechanism over all the test functions in 30 dimensions. For SLPSO, the default parameter settings were used. In Non-SLPSO, all the four learning operators have an equal selection ratio of 0.25 during the whole evolutionary process. Table 7.2

Table 7.2: Comparison with random selection for the four learning operators regarding the mean value

f	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9
Random	1.52e-13	7.47e-10	7.08e-07	1.34e-04	0.0168504	3.82e-04	8.99e-08	23.5034	1.39e-07
Adaptive	6.32e-50	0	0	4.50e-15	0.00816972	3.82e-04	4.05e-14	7.34554	3.32e-28
f	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}	f_{16}	f_{17}	f_{18}
Random	30.7624	1.32e-04	2.49e-15	90.7131	90.3314	3.82e-04	0.001151	2.18e-05	4.21e-05
Adaptive	0.185749	0.369122	1.57e-32	20.0509	20.4416	3.82e-04	3.29e-14	0	0
f	f_{19}	f_{20}	f_{21}	f_{22}	f_{23}	f_{24}	f_{25}	f_{26}	f_{27}
Random	4.31e-04	3.94842	0.015133	0.088096	1.03e-12	214.611	6556.43	3.67856	1.22e-04
Adaptive	6.47e-04	5.46e-04	0.01918	0.122745	5.87e-46	115.579	4575.08	4.50667	0
f	f_{28}	f_{29}	f_{30}	f_{31}	f_{32}	f_{33}	f_{34}	f_{35}	f_{36}
Random	6954.29	20.8023	161.818	4.19e-05	2.88e-05	157.64	34.366	20.7782	65.9935
Adaptive	4948.79	20.6376	105.687	1.29e-13	1.21e-13	114.67	30.4103	20.5754	8.20752
f	f_{37}	f_{38}	f_{39}	f_{40}	f_{41}	f_{42}	f_{43}	f_{44}	f_{45}
Random	228.201	14676	0.642808	336.844	420.118	607.911	913.073	433.022	296.972
Adaptive	0.0694798	14277.4	1.31e-13	290.464	414.28	986.578	1179.03	399.39	307.761

presents the results on the 45 functions for SLPSO and Non-SLPSO.

From Table 7.2, it can be seen that the results of SLPSO are much better than that of Non-SLPSO on 36 functions out of the total 45 test problems. The reason for the problems where the results of SLPSO are worse than those of Non-SLPSO is because improper parameter values were used. This can be seen from the comparison of SLPSO with the default configurations and the optimal configurations in the following section where the performance of SLPSO is greatly improved when the optimal configuration are used. The comparison result shows that the adaptive mechanism works well on most problems. And it also confirms that different problems need different strategies to solve. To achieve the best performance, it is necessary to tune the selection ratios of the four learning operators so that the best learning strategy can effectively play a role.

7.2.3 Parameter Sensitivity Analysis of SLPSO

To find out how the parameters affect the performance of SLPSO, an experiment on parameter sensitivity analysis of SLPSO was also conducted on all the 45 problems in 30 dimensions. The key parameters include the update frequency

Table 7.3: Effect of the update frequency

U_f	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9
1	9.49e-33	0	0	1.37e-14	0.0198629	19.7401	5.36e-14	4.19863	3.38e-17
3	1.12e-29	0	0	1.09e-14	0.0179618	7.89627	5.89e-14	6.19834	2.45e-16
7	1.64e-22	0	0	8.19e-13	0.0261546	31.5839	1.10e-11	10.7329	1.17e-12
10	5.01e-19	2.37e-16	1.37e-13	1.33e-09	0.0186557	11.8442	6.13e-11	12.3649	5.08e-11
U_f	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}	f_{16}	f_{17}	f_{18}
1	0.00855641	0.0118861	1.57e-32	66.6667	169.293	19.7401	5.16e-14	0	0
3	0.0219023	3.40e-04	5.66e-29	66.6667	178.059	11.8442	6.23e-14	0	5.39e-27
7	1.07724	3.79e-05	2.47e-21	66.6667	231.113	11.8442	2.24e-12	0	2.60e-22
10	3.12558	4.91e-05	6.24e-19	66.6667	226.873	3.82e-04	1.73e-11	0	2.26e-20
U_f	f_{19}	f_{20}	f_{21}	f_{22}	f_{23}	f_{24}	f_{25}	f_{26}	f_{27}
1	7.20e-04	23.6881	0.0194899	0.129472	1.69e-29	178.468	5527.69	8.40049	4.01e-30
3	8.57e-04	7.89635	0.0210841	0.123632	2.95e-28	177.628	5628.55	5.37123	1.68e-25
7	7.11e-04	3.94843	0.0187445	0.104331	6.68e-20	191.285	5770.61	5.01039	2.78e-21
10	6.33e-04	7.89636	0.0196384	0.13006	4.36e-18	208.603	6123.21	6.50096	2.96e-19
U_f	f_{28}	f_{29}	f_{30}	f_{31}	f_{32}	f_{33}	f_{34}	f_{35}	f_{36}
1	6119.39	20.5861	136.602	1.50e-13	1.33e-13	149.809	31.6188	20.5105	11.131
3	6502.66	20.6557	135.463	1.42e-13	1.38e-13	127.238	32.8685	20.5092	15.1888
7	6380.69	20.7836	136.813	1.46e-13	1.31e-13	127.436	32.9158	20.7317	15.7967
10	6550.11	20.8193	133.155	1.61e-13	1.67e-13	122.834	33.4664	20.8005	16.6575
U_f	f_{37}	f_{38}	f_{39}	f_{40}	f_{41}	f_{42}	f_{43}	f_{44}	f_{45}
1	0.00314017	20074.9	1.61e-13	270	456.783	1074.81	1313.41	520.227	322.231
3	0.00718157	18885.6	1.55e-13	270	443.597	1069.83	1464.8	511.16	344.807
7	0.132277	17770.5	2.18e-13	270	451.379	935.929	1524.22	499.565	311.218
10	0.745756	18413.8	2.25e-13	250.716	484.477	912.493	1472.35	536.127	333.969

(U_f), the learning probability (P_l), and the number of particles that learn from the *abest* position (M, which is replaced with percentages of population size for convenience here). The default values of the three parameters are set to 10,1.0, and 1.0, respectively. To separately test the effect of a particular parameter, we used the default values of the other two parameters. For example, to test the effect of U_f, we choose a set of values for U_f and use the default values for P_l =1.0 and M =1.0, respectively.

Three groups of experiments with U_f in the set [1,3,7,10], P_l in the set [0.05,0.1, 0.3,0.7,1.0], and M in the set [0.0,0.1,0.3,0.7,1.0] were carried out separately and the corresponding results are summarized in Table 7.3, Table 7.4, and Table 7.5, respectively. Figure 7.5 shows the statistical results of the number of problems where SLPSO achieves the best result with each particular parameter.

For the parameter U_f, as already mentioned in Section 5.3.3 in Chapter 5, the optimal value of U_f for a specific problem depends on the property of that

Table 7.4: Effect of the learning probability

P_l	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9
0.05	1.27e-61	0.0994959	0.0333333	1.95e-04	0.0184491	367.159	2.24e-14	6.31095	3.02e-30
0.1	1.99e-86	1.18e-16	1.78e-16	5.33e-07	0.0148208	228.981	2.03e-14	3.37152	1.73e-50
0.3	1.43e-62	0	0	1.42e-15	0.0195018	43.4278	2.98e-14	4.86947	4.05e-33
0.7	1.59e-27	0	0	6.87e-15	0.0140862	3.94833	5.44e-14	10.5145	2.21e-15
1	5.01e-19	2.37e-16	1.37e-13	1.33e-09	0.0186557	11.8442	6.13e-11	12.3649	5.08e-11
P_l	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}	f_{16}	f_{17}	f_{18}
0.05	0.0361189	1.16228	4.04e-32	30	56.6667	403.179	1.20e-14	0.709638	8.94e-30
0.1	0.00513775	0.204989	1.69e-32	26.6667	56.6667	197.359	1.45e-14	0.203381	0
0.3	0.00145181	0.0144136	1.57e-32	92.6254	123.333	58.4005	2.33e-14	0	0
0.7	0.176488	8.74e-05	2.27e-27	96.8129	144.652	23.688	5.36e-14	0	5.46e-28
1	3.12558	4.91e-05	6.24e-19	66.6667	226.873	3.82e-04	1.73e-11	0	2.26e-20
P_l	f_{19}	f_{20}	f_{21}	f_{22}	f_{23}	f_{24}	f_{25}	f_{26}	f_{27}
0.05	2.08e-04	390.847	0.0101869	0.0852683	8.16e-32	84.3764	3876.63	3.46848	2.92e-28
0.1	2.04e-04	213.189	0.0104484	0.0435477	1.32e-64	101.493	4239.88	3.05643	2.82e-30
0.3	2.96e-04	35.5319	0.0125978	0.0615876	1.33e-59	148.328	4626.1	3.48268	0
0.7	5.85e-04	3.94839	0.0171477	0.0985399	1.66e-26	166.257	5965.69	4.5377	5.90e-27
1	6.33e-04	7.89636	0.0196384	0.13006	4.36e-18	208.603	6123.21	6.50096	2.96e-19
P_l	f_{28}	f_{29}	f_{30}	f_{31}	f_{32}	f_{33}	f_{34}	f_{35}	f_{36}
0.05	4168.09	20.7942	91.8854	6.44e-14	1.42611	84.7878	28.0658	20.7632	6.94265
0.1	4483.06	20.7503	81.085	6.06e-14	0.431149	95.3393	27.8497	20.7512	2.94607
0.3	5142.13	20.7779	122.312	9.09e-14	9.76e-08	102.124	31.2341	20.7244	3.51637
0.7	5648.19	20.7756	125.938	1.25e-13	1.12e-13	129.741	32.3224	20.7571	11.5467
1	6550.11	20.8193	133.155	1.61e-13	1.67e-13	122.834	33.4664	20.8005	16.6575
P_l	f_{37}	f_{38}	f_{39}	f_{40}	f_{41}	f_{42}	f_{43}	f_{44}	f_{45}
0.05	5.42e-05	4541.2	6.25e-14	274.851	432.941	1069.05	1182.15	529.11	337.274
0.1	3.17e-06	6893.24	6.63e-14	216.667	400	1044.61	1132.05	421.245	304.6
0.3	2.07e-05	10695	9.28e-14	326.472	430	1005.83	1243.52	399.053	300
0.7	0.0452609	16964.4	1.53e-13	340.283	423.333	897.496	1374.42	438.195	296.685
1	0.745756	18413.8	2.25e-13	250.716	484.477	912.493	1472.35	536.127	333.969

problem. If the value of U_f is too large, the algorithm may suffer from learning outdated information. On the other hand, if the value of U_f is too small, the algorithm may not have enough knowledge to estimate the optimal selection ratio properly. Table 7.3 shows that SLPSO does need an optimal value of U_f to achieve the best performance on each test function, and the optimal value of U_f is different for different test functions.

For the second and the third group of experiments, similar observations can be seen as the results of the first group of experiments: the optimal value for a particular parameter is problem dependant. Different problems need different optimal parameter values to be solved and there is no any rule that can be applied to find out the optimal parameter settings for a general problem.

From the general view of results of the three groups of experiments on the

Table 7.5: Effect of the number of particles that learn to the *abest* position

M	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9
0	0.0136607	0.0119123	0.0164126	0.336143	0.0313058	0.271441	0.0279166	34.741	0.0279932
0.1	0.00568289	0.0100958	0.0131758	0.274608	0.0231501	0.0278327	0.0162456	24.772	0.0236951
0.3	0.0011778	0.00846347	0.0108852	0.177459	0.0211172	0.00953315	0.00809967	26.0393	0.0118511
0.7	1.75e-05	9.36e-04	0.00392394	0.049107	0.0184869	7.24e-04	0.00101772	21.847	0.00130994
1	5.01e-19	2.37e-16	1.37e-13	1.33e-09	0.0186557	11.8442	6.13e-11	12.3649	5.08e-11
M	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}	f_{16}	f_{17}	f_{18}
0	4147.61	0.738217	1.39e-04	162.256	113.263	0.413042	0.0120229	0.0234658	0.00282844
0.1	3430.35	0.550669	1.71e-04	107.942	93.792	0.0342727	0.0107088	0.00796089	0.0028595
0.3	2239.15	0.341625	3.42e-05	90.9429	162.531	0.00837046	0.00748414	0.00482638	0.00173339
0.7	1102.69	0.0769767	2.04e-07	38.7966	62.9094	8.78e-04	0.00297164	8.87e-04	1.74e-04
1	3.12558	4.91e-05	6.24e-19	66.6667	226.873	3.82e-04	1.73e-11	0	2.26e-20
M	f_{19}	f_{20}	f_{21}	f_{22}	f_{23}	f_{24}	f_{25}	f_{26}	f_{27}
0	0.0213727	0.230281	0.0656114	0.44925	0.0607009	156.371	5157.51	7.17576	0.0198961
0.1	0.0119308	0.0311916	0.0511325	0.442512	0.0202455	186.352	5679.35	6.36836	0.00844045
0.3	0.00591048	0.0100897	0.0430735	0.352748	0.00662235	227.502	6549.84	6.96851	0.00588083
0.7	0.00210027	0.00155953	0.0285277	0.221568	9.68e-05	201.066	7021.61	6.57414	8.95e-04
1	6.33e-04	7.89636	0.0196384	0.13006	4.36e-18	208.603	6123.21	6.50096	2.96e-19
M	f_{28}	f_{29}	f_{30}	f_{31}	f_{32}	f_{33}	f_{34}	f_{35}	f_{36}
0	5525.13	20.8644	136.2	0.00177881	0.0178188	126.27	33.0961	20.8554	112.264
0.1	5521.1	20.8769	151.893	0.00227756	0.00662248	130.141	33.2139	20.8399	96.9137
0.3	5980.5	20.8699	157.813	0.00117422	0.00527378	149.129	32.8707	20.836	79.4044
0.7	7458.51	20.8419	159.862	1.81e-04	0.00118309	145.689	32.5435	20.8004	53.4771
1	6550.11	20.8193	133.155	1.61e-13	1.67e-13	122.834	33.4664	20.8005	16.6575
M	f_{37}	f_{38}	f_{39}	f_{40}	f_{41}	f_{42}	f_{43}	f_{44}	f_{45}
0	7298.93	20095.6	10.6821	359.135	427.847	880.05	1445.78	503.007	332.696
0.1	4289.14	18888.7	13.1425	308.301	424.303	661.048	1353.64	446.083	316.515
0.3	2852.46	25160.9	8.43714	292.461	429.486	746.823	1496.26	475.221	331.183
0.7	1366.26	26474.5	1.51633	321.861	417.496	660.098	1429.16	451.142	312.348
1	0.745756	18413.8	2.25e-13	250.716	484.477	912.493	1472.35	536.127	333.969

45 test problems in Figure 7.5, we can see that SLPSO with $U_f = 1$ achieves the best performance on most problems, that SLPSO with $P_l = 0.1$ achieves the best performance on most problems, and that SLPSO with $M = 1$ achieves the best performance on most problems.

Although we have an overview of how the three parameters affect the performance of SLPSO, the corresponding optimal value of each parameter above may not be the real optimal parameter setting for a particular problem. It is difficult to obtain the real optimal parameter setting for a general problem for several reasons. First, we cannot test all the possible values of the three parameters as one of them is continuous, i.e., P_l. In addition, the value of M is population size dependent. Second, there may be relationships among the three parameters, i.e., they are not independent for SLPSO to achieve the best performance on a general problem.

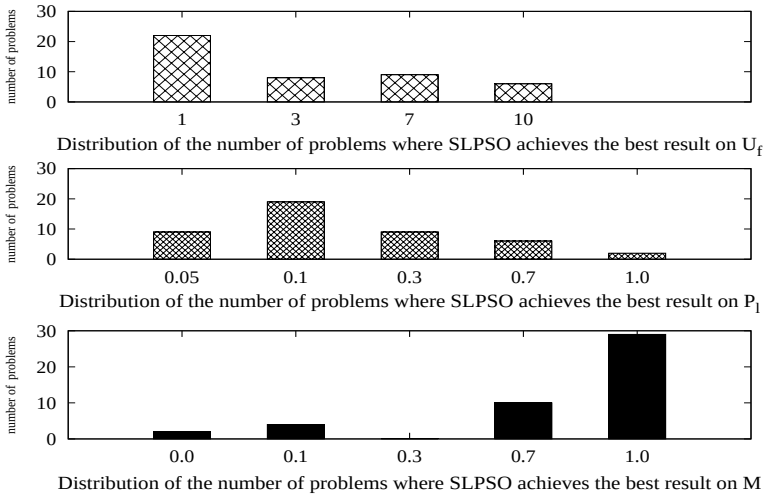


Figure 7.5: Distribution of the number of problems where SLPSO achieves the best result with each particular parameter.

Taking the Sphere function (f_1) as an example. The corresponding optimal values of U_f, P_l, and M are 1, 0.05, and 1, respectively. However, the direct combination of the three optimal values may not help SLPSO achieve the best performance on the Sphere function. The evidence can be seen in the next section.

7.2.4 Comparison with the Optimal Configurations

In this section, we have three objectives: the first is to test whether the three key parameters of SLPSO are interdependent or not; the second objective is to find out what the optimal configuration is for each test problem; the last one is to investigate how big the performance difference between SLPSO with default configurations and SLPSO with the optimal configurations could be for all the 45 test problems.

Although we do not know the real optimal configurations of SLPSO for a general problem, we can test the performance of SLPSO based on the combination of the values given above for all the three parameters. The experimental results regarding the best results on each problem are shown in Table 7.6 and the cor-

Table 7.6: Comparison with SLPSO with optimal configurations in terms of mean values

f	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9
Optimal	9.42e-106	0	0	0	0.00632859	3.82e-04	1.19e-14	1.04193	1.73e-50
Default	6.32e-50	0	0	4.50e-15	0.00816972	3.82e-04	4.05e-14	7.34554	3.32e-28
f	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}	f_{16}	f_{17}	f_{18}
Optimal	9.15e-05	3.79e-05	1.57e-32	2.5771	6.49348	3.82e-04	1.20e-14	0	0
Default	0.185749	0.369122	1.57e-32	20.0509	20.4416	3.82e-04	3.29e-14	0	0
f	f_{19}	f_{20}	f_{21}	f_{22}	f_{23}	f_{24}	f_{25}	f_{26}	f_{27}
Optimal	1.62e-04	5.46e-04	0.00967437	0.0435477	4.90e-68	77.5683	3008.42	0.965547	0
Default	6.47e-04	5.46e-04	0.01918	0.122745	5.87e-46	115.579	4575.08	4.50667	0
f	f_{28}	f_{29}	f_{30}	f_{31}	f_{32}	f_{33}	f_{34}	f_{35}	f_{36}
Optimal	2347.94	20.3446	77.4264	5.87e-14	1.00e-13	68.0718	22.6206	20.3093	2.94607
Default	4948.79	20.6376	105.687	1.29e-13	1.21e-13	114.67	30.4103	20.5754	8.20752
f	f_{37}	f_{38}	f_{39}	f_{40}	f_{41}	f_{42}	f_{43}	f_{44}	f_{45}
Optimal	3.56e-08	2751.54	6.25e-14	154.843	387.307	502.41	400.012	241.935	289.209
Default	0.0694798	14277.4	1.31e-13	290.464	414.28	986.578	1179.03	399.39	307.761

responding optimal combinations of the three parameters for each problem are summarized in Table 7.7. It should be noticed that we take the parameters of the optimal combinations as the real optimal configurations of SLPSO for each test problem even if they may not be. Table 7.8 shows the comparison of the success rate between SLPSO with the optimal configurations and SLPSO with the default configurations on each problem.

From the results in Table 7.6, the performance of SLPSO with the optimal configurations is better than that of SLPSO with the default configurations on most test problems. This result shows that it is necessary to further study how to effectively set up the parameters of SLPSO for general problems.

Comparing the optimal configurations for each problem obtained in this section with the results in the above section, we can see that the three key parameters do have inter-relationships. Taking the Sphere function (f_1) as an example. The optimal combination of the three parameters are 7, 0.1, and 1 for U_f, P_l, and M, respectively, which are different from the above experimental results where the corresponding optimal values are 1, 0.05, and 1, respectively.

However, from Table 7.8, it can be seen that SLPSO with the default configurations achieves the equal success rate compared with SLPSO with the optimal

Table 7.7: The optimal configurations for the 45 problems

Parameter	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}
U_f	7	1	1	3	1	7	10	3	10	3	7	1	1	1	10
P_l	0.1	0.3	0.7	0.1	0.7	0.3	0.05	0.3	0.1	0.3	1	0.3	0.05	0.05	0.3
M	1	1	1	0.7	0.7	0.7	0.3	1	1	1	1	1	0.7	0.7	0.7
Parameter	f_{16}	f_{17}	f_{18}	f_{19}	f_{20}	f_{21}	f_{22}	f_{23}	f_{24}	f_{25}	f_{26}	f_{27}	f_{28}	f_{29}	f_{30}
U_f	10	1	1	7	10	7	10	7	7	10	10	1	7	1	1
P_l	0.05	0.7	0.1	0.05	0.1	0.05	0.1	0.1	0.05	0.1	0.05	0.3	0.1	0.05	0.1
M	1	1	1	0.1	0.3	0.1	1	1	0.1	0.1	0.1	1	0.1	1	0
Parameter	f_{31}	f_{32}	f_{33}	f_{34}	f_{35}	f_{36}	f_{37}	f_{38}	f_{39}	f_{40}	f_{41}	f_{42}	f_{43}	f_{44}	f_{45}
U_f	1	7	7	7	1	10	3	10	10	1	1	10	10	1	1
P_l	0.05	0.3	0.05	0.05	0.05	0.1	0.3	0.05	0.05	0.1	0.1	0.3	0.05	0.05	0.3
M	0.1	1	0.1	0.3	1	1	1	0	1	0.7	0.3	0.3	0.7	0.3	0.3

Table 7.8: Comparison with SLPSO with the optimal configurations in terms of the success rate

Configuration	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}
Optimal	1	1	1	1	0.8	1	1	0.27	1	1	0.03	1	0.83	0.53	1
Default	1	1	1	1	0.73	1	1	0.1	1	0.67	0	1	0.7	0.7	1
	f_{16}	f_{17}	f_{18}	f_{19}	f_{20}	f_{21}	f_{22}	f_{23}	f_{24}	f_{25}	f_{26}	f_{27}	f_{28}	f_{29}	f_{30}
Optimal	1	1	1	0	1	0	0	1	0	0	0.33	1	0	0	0
Default	1	1	1	0	1	0	0	1	0	0	0	1	0	0	0
	f_{31}	f_{32}	f_{33}	f_{34}	f_{35}	f_{36}	f_{37}	f_{38}	f_{39}	f_{40}	f_{41}	f_{42}	f_{43}	f_{44}	f_{45}
Optimal	1	1	0	0	0	0.33	1	0	1	0.2	0	0	0	0	0
Default	1	1	0	0	0	0.17	0.77	0	1	0.03	0	0	0	0	0

configurations on most test problems. In other words, the parameter tuning methods for the three parameters work well for the SLPSO algorithm. One interesting observation from Table 7.6 and Table 7.8 is that although the mean value of SLPSO with the default configuration on function f_{14} is larger than that of SLPSO with the optimal configuration, but the success rate of the former configuration is 0.7, which is higher than the success rate 0.53 of the later configuration.

7.2.5 The Learning Strategy for the *abest* Position

In order to test how much benefit SLPSO can get from the learning strategy introduced for the *abest* position in Section 5.4, we define a learning to be a success learning if the *abest* position gets improvement by learning from an improved particle. The success learning rate (l_r) is defined by the rate of the number of successful learnings to the total number of learnings when the stop criterion is

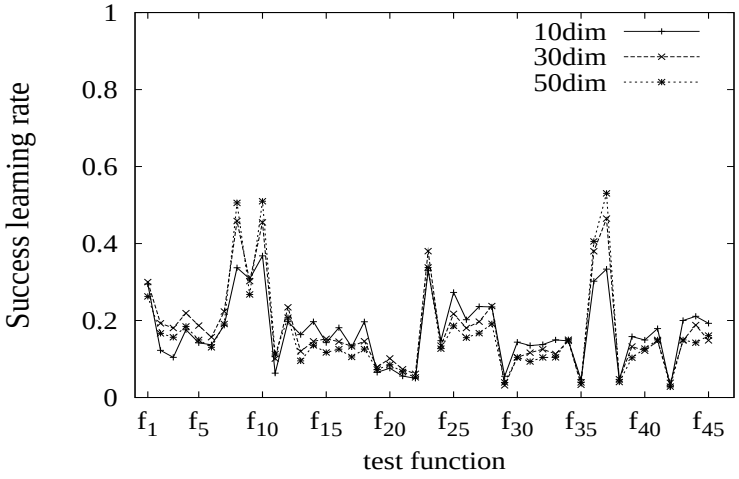


Figure 7.6: Success learning rate of SLPSO for the 45 problems in 10, 30, and 50 dimensions

satisfied. Experiments were conducted on the 45 problems in 10, 30, and 50 dimensions, respectively, and the default configurations of SLPSO were used in this section. The experimental results are summarized in Figure 7.6.

From Figure 7.6, it can be seen that the success learning rate is between 0.1 and 0.3 on most problems, where the average value is 0.172, 0.178, and 0.166 in 10, 30, and 50 dimensions, respectively. This means, for the *abest* position, there is one successful learning every six times for a general problem, which is a satisfied level. For some problems, e.g., functions f_8, f_{10}, and f_{37},this figure even reaches about 0.5. The results show that the idea of attempting to extract useful information from improved particles works well and it may be an effective method to solve the "two step forward, one step back" problem discussed in Section 5.4. In a word, we can conclude that the learning strategy for the *abest* position is favorable to the search.

7.2.6 Common Selection Ratios

So far we have recognized that the adaptive learning mechanism is beneficial. Although we have carried out the analysis on the four learning operators for each particle in a small-scale search space, it is not yet clear that how the four operators would perform in a high dimensional search space at the population level. To answer this question, we analyze the behavior of each learning operator on some selected problems of 30 dimensions over 30 runs. To clearly show the learning behavior of the four operators without impact from other factors, we disabled the restart mechanism in SLPSO in this set of experiments. Figure 7.7 and Figure 7.8 only present the results on 20 problems since similar observations can be obtained on other functions. From Figure 7.7 and Figure 7.8, several observations can be made and are described below.

First, the learning operators have very different performances on a specific problem and their performances also vary on different problems. On many problems, the best learning operator changes from the beginning to the end of evolution. For example, the best learning operator is the jumping-out operator (operator b) for the Schwefel_2_21 function (f_{11}) at the early stage. However, its selection ratio decreases after $fes = 20000$ until the end. On the contrast, the selection ratio of the exploration operator (operator c) increases and becomes the best learning operator after about 50000 evaluations. There is a period from $fes = 40000$ to $fes = 50000$ where the convergence operator (operator d) turns out to be the best learning operator. The exploitation operator (operator a) may be not suitable for the Schwefel_2_21 function as its selection ratio decreases from the beginning and remains at a very low level till the end.

Second, for some functions, the best learning operator does not change during the whole evolution process, e.g., the convergence operator in the Sphere function (f_1) and the exploration operator in the R_Schwefel function (f_{25}). The correspond-

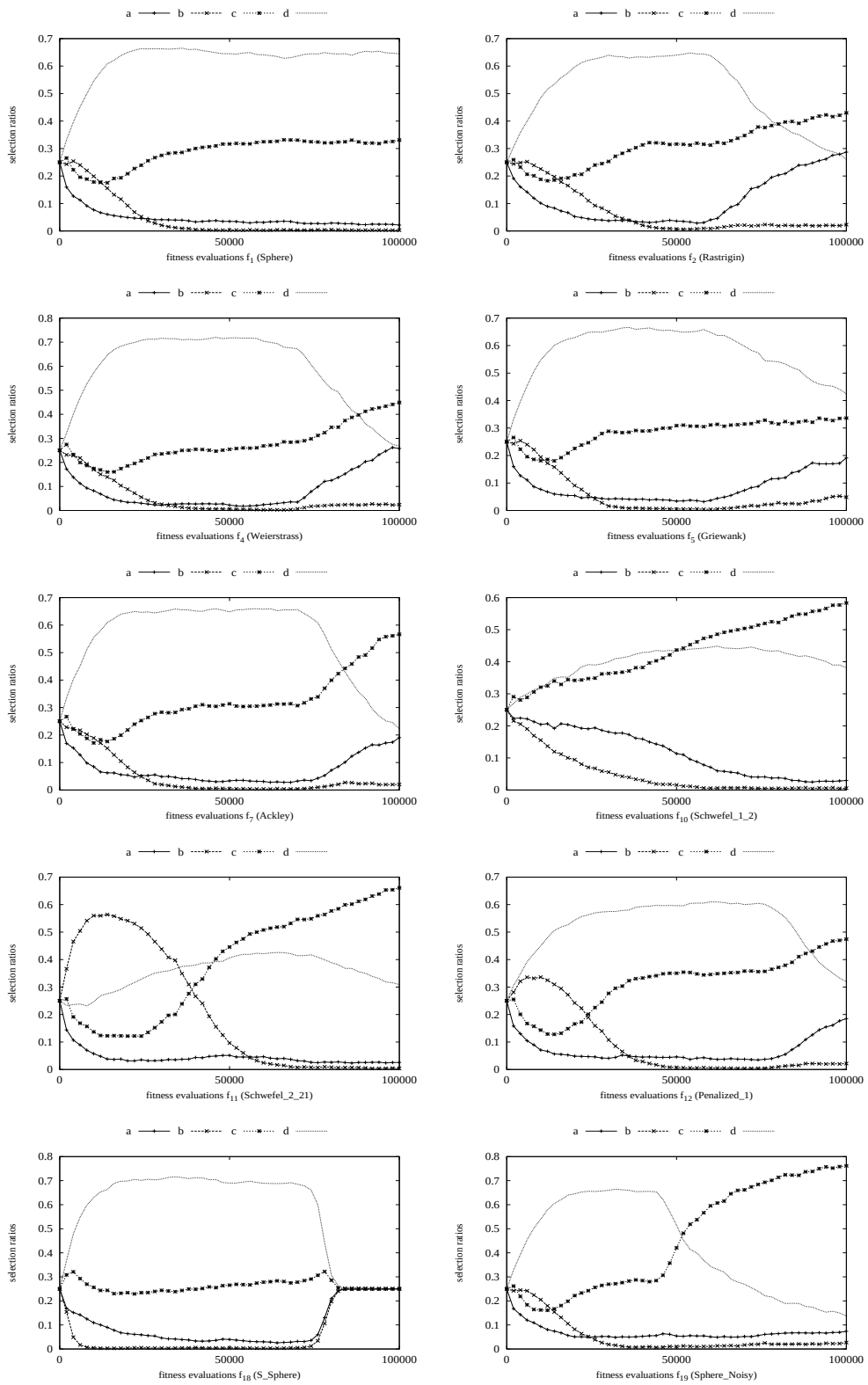


Figure 7.7: Common selection ratios of the four learning operators for ten selected functions, where *a*,*b*,*c*, and *d* represent the exploitation, jumping-out,exploration and convergence operators defined in Section 5.2, respectively.

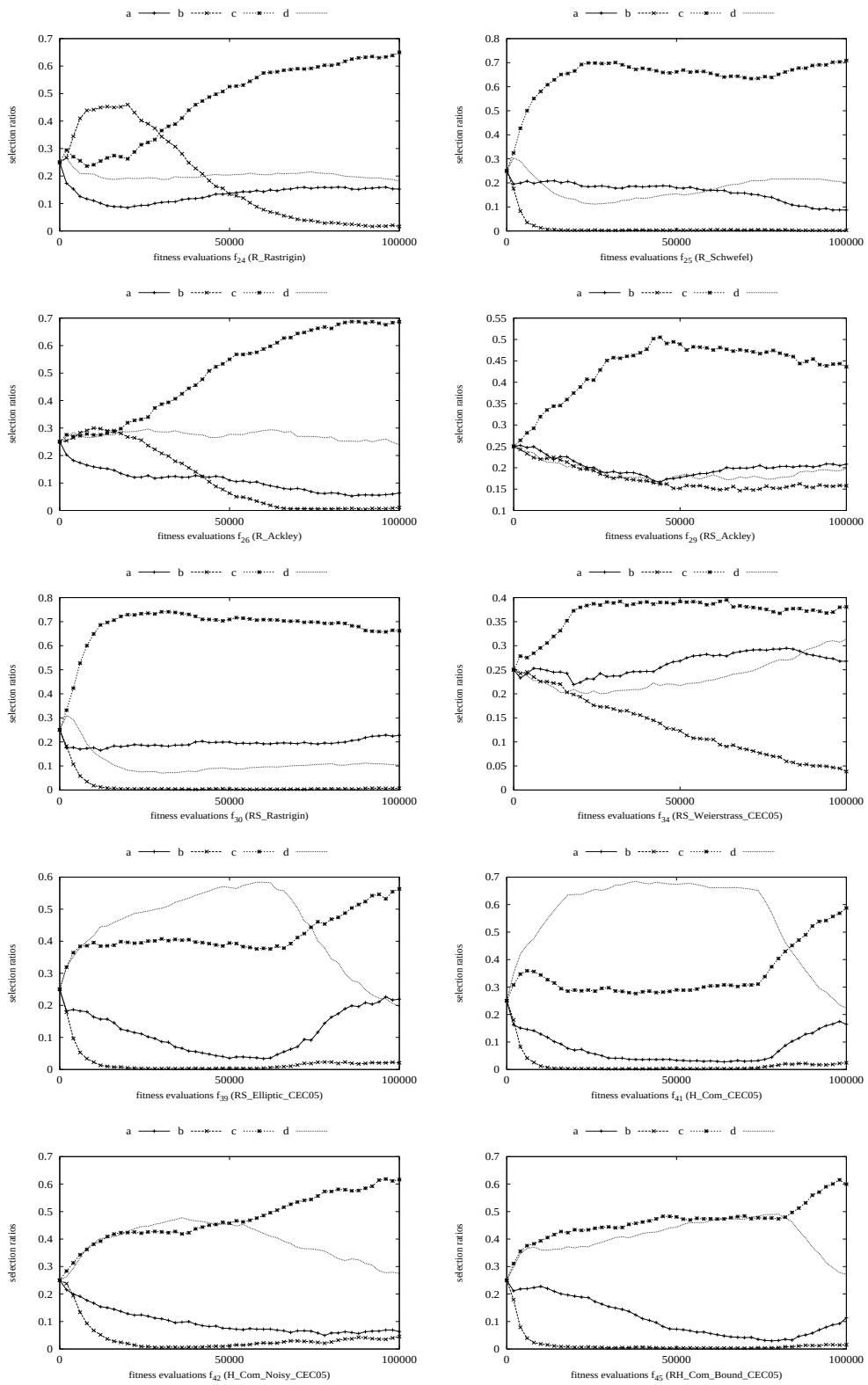


Figure 7.8: Common selection ratios of the four learning operators for ten selected functions, where *a*,*b*,*c*, and *d* represent the exploitation, jumping-out,exploration and convergence operators defined in Section 5.2, respectively.

ing selection ratios remain at the highest level during the whole evolution process on these two functions.

In addition, the convergence status appears at the population level for the S_Sphere function (f_{18}). From the graph of function f_{18}, we can see that the selection ratios of the four learning operators return to the initial state where they have the same value as they start. The convergence status appears only if none of the four learning operators can help particles to move to better areas. Of course, when a whole swarm converges to the global optimum, this phenomenon will show.

In general, we can get the following observations: 1) due to the advantages of the convergence operator discussed in Section 5.6, particles get the greatest benefit from the convergence operator on functions f_1, f_2,f_4, f_5, f_7, f_{12}, f_{18}, f_{19}, and f_{41}; 2) although the jumping-out operator has the lowest selection ratio on most problems, it does help the search during a certain period on some problems, e.g., f_1, f_2, f_{11}, f_{12}, f_{19}, f_{24},and f_{26}; 3) particles always get benefit from the exploration operator and even the largest benefit from it on some functions, e.g.,f_{25}, f_{26}, f_{29}, f_{30}, f_{34}, and f_{45}; 4) the exploitation operator may work for a short period after particles jump into a new local area as its selection ratio never reaches the highest level.

The results of this section confirm the fact that different problems need different kinds of intelligence to solve them and the need of using an adaptive method to automatically switch to an appropriate learning operator at different evolutionary stages. We also can draw the conclusion that the individual level of intelligence works well for most problems.

Table 7.9: Comparison results of means in 10 dimensions

f	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}
SLPSO	7.96e-34	**0.0307**	**0.00626**	**0**	0.0274	0.00391	6.84e-15	1.89	2.60e-12	0.0551	0.0484	2.82e-30	**63**	**10.2**	3.99
APSO	1.50e-173	0.199	0.333	0.0292	0.11	**1.27e-04**	4.35e-15	**0.107**	**4.23e-93**	4.99e-50	**2.83e-33**	1.74e-32	308	234	**1.27e-04**
CLPSO	1.05e-45	0.0663	0.367	0	**0.00412**	19.7	4.35e-15	4.2	5.22e-27	0.042	0.0365	**1.57e-32**	74.9	37.1	23.7
CPSOH	6.95e-46	0.199	0.3	2.37e-16	0.0321	551	9.92e-15	0.181	2.62e-23	6.23e-15	8.51e-13	1.57e-32	250	378	451
FIPS	9.81e-53	2.46	2.32	0	0.0414	398	**4.00e-15**	0.625	1.42e-28	2.19e-20	4.71e-19	1.57e-32	327	478	456
SPSO	**3.95e-184**	10	8.6	0.418	0.0674	726	0.534	0.534	2.12e-87	**6.57e-52**	1.42e-21	0.0207	206	202	862
f	f_{16}	f_{17}	f_{18}	f_{19}	f_{20}	f_{21}	f_{22}	f_{23}	f_{24}	f_{25}	f_{26}	f_{27}	f_{28}	f_{29}	f_{30}
SLPSO	5.65e-15	**5.56e-04**	**0**	1.76e-05	0.197	0.00487	**0.0305**	9.73e-25	15.3	1.08e+03	0.513	6.08e-22	1.38e+03	**20.1**	15.9
APSO	3.73e-10	0.365	1.56e-28	1.93e-05	**1.44e-04**	0.00693	0.304	2.63e-153	25.7	1.5e+03	2.16	3.69e-27	1.79e+03	20.4	27.3
CLPSO	**3.29e-15**	0.0995	0	1.15e-05	19.7	0.00438	0.0356	2.58e-16	**11.4**	1.21e+03	0.00174	5.68e-17	1.39e+03	20.4	**10.4**
CPSOH	1.16e-14	1.73	0	1.39e-05	577	0.0047	0.373	1.44e-45	37	2.09e+03	5.05	**1.07e-30**	2.36e+03	20.3	41.4
FIPS	1.08	3.8	71.8	1.08e-05	381	**0.00403**	1.86	6.69e-48	16.1	**919**	**4.00e-15**	61.4	**1.1e+03**	20.4	14.2
SPSO	1.29	14.1	3.24e-29	**1.05e-05**	857	0.457	9.02	**3.73e-163**	17.4	1.29e+03	0.94	3.64e-29	1.53e+03	20.3	15.6
f	f_{31}	f_{32}	f_{33}	f_{34}	f_{35}	f_{36}	f_{37}	f_{38}	f_{39}	f_{40}	f_{41}	f_{42}	f_{43}	f_{44}	f_{45}
SLPSO	**0**	**0.00434**	16.1	**4.2**	**20.3**	15.1	8.65e-04	71.1	1.89e-15	68.1	**423**	**650**	**843**	982	537
APSO	3.79e-15	0.298	27.3	5.65	20.4	**3.21**	**8.15e-14**	**5.32e-06**	9.47e-15	235	682	699	1.49e+03	1.56e+03	979
CLPSO	0	0.0995	**10.4**	4.23	20.4	3.46	0.0416	1.09	**0**	**28.5**	488	662	911	**892**	**437**
CPSOH	4.74e-14	1.23	41.3	7.49	20.3	40	1.73e-08	136	5.31e-14	290	1.24e+03	1.27e+03	1.79e+03	1.76e+03	1.68e+03
FIPS	71.8	2.8	14.2	5.05	20.4	1.69e+07	26.1	38.7	4.65e+04	716	949	768	1.16e+03	1.28e+03	1.07e+03
SPSO	6.82e-14	11.2	15.2	4.7	20.4	674	5.61e-13	1.94e-04	4.36e-14	759	849	830	1.31e+03	1.21e+03	909

7.3 Comparison with Variant PSO Algorithms

In order to compare the performance of SLPSO with other PSO algorithms, we carried out experiments on the 45 problems with the six involved PSO algorithms introduced in Section 7.1.

7.3.1 Comparison of Means

In this section, experiments on comparison of SLPSO with the other five peer PSO algorithms were conducted. Each algorithm was run 30 independent times over the 45 problems in 10, 30, and 50 dimensions, respectively. The corresponding results are provided in Table 7.9, Table 7.10, and Table 7.11, where the best result of each problem is shown in bold.

From the results shown in the tables, we can see that SLPSO is the best performer among the six algorithms. It achieves the best result on 16 problems in 10 dimensions and on 22 problems in 30 and 50 dimensions, respectively. So, the total number of problems on which SLPSO achieves the best result is more than the total number of problems on which any of the other five algorithms achieves the best result.

Table 7.10: Comparison results of means in 30 dimensions

f	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}
SLPSO	6.32e-50	**0**	**0**	**4.50e-15**	0.00817	**3.82e-04**	4.05e-14	**7.35**	3.32e-28	0.186	0.369	**1.57e-32**	**20.1**	**20.4**	**3.82e-04**
APSO	1.64e-51	5.7	2.3	0.27	0.013	686	0.0771	23.4	4.56e-32	0.00991	0.203	0.0173	73.3	510	318
CLPSO	3.84e-13	2.36e-04	0.302	1.67e-06	**2.53e-08**	39.5	4.05e-07	22.9	1.09e-08	1.48e+03	8.69	4.47e-14	40.1	75.2	15
CPSOH	1.64e-30	10.8	10.9	1.57e-07	0.0235	2.67e+03	**2.91e-14**	18.4	1.72e-14	309	**1.69e-04**	4.61e-32	264	794	2.27e+03
FIPS	5.36e-12	75	82.4	2.06e-04	0.00162	2.46e+03	5.23e-07	24.7	1.17e-07	321	0.0541	2.27e-13	306	166	2.37e+03
SPSO	**1.15e-69**	46.1	48	2.31	0.0144	3.79e+03	1.35	17.4	**7.24e-37**	**4.54e-04**	1.41	0.287	72	93.7	3.19e+03
f	f_{16}	f_{17}	f_{18}	f_{19}	f_{20}	f_{21}	f_{22}	f_{23}	f_{24}	f_{25}	f_{26}	f_{27}	f_{28}	f_{29}	f_{30}
SLPSO	**3.29e-14**	**0**	**0**	6.47e-04	**5.46e-04**	0.0192	**0.123**	5.87e-46	116	**4.58e+03**	4.51	**0**	**4.95e+03**	**20.6**	106
APSO	1.07	5.58	1.14e-26	2.40e-04	788	0.0113	5.72	4.38e-42	113	7.58e+03	3.76	1.05e-25	7.85e+03	21.2	152
CLPSO	4.77e-05	0.465	1.28e-12	2.22e-04	23.7	0.0108	0.202	7.56e-05	134	7.62e+03	2.25	3.47e-04	7.88e+03	21	150
CPSOH	0.36	25	1.89	1.87e-04	2.76e+03	0.0101	13.1	1.56e-28	158	8.48e+03	8.25	4.91	8.45e+03	20.9	245
FIPS	8.35e-06	50.4	14.6	**1.49e-04**	2.56e+03	**0.00891**	79.3	2.29e-10	184	5.88e+03	**8.61e-06**	338	5.98e+03	21	180
SPSO	1.1	58.2	1.01e-28	1.49e-04	3.48e+03	0.966	49.4	**1.10e-56**	**84.9**	5.33e+03	1.61	9.56e-28	5.79e+03	20.9	**80.7**
f	f_{31}	f_{32}	f_{33}	f_{34}	f_{35}	f_{36}	f_{37}	f_{38}	f_{39}	f_{40}	f_{41}	f_{42}	f_{43}	f_{44}	f_{45}
SLPSO	1.29e-13	**1.21e-13**	115	30.4	**20.6**	**8.21**	0.0695	1.43e+04	1.31e-13	**290**	414	987	1.18e+03	399	308
APSO	**7.20e-14**	5.74	133	30.4	21.2	29.9	**0.0182**	946	**7.39e-14**	363	462	656	1.71e+03	**312**	**300**
CLPSO	8.01e-13	0.199	142	30.1	21	10.4	3.38e+03	1.12e+04	4.06e-09	291	**400**	838	1.5e+03	492	345
CPSOH	0.0426	35.5	221	**28.2**	20.9	1.73e+03	1.34e+03	1.07e+04	1.24	639	1.22e+03	1.35e+03	2.08e+03	649	619
FIPS	130	48.1	190	37.5	21	3.56e+06	172	770	8.87e+03	477	451	471	**430**	520	534
SPSO	1.63e-13	47.9	**82.4**	32.5	21	29.1	0.616	**540**	51	418	403	**426**	545	520	586

Table 7.11: Comparison results of means in 50 dimensions

f	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}
SLPSO	1.05e-73	**0**	**0**	**2.08e-14**	0.00583	**6.36e-04**	6.26e-14	**9.85**	5.17e-33	**0.00524**	0.26	**1.57e-32**	48.3	**110**	**6.36e-04**
APSO	9.23e-58	21.3	14.4	0.271	0.0132	2.15e+03	0.052	44.7	2.6	7.27e+03	7.24	0.00346	143	483	2.66e+03
CLPSO	1.01e-17	0.0337	0.46	5.06e-11	**2.22e-13**	7.9	1.10e-09	41.3	1.59e-11	6.31e+03	6.2	1.82e-18	**9.83**	326	15.8
CPSOH	2.97e-44	30.5	22.3	4.56e-05	0.0171	5.6e+03	**3.76e-14**	38.3	1.70e-20	855	**1.14e-04**	1.60e-32	441	1e+03	5.23e+03
FIPS	2.03e-11	201	220	2.42e-04	2.14e-05	6.75e+03	8.21e-07	44.2	8.44e-08	7.86e+03	0.741	4.74e-12	78.2	182	6.79e+03
SPSO	**5.06e-91**	96.6	85.9	3.44	0.0139	6.67e+03	1.32	35.7	**5.58e-50**	0.222	4.33	0.88	223	330	6.34e+03
f	f_{16}	f_{17}	f_{18}	f_{19}	f_{20}	f_{21}	f_{22}	f_{23}	f_{24}	f_{25}	f_{26}	f_{27}	f_{28}	f_{29}	f_{30}
SLPSO	**5.40e-14**	**0**	**0**	0.00121	**9.58e-04**	0.0195	0.263	8.78e-70	210	**1.01e+04**	4.09	**0**	1.45e+04	**20.6**	222
APSO	8.03	22.8	1.13e-25	6.28e-04	1.98e+03	0.0152	34.5	7.61e-50	232	1.59e+04	4.79	1.61e-25	1.93e+04	21.3	437
CLPSO	2.26e-07	0.432	4.19e-17	3.77e-04	19.7	0.0111	**0.194**	1.02e-07	302	1.43e+04	1.08	7.38e-08	1.8e+04	21.2	306
CPSOH	1.37	117	29.2	4.03e-04	5.42e+03	0.012	36.2	2.27e-41	382	1.61e+04	8	115	1.83e+04	21.1	487
FIPS	7.51e-07	142	2.47e-11	3.31e-04	7.13e+03	**0.0105**	202	1.79e-09	373	1.26e+04	**2.06e-05**	84.7	1.39e+04	21.2	369
SPSO	1.23	111	3.21e-28	**3.29e-04**	5.91e+03	1.06	104	**6.89e-72**	**135**	1.05e+04	2.01	1.83e-27	**1.34e+04**	21.1	**129**
f	f_{31}	f_{32}	f_{33}	f_{34}	f_{35}	f_{36}	f_{37}	f_{38}	f_{39}	f_{40}	f_{41}	f_{42}	f_{43}	f_{44}	f_{45}
SLPSO	1.88e-13	**2.01e-13**	222	57.7	**20.6**	**0.315**	**0.0284**	7.17e+04	**1.99e-13**	**407**	563	1.33e+03	1.76e+03	1.72e+03	1.65e+03
APSO	1.23e-13	42.8	437	59.3	21.3	1.03e+08	10.5	4.35e+04	1.56e+06	512	716	1.9e+03	2.13e+03	2.21e+03	2.19e+03
CLPSO	**7.77e-14**	0.365	306	55.9	21.2	14.1	1.23e+04	3.92e+04	2.44e-13	430	446	1.42e+03	2.09e+03	2.12e+03	2.09e+03
CPSOH	29.2	115	487	**51.6**	21.1	1.10e+05	4e+03	1.59e+04	2.03e-07	684	1.29e+03	1.47e+03	2.2e+03	2.22e+03	2.23e+03
FIPS	2.42e-11	131	369	70.5	21.2	58.7	5.66e+03	1.29e+04	1.57e+06	418	468	630	**536**	**515**	**637**
SPSO	3.32e-13	106	**128**	62	21.1	41	2.6	**6.83e+03**	2.05e-13	549	**433**	**467**	560	806	828

Among the five unimodal problems, functions f_1 and f_9 are successfully solved by all algorithms in 10, 30, and 50 dimensions in terms of the given accuracy level, except that APSO failed on function f_9 in 50 dimensions. For function f_8, although SLPSO does not obtain the best result among the six algorithms in 10 dimensions, it achieves the best results in 30 and 50 dimensions. SPSO achieves the best result on function f_{10} in 10 and 30 dimensions, while SLPSO shows the best performance on function f_{10} in 50 dimensions. For function f_{11}, CPSOH shows the best results on all the three dimensional cases.

Because of the capability of maintaining diversity within SLPSO, it is the only algorithm that successfully found the global optima for the Rastrigin (f_2) and non-

Table 7.12: The number of problems where the best result achieved by each algorithm over the 45 problems in 10, 30, and 50 dimensions

Dim	SLPSO	APSO	CLPSO	CPSOH	FIPS	SPSO
10	16	9	12	2	6	4
30	22	5	2	3	4	9
50	22	0	4	3	5	11

continuous Rastrigin (f_3) functions in 30 and 50 dimensions, and it also achieves the best performance on the Schwefel function (f_6) in all tested dimensions. For composition functions f_{13} and f_{14}, SLPSO achieves much better results than the other five algorithms on all dimensional cases except f_{13} in 50 dimensions. For the other composition functions (f_{40}-f_{45}), the performance of SLPSO is not the best one among the six algorithms in problems with 30 and 50 dimensions. But it does not mean that SLPSO is not suitable to solve this kind of problems. Comparing the results of SLPSO with the optimal configurations in Table 7.6 and the best results achieved by the other five algorithms in Table 7.10 on the eight composition functions (f_{13}, f_{14}, and f_{40}-f_{45}), it is easy to see that SLPSO with the optimal configurations outperforms all the other algorithms except on function f_{42}. Generally speaking, among the six algorithms, FIPS and SPSO have relatively better performances on the composition problems. The detailed comparison of SLPSO with the other algorithms on the modified multi-modal problems will be discussed later in Section 7.3.4.

Table 7.12 shows the number of problems where the best result achieved by each algorithm over the 45 problems in 10, 30, and 50 dimensions, respectively. Based on the results in Table 7.12, the performance of APSO and CLPSO dramatically decreases with the increasing of the number of dimensions. But, the case of SPSO is opposite to APSO and CLPSO: its performance increases with the increasing of the number of dimensions, which makes it the second best algorithm. FIPS and CPSOH have better robustness compared with APSO and CLPSO.

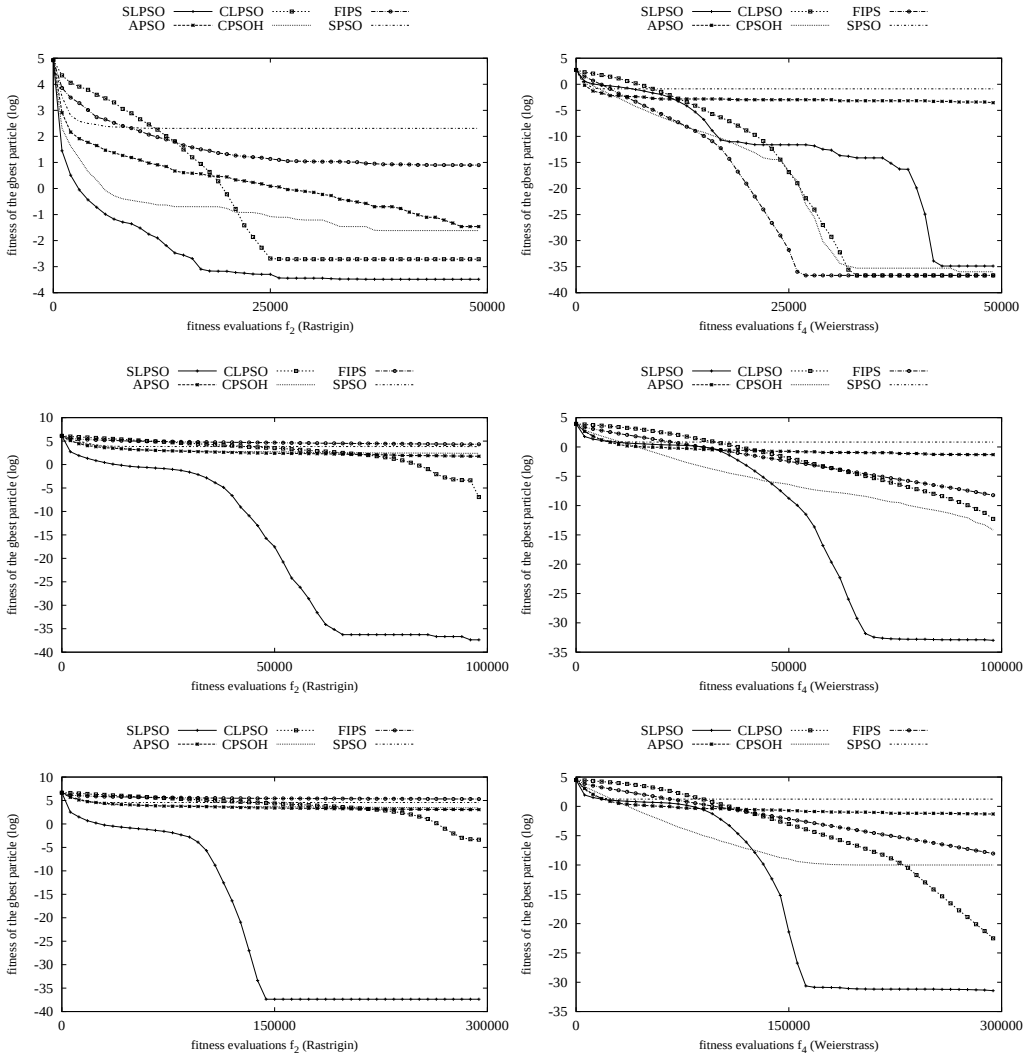


Figure 7.9: The convergence process of the six algorithms on the functions Rastrigin (left), and Weierstrass (right) in 10 (top), 30 (middle), and 50 (bottom) dimensions

7.3.2 Comparison Regarding the Convergence Speed

In this section, we aim to not only compare SLPSO with the other five algorithms regarding the convergence speed, but also find the evidence to show the advantage of the restart mechanism in SLPSO at the population level. For this purpose, we just chosen six problems for comparison, which are the Rastrigin (f_2), Weierstrass (f_4), Schwefel (f_6), Rosenbrock (f_8), R_Com (f_{14}), and RH_Com_Bound_CEC05(f_{45})

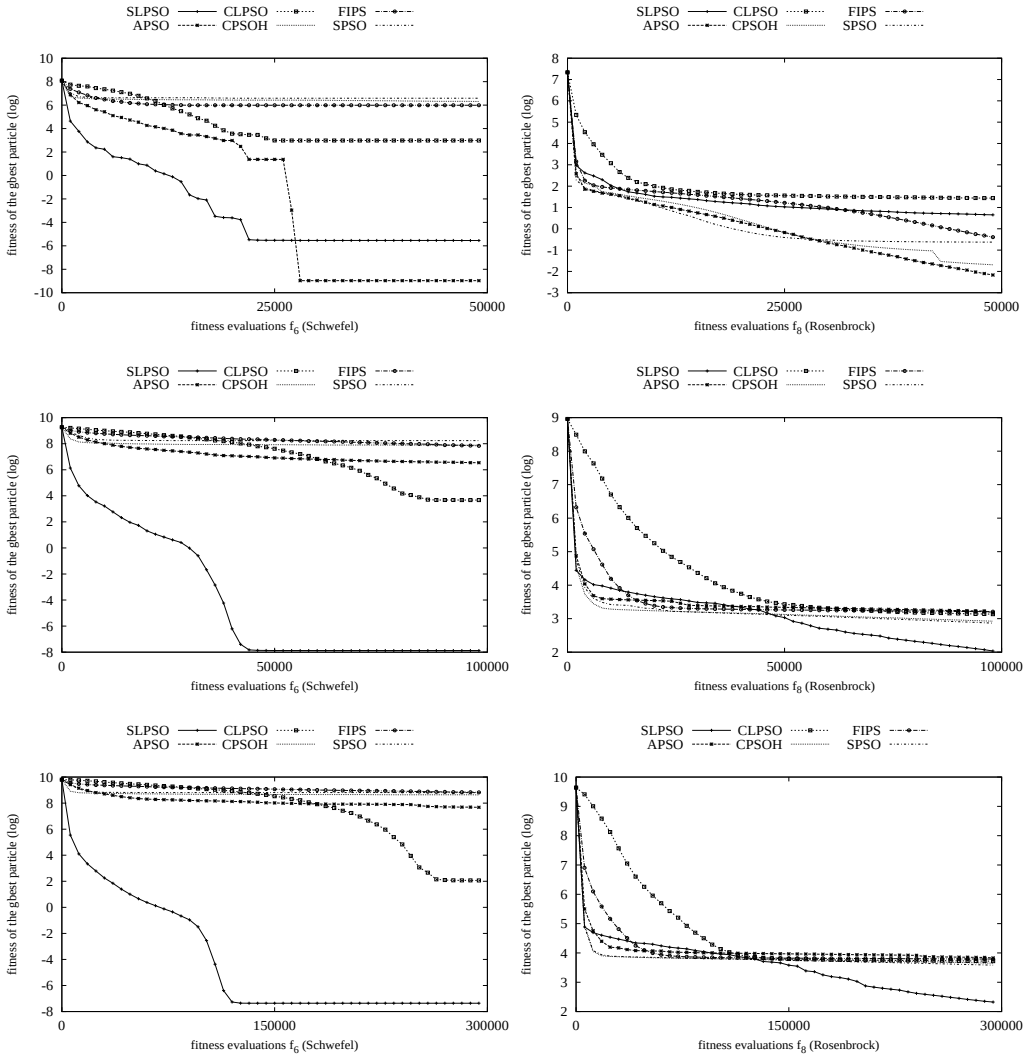


Figure 7.10: The convergence process of the six algorithms on the functions Schwefel (left), and Rosenbrock (right) in 10 (top), 30 (middle), and 50 (bottom) dimensions

functions. Figure 7.9, Figure 7.10, and Figure 7.11 present the comparison results regarding the convergence speed on the 6 selected problems in 10 (on the top), 30 (in the middle), and 50 (at the bottom) dimensions.

From the graphs of the convergence speed process, it can be seen that the convergence speed of SLPSO is the fastest among the six algorithms. It should be noticed that this may not be the fact for other functions. Since there are

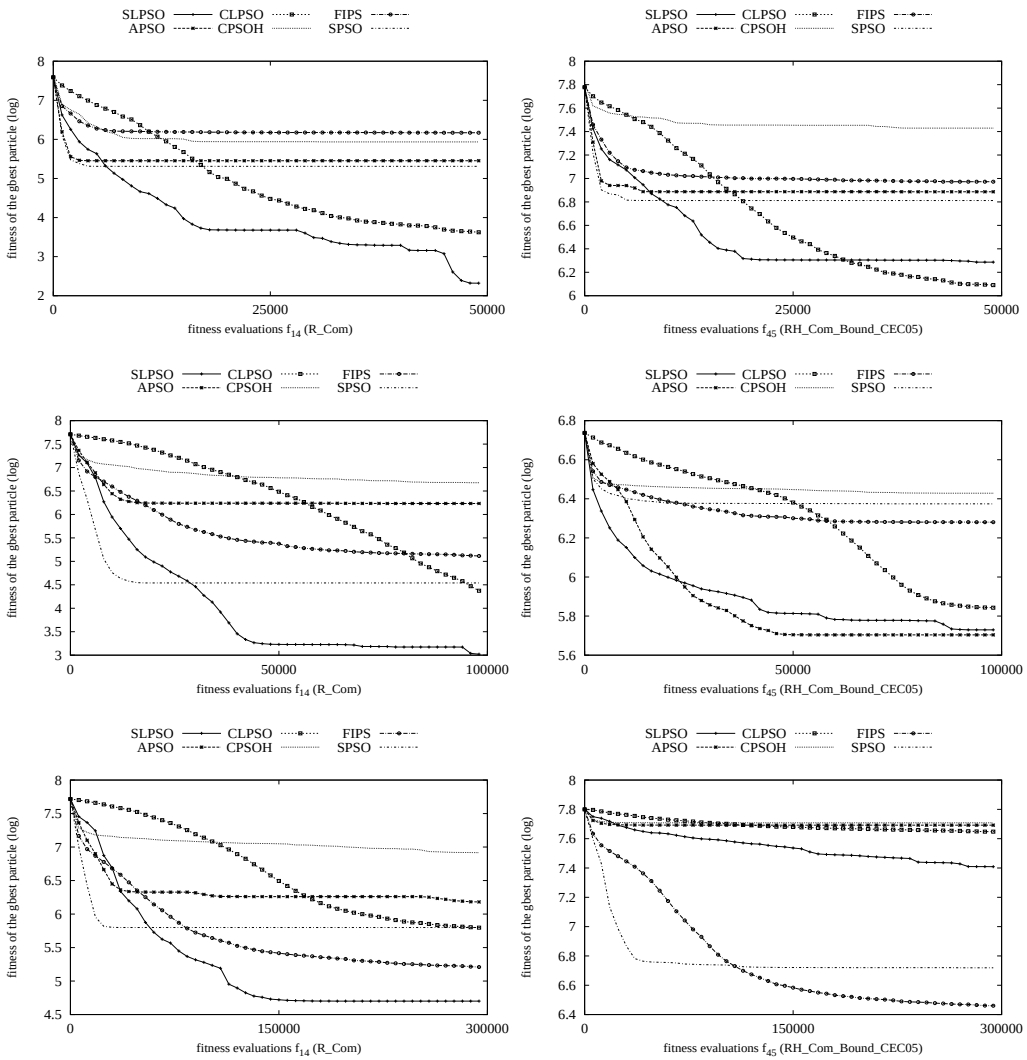


Figure 7.11: The convergence process of the six algorithms on the functions R_Com (left), and RH_Com_Bound_CEC05 (right) in 10 (top), 30 (middle), and 50 (bottom) dimensions

too many figures of the convergence speed for the six algorithms across the three dimensional cases, the remaining figures of the other 39 functions are not provided in this section.

For most algorithms, generally speaking, there is one knee point during the whole evolutionary process, which can be seen from the convergence curves in Figure 7.9, Figure 7.10, and Figure 7.11. A knee point in a convex or concave

curve can be roughly understood as the point that has the farthest distance to the line segment defined by the beginning point and the ending point of that curve. Taking CLPSO on the Rosenbrock function in Figure 7.10 as an example. The convergence curve quickly drops down to a certain level at the exploration stage, and slowly decreases or remains at that level when it is in the convergence stage. This is a common phenomenon for most algorithms. However, there may be several knee regions for SLPSO due to the restart mechanism. Taking the R_Com function (f_{14}) in 10 dimensions in Figure 7.11 as an example. There are three knee points. The first knee point shows at about $fes = 17000$. Then, the swarm is in a temporal convergence stage. Due to the restart mechanism, the swarm jumps to a new better local optimum at about $fes = 30000$. As a result, the second knee appears. The third knee takes place at about $fes = 45000$ and brings the swarm to the location that is near the global optimum. The height of the second best peak of the R_Com function is 100 and the average best results obtained by SLPSO is 10.2 in Table 7.9, which is very close to the global optimum. Similar observations for SLPSO can also be seen from the other figures, except on the Rosenbrock function in Figure 7.10.

The above evidence shows that the restart mechanism does help SLPSO escape from local optima and find the global optimum.

7.3.3 Comparison Regarding the Success Rate

According to the accuracy level given for each problem in Table 3.5, we present the comparison of the statistical results of the six algorithms in terms of the success rate. Figure 7.12 shows the distribution of the success rate of the algorithms over the 45 test problems, and Figure 7.13 shows the number of problems that are solved, partially solved, and never solved by the six algorithms in 10, 30, and 50 dimensions. A problem is solved if an algorithm reaches the corresponding given

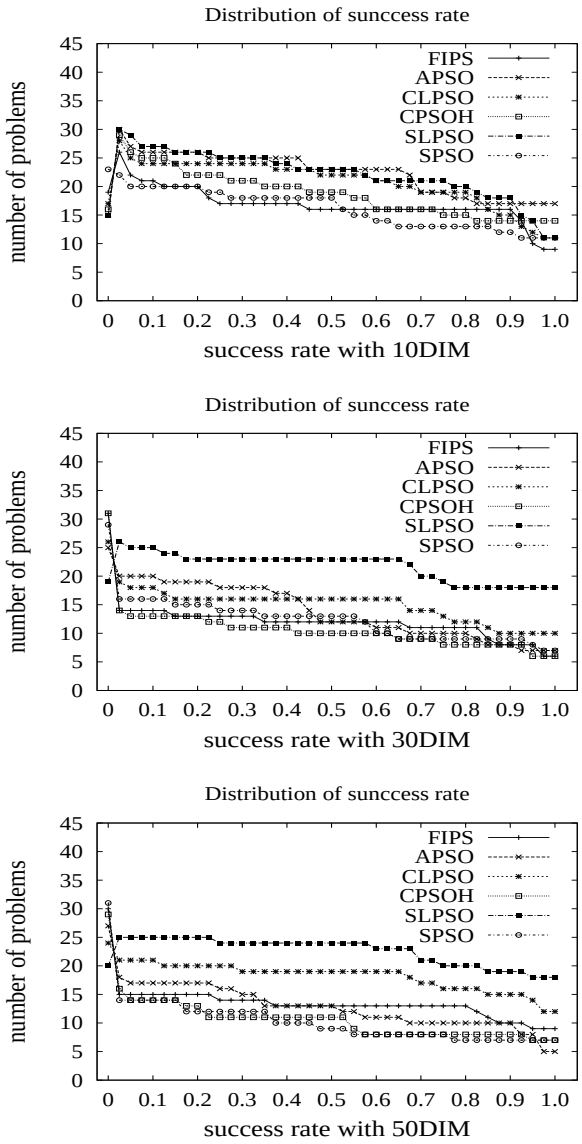


Figure 7.12: Distribution of the success rate of the six algorithms on problems in 10,30, and 50 dimensions.

accuracy level for all runs. A partially solved problem means that some runs of an algorithm achieve the given accuracy level. A never solved problem means none of the 30 runs reaches the given accuracy level.

In Figure 7.12, each pointed curve means the number of problems where the success rate is equal to or greater than a specific value for an algorithm in 10, 30, and 50 dimensions, except the point with the success rate value 0, which denotes

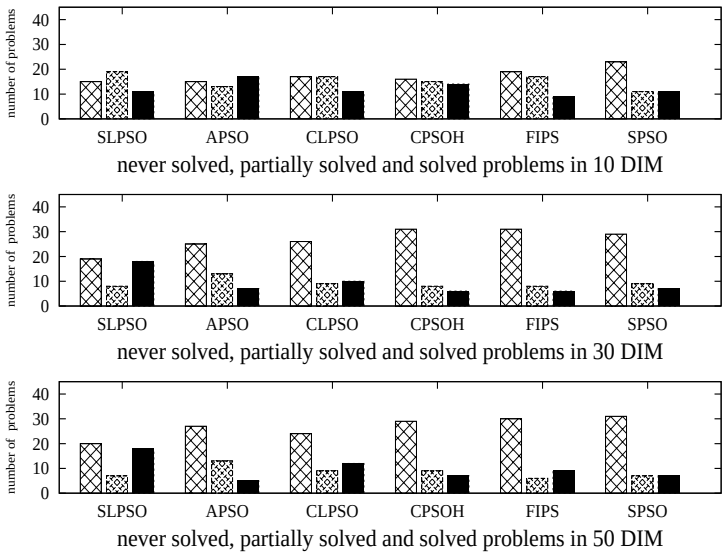


Figure 7.13: The number of problems that are solved, partially solved, or never solved by the six algorithms in 10, 30, and 50 dimensions.

the number of problems where the success rate is greater than 0. From the results, we can see that the performance of SLPSO is slightly better than APSO and CLPSO in 10 dimensions, but it is much better than the other five algorithms in 30 and 50 dimensions. CLPSO is the second best algorithm in terms of the success rate.

In Figure 7.13, the number of never solved, partially solved, and solved problems is calculated by the number of problems where the success rate is equal to 0, within (0, 1), or equal to 1, respectively, for each algorithm. The net bar, dotted bar, and black bar for each algorithm represent the number of problems that are never solved, partially solved, and solved, respectively. As in Figure 7.12, it can be seen that SLPSO has similar performance to other algorithms on problems in 10 dimensions, but has much better performance than the other algorithms on problems in 30 and 50 dimensions. The number of problems solved by SLPSO is about 20 in 30 and 50 dimensions, which is about twice as the number of problems solved by the second best algorithm (CLPSO).

Generally speaking, the difficulty of a problem will increase when the number

of dimensions increases. So, an algorithm's performance will decrease. From the results in Figure 7.13, we can clearly see this trend for some algorithms, e.g., APSO and CPSOH, where the number of never solved problems increases while the number of solved problems deceases when the number of dimensions increases. Compared with the other five algorithms, the performance decrease of SLPSO is not so obvious where the number of never solved problems slightly increases with the increasing number of dimensions.

7.3.4 Comparison Regarding the Robustness

In order to compare the robustness of the six algorithms, we present the comparison in two aspects in this section: the performance decrease on a problem with different modifications and with different number of dimensions.

Comparison Regarding the Performance Decrease on Modified Problems

In order to further investigate the effect of noise, rotation, and landscape shifting on the performance of an algorithm, we chose four traditional problems in the first group: the Sphere (f_1), Rastrigin (f_2), Ackley (f_7), and Schwefel (f_6) functions. For all original problems in this section, we extend them to the other four different kinds of problems by (a) adding noise, (b) shifting the landscape, (c) rotating the landscape, and (d) combining shifting and rotating of the landscape. Figure 7.14 shows the comparison regarding the mean values on the four traditional functions with different modifications in 10 (left), 30 (middle), and 50 (right) dimensions.

From Figure 7.14, several observations can be obtained: 1) different modifications on a specific problem bring in different difficulties; 2) the same modification on different problems also brings in different difficulties; 3) the same modification on a specific problem brings in different difficulties for different algorithms.

For the Sphere function (f_1), shifting the global optimum does not affect the

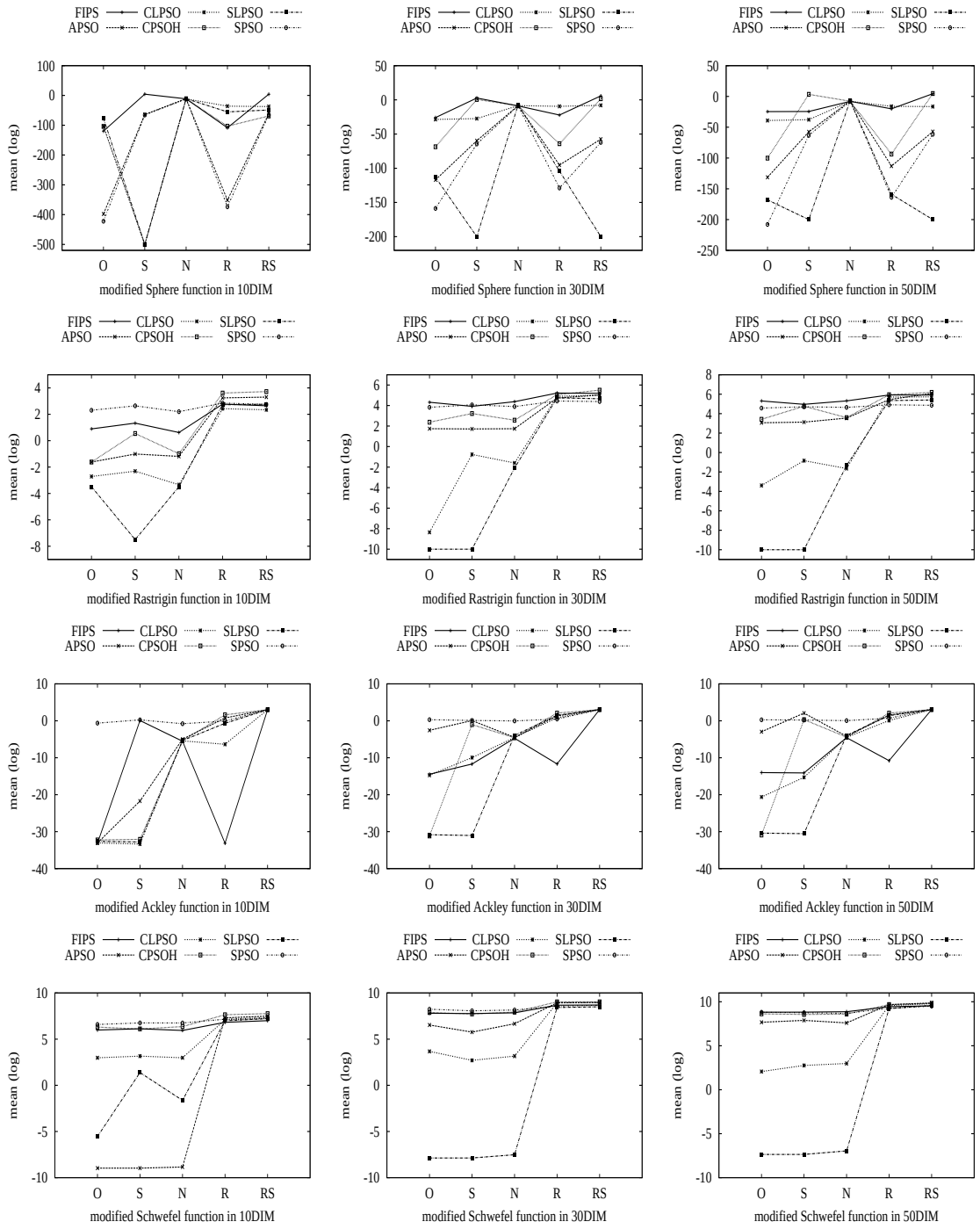


Figure 7.14: Comparison regarding the performance decrease on modified problems, where "O", "N", "S", "R", and "RS" represent the original problems, the modified problems by adding noise, shifting, rotating, and combination of shifting and rotating, respectively.

performance of SLPSO, but seriously affects the performance of the other five algorithms. Adding noise produces the similar level of difficulties for all the six algorithms in all the three dimensional cases. Rotating does not cause too much difficulties for the six algorithms. Due to the effect of shifting, the performance of the algorithms except SLPSO deceases on the rotated shifted Sphere function.

Similar observations can be made on the Rastrigin and Schwefel functions: shifting and adding noise do not affect most algorithms too much, but rotating the fitness landscape seriously affects the performance of all the six algorithms.

For the Ackley function, shifting does not raise the challenge for SLPSO, FIPS, and SPSO, but raises the challenge for the other three algorithms. Adding noise leads them to have similar performance. Although rotating does not affect the performance of FIPS, the combination of rotating and shifting traps all the six algorithms into the same local optimum.

From the experimental results, we can see that the modifications do raise the challenge to most algorithms and make the problems harder to solve than the original problems: shifting makes algorithms unable to take "useful" information from other dimensions; adding noise makes it harder for algorithms to detect the global optimum; rotating changes the shape of the original fitness landscape. Among the six algorithms, the interesting thing is that SLPSO is the only algorithm that is not sensitive to the modification of shifting the global optimum to a random location on the four test functions. In other words, SLPSO has a better capability than the other five algorithms to deal with the shifting modification.

Comparison Regarding the Performance Deterioration Rate with Increasing Dimensions

To investigate how much the performance of an algorithm decreases with the increasing of the number of dimensions, we define the following metric: the

Table 7.13: Performance deterioration rate of the six algorithms

f	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9
SLPSO	(-,-)	(0,0)	(-,-)	(-,-)	(0.3,0.21)	(-,-)	(-,-)	(3.9,5.2)	(-,-)
APSO	(-,-)	(29,1e+2)	(6.9,43)	(9.3,9.3)	(0.12,0.12)	(-,3.1)	(-,0.67)	(2e+2,4e+2)	(-,-)
CLPSO	(-,-)	(0.0036,0.51)	(0.82,1.3)	(-,-)	(-,-)	(2,0.4)	(-,-)	(5.5,9.8)	(-,-)
CPSOH	(-,-)	(54,1.5e+2)	(36,74)	(-,-)	(0.73,0.53)	(4.8,10)	(-,-)	(1e+2,2e+2)	(-,-)
FIPS	(-,-)	(31,82)	(36,95)	(-,-)	(0.039,5e-4)	(6.2,17)	(-,-)	(40,71)	(-,-)
SPSO	(-,-)	(4.6,9.6)	(5.6,10)	(5.5,8.2)	(0.21,0.21)	(5.2,9.2)	(2.5,2.5)	(33,67)	(-,-)
f	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}	f_{16}	f_{17}	f_{18}
SLPSO	(3.4,0.095)	(7.6,5.4)	(-,-)	(0.32,0.77)	(2,11)	(10e-5,2e-4)	(-,-)	(-,-)	(-,-)
APSO	(-,-)	(-,36)	(-,0.2)	(0.24,0.47)	(2.2,2.1)	(-,8.4)	(-,7.5)	(15,62)	(-,-)
CLPSO	(4e+4,2e+5)	(1e+2,2e+2)	(-,-)	(0.54,0.13)	(2,8.8)	(0.63,0.67)	(-,0.0047)	(4.7,4.3)	(-,-)
CPSOH	(-,2.8)	(-,0.68)	(-,-)	(1.1,1.8)	(2.1,2.7)	(5,12)	(-,3.8)	(14,68)	(-,15)
FIPS	(-,24)	(-,14)	(-,-)	(0.94,0.24)	(0.35,0.38)	(5.2,15)	(7e-6,6e-7)	(13,37)	(0.2,3e-13)
SPSO	(-,-)	(-,3.1)	(14,42)	(0.35,1.1)	(0.46,1.6)	(3.7,7.4)	(0.85,0.95)	(4.1,7.9)	(-,-)
f	f_{19}	f_{20}	f_{21}	f_{22}	f_{23}	f_{24}	f_{25}	f_{26}	f_{27}
SLPSO	(37,68)	(3e-3,5e-3)	(3.9,4)	(4,8.6)	(-,-)	(7.6,14)	(4.2,9.3)	(8.8,8)	(-,-)
APSO	(12,32)	(-,2.5)	(1.6,2.2)	(19,1e+2)	(-,-)	(4.4,9)	(5.1,11)	(1.7,2.2)	(-,-)
CLPSO	(19,33)	(1.2,1)	(2.5,2.5)	(5.7,5.4)	(-,0.0014)	(12,27)	(6.3,12)	(1e+3,6e+2)	(-,2e-4)
CPSOH	(13,29)	(4.8,9.4)	(2.1,2.5)	(35,97)	(-,-)	(4.3,10)	(4.1,7.7)	(1.6,1.6)	(-,23)
FIPS	(14,31)	(6.7,19)	(2.2,2.6)	(43,1e+2)	(-,-)	(11,23)	(6.4,14)	(-,2.4)	(5.5,1.4)
SPSO	(14,31)	(4.1,6.9)	(2.1,2.3)	(5.5,12)	(-,-)	(4.9,7.8)	(4.1,8.2)	(1.7,2.1)	(-,-)
f	f_{28}	f_{29}	f_{30}	f_{31}	f_{32}	f_{33}	f_{34}	f_{35}	f_{36}
SLPSO	(3.6,11)	(1,1)	(6.7,14)	(-,-)	(-,-)	(7.1,14)	(7.2,14)	(1,1)	(0.54,0.02)
APSO	(4.4,11)	(1,1)	(5.6,16)	(-,-)	(19,1e+2)	(4.9,16)	(5.4,10)	(1,1)	(9.3,3e+7)
CLPSO	(5.7,13)	(1,1)	(14,29)	(-,-)	(2,3.7)	(14,29)	(7.1,13)	(1,1)	(3,4.1)
CPSOH	(3.6,7.7)	(1,1)	(5.9,12)	(-,7e+2)	(29,93)	(5.3,12)	(3.8,6.9)	(1,1)	(43,2e+3)
FIPS	(5.5,13)	(1,1)	(13,26)	(1.8,3e-13)	(17,47)	(13,26)	(7.4,14)	(1,1)	(0.21,3e-06)
SPSO	(3.8,8.8)	(1,1)	(5.2,8.2)	(-,-)	(4.3,9.5)	(5.4,8.4)	(6.9,13)	(1,1)	(0.04,0.06)
f	f_{37}	f_{38}	f_{39}	f_{40}	f_{41}	f_{42}	f_{43}	f_{44}	f_{45}
SLPSO	(-,0.41)	(2e+2,1e+3)	(-,-)	(4.3,6)	(0.98,1.3)	(1.5,2)	(1.4,2.1)	(0.41,1.8)	(0.57,3.1)
APSO	(-,5.7e+02)	(-,46)	(-,-)	(1.5,2.2)	(0.68,1.1)	(0.94,2.7)	(1.1,1.4)	(0.2,1.4)	(0.31,2.2)
CLPSO	(8e+4,3e+5)	(1e+4,3e+4)	(-,-)	(10,15)	(0.82,0.91)	(1.3,2.1)	(1.6,2.3)	(0.55,2.4)	(0.79,4.8)
CPSOH	(-,3)	(79,1e+2)	(-,1e-7)	(2.2,2.4)	(0.98,1)	(1.1,1.2)	(1.2,1.2)	(0.37,1.3)	(0.37,1.3)
FIPS	(6.6,2e+2)	(20,3e+2)	(0.2,34)	(0.7,0.6)	(0.5,0.5)	(0.6,0.8)	(0.3,0.5)	(0.4,0.4)	(0.5,0.6)
SPSO	(-,4.2)	(-,13)	(-,4e-15)	(0.5,0.7)	(0.4,0.5)	(0.5,0.6)	(0.4,0.4)	(0.4,0.7)	(0.6,0.9)

performance deterioration rate (PDR). To calculate the PDR, we need to set up a base value, which is the mean values obtained by each algorithm on problems in 10 dimensions. Then, we calculate the PDR of the mean values in higher dimensions over the base values for each problem. In order to reasonably calculate the PDR on different problems for each algorithm, there is a condition to calculate the PDR for an algorithm on a specific problem: it is computed only if the base value does not reach the corresponding accuracy level. If the condition is not satisfied, we take the mean value of 30 dimensions as a new base value. If the condition still does not hold, the corresponding algorithm quits the comparison.

Table 7.13 shows the PDR of each algorithm over the 45 test problems, where

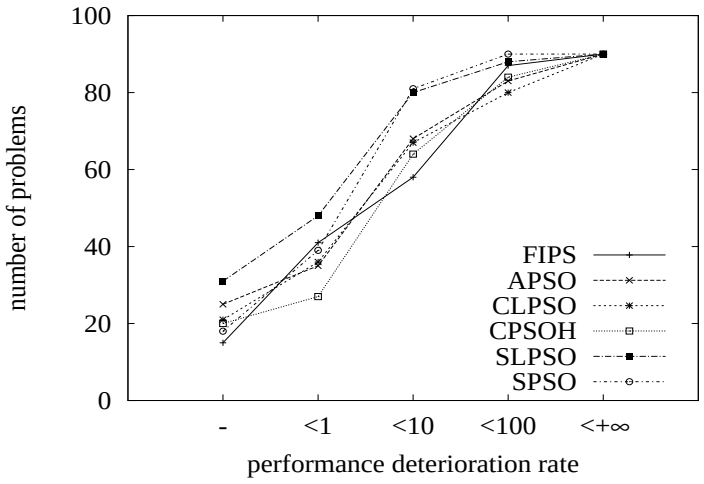


Figure 7.15: Distribution of the PDR, "–" means the number of problems where algorithms have achieved the given accuracy level in the base dimensions

the symbol "–" means that the corresponding algorithm does not take part in the comparison as its base value reaches the given accuracy level on that problem. Each result contains two values, which are the PDR of an algorithm on that problem in 30 (the former one) and 50 (the later one) dimensions, respectively. If the PDR is less than 1.0, it means the algorithm's performance in higher dimensions is better than the performance in its base dimension; otherwise, the larger the PDR, the heavier the performance decrease.

Figure 7.15 summarizes the results in Table 7.13, which shows the distribution of the PDR. The start point is the number of problems that do not take part in the comparison. The pointed curves represent the number of problems where the PDR is less than a given value over the 30 and 50 dimensional cases for the six algorithms. It also includes the number of problems that do not take part in the comparison. From Figure 7.15, it can be seen that SLPSO has the largest number of cases where the PDR is less than 1. It also shows that the performance of SLPSO and SPSO is better than the other four algorithms in terms of the PDR when the value of PDR is less than 10.

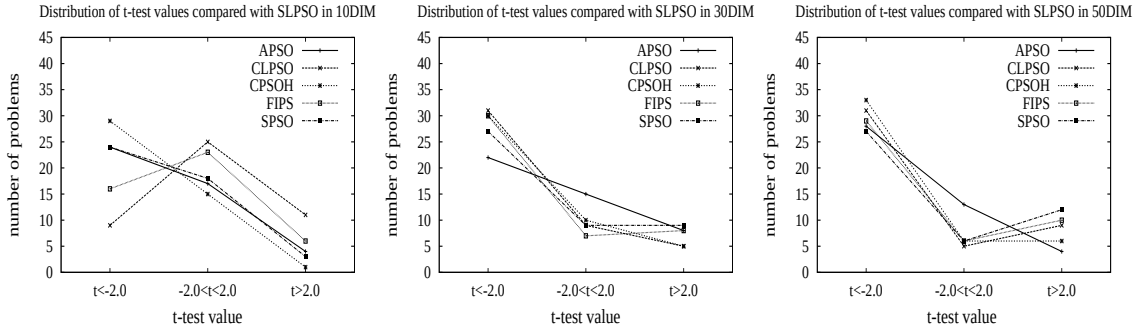


Figure 7.16: Distribution of the t-test results compared with SLPSO in 10, 30, and 50 dimensions.

7.3.5 Comparison Regarding the t-Test Results

In order to investigate how much the performance of SLPSO is better or worse than the performance of the other five algorithms at the statistical level on each test problem, a two tailed t-test operation was performed in this section. The performance difference is significant between two algorithms if the absolute value of the t-test result is greater than 2.0. For all the t-test results in this chapter, the suffix "+", "∼", or "−" is attached to the end of each result, which represents that the performance of SLPSO is significantly better than, statistically equivalent to, or significantly worse than the performance of its rival, respectively. The results are shown in Table 7.14 and summarized in Figure 7.16.

Compared with the other five algorithms on problems in 10 dimensions, it can be seen that the performance of CLSPO is slightly better than that of SLPSO. The t-test results of 11 problems with CLPSO are significantly better than that of SLPSO, and the t-test results of 9 problems with SLPSO is significantly better than those of CLPSO. However, SLPSO outperforms the other four algorithms in terms of the t-test results.

Compared with the other five algorithms on problems in 30 and 50 dimensions, it can be seen that the performance of SLPSO is significantly better than any of the

Table 7.14: t-Test results of comparing SLPSO with the other five algorithms in 10, 30, and 50 dimensions

	10Dim					30Dim					50Dim				
f	APSO	CLPSO	CPSOH	FIPS	SPSO	APSO	CLPSO	CPSOH	FIPS	SPSO	APSO	CLPSO	CPSOH	FIPS	SPSO
f_1	$1^{\sim}$	$1^{\sim}$	$1^{\sim}$	$1^{\sim}$	$1^{\sim}$	$-1.3^{\sim}$	-7.5^{+}	$-1.8^{\sim}$	-6.9^{+}	$-1.2^{\sim}$	$-2^{\sim}$	-10^{+}	-3.1^{+}	-20^{+}	$1^{\sim}$
f_2	-2.2^{+}	$-0.73^{\sim}$	-2.2^{+}	-6.9^{+}	-12^{+}	-14^{+}	-5.4^{+}	-9.2^{+}	-27^{+}	-17^{+}	-2.8^{+}	$-1^{\sim}$	-10^{+}	-48^{+}	-22^{+}
f_3	-3.3^{+}	-3.2^{+}	-3^{+}	-5^{+}	-20^{+}	-5.5^{+}	-2.5^{+}	-9^{+}	-28^{+}	-20^{+}	-2.2^{+}	-4^{+}	-5.4^{+}	-53^{+}	-10^{+}
f_4	-3.1^{+}	$0^{\sim}$	$-1^{\sim}$	$0^{\sim}$	-4.3^{+}	-4.6^{+}	-6.5^{+}	$-1.5^{\sim}$	-19^{+}	-8.5^{+}	-7.1^{+}	-10^{+}	$-1.3^{\sim}$	-26^{+}	-9.3^{+}
f_5	-9.2^{+}	7.8^{-}	$-0.98^{\sim}$	$-1.5^{\sim}$	-6.7^{+}	-2^{+}	4.3^{-}	-5^{+}	2.4^{-}	-2^{+}	$-1.6^{\sim}$	3^{-}	-2.5^{+}	3^{-}	-2^{+}
f_6	$1.2^{\sim}$	-2.4^{+}	-13^{+}	-11^{+}	-16^{+}	-2.1^{+}	-3.8^{+}	-20^{+}	-16^{+}	-27^{+}	-4.1^{+}	$-1.4^{\sim}$	-39^{+}	-38^{+}	-30^{+}
f_7	4.1^{-}	4.1^{-}	-3.4^{+}	4.9^{-}	-4^{+}	$-1.4^{\sim}$	-12^{+}	-10^{+}	-22^{+}	-7^{+}	$-1^{\sim}$	-19^{+}	9.4^{-}	-27^{+}	-9.4^{+}
f_8	5.1^{-}	-5^{+}	4.6^{-}	3.6^{-}	3.1^{-}	-9.3^{+}	-47^{+}	-44^{+}	-84^{+}	-39^{+}	-8.5^{+}	-18^{+}	-14^{+}	-21^{+}	-11^{+}
f_9	$1^{\sim}$	$1^{\sim}$	$1^{\sim}$	$1^{\sim}$	$1^{\sim}$	$-1.7^{\sim}$	-13^{+}	$-1.7^{\sim}$	-15^{+}	$-1.2^{\sim}$	$-1.8^{\sim}$	-22^{+}	$-1^{\sim}$	-18^{+}	$1^{\sim}$
f_{10}	$1.2^{\sim}$	$0.25^{\sim}$	$1.2^{\sim}$	$1.2^{\sim}$	$1.2^{\sim}$	-5.7^{+}	-24^{+}	$-1.4^{\sim}$	-11^{+}	-2.3^{+}	$-1.8^{\sim}$	-24^{+}	-2^{+}	-21^{+}	-8.1^{+}
f_{11}	$1.8^{\sim}$	$0.42^{\sim}$	$1.8^{\sim}$	$1.8^{\sim}$	$1.8^{\sim}$	-6.6^{+}	-38^{+}	-2.3^{+}	-12^{+}	-9.4^{+}	-20^{+}	-33^{+}	5^{-}	-8.2^{+}	-7.5^{+}
f_{12}	$1^{\sim}$	$1^{\sim}$	$1^{\sim}$	$1^{\sim}$	$-1.4^{\sim}$	-2.4^{+}	-6.6^{+}	$-1.4^{\sim}$	-5.7^{+}	-3.4^{+}	$-1^{\sim}$	-9^{+}	$-1.4^{\sim}$	-8.7^{+}	-3.5^{+}
f_{13}	-3.5^{+}	$-1^{\sim}$	-3.2^{+}	-5.8^{+}	-2.5^{+}	-3.5^{+}	-4.2^{+}	-7^{+}	-6.8^{+}	-2.2^{+}	$-2^{\sim}$	$1.8^{\sim}$	-6.7^{+}	$-0.98^{\sim}$	-3.2^{+}
f_{14}	-3.4^{+}	-2.6^{+}	-3.6^{+}	-6.2^{+}	-2.8^{+}	-3.9^{+}	-3.3^{+}	-6.2^{+}	-4^{+}	$-2^{\sim}$	-3.2^{+}	-3.2^{+}	-7.8^{+}	$-1.2^{\sim}$	-3.2^{+}
f_{15}	$1^{\sim}$	-2^{+}	-10^{+}	-13^{+}	-14^{+}	-2.2^{+}	-2.1^{+}	-22^{+}	-14^{+}	-23^{+}	-3.7^{+}	-2.1^{+}	-29^{+}	-37^{+}	-32^{+}
f_{16}	$-1^{\sim}$	4.9^{-}	-5.2^{+}	$-1.8^{\sim}$	-3.8^{+}	$-1.7^{\sim}$	-2.8^{+}	-2.8^{+}	$-1.3^{\sim}$	-6.6^{+}	-5^{+}	-3.7^{+}	-8^{+}	-5.5^{+}	-9.5^{+}
f_{17}	-3.3^{+}	$-1.8^{\sim}$	-4.6^{+}	-7.2^{+}	-8^{+}	-13^{+}	-3.5^{+}	-15^{+}	-20^{+}	-23^{+}	-2.5^{+}	-3.1^{+}	-21^{+}	-43^{+}	-25^{+}
f_{18}	-4^{+}	$0^{\sim}$	$0^{\sim}$	$-1.3^{\sim}$	$-1.2^{\sim}$	-2.5^{+}	-8.9^{+}	-4.1^{+}	$-1.2^{\sim}$	-4.2^{+}	-2.4^{+}	-11^{+}	-2.7^{+}	-2.3^{+}	-4.3^{+}
f_{19}	$-0.38^{\sim}$	4.9^{-}	$1.9^{\sim}$	5.7^{-}	5.5^{-}	-3.1^{+}	-7.1^{+}	-2.5^{+}	2^{-}	$1.6^{\sim}$	5^{-}	8.4^{-}	8^{-}	8.9^{-}	8.9^{-}
f_{20}	$1.7^{\sim}$	-2.4^{+}	-14^{+}	-12^{+}	-14^{+}	-2.6^{+}	-2.3^{+}	-33^{+}	-18^{+}	-30^{+}	-4.4^{+}	-2.4^{+}	-45^{+}	-35^{+}	-26^{+}
f_{21}	$-1.5^{\sim}$	2.5^{-}	$0.73^{\sim}$	4.9^{-}	-2.9^{+}	-3.1^{+}	-5.2^{+}	$-1.7^{\sim}$	3.9^{-}	-6.3^{+}	3.5^{-}	8.3^{-}	7.3^{-}	8.9^{-}	-6.1^{+}
f_{22}	-2.1^{+}	$-0.14^{\sim}$	-3.1^{+}	-6.5^{+}	-9.7^{+}	-14^{+}	-2.8^{+}	-8.9^{+}	-26^{+}	-26^{+}	-2.6^{+}	2.7^{-}	-9.1^{+}	-46^{+}	-14^{+}
f_{23}	$1.2^{\sim}$	$-1.1^{\sim}$	$1.2^{\sim}$	$1.2^{\sim}$	$1.2^{\sim}$	$-1^{\sim}$	-5.2^{+}	-2.3^{+}	-7.9^{+}	$-1.2^{\sim}$	-2.1^{+}	-5.4^{+}	$-1.4^{\sim}$	-16^{+}	$1^{\sim}$
f_{24}	-4.5^{+}	2.6^{-}	-6.4^{+}	$-0.42^{\sim}$	$-1^{\sim}$	-5.3^{+}	-11^{+}	-7.8^{+}	-26^{+}	$-0.77^{\sim}$	$-0.97^{\sim}$	-8.3^{+}	-9^{+}	-16^{+}	5.6^{-}
f_{25}	-3.4^{+}	$-1.2^{\sim}$	-7.6^{+}	$1.5^{\sim}$	$-1.9^{\sim}$	-13^{+}	-16^{+}	-17^{+}	-8.9^{+}	-7.6^{+}	-8.1^{+}	-7.1^{+}	-9.5^{+}	-4.3^{+}	$-0.69^{\sim}$
f_{26}	-7.8^{+}	3.9^{-}	-3.6^{+}	3.9^{-}	$-1.7^{\sim}$	-11^{+}	-6.8^{+}	-4.5^{+}	6.2^{-}	-3^{+}	$-1.5^{\sim}$	7.3^{-}	-2.5^{+}	10^{-}	5.1^{-}
f_{27}	$1^{\sim}$	$-1.3^{\sim}$	$1^{\sim}$	$-1.4^{\sim}$	$1^{\sim}$	$-1.4^{\sim}$	-7^{+}	$-1.4^{\sim}$	$-1.7^{\sim}$	-3.7^{+}	-4.4^{+}	-9.3^{+}	-4.2^{+}	$-1^{\sim}$	-6.8^{+}
f_{28}	-4.5^{+}	$-0.25^{\sim}$	-9.6^{+}	3.9^{-}	$-1.8^{\sim}$	-16^{+}	-19^{+}	-20^{+}	-11^{+}	-11^{+}	-7.4^{+}	-16^{+}	-12^{+}	2.1^{-}	4.4^{-}
f_{29}	-12^{+}	-12^{+}	-5.6^{+}	-14^{+}	-8.6^{+}	-24^{+}	-23^{+}	-18^{+}	-21^{+}	-19^{+}	-24^{+}	-21^{+}	-16^{+}	-21^{+}	-19^{+}
f_{30}	-5.9^{+}	4.6^{-}	-7.2^{+}	$1.1^{\sim}$	$0.14^{\sim}$	-8.9^{+}	-12^{+}	-18^{+}	-23^{+}	$-0.43^{\sim}$	-5.6^{+}	-6.1^{+}	-13^{+}	-12^{+}	6.2^{-}
f_{31}	$-1.4^{\sim}$	$0^{\sim}$	-9.9^{+}	$-1.3^{\sim}$	-8.2^{+}	-2.3^{+}	-6.4^{+}	$-1^{\sim}$	$-1.6^{\sim}$	-7.5^{+}	6.5^{-}	12^{-}	-2.7^{+}	-2.3^{+}	$-1.2^{\sim}$
f_{32}	-3.5^{+}	$-1.7^{\sim}$	-3.3^{+}	-8.5^{+}	-11^{+}	-19^{+}	-2.3^{+}	-18^{+}	-25^{+}	-19^{+}	-2.6^{+}	-3.6^{+}	-25^{+}	-45^{+}	-26^{+}
f_{33}	-5.5^{+}	4.4^{-}	-7.1^{+}	$1.2^{\sim}$	$0.5^{\sim}$	-5.9^{+}	-15^{+}	-14^{+}	-27^{+}	$-1.8^{\sim}$	-5.6^{+}	-6.1^{+}	-13^{+}	-12^{+}	6.2^{-}
f_{34}	-4.2^{+}	$-0.14^{\sim}$	-7.7^{+}	$-2^{\sim}$	$-1.5^{\sim}$	-7.7^{+}	-12^{+}	-6.9^{+}	-23^{+}	-9.2^{+}	$-1.3^{\sim}$	2.4^{-}	5.4^{-}	-16^{+}	-5.9^{+}
f_{35}	-3.1^{+}	-6.2^{+}	$-0.31^{\sim}$	-7.7^{+}	-4^{+}	-30^{+}	-26^{+}	-19^{+}	-24^{+}	-22^{+}	-27^{+}	-22^{+}	-17^{+}	-22^{+}	-20^{+}
f_{36}	2.5^{-}	2.5^{-}	-3.3^{+}	-2.3^{+}	$-1.4^{\sim}$	-4.7^{+}	-3.5^{+}	$-1.2^{\sim}$	$-1.3^{\sim}$	-3.6^{+}	$-1^{\sim}$	-4.7^{+}	-3.5^{+}	-12^{+}	-7.1^{+}
f_{37}	$1.2^{\sim}$	-2.8^{+}	$1.2^{\sim}$	$-1^{\sim}$	$1.2^{\sim}$	-5.6^{+}	-31^{+}	-3.5^{+}	-11^{+}	$-1^{\sim}$	-2.3^{+}	-48^{+}	-8.9^{+}	-18^{+}	-2.2^{+}
f_{38}	4.6^{-}	4.5^{-}	$-1^{\sim}$	$1^{\sim}$	4.6^{-}	6.4^{-}	-14^{+}	-3.2^{+}	7.2^{-}	6.7^{-}	2.1^{-}	11^{-}	16^{-}	20^{-}	23^{-}
f_{39}	$-1.7^{\sim}$	$1^{\sim}$	-12^{+}	$-1^{\sim}$	-7.5^{+}	$-2^{\sim}$	-9.4^{+}	$-1^{\sim}$	$-1^{\sim}$	$-1^{\sim}$	$-1^{\sim}$	-2.4^{+}	$-1.2^{\sim}$	$-1.8^{\sim}$	$-0.28^{\sim}$
f_{40}	-2.8^{+}	$1.4^{\sim}$	-3^{+}	-12^{+}	-15^{+}	-6.6^{+}	-5^{+}	-7.2^{+}	-11^{+}	-6.3^{+}	-2.7^{+}	$-0.95^{\sim}$	-4.9^{+}	$-0.54^{\sim}$	-2.9^{+}
f_{41}	-4^{+}	$-1.7^{\sim}$	-13^{+}	-8.5^{+}	-6.5^{+}	$-1.6^{\sim}$	$-1.4^{\sim}$	-13^{+}	-2.3^{+}	$-1.7^{\sim}$	$-1.4^{\sim}$	2.1^{-}	-11^{+}	$1.8^{\sim}$	2.4^{-}
f_{42}	$-0.63^{\sim}$	$-0.19^{\sim}$	-9.5^{+}	$-1.7^{\sim}$	-2.4^{+}	$-1.5^{\sim}$	-8.9^{+}	-13^{+}	$0.91^{\sim}$	2.7^{-}	-8.7^{+}	$-1.4^{\sim}$	$-1.8^{\sim}$	13^{-}	19^{-}
f_{43}	-4^{+}	$-0.47^{\sim}$	-6.9^{+}	$-2^{\sim}$	-2.9^{+}	-12^{+}	-13^{+}	-61^{+}	$-1^{\sim}$	$-1.8^{\sim}$	-4.7^{+}	-5^{+}	-7.2^{+}	14^{-}	11^{-}
f_{44}	-3.9^{+}	$0.62^{\sim}$	-5.7^{+}	$-2^{\sim}$	$-1.4^{\sim}$	$-1.9^{\sim}$	-6.6^{+}	-19^{+}	-7.4^{+}	-6.7^{+}	-5.4^{+}	-5^{+}	-6.5^{+}	13^{-}	6.3^{-}
f_{45}	-3^{+}	$1.7^{\sim}$	-8^{+}	-4.5^{+}	-2.7^{+}	$-1.7^{\sim}$	-2.5^{+}	-31^{+}	-8.8^{+}	-14^{+}	-6.7^{+}	-5.5^{+}	-7.3^{+}	8.7^{-}	5.7^{-}
+	24	9	29	16	24	22	31	30	30	27	28	31	33	29	27
−	4	11	1	6	3	8	5	5	8	9	4	9	6	10	12
∼	17	25	15	23	18	15	9	10	7	9	13	5	6	6	6

other algorithms on at least 22 out of the 45 problems in 30 dimensions and 28 out of the 45 problems in 50 dimensions. From Figure 7.16, it is obvious to see that the number of problems where SLPSO achieves significantly better results than the other algorithms is much larger than the number of problems where SLPSO performs significantly worse than the other algorithms. Compared among the results in the three dimensional cases, we can also see that the performance of SLPSO increases when the number of dimensions increases.

Figure 7.17 shows an overview of how much the performance of SLPSO is better than that of the other five algorithms on the 45 test problems in terms of

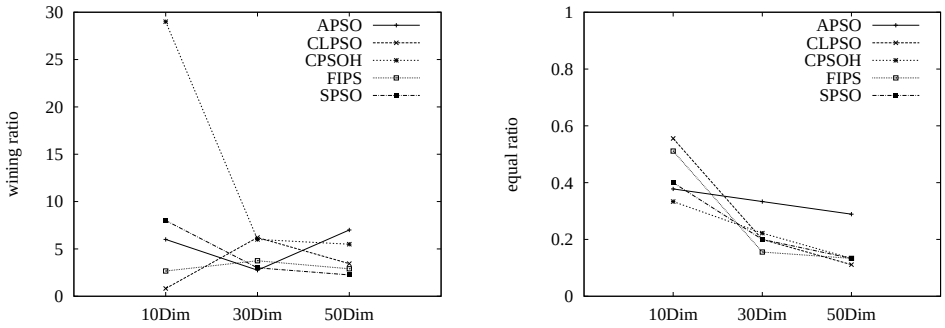


Figure 7.17: The winning ratio and equal ratio of SLPSO compared with the other five algorithms.

the winning ratio and equal ratio defined in Section 3.3. The average winning ratios against algorithms APSO, CLPSO, CPSOH, FIPS, and SPSO are 5.25, 3.49, 13.5, 3.11, and 4.42, respectively, over the three dimensional cases. The smallest average winning ratio is 3.11, which is against FIPS. It means that on about three problems out of every four problems, SLPSO achieves significantly better results than FIPS. The average equal ratio against algorithms APSO, CLPSO, CPSOH, FIPS, and SPSO is 0.33, 0.29, 0.22, 0.26, and 0.33, respectively. The largest average equal ratio is 0.33, which means that there are at most 33 percentages of the total 45 test problems where SLPSO achieves similar performance as its peer algorithms. Together, the results in Figure 7.17 show the overall performance of SLPSO is significantly better than any of other peer algorithms on the 45 test problems.

7.4 Summary

From the experimental results of the working mechanism test on SLPSO, we summarize the results as following: 1) the adaptive learning scheme enables each particle to "intelligently" choose the optimal learning strategy according to its own local environments; 2) each particle is able to monitor its own evolutionary status by using the monitoring mechanism; 3) compared with the non-adaptive learning

method, particles do need intelligence to choose the best learning strategy; 4) the inter relationships among the three key parameters of SLPSO make it difficult to find out the optimal configurations for general problems; 5) the parameters tuning method works well for general problems although there is a big performance difference between SLPSO with the default configuration and SLPSO with the optimal configuration; 6) the learning method for the *abest* position is helpful for the *abest* position to extract useful information from improved particles; 7) the performance of the four learning operators depends on the problem to be solved.

Based on the above comparison of SLPSO with the other five algorithms over the 45 test problems over the three dimensional cases in this chapter, we summarize the results as following: 1) SLPSO achieves the best mean values among the six algorithms on the largest number of problems; 2) regarding the convergence speed, the self-restart mechanism does help SLPSO escape from local optima and it is the characteristics which only SLPSO has among the six algorithms; 3) although the performance of SLPSO is not the best in terms of the success rate on problems in 10 dimensions, it achieves the best performance on problems in 30 and 50 dimensions; 4) like the other algorithms, the performance of SLPSO is affected by the modifications on the tested problems. But, it is the only algorithm that is not sensitive to the shifting modification; 5) increasing the number of dimensions brings the smallest challenge to SLPSO in the six algorithms; 6) SLPSO outperforms all the other algorithms in terms of the winning ratio according to the *t*-test results.

Although SLPSO achieves the best performance among the six algorithms on the 45 test problems, compared with SLPSO with the optimal configurations, there is much room for SLPSO to be improved regarding how to tune the three key parameters, i.e., the update frequency (U_f), the learning probability (P_l), and the number of particles that learn to the *abest* position (M).

Chapter 8

Experimental Study of CPSO

In order to test the performance of the CPSO algorithm introduced for DOPs in Chapter 4, we first conduct the experimental study based on the MPB problem [13] and compare the performance of CPSO with several state-of-the-art PSO algorithms that were developed for DOPs in the literature, including two PSO algorithms proposed by Blackwell and Branke in [10], the collaborative model introduced by Lung and Dumitrescu [74], the speciation PSO proposed by Parrott and Li in [84] as well as an improved version of SPSO with regression (rSPSO) [5]. Beside the experimental study on the MPB problem, we also carry out experiments on the GDBG benchmark proposed recently in [64, 68].

Based on the experimental results, an algorithm performance analysis regarding the weakness and the strength of investigated algorithms is carried out. This chapter also carries out experiments on the sensitivity analysis with respect to several key parameters, such as the population size of the initial swarm and the maximum size of each sub-swarm on the performance of CPSO for DOPs.

For the MPB problem [13], three groups of experiments were carried out in this chapter. The objective of the first group of experiments is to investigate the work mechanism of CPSO, analyze the sensitivity of key parameters, and study the effect of the training process used in the original CPSO [69]. In the second group

of experiments, the performance of CPSO is compared with a number of PSO algorithms taken from the literature. The involved algorithms include mCPSO [10], mQSO [10], SPSO [84], rSPSO [5], and CESO [74]. All the results of the peer algorithms shown in this chapter are provided in the papers where they were proposed. Finally, we give the comparison results between CPSO and PSO with the *lbest* model in the third group of experiments.

As for the GDBG benchmark [68], we carried out experiments following the instruction in the competition of " Evolutionary Computation in Dynamic and Uncertain Environments" [68] held with CEC'2009 and compared with a simple GA algorithm and PSO with the *gbest* model as well as the winner of the competition.

8.1 Experimental Setup

For the convenience of description, the configuration of CPSO is represented by $C(M, N)$, where M is the initial swarm size of the cradle swarm and N is the value of *max_subsize*. In CPSO, the acceleration constants η_1 and η_2 were both set to 1.7. The inertia weight ω is linearly decreased from $\omega_{max} = 0.6$ to $\omega_{min} = 0.3$ using Eq. (6.3) for sub-swarms to perform local search.

The MPB problem proposed by Branke [13] and the GDBG benchmark were used to test the performance of CPSO. The default settings and definitions of the two benchmarks used in the experiments can be found in Table 4.1 and Table 4.3 respectively, which are the same for all the involved algorithms.

The performance measure on the MPB benchmark is the offline error, which is defined in Eq. (4.11). For each run, there were 100 environmental changes, i.e., $K = 100$. All the results reported are based on the average over 50 independent runs with different random seeds.

The performance metric on the GDBG benchmark is defined in [68], which is

described in Section 4.4.3 in Chapter 4. All the results reported are based on the average over 20 independent runs with different random seeds, where each run contains 60 environmental changes.

8.2 Experimental Study on the MPB Problem

In this section, we first perform experiments to analyze the performance of CPSO in the following aspects: the working mechanism of CPSO, the parameter sensitivity analysis, and the training process. Then, we compare CPSO with the other algorithms with different problem configurations, including the shift severity, the number of peaks, and the environmental change frequency. Finally, we carry out experiments to compare the performance between CPSO and a basic PSO algorithm with the *lbest* model.

8.2.1 Testing the Working Mechanism of CPSO

In order to understand the work mechanism of CPSO, in this section experiments were carried out to investigate the dynamic behaviour of internal features of CPSO, including the number of sub-swarms, the number of total particles, and the number of converged sub-swarms, during the solving process. In the experiments, the configuration of $C(70, 3)$ was applied for CPSO. Figure 8.1 shows the average dynamic behavior of these features and the offline error against the solving process for five environmental changes based on the default settings of the MPB problem over 50 runs.

Figures 8.1(a) and 8.1(b) clearly present the changes of the total number of particles and the total number of sub-swarms during the evolution process. When CPSO is configured to $C(70, 3)$, we can estimate that the total number of initial sub-swarms produced by the clustering method should be about 25, which can

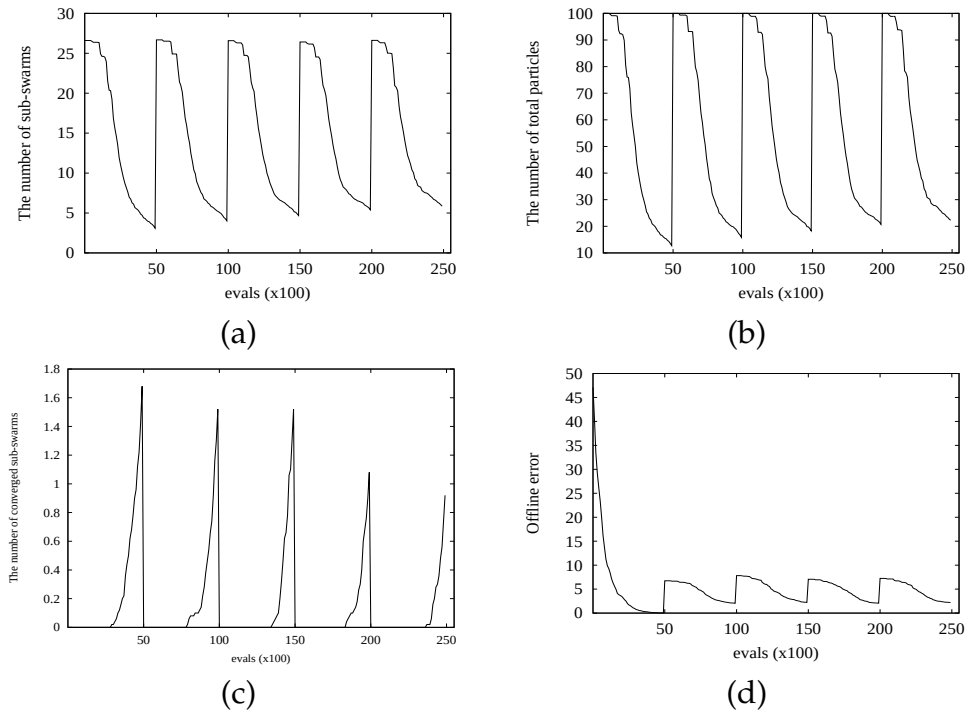


Figure 8.1: The dynamic behaviour of CPSO regarding (a) the number of sub-swarms, (b) the total number of particles, (c) the number of converged sub-swarms, and (d) the offline error for five environmental changes.

be observed from Figure 8.1(a) after an environmental change has just occurred. Since there are ten peaks in the whole fitness landscape, we need to remove some redundant sub-swarms to achieve the best performance. This is automatically conducted by the overlapping check mechanism. When approaching the next environment, some sub-swarms converge to some different peaks and they are recorded by *clst*, which can be seen in Figure 8.1(c).

In order to visualize the search trajectories of all particles in different evolutionary stages, we give the *pbest* locations of particles at different *evals* within a single environmental change of a typical run of CPSO on a 2-dimensional fitness landscape in Figure 8.2. In Figure 8.2, the cross and black square points are the *pbest* positions and the locations of ten peaks, respectively. From left to right and from top to bottom, the six images in Figure 8.2 show the movements of *pbest*

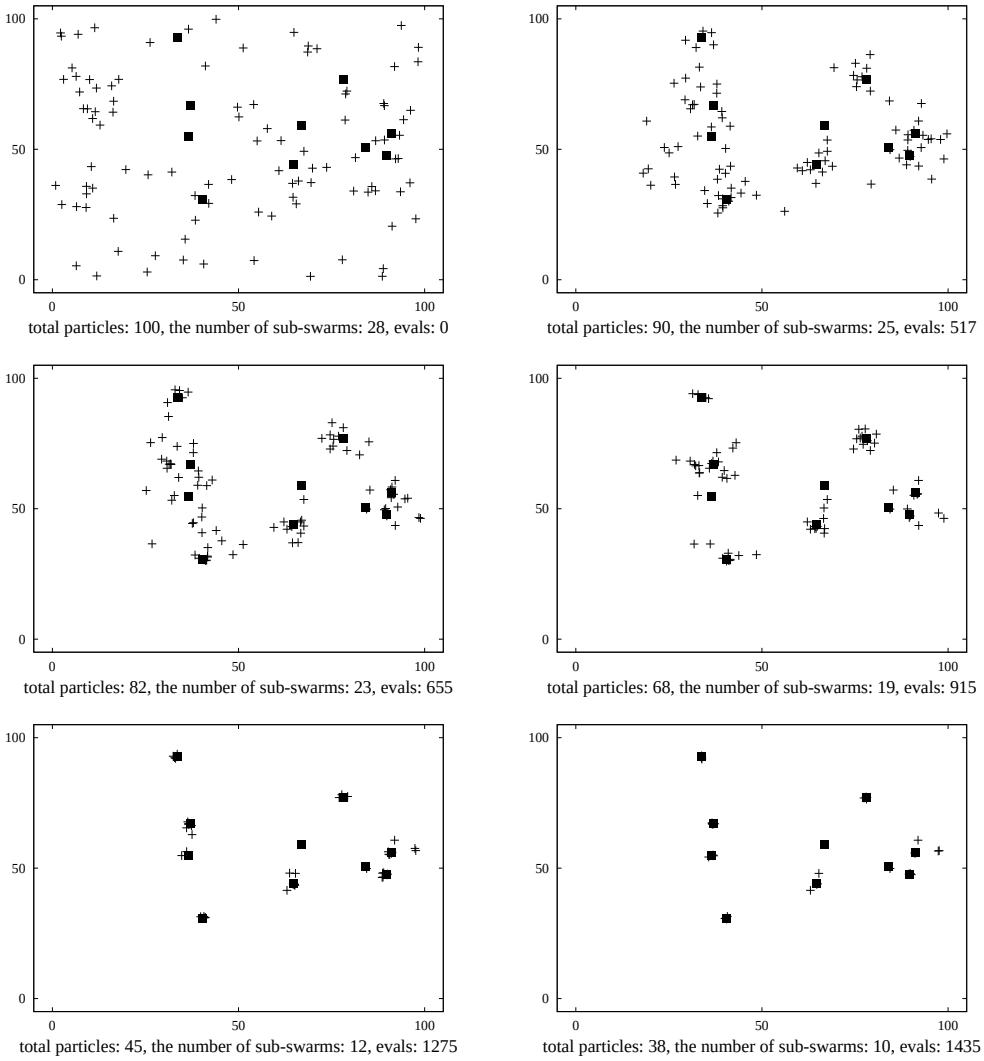


Figure 8.2: The *pbest* locations at different *evals* within a single environmental change of a typical run of CPSO on a 2-dimensional fitness landscape.

positions during the evolution progress. Overlapping and overcrowding happen when more than one sub-swarms move toward a same peak. At this moment, overlapping and overcrowding check will take effect to remove the redundant particles. The mechanism can be seen by the changing distribution with the deceasing number of particles and sub-swarms. When *evals* reaches 1275 in Figure 8.2, the number of total particles reduces to 45 and the number of sub-swarms decreases from 28 to 12. Finally, just before the environmental change occurs, the

Table 8.1: Offline error of different parameter configurations

C(M,N)	M=10	M=30	M=50	M=70	M=100	M=120	M=150	M=200
N=2	**3.79**	**1.77**	**1.39**	1.48	2.31	3.12	3.95	5.05
N=3	4.47	1.98	1.41	**1.06**	1.3	1.85	3.1	4.28
N=4	4.82	2.47	1.77	1.47	**1.11**	**1.2**	1.67	3.36
N=5	6.05	2.76	1.9	1.61	1.27	1.33	**1.47**	2.67
N=6	6.61	3.64	2.21	1.85	1.26	1.21	1.58	2.59
N=7	5.91	3.41	2.57	2.05	1.74	1.44	1.49	**1.94**
N=10	8.27	4.63	3.44	2.7	2.04	1.83	1.88	2.03
N=12	7.82	4.82	3.45	2.88	2.22	2.15	1.9	2.13
N=15	8.65	5.73	3.87	3.29	2.94	2.44	2.32	2.28

positions in *clst* and the best particles in sub-swarms will be recorded for tracking the movement of peaks in the next environment.

From the first image of the initial stage to the last image of the final stage in Figure 8.2, it can also be seen that particles gradually move toward sub-regions where the ten peaks are located. Finally, they converge to these peaks. This observation validates our previous analysis that the training process in the original CPSO is not necessary.

8.2.2 Effect of Varying the Configurations

The aim of this set of experiments is to examine the effect of different configurations on the performance of CPSO. The default parameter settings of the MPB problem were used in the experiments. CPSO was run 50 times with each of the combined values of *max_subsize*(N) in {2, 3, 4, 5, 7, 10, 15} and the initial size of the cradle swarm (M) in {10, 30, 50, 70, 100, 120, 150, 200}. The offline error is shown in Table 8.1, where the best result over all values of *max_subsize* for each fixed number of initial population size of the cradle swarm is shown in bold font. Table 8.2 shows the number of sub-swarms created from the cradle swarm using the clustering method. Table 8.3 gives the results of the number of survived

Table 8.2: The number of sub-swarms created by the clustering method

C(M,N)	M=10	M=30	M=50	M=70	M=100	M=120	M=150	M=200
N=2	5	15	25	35	50	60	75	100
N=3	4	10.4	17.4	24.4	34.7	41.5	51.8	69
N=4	3.07	8.27	13.5	18.7	26.6	31.8	39.7	52.7
N=5	2.39	6.73	11	15.3	21.7	25.9	32.3	42.8
N=6	2.29	5.75	9.52	13	18.3	21.9	27.2	36.1
N=7	2.14	5.29	8.39	11.3	16	19	23.6	31.2
N=10	1.48	3.72	5.95	8.15	11.4	13.6	16.8	22.2
N=12	1.44	3.5	5.33	6.92	9.76	11.4	14.2	18.6
N=15	1.46	2.61	4.49	5.76	7.91	9.29	11.5	15.1

sub-swarms before the environment changes. The survived sub-swarms include those converged sub-swarms and non-converged sub-swarms at the last generation before a change occurs. The number of peaks found by CPSO is presented in Table 8.4. If a peak is within the radius of a survived sub-swarm, then the peak is considered to be found by CPSO. Strictly speaking, we cannot assume that a peak has been found just because it is within a sub-swarm's radius. We use this simple approximate measure to consider whether a peak is found or not since it works for the purpose of our experiments here, i.e., to compare the relative performance of different configurations of the initial swarm size M and *max_subsize* (N) in terms of locating peaks. This performance measure is derived from the measure used in [84].

From Table 8.1, it can be seen that different configurations of CPSO significantly affect the performance of CPSO. When the maximum sub-swarm size, i.e., N, is fixed to a specific value, setting the initial size of the cradle swarm, i.e., M, to a too large or too small value will affect the performance of CPSO, vice versa. The optimal configuration of CPSO for the default settings of the MPB problem with ten peaks is $C(70, 3)$, which enables CPSO to achieve the smallest offline error of 1.06.

Table 8.3: The number of survived sub-swarms

C(M,N)	M=10	M=30	M=50	M=70	M=100	M=120	M=150	M=200
N=2	3.68	7.48	10.7	14.4	21.8	28.3	39.8	63
N=3	3.45	5.44	6.97	8.45	10.8	13.2	21.9	41.5
N=4	2.56	4.68	5.71	6.63	7.72	8.5	10	19.5
N=5	2.08	4	5.12	5.86	7	7.41	8.47	12.4
N=6	1.98	3.59	4.82	5.48	6.62	7.01	7.49	9.98
N=7	1.88	3.39	4.53	5.08	5.89	6.61	7.06	8.4
N=10	1.33	2.68	3.55	4.21	4.99	5.53	6	6.57
N=12	1.31	2.55	3.31	3.78	4.64	5.01	5.57	6.25
N=15	1.34	2.07	2.97	3.38	4.07	4.54	4.85	5.76

Table 8.4: The number of peaks found by CPSO

C(M,N)	M=10	M=30	M=50	M=70	M=100	M=120	M=150	M=200
N=2	3.67	5.92	6.82	7.27	7.52	7.7	7.9	8
N=3	3.13	5.18	6.39	7.15	7.71	7.88	8.33	8.72
N=4	2.79	4.53	5.69	6.36	6.99	7.33	7.76	8.27
N=5	2.42	4.11	5.03	5.85	6.5	6.98	7.36	8.02
N=6	2.33	3.9	4.89	5.46	6.17	6.77	7.05	7.73
N=7	2.24	3.58	4.55	5.03	5.74	6.06	6.49	7.22
N=10	1.65	3.01	3.88	4.39	4.87	5.29	5.71	6.26
N=12	1.66	2.93	3.66	4.03	4.79	4.98	5.56	6.14
N=15	1.76	2.48	3.33	3.65	4.23	4.59	5.03	5.48

As discussed before, the value of *max_subsize* directly determines the number of sub-swarms that are generated by the clustering method. For example, if we take the extreme value of *max_subsize* which is equal to the size of the cradle swarm, only one sub-swarm may be obtained. It can be seen from Table 8.2, where the larger the value of *max_subsize* under a fixed size of the initial cradle swarm, the smaller number of sub-swarms that are created. Too large or too small *max_subsize* will cause too few or too many sub-swarms created, which may be far away from the real number of peaks in the search space. This is why the performance of CPSO greatly depends on the value of *max_subsize*. On the other hand, from Table 8.2 it can be observed that when the value of *max_subsize* is fixed,

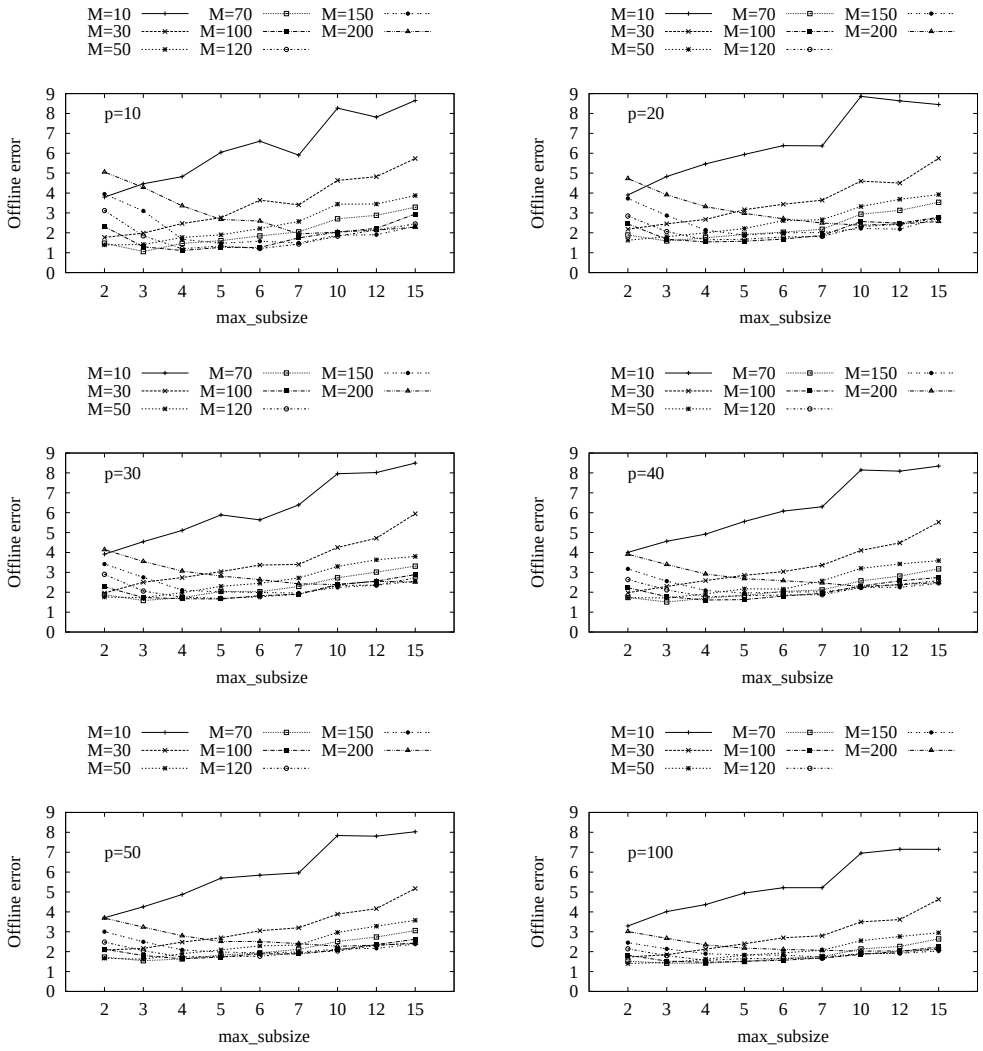


Figure 8.3: The offline error of CPSO with different configurations on the MPB problems with different number of peaks.

the larger the initial population size of the cradle swarm, the larger the number of sub-swarms that are created.

By observing Table 8.1 and Table 8.3, it can be seen that when the number of survived sub-swarms remains at the level of 7 to 10, which is slightly smaller than the total number of peaks, i.e., 10, CPSO achieves relatively smaller offline errors that are below 1.5. Although the number of peaks found by CPSO is less than the total number of peaks in the search space, it is reasonable. There may be two

possible reasons: first, some peaks of low heights may be covered by the peaks with higher heights and are invisible in the fitness landscape. Second, actually, it is very hard for an algorithm to track all peaks even though they are all visible, especially when there are many peaks in the fitness landscape since the swarm size is limited to track all these peaks.

Comparing the results in Table 8.1 and Table 8.4, it can be seen that the larger the initial size for the cradle swarm, the more peaks that are found by CPSO. Intuitively, the performance of CPSO should get better with a larger initial size for the cradle swarm since the clustering technique in CPSO can obtain more clusters (and peaks) in the fitness landscape. But, this result is not seen in Table 8.1 as expected. This occurs because the change frequency was set to 5000, which may not be big enough to achieve the best performance of CPSO. If the change frequency is increased, we will get better results as expected. The experimental results will be shown later in the following experiments.

Figure 8.3 presents the performance of CPSO with different configurations on the MPB problems with the number of peaks set in {10, 20, 30, 40, 50, 100}. From Figure 8.3, similar observations can be obtained on the four problems with different number of peaks. Just as shown in Table 8.1, to achieve the best performance, CPSO needs an optimal configuration. For example, on the ten peaks problem, when the initial population size M is set to a specific number (e.g., $M = 150$), the offline error first decreases as the value of *max_subsize* (N) increases from 2 to a turning point 7. Then, after the turning point, the offline error increases. In addition, it can be observed from Figure 8.3 that the turning point is different for different configurations. For example, for the ten peaks problem, the turning point is $N = 7$ for $M = 150$, $N = 4$ for $M = 100$, etc. It is understandable. In order to achieve the best performance, CPSO needs to adjust the value of *max_subsize* to adapt to the environments when a specific initial population size M is given.

Table 8.5: Results of CPSO with different number of iterations for training

Training iterations	0	1	3	5	7	9
Offline error	1.06	1.56	1.65	1.77	1.697	1.81
subswarms produced	24.4	26.01	26.33	26.55	26.64	26.62
Survived subswarms	8.45	12.60	14.56	15.38	16.49	17.24
Real peaks found	7.5	7.3	7.18	6.96	6.98	6.79

From Figure 8.3, interestingly, the optimal configuration of CPSO is $C(70,3)$ on the MPB problems with different number of peaks.

8.2.3 Effect of the Training Process

In this set of experiments, we test the effect of the training process used in the original CPSO on the performance of CPSO on the MPB problems. Here, the same training method in [69] was applied in CPSO, where the neighborhood of a particle is defined as the nearest particle to that particle. This unique particle in the neighborhood of a particle is used for the particle's velocity update in Eq. (2.1).

As analyzed above, the training process in [69] does not help the search for CPSO. In order to give an explanation from the experimental view, experiments were conducted based on the configuration of $C(70,3)$ for CPSO with different number of iterations for the training process. The comparison results are shown in Table 8.5.

From Table 8.5, it can be seen that the results of CPSO with training are much worse than the results obtained by CPSO without training, where the smallest offline error achieved is 1.06. The training process may cause too many isolated pairs of close particles moving together since the neighborhood of a particle is composed of only the nearest particle. It can be seen from Table 8.5 that too many sub-swarms are generated by the clustering method due to the training consequence. Just as pointed out above, the training process is not necessary since

Table 8.6: Offline error of algorithms on the MPB problems with different shift severities

s	$C(70,3)$	mCPSO	mQSO	CESO	rSPSO	SPSO
0.0	0.80	1.18	1.18	0.85	0.74	0.95
	±0.21	±0.07	±0.07	±0.02	±0.08	±0.08
1.0	1.056	2.05	1.75	1.38	1.50	2.51
	±0.24	±0.07	±0.06	±0.02	±0.08	±0.09
2.0	1.17	2.80	2.40	1.78	1.87	3.78
	±0.22	±0.07	±0.06	±0.02	±0.05	±0.09
3.0	1.36	3.57	3.00	2.03	2.4	4.96
	±0.28	±0.08	±0.06	±0.03	±0.08	±0.12
4.0	1.38	4.18	3.59	2.23	2.90	2.56
	±0.29	±0.09	±0.10	±0.05	±0.08	±0.13
5.0	1.58	4.89	4.24	2.52	3.25	6.76
	±0.32	±0.11	±0.10	±0.06	±0.09	±0.15
6.0	1.53	5.53	4.79	2.74	3.86	7.68
	±0.29	±0.13	±0.10	±0.10	±0.11	±0.16

sub-swarms produced from the cradle swarm without training can also achieve the same objective of training the cradle swarm in [69]. In addition, we can take the advantage of assigning the computing resources for training the cradle swarm to sub-swarms to perform local search. Therefore, the training process in [69] has been removed in the updated version of CPSO.

8.2.4 Effect of Varying the Shift Severity

In this group of experiments, we compare the performance of CPSO with mCPSO, mQSO, SPSO, rSPSO and CESO on the MPB problems with different problem settings regarding the shift severity. The experimental results regarding the offline error and standard deviation are shown in Table 8.6. The experimental results of the peer algorithms are taken from the corresponding papers with the configuration that enables them to achieve their best performance. We choose the optimal configuration of $C(70,3)$ for CPSO. Other parameter settings are the same as above.

From Table 8.6, it can be seen that the results achieved by CPSO with the optimal configuration are much better than the results of the other five algorithms on the MPB problems with different shift severities. As we know, the peaks are more and more difficult to track with the increasing of the shift length. Naturally, the performance of all algorithms degrades when the shift length increases. However, the offline error of CPSO is only slightly affected in comparison with the other five algorithms. This result shows that CPSO is very robust to locate and track multiple optima even in severely changing environments.

8.2.5 Effect of Varying the Number of Peaks

This set of experiments investigate how CPSO scales with the number of peaks on the MPB problem. The performance of CPSO is also compared with that of the peer algorithms mCPSO, mQSO,SPSO, rSPSO, and CESO. The number of peaks was set to different values in the range form 1 to 200. The configuration of $C(70, 3)$ was chosen for CPSO on the MPB problems with different number of peaks, except $C(70, 3)$ on the MPB problem with ten peaks in the experiments. Table 8.7 presents the experimental results in terms of the offline error and standard deviation of eight algorithms, where the results of the other seven algorithms are provided by the corresponding papers with their optimal configuration that enables them to achieve their best performance. In Table 8.7, mCPSO* and mQSO* denote mCPSO without anti-convergence and mQSO without anti-convergence, respectively.

From Table 8.7, it can be seen that the performance of CPSO is not influenced too much when the number of peaks is increased. Generally speaking, increasing the number of peaks makes it harder for algorithms to track the optima. However, interestingly, the offline error decreases when the number of peaks is larger than 20 for CPSO. Similar trend can also be observed from the results of the other seven algorithms. It seems contrary to our prediction. The reason behind this is that

Table 8.7: The offline error of algorithms on the MPB problems with different number of peaks

peaks	CPSO	mCPSO	mQSO	mCPSO*	mQSO*	CESO	rSPSO	SPSO
1	0.14	4.93	5.07	4.93	5.07	1.04	1.42	2.64
	±0.11	±0.17	±0.17	±0.17	±0.17	±0.00	±0.06	±0.10
2	0.20	3.36	3.47	3.36	3.47	-	1.10	2.31
	±0.19	±0.26	±0.23	±0.26	±0.23	-	±0.03	±0.11
5	0.72	2.07	1.81	2.07	1.81	-	1.04	2.15
	±0.30	±0.08	±0.07	±0.11	±0.07	-	±0.03	±0.07
7	0.93	2.11	1.77	2.11	1.77	-	1.21	1.98
	±0.30	±0.11	±0.07	±0.11	±0.07	-	±0.05	±0.04
10	1.056	2.08	1.80	2.05	1.75	1.38	1.50	2.51
	±0.24	±0.07	±0.06	±0.07	±0.06	±0.02	±0.08	±0.09
20	1.59	2.64	2.42	2.95	2.74	1.72	2.20	3.21
	±0.22	±0.07	±0.07	±0.08	±0.07	±0.02	±0.07	±0.07
30	1.58	2.63	2.48	3.38	3.27	1.24	2.62	3.64
	±0.17	±0.08	±0.07	±0.11	±0.11	±0.01	±0.07	±0.07
40	1.51	2.67	2.55	3.69	3.60	1.30	2.76	3.85
	±0.12	±0.07	±0.07	±0.11	±0.08	±0.02	±0.08	±0.08
50	1.54	2.65	2.50	3.68	3.65	1.45	2.72	3.86
	±0.12	±0.06	±0.06	±0.11	±0.11	±0.01	±0.08	±0.08
100	1.41	2.49	2.36	4.07	3.93	1.28	2.93	4.01
	±0.08	±0.04	±0.04	±0.09	±0.08	±0.02	±0.06	±0.07
200	1.24	2.44	2.26	3.97	3.86	-	2.79	3.82
	±0.06	±0.04	±0.03	±0.08	±0.07	-	±0.05	±0.05

when the number of peaks increases, there will be more local optima that have a similar height as the global optima and hence, there will be a higher probability for algorithms to find relatively better local optima.

Comparing the results of CPSO with the other seven algorithms, the offline error achieved by CPSO is much less than that achieved by all the other algorithms when the number of peaks is less than 20. Although the results of CPSO are slightly worse than the results of CESO when the number of peaks exceeds 30, they are much better than the results of the other six algorithms. In addition, if we increase the value of the change frequency, CPSO can achieve much better results, which can be seen in the following section.

Table 8.8: The offline error of CPSO on the MPB problems with different number of peaks and the change frequency of 10000

peaks	M=10	M=30	M=50	M=70	M=100	M=120	M=150	M=200
1	0.008	0.010	0.017	0.022	0.069	0.11	0.494	0.494
2	0.354	0.175	0.0931	0.0929	0.107	0.152	0.447	0.447
5	0.942	0.969	0.375	0.22	0.375	0.321	0.522	0.522
7	1.51	1.02	0.708	0.563	0.401	0.468	0.732	0.732
10	2.39	1.16	0.907	0.625	0.638	0.594	0.873	0.873
20	2.19	1.53	1.22	1.06	0.922	0.809	1.04	1.04
30	2.24	1.57	1.36	1.02	1	0.96	1.12	1.12
40	2.21	1.54	1.31	1.05	0.915	0.964	1.16	1.16
50	2.11	1.47	1.31	1.05	0.982	0.961	1.18	1.18
100	1.79	1.3	1.13	0.945	0.925	0.932	1.14	1.14
200	1.53	1.09	0.941	0.802	0.773	0.843	1.03	1.03

From Table 8.7, it can also be seen that CESO outperforms all algorithms including CPSO when the number of peaks is larger than 30. As we know, a large number of peaks needs more sub-swarms to locate and track. It means that an algorithm with a good diversity maintaining mechanism may perform well to find more relatively better local optima. CESO just benefits from such sort of algorithms: the CDE algorithm [111] is used as a component algorithm in CESO to maintain the population diversity. However, to locate and track more local optima for CPSO, we need a larger initial swarm and big enough change frequency to globally locate optima in the whole fitness landscape. It can be seen from Table 8.8 (to be described below) that CPSO achieves much better results when the initial swarm size is increased to 120 for many peaks problems (i.e., problems with more than 10 peaks).

8.2.6 Effect of Varying the Environmental Change Frequency

This set of experiments investigate the effect of different environmental changing speeds on the performance of CPSO. The *max_subsize* was set to 3 in this set of

experiments. Table 8.8 presents the results of CPSO with different initial size of the cradle swarm on the MPB problems with different number of peaks when the value of the change frequency is increased from the default setting of 5000 to 10000.

From Table 8.8, it can be seen that the performance of CPSO gets much better when we increase the value of the change frequency. The smallest offline error of CPSO on the MPB problems with different number of peaks is less than 1.0 in Table 8.8. For example, the offline error of CPSO with $M = 100$ on the MPB problem with ten peaks is now 0.638, which is much lower than the value of 1.3 for CPSO with the same configuration but on the MPB problem with the change frequency set to 5000, as seen from Table 8.1. This result is reasonable since increasing the value of the change frequency gives CPSO more time to search before the next environmental change occurs.

8.2.7 Comparison of CPSO and PSO$_{lbest}$

In this set of experiments, we test the aforementioned advantages of CPSO over PSO with the *lbest* model, i.e., PSO$_{lbest}$. Both CPSO and PSO$_{lbest}$ were run under a fair algorithm setting. The same *gbest* learning strategy used in CPSO was used in PSO$_{lbest}$. For a particle in PSO$_{lbest}$, we define the *max_subsize* nearest particles as its neighborhood. The best particle of its neighborhood is assigned to *gbest*, which is used in Eq. (2.1). The same strategy of linearly decreasing ω was used for PSO$_{lbest}$. For both algorithms, the initial swarm size was set to 100 and the default problem settings were used. Table 8.9 presents the experimental results of the two algorithms with different settings of *max_subsize*.

From Table 8.9, it can be seen that all the results of CPSO are much better than the results of PSO$_{lbest}$. The comparison results obviously show the advantages of CPSO over PSO$_{lbest}$. Together with the above experimental results, CPSO shows

Table 8.9: The offline error of CPSO and PSO_{lbest}

	N=2	N=3	N=4	N=5	N=6	N=7	N=10	N=12	N=15
CPSO	2.43	1.3	1.06	1.44	1.46	1.51	2.21	2.45	2.55
PSO_{lbest}	9.85	9.40	8.51	8.74	8.25	8.27	8.14	9.22	8.70

its outstanding capability of tracking multiple optima, overcrowding control, and adaptively adjusting the number of particles needed when solving problems in dynamic environments.

8.3 Experimental Study on the GDBG Benchmark

In order to investigate how CPSO performs on complex problems, we test CPSO on the benchmark cases in the real space in the GDBG system [64], which has been introduced in Chapter 4. In this section, two groups of experiments were conducted. We first provide experimental results of varying the configurations of CPSO on the six test functions with the small step change. The second group of experiments were carried out based on the instruction of the competition of " Evolutionary Computation in Dynamic and Uncertain Environments" [68] held with CEC'2009 and CPSO is compared with PSO with the *gbest* model (PSO_{gbest}) and a simple genetic algorithm (SGA) as well as the winner jDE [15] of the competition, which is a self-adaptive differential evolution algorithm. The setting for the GDBG benchmark used in this section can be seen in Section 4.3 in Chapter 4.

In the first group of experiments, the configuration (C(M,N)) of CPSO was tested in the combination of M in the set of {10,30,50,70,100,150,200} and N in the set of {2,5,7,10,12,15}. For the other parameters, the default settings were used as on the MPB problem, where, the acceleration constants η_1 and η_2 were both set to 1.7. The inertia weight ω is linearly decreased from $\omega_{max} = 0.6$ to $\omega_{min} = 0.3$ using

Eq. (6.3) for sub-swarms to perform local search. The value of $R_{operlap}$ was set to 0.7.

In the second group of experiments, both PSO_{gbest} and SGA use the restart with elitism scheme. That is, when an environment change is detected, the population is re-initialized and the best individual in the previous generation is replaced into the restarted population. Ten sub-swarms/sub-populations were used in PSO_{gbest} and SGA. The multi-parent crossover operator proposed in [108] was used in SGA.

For PSO_{gbest}, the acceleration constants η_1 and η_2 were both set to 1.49618 and the inertia weight $\omega = 0.729844$, as suggested in the literature [19]. The total population size for both PSO_{gbest} and SGA was set to 100. Hence, the population size of each sub-swarm was 10. The crossover and mutation probabilities in SGA were set to 0.8 and 0.02, respectively.

All the involved algorithms in this section use multi-population methods. However, the means of generating sub-swarms is different. CPSO uses the method described in Chapter 6, SGA and PSO use a fixed number of sub-swarms/sub-populations without any further techniques, e.g., avoiding overlapping search mechanism, convergence checking mechanism, or adaptively adjusting the number of sub-swarms. jDE [15] also uses a fixed number of sub-populations without any information sharing between sub-populations, except the overlapping search between two best individuals of two sub-populations.

8.3.1 Effect of Varying the Configurations

The corresponding offline errors of the above combinatorial configurations of CPSO on the six functions with the small step change are shown in Table 8.10, where the best result over all values of *max_subsize* (N) for each fixed number of initial population size (M) of the cradle swarm is shown in bold font.

Table 8.10: Offline error of different configurations on the six functions with small step change

F	N	M=10	M=30	M=50	M=70	M=100	M=150	M=200
$F_1(p = 10)$	N=2	2.7	0.976	0.963	0.939	0.942	1.03	0.803
	N=5	**1.8**	**0.286**	**0.178**	**0.0957**	**0.0219**	**0.00988**	0.00313
	N=7	1.94	0.711	0.185	0.134	0.0692	0.0635	**0.00251**
	N=10	3.63	1.26	0.639	0.283	0.125	0.115	0.0504
	N=12	3.45	1.03	0.771	0.389	0.161	0.196	0.0867
	N=15	3.91	2.21	1.06	0.795	0.363	0.185	0.142
$F_1(p = 50)$	N=2	**4.15**	**1.44**	0.901	0.719	0.643	0.662	0.542
	N=5	4.6	1.44	**0.8**	**0.578**	**0.25**	**0.147**	**0.0645**
	N=7	4.44	2.77	1.31	0.889	0.518	0.21	0.174
	N=10	6.33	3.21	1.82	1.36	0.736	0.458	0.3
	N=12	6.92	3.32	2.31	1.66	1.09	0.625	0.501
	N=15	6.75	4.61	2.96	2.11	1.55	0.776	0.47
F_2	N=2	**14.8**	**5.98**	3.61	3.31	3.19	3.27	3.48
	N=5	15	6.25	**3.59**	**2.48**	**1.19**	**0.853**	**0.476**
	N=7	15.4	8.14	4.77	4.17	2.16	1.43	1.18
	N=10	19.4	13	7.71	4.92	3.18	1.79	0.871
	N=12	19.3	13.2	8.14	6.48	3.72	2.29	1.74
	N=15	17.9	15.4	11.1	8.61	5.83	3.96	2.34
F_3	N=2	455	346	279	298	230	221	211
	N=5	214	64.1	44.8	32.4	26.4	20.2	17.7
	N=7	198	63.6	38.8	25.8	19.3	15.7	14.2
	N=10	133	**50.2**	**30.8**	24.2	**15.7**	**10.9**	**10.4**
	N=12	159	54.2	33.7	**22.6**	17.2	13.3	11.9
	N=15	**127**	57.2	35.2	24.3	18.8	15	11.1
F_4	N=2	20.5	8.81	8.52	5.78	5.76	6.02	5.6
	N=5	20.1	**7.28**	**4.1**	**2.39**	**1.96**	**0.913**	**0.875**
	N=7	21.9	10.8	6.19	3.53	2.57	1.03	0.962
	N=10	21.6	14.8	8.98	6.54	3.32	2.36	1.36
	N=12	22.1	12.3	10.6	6.85	6.04	3.37	2.48
	N=15	**19.5**	21.3	11.8	8.44	6.11	4.45	2.61
F_5	N=2	38.3	14.3	13.5	10.8	9.25	6.58	7.24
	N=5	16.6	**5.24**	**1.91**	**1.16**	**0.851**	**0.339**	**0.256**
	N=7	16.1	7.23	3.38	2.61	1.68	0.909	0.572
	N=10	**15.6**	9.91	5.54	4.35	2.9	1.64	1.16
	N=12	18.3	11.4	7.38	6.41	4.4	2.33	1.44
	N=15	20.8	15.3	9.29	8.01	4.48	3.3	2.7
F_6	N=2	35.2	21	17.6	14.6	13.4	13.4	14.6
	N=5	20.5	**9.26**	**8.38**	**5.47**	**3.58**	**3.71**	**2.62**
	N=7	**20.1**	13.1	8.5	6.03	5.14	3.94	3.2
	N=10	23.7	15.6	10.8	9.46	7.04	5.09	4.5
	N=12	24.5	14.4	10.9	9.94	7.76	6.28	5.91
	N=15	21.5	19.5	15.7	12.5	8.64	5.98	3.84

As can be seen from the configuration study on the MPB problem, from Table 8.10, different configurations also significantly affect the performance of CPSO on the functions. From the results of the six functions in Table 8.10, it can be seen that the offline error decreases to a certain level and then increases as the value of M increases for each fixed value of N and similar observation can be seen with increasing the value of N for each fixed value of M. The interesting thing that can be observed for the six functions is that setting the value of N to 5 achieves the

best results on most test cases for the six functions with small step change.

Generally speaking, the larger the initial swarm size, the better the results will be achieved by CPSO. It is because, compared with the MPB problem, the GDBG benchmark is more difficult to solve, especially the test cases based on composition functions, e.g., F_2-F_6. Consequently, we obtain different observations from the configuration study on the MPB problem, where the optimal results obtained by CPSO on the six functions are based on the largest size of the cradle swarm with value of 200.

8.3.2 Comparison of CPSO with Peer Algorithms

In this section, we present the comparison of the performance of CPSO with other peer algorithms on the GDBG benchmark. All the results of jDE are provided from the paper [15]. Table 8.11 presents the results of the offline error and STD of CPSO, jDE, SGA, and PSO on each test case, where the best offline error result for each test case is shown in bond font. Table 8.12 shows the average performance of CPSO, jDE, SGA, and PSO on each test case and the final marks they got, respectively. From Tables 8.11 and 8.12, several results can be observed and are described below.

First, in comparison with the results in the previous group of experiments, it can be seen that the performance of CPSO degrades in this group of experiments. This is because the DOPs in this group of experiments are much harder for CPSO to locate and track the global optima than the MPB problem. Due to the huge number of local optima and the higher dimensions of the GDBG benchmark problems, the complexity of some test functions (e.g., F_2 - F_6) is much higher than the MPB problem.

Second, from the comparison results of Table 8.11 and Table 8.12, it can be seen that CPSO performs much better than SGA and SPSO on most test cases over

Table 8.11: Offline errors of CPSO, jDE, SGA, and PSO_{gbest} on all the test cases

F	Algorithm	Error	T_1	T_2	T_3	T_4	T_5	T_6	T_7
$F_1(p=10)$	CPSO	e_{off}	**0.00313**	**1.04**	3.62	0.0628	**1.42**	**0.468**	**1.88**
		STD	0.0063	0.505	1.65	0.0636	0.48	0.253	0.793
	jDE	e_{off}	0.028813	3.5874	**2.999**	**0.0153**	2.177	1.1457	3.51
		STD	0.442	7.838	7.12	0.288	4.38812	5.72	7.898
	SGA	e_{off}	0.58614	5.75052	8.46	2.12647	3.56845	6.9611	7.69319
		STD	1.99069	10.1057	12.0128	3.54302	5.53126	14.9035	11.5626
	PSO_{gbest}	e_{off}	2.99149	7.17879	7.25077	6.58312	3.33609	13.0866	11.36
		STD	5.77366	10.1054	9.43427	9.33229	5.56398	17.8091	13.2116
$F_1(p=50)$	CPSO	e_{off}	**0.0645**	**2.45**	5.23	**0.0745**	**0.757**	**0.345**	**4.26**
		STD	0.0317	0.511	1.55	0.0146	0.104	0.0661	0.979
	jDE	e_{off}	0.172	4.086	**4.292**	0.0877	0.948	1.765	4.369
		STD	0.763932	6.4546	6.74538	0.24613	1.76552	5.82652	6.9321
	SGA	e_{off}	1.27703	6.30659	11.0064	1.87469	1.7742	7.66115	8.70514
		STD	2.17841	8.32954	10.4983	2.16038	2.59909	14.518	9.03533
	PSO_{gbest}	e_{off}	2.75578	7.40292	9.83031	2.87054	1.88839	11.9457	12.0617
		STD	4.09429	8.25633	9.7497	4.97619	3.38957	17.7187	10.6151
F_2	CPSO	e_{off}	**0.476**	**24.5**	**19.4**	**0.587**	**56.8**	**1.31**	**2.2**
		STD	0.482	11	8.83	0.328	11	0.432	0.871
	jDE	e_{off}	0.963	43.0004	50.19	0.793	67.05	3.366	13.25
		STD	3.08329	114.944	124.015	2.53425	130.146	12.9738	45.7797
	SGA	e_{off}	12.5983	115.969	86.9695	12.726	133.18	14.6082	21.5497
		STD	23.7612	183.078	155.181	15.4225	173.885	25.6411	49.0415
	PSO_{gbest}	e_{off}	19.1488	120.912	78.2692	30.141	126.76	32.3515	29.9937
		STD	28.54	183.7	146.534	53.1909	175.519	57.3215	56.8522
F_3	CPSO	e_{off}	**10.4**	813	659	439	737	577	337
		STD	3.93	16	30.5	20	39.5	35.3	13.8
	jDE	e_{off}	11.3927	**558.497**	**572.105**	**65.7409**	**475.768**	**243.27**	**153.673**
		STD	58.1106	384.621	386.09	208.925	379.89	384.98	286.379
	SGA	e_{off}	52.6124	786.113	629.924	441.974	670.328	547.149	395.602
		STD	119.283	228.672	325.499	428.704	316.443	427.392	367.686
	PSO_{gbest}	e_{off}	670.452	902.23	851.247	813.555	856.449	883.029	715.163
		STD	163.321	116.601	121.802	269.267	148.93	265.821	337.618
F_4	CPSO	e_{off}	**0.875**	53.1	52.2	**0.89**	113	**1.66**	**5.48**
		STD	0.526	17.1	14.9	0.323	19.6	0.535	2.01
	jDE	e_{off}	1.48568	**49.5044**	**51.9448**	1.50584	**69.4395**	2.35478	11.7425
		STD	4.47652	135.248	141.78	4.10062	144.041	5.78252	39.4469
	SGA	e_{off}	23.9116	287.587	223.017	20.913	286.289	33.924	73.9041
		STD	59.4481	260.582	242.403	46.0074	229.639	93.3312	143.608
	PSO_{gbest}	e_{off}	3.9761	211.229	178.004	40.2054	219.112	50.5672	44.1123
		STD	48.2643	249.102	224.357	74.3142	230.775	98.1716	98.6696
F_5	CPSO	e_{off}	0.256	0.605	0.794	0.162	2.56	**0.201**	67
		STD	0.165	0.174	0.284	0.077	0.67	0.0825	0.116
	jDE	e_{off}	**0.159877**	**0.3339**	**0.3579**	**0.1081**	**0.4093**	0.2296	**0.4342**
		STD	1.02554	1.64364	1.83299	0.826746	1.90991	0.935494	2.22792
	SGA	e_{off}	11.3323	14.6559	13.9869	11.1733	15.6641	11.4771	13.752
		STD	7.87798	9.37542	8.49015	7.33382	10.2018	6.8062	8.39936
	PSO_{gbest}	e_{off}	13.4091	8.81476	9.10639	20.4131	4.42864	25.2147	19.4093
		STD	37.808	29.9502	40.1531	61.7999	14.1769	75.0437	76.5314
F_6	CPSO	e_{off}	**2.62**	11.2	17.3	**5.96**	36.5	**5.4**	**5.39**
		STD	0.868	2.01	8.19	2.29	12.6	0.756	2.37
	jDE	e_{off}	6.22948	**10.3083**	**10.954**	6.78734	**14.95**	7.8028	10.736
		STD	10.4373	13.2307	23.2974	10.1702	45.208	10.9555	14.7267
	SGA	e_{off}	17.7831	64.034	37.3423	26.1903	52.776	24.8264	13.752
		STD	28.4897	147.026	95.6551	52.2777	125.891	51.1886	8.39936
	PSO_{gbest}	e_{off}	54.4643	73.5971	68.8196	89.4687	82.2801	98.8683	94.9537
		STD	90.1469	159.67	150.61	150.028	192.617	160.007	177.889

Table 8.12: Overall performance of CPSO, jDE, SGA, and PSO$_{gbest}$ on GDBG benchmark

Algorithm	F	T_1	T_2	T_3	T_4	T_5	T_6	T_7	mark
CPSO	$F_1(10)$	0.0145	0.0142	0.0135	0.0143	0.0139	0.0141	0.00936	0.0939062
	$F_1(50)$	0.0145	0.0139	0.0132	0.0142	0.0142	0.0141	0.00898	0.092998
	F_2	0.021	0.0166	0.0164	0.0206	0.0143	0.0195	0.0134	0.121754
	F_3	0.0166	9.10e-04	0.00223	0.00831	0.00198	0.0044	0.0056	0.0400751
	F_4	0.0206	0.0147	0.0142	0.0202	0.0122	0.0192	0.0123	0.113448
	F_5	0.0199	0.0198	0.0197	0.0199	0.0188	0.0196	0.0131	0.130943
	F_6	0.0187	0.0144	0.0133	0.0155	0.0149	0.0153	0.0116	0.103629
jDE	$F_1(10)$	0.0148	0.0137	0.0138	0.0147	0.0139	0.0141	0.00911	0.0942
	$F_1(50)$	0.0147	0.0136	0.0135	0.0147	0.0144	0.0139	0.00899	0.0937
	F_2	0.0211	0.0135	0.0131	0.021	0.0124	0.0178	0.0102	0.109
	F_3	0.0157	0.00298	0.00281	0.0128	0.00441	0.00735	0.00549	0.0515
	F_4	0.0207	0.0131	0.0135	0.0199	0.0124	0.018	0.0102	0.108
	F_5	0.0218	0.0209	0.0209	0.0222	0.0213	0.0207	0.0138	0.142
	F_6	0.017	0.0139	0.0142	0.0153	0.0155	0.014	0.00943	0.0994
PSO$_{gbest}$	$F_1(10)$	0.0133	0.0126	0.0124	0.0124	0.0134	0.0112	0.00753	0.0829
	$F_1(50)$	0.0135	0.0126	0.012	0.0133	0.0139	0.0115	0.0076	0.0844
	F_2	0.0118	0.00828	0.0103	0.00965	0.0097	0.00939	0.00745	0.0666
	F_3	0.000693	0.000343	0.000415	0.000409	0.000813	0.000327	0.00122	0.00422
	F_4	0.0115	0.0067	0.00766	0.00912	0.0079	0.00858	0.00749	0.059
	F_5	0.0135	0.0156	0.0157	0.0114	0.0185	0.011	0.0092	0.095
	F_6	0.00769	0.00899	0.00942	0.00612	0.0118	0.00625	0.00615	0.0564
SGA	$F_1(10)$	0.0137	0.0126	0.0119	0.0129	0.013	0.0121	0.00809	0.0842
	$F_1(50)$	0.0135	0.0126	0.0116	0.013	0.0135	0.0119	0.00803	0.0842
	F_2	0.0106	0.00619	0.00746	0.00847	0.00735	0.00809	0.00631	0.0545
	F_3	0.00601	0.000614	0.00146	0.00279	0.00164	0.00192	0.00187	0.0163
	F_4	0.00921	0.00354	0.00517	0.00724	0.00463	0.00688	0.00479	0.0415
	F_5	0.00932	0.00818	0.00887	0.00745	0.011	0.00731	0.00603	0.0582
	F_6	0.00863	0.00577	0.00756	0.00617	0.00898	0.00613	0.00409	0.0473
Overall performance	CPSO: 69.6753, jDE: 69.7269, SGA: 38.0564, PSO: 44.8442								

all change types, except on F_3 under some change types. SGA may benefit from its better diversity maintenance than CPSO on these test cases of F_3. Compared with jDE algorithm, CPSO outperforms jDE on 28 cases out of the total 49 cases in terms of the average offline errors. In addition, the variance of each test case obtained by jDE is much larger than that of CPSO. In other words, CPSO is more robust than jDE to adapt to the dynamic environments. Although jDE obtains the highest total mark among the four algorithms in Table 8.12, the value is only a little higher than the total mark obtained by CPSO where the difference is 0.0516. It should be noticed that the mark on some test cases obtained by jDE is higher than the mark of corresponding test cases achieved by CPSO although the mean values are worse than the mean values of CPSO, e.g., F_1 under change type T_1.

Third, from the tables, it can be seen that the challenge of different change

types is quite different. The small step change is the easiest for algorithms on most test cases. The large step and chaotic change types bring in the biggest challenge on most test cases. The dimensional change is also difficult to optimize for algorithms.

Finally, the results also show that different problems have a different difficulty level for algorithms. Since F_1 has a smooth fitness landscape, it is the simplest function to optimize. The composition problems are difficult for algorithms to get the global optima. F_3 is the most complicated one among all the test problems.

8.4 Summary

The multi-swarm method is an effective technique to locate and track multiple peaks in dynamic environments. However, to make full use of this technique, several issues should be considered, e.g., how to guide particles to move toward different promising sub-regions, how to define the area of each sub-region, how to determine the number of sub-swarms needed, and how to generate sub-swarms.

To effectively solve these issues, we propose a hierarchical clustering method to create sub-swarms. This method is able to automatically create sub-swarms based on the distribution of initial particles in the fitness landscape, and the number of sub-swarms and the size of each sub-swarm are also automatically determined by the fitness landscape. As a result, the search region of each sub-swarm is also automatically obtained according to the distribution of particles in their local fitness landscape. In addition, to avoid overlapping search and save computational resources, an effective overlapping check mechanism and an overcrowd detecting method are proposed.

Generally speaking, considering the above issues, CPSO can effectively locate and track multiple optima in dynamic environments. The experimental results indicate that the proposed CPSO can be a good optimizer in dynamic environments,

especially for a dynamic fitness landscape with multiple changing peaks.

Chapter 9

Conclusions and Future Work

This chapter summarizes the thesis based on the general issues of applying EAs for solving global optimization problems and dynamic optimization problems, including the main technical contributions, conclusions, future work, and discussion of how to increase an individual's intelligence in EAs.

9.1 Technical Contributions

The intelligent system developed by considering some challenging issues discussed in Chapter 1 for EAs in global optimization is one of the main contributions in this thesis. In order to assign to each particle enough intelligence to cope with different situations, we implemented several approaches to enhancing the performance of EAs. For example, the self-learning strategy, the evolutionary status monitoring mechanism, the re-start strategy, and the repelling scheme. These ideas were implemented into the PSO algorithm and substantially improve the performance of the PSO algorithm.

Another important contribution is the clustering idea for tracking and locating multiple optima in dynamic environments. Several challenging issues in dynamic environments are comprehensively discussed in the thesis when ap-

plying multi-population methods, e.g., how to guide particles to move toward different promising sub-regions, how to define the area of each sub-region, how to determine the number of sub-swarms needed, and how to generate sub-swarms. The clustering idea was proposed to alleviate the sufferings of EAs in dynamic environments.

The following subsections summarize the main technical contributions for both global optimization and dynamic optimization in this thesis.

9.1.1 Techniques Developed for Global Optimization

Some ideas proposed for EAs in global optimization were implemented by the PSO optimization tool, which is called SLPSO. In this intelligent system, several novel ideas for some challenging issues are systematically organized together to make SLPSO effectively to deal with different complex situations.

In SLPSO, each particle has four learning sources produced by four learning operators. The four learning sources are the *abest* position, its individual *pbest* position, the *pbest* of a random particle ($pbest_{rand}$) whose *pbest* is better than the particle's own *pbest*, and a random position *prand*. The four learning objectives have different properties, which guide the particle to converge to the current global best position, exploit a local optimum, explore new promising areas, and jump out of a local optimum, respectively. In order to enable particles to automatically choose the appropriate learning objective at the appropriate moment during the search process, an adaptive selection mechanism, which is based on the assumption that the most successful operator used in the recent past iterations may also be successful in the future several iterations, is introduced. For each particle, one of the four learning operators is selected according to their selection ratios. For all particles, the selection ratio of each operator is equally initialized to 1/4 and is updated according to its relative performance. The operator that results

in a higher relative performance will have its selection ratio increased. Gradually, the most suitable operator will be chosen automatically for a particle and that operator will control the particle's search behavior according to its local fitness landscape at the corresponding evolutionary stage.

To increase diversity, we introduce another mechanism to check particles' evolutionary status by introducing the monitoring selection ratios of the four learning operators. If the variance of the four monitoring selection ratios is less than 0.05, the corresponding particle will be re-initialized with a random position. This mechanism enables SLPSO to automatically regain diversity by restarting those particles that have converged to local optima.

There are some other heuristic rules applied in SLPSO, e.g., extracting useful information from improved particles for the *abest* position to avoid the "two step forward, one step back" problem, using an external memory to encourage each particle to explore new promising sub-regions, and introducing multiple-swarm methods to avoid the attraction to the exploited *abest* positions.

In order to make SLPSO effectively work on a general problem, several techniques are introduced. First, a bias selection is given to those badly performing particles to improve the whole swarm's performance in the exploration operator where a particle learns from a random $pbest_{rand}$ position only if the $pbest_{rand}$ position is better than its own historical best position *pbest*. Second, to reduce the risk of punishing the current best operator for a particle because of the temporal bad performance of that operator in a short period, the selection ratios of the four learning operators are updated only if a particle is not improved for U_f successive iterations instead of every U_f iterations. Third, a learning probability P_l is introduced to solve the time-consuming problem and also reduce the probability of learning un-useful information when the *abest* position learns from an improved particle. Fourth, for a fair competition among the four learning operators, the

scheme of controlling the number of particles that learn from the *abest* position is introduced. Finally, the parameter tuning method for the three parameters (U_f, P_l, and M) is developed to set up SLPSO for solving a general problem.

9.1.2 Techniques Developed in Dynamic Environments

In order to track and locate multiple optima in dynamic environments, a single linkage hierarchical clustering method was applied to create clusters of particles to form sub-swarms. With this clustering method, the proper number of sub-swarms is automatically determined and the search area of each sub-swarm is also automatically calculated. When a sub-swarm is created, it will undergo the local search process to exploit the promising sub-regions for optimal solutions. In order to speed up the searching of a sub-swarm, a new learning mechanism was introduced for its global best particle to learn from those particles that are improved during the local search process. In order to address environmental changes directly, a restart scheme with reservation of best positions found in the previous environment was also applied in CPSO. In addition, an effective overlapping check mechanism and an overcrowd detecting method are proposed to automatically remove redundant particles during the search progress.

9.2 Conclusions

In this section, we will summarize the conclusions based on the experimental results in this thesis. From the experimental results, we can conclude the ideas proposed in this thesis greatly improve the performance of PSO for both global optimization problems and DOPs.

9.2.1 Self-learning PSO

In order to investigate the performance of the proposed SLPSO, we carried out experiments on some challenging test problems, such as shifted, noisy, rotated shifted, and hybrid composition functions in this thesis. From the behavior analysis and comparison results on the 45 test problems, three conclusions can be drawn for the SLPSO algorithm. First, the adaptive learning mechanism can well balance the behavior of exploitation and exploration at the individual level where particles are independent to adapt to their local fitness landscapes in different sub-regions. The adaptive framework enables the four complementary learning operators with different properties to cooperate with each other to guide particles' search behavior. Each particle can choose an appropriate learning objective at an appropriate moment according to the property of its local search space for achieving the best performance of SLPSO.

Second, the restart mechanism can precisely check a particle's convergence status by checking the monitoring selection ratios of the four learning operators. This mechanism enables SLPSO to automatically regain the population diversity when particles converge to local optimum.

Third, SLPSO significantly enhances the performance of PSO in terms of the convergence speed and the solution accuracy comparing with other peer algorithms. From the experimental results in Chapter 7, SLPSO performs much better than some state-of-the-art algorithms in two aspects: one is the comparison of performance on the 45 problems in terms of the t-test results; the other is the comparison of performance decrease on modified problems and on problems with high dimensions (30, and 50 dimensions).

To summarize the research work done in this thesis for global optimization problems, the techniques developed based on the idea of individual level of intelligence, including the four learning strategies, the adaptive selection framework,

the self-restart scheme, and the idea of extracting useful information for the *abest* position, compose an intelligent system to effectively alleviate the exploration-exploitation dilemma in EAs.

9.2.2 Clustering PSO

In order to justify the proposed CPSO, experiments were carried out to compare the performance of CPSO with several advanced PSO algorithms that are the state-of-the-art algorithms for DOPs in the real space (i.e., the SPSO and rSPSO algorithms [10, 5], the mCPSO and mQSO algorithms with and without anti-convergence [84], CESO [74]) on the widely used MPB generator [13]. In addition, to test how CPSO performs on complex problems, CPSO was compared with the winner (jDE [15]) of the competition of " Evolutionary Computation in Dynamic and Uncertain Environments" [68] held with CEC'2009 on the GDBG benchmark.

From the experimental results, the following conclusions can be drawn on the dynamic test environments. CPSO greatly improves the performance of PSO in terms of tracking and locating multiple optima in a dynamic fitness landscape with multiple changing peaks by introducing the clustering method. The performance of CPSO scales well regarding the change severity in the fitness landscape on the MPB problem in comparison with other peer PSO algorithms. CPSO performs much better than mCPSO, mQSO, SPSO, rSPSO, and CESO in locating and tracking multiple changing peaks in dynamic environments of the MPB problem, especially in severely changing environments. The performance of CPSO also scales well regarding the number of peaks in dynamic environments. When the number of peaks in the fitness landscape is relatively small (e.g., less than 20), CPSO outperforms all the other peer algorithms. In comparison with jDE, SGA, and PSO with the *gbest* model on the GDBG benchmark, CPSO outperforms SGA and PSO algorithms. The performance of CPSO is a bit worse than that of jDE

in terms of the total mark obtained on the 49 cases of the GDBG benchmark. However, the number of cases where CPSO achieves the best mean is larger than that achieved by jDE. In other words, CPSO and jDE are matched with each other on the GDBG benchmark.

To summarize the research work that has been done for DOPs, the basic idea employed in the CPSO algorithm is an effective way to apply multi-population methods to solve DOPs by taking into account some fundamental open issues, e.g., how to determine the number of sub-populations needed, how to define the search area of each sub-population, and how to create sub-populations.

9.3 Future Work

In this section, we mainly discuss some future work that need to be done for both the SLPSO algorithm and the CPSO algorithm.

9.3.1 Self-learning PSO

Although the current research of SLPSO has achieved its two main objectives: accelerating the convergence speed and preventing PSO from being trapped in local optima, there are still some remaining issues that we would like to address in the future.

The first issue is how to update the current SLPSO to a fully self-adaptive version. The self-adaptive structures should be able to self-adaptively tune not only parameters in SLPSO, such as the update frequency (U_f), the learning probability (P_l), and the number of particles that learn form the *abest* position (M), but also the inertia weight (ω) as well as acceleration constrains (η). Currently, our tuning mechanism for the three key parameters in SLPSO is non-adaptive. Although different particles use different parameter settings, it cannot adaptively

tune these values. In the future, the self-adaptive mechanism should be able to adaptively find suitable parameter values without any a priori knowledge. Second, exploring more effective learning operators is also an important future work. More complimentary learning operators can enable particles to have more intelligence to cope with more complicated situations. Third, how to make full use of the external memory for each particle is also important work to enhance the performance of SLPSO. Another interesting issue is to compare the performance of SLPSO with some state-of-the-art non-PSO algorithms (e.g., the covariance matrix adaptation evolution strategy (CMA-ES) [36] and some differential evolution (DE) [105] algorithms). Finally, how to cooperate the learning operators in a more effective way should be further investigated.

9.3.2 Clustering PSO

There are several relevant works to pursue in the future. First, although the clustering method applied in CPSO is effective to generate sub-swarms, it is still difficult to create accurate sub-swarms, especially for the situation when a single peak is covered by only one particle. More work should be done to solve this problem.

Second, CPSO can not explore an untouched area in the search space when no particles cover that area since all sub-swarms only concern exploitation in their own local search areas in CPSO. Introducing new local search algorithms may solve this problem to improve the exploration capability of CPSO.

Third, in the CPSO studied in this paper, when a sub-swarm becomes converged or overcrowded, we just remove the sub-swarm or the overcrowded particles from the sub-swarm. Hence, the whole population size may become smaller and smaller during the solving process. However, according to our preliminary experiments, simply adding corresponding number of random particles into the

cradle swarm does not work well since they may be attracted to those existing sub-swarms. It is worth investigating how to effectively add particles into the cradle swarm to maintain the whole population size at an optimal value.

How to deal with the environmental changes is another important issue. We need to introduce more effective methods rather than a simple restart with elitism scheme to address the dynamism in DOPs.

Finally, it would be also interesting to combine other techniques into CPSO to further improve its performance in dynamic environments. For example, the *max_subsize* parameter has a great impact on the performance of CPSO. More research could be conducted to look at adaptation or self-adaptation of *max_subsize* and the population size to adjust them according to the environmental dynamics and the properties of the base function. To the opposite direction, extending the ideas applied in CPSO, e.g., the clustering idea and the learning technique for updating the global best particle of a sub-swarm, to other algorithms may also be valuable to improve their performance in dynamic environments.

9.4 Discussion on Creating Individual Level of Intelligence

EAs are inspired by natural evolution. The properties of parallel computation, self-learning, and self-adaptation enable EAs to be an ideal tool for solving complex optimization problems, such as the complicated optimization problems which are considered to be impractical to be solved by traditional methods.

Problems come from nature, and solutions are provided by nature as well. Nature, the greatest problem solver, gives us inspiration to find solutions to solve different kinds of problems. EAs are the techniques inspired by nature and have been successfully applied to solve many real world problems. However, this

research field is still in a very young stage. There are many challenging aspects to be comprehensively studied, e.g., the theoretical ground, applications to real-world problems, and novel effective algorithms. In the end of this thesis, we would like to briefly discuss several principles regarding how to increase an individual's intelligence in EAs.

First, a set of search patterns are needed to cope with different situations. Most EAs so far only use a single search pattern. The monotonic learning pattern may cause the lack of intelligence, which makes an algorithm unable to deal with different complex situations. In order to bring more intelligence to deal with different situations, the algorithm needs a set of learning patterns.

Second, the individual level of intelligence is needed for individuals to deal with different sub-regions of the search space. Due to different structures of sub-regions of the search space, individuals in different local search spaces need different strategies to effectively explore the local search spaces where they reside.

Third, memory can enable individuals to learn knowledge from previous search and to use the knowledge to guide the search to promising regions in the whole search space. Memory schemes can assign to individuals learning capability, which is one of the most important features of intelligent computing. Therefore, it is important for algorithms to have the memory mechanism.

Finally, for DOPs, locating and tracking a set of relatively good peaks is an effective method for algorithms in dynamic environments. Experimental results show that the objective in dynamic environments should be a set of optima instead of a single global optimum.

We would like to imagine that a truly intelligent algorithm should be the one that can self-evolve according to the surrounding environments. In other words, as human beings' brain, the algorithm should have the capability of learning and using the knowledge learned to solve problems. Although there are too many

challenges on the way, we would spare no effort toward that goal , developing the truly intelligent algorithm.

Bibliography

[1] P. Angeline, "Evolutionary optimization versus particle swarm optimization: Philosophy and performance differences," in *Proc. 7th Conf. Evolutionary Programming*, 1998, pp. 601-610.

[2] T. B*ä*ck and H.-P. Schwefel. "An overview of evolutionary algorithms for parameter optimization," *Evol. Comput.*,vol. 1, 1993, pp. 1-23.

[3] F. van den Bergh. "An analysis of particle swarm optimizers," *PhDThesis*, Department of Computer Science, University of Pretoria,South Africa, 2002.

[4] S. Bird and X. Li, "Adaptively choosing niching parameters in a PSO", in *Proc. 2006 Genetic Evol. Comput. Conf.*, 2006, pp. 3-10.

[5] S. Bird and X. Li, "Using regression to improve local convergence," in *Proc. 2007 IEEE Congr. Evol. Comput.*, 2007, pp. 592-599.

[6] T. M. Blackwell and P. Bentley, "Don't push me! Collision-avoiding swarms," in *Proc. 2002 Congr. Evol. Comput.*, vol. 2, 2002, pp. 1691-1696.

[7] T. M. Blackwell. Swarms in dynamic environments. *Proc. of the2003 Genetic and Evolutionary Computation Conference*, LNCS 2723, 2003, pp. 1-12.

[8] T. M. Blackwell, "Particle swarms and population diversity II: Experiments," in *Proc. 2003 Genetic Evol. Comput. Workshops*, 2003, pp. 108-112.

[9] T. M. Blackwell and J. Branke, "Multi-swarm optimization in dynamic environments," in *EvoWorkshops 2004: Appl. Evol. Comput.*, 2004, LNCS, vol. 3005,

pp. 489-500.

[10] T. M. Blackwell and J. Branke, "Multiswarms, exclusion, and anti-convergence in dynamic environments," *IEEE Trans. Evol. Comput.*, vol. 10, no. 4, 2006, pp. 459-472.

[11] J. Branke, T. Kaußler, C. Schmidth, and H. Schmeck, "A multi-population approach to dynamic optimization problems" in *Proc. 4th Int. Conf. Adaptive Computing in Design and Manufacturing*, 2000, pp. 299-308.

[12] J. Branke, *Evolutionary Optimization in Dynamic Environments*, Kluwer Academic Publishers, 2002.

[13] J. Branke, "Memory enhanced evolutionary algorithms for changing optimization problems," in *Proc. 1999 Congr. Evol. Comput.*, 1999, vol. 3, pp. 1875-1882.

[14] D. Bratton and J. Kennedy, "Defining a standard for particle swarm optimization," in IEEE Swarm Intel. Symp., 2007, pp. 120-127.

[15] J. Brest, A. Zamuda, B. Boskovic, M. S. Maucec, V. Zumer, "Dynamic optimization using Self-Adaptive Differential Evolution," in *Proc. 2009 Congr. Evol. Comput.*, 2009, pp. 415-422.

[16] R. Brits, A. P. Engelbrecht, and F. van den Bergh, "Solving systems of unconstrained equations using particle swarm optimization," in *Proc. 2002 IEEE Conf. Syst., Man, Cybern.*, 2002, pp. 102-107.

[17] R. Brits, A. Engelbrecht, and F. van den Bergh, "A niching particle swarm optimizer," in *Proc. 4th Asia-Pacific Conf. Simulated Evolution and Learning*, vol. 2, 2002, pp. 692-696.

[18] L. T. Bui, H. A. Abbass, and J. Branke, "Multiobjective optimization for dynamic environments," in *Proc. 2005 Congr. Evol. Comput.*, 2005, vol. 3, pp. 2349-2356.

[19] M. Clerc and J. Kennedy, "The particle swarm: Explosion, stability and convergence in a multi-dimensional complex space," *IEEE Trans. Evol. Comput.*, vol. 6, no. 1, 2002, pp. 58-73.

[20] M. Clerc, *Particle swarm optimization*. ISTE, London,UK, 2006.

[21] H. G. Cobb and J. J. Grefenstette, "Genetic algorithms for tracking changing environments," in *Proc. 5th Int. Conf. Genetic Algorithms*, 1993, pp. 523-530.

[22] A. Colorni, M. Dorigo, and V. Maniezzo, "Distributed optimization by ant colonies," in European Conference on Artificial Life, 1991, pp. 134-142.

[23] L. Dacosta, A. Fialho, M. Schoenauer, and M. Sebag, "Adaptive operator selection with dynamic multi-armed bandits," in *GECCO '08: Proc. 10th Conf. Genetic and Evol. Comput.*, 2008, pp. 913-920.

[24] S. Das, A. Abraham, U. K. Chakraborty, A. Konar, "Differential Evolution Using a Neighborhood-Based Mutation Operator," *IEEE Trans. Evol. Comput.*, vol. 13, no. 3, June 2009, pp. 526-553.

[25] F. van den Bergh. "An Analysis of Particle Swarm Optimizers," *PhDthesis*, Department of Computer Science, University of Pretoria, South Africa, 2002.

[26] R. C. Eberhart and J. Kennedy, "A new optimizer using particle swarm theory," in *Proc. 6th Int. Symp. Micro Machine and Human Science*, 1995, pp. 39-43.

[27] A. E. Eiben, R. Hinterding, and Z. Michalewicz, "Parameter control in evolutionary algorithms," *IEEE Trans. Evol. Comput.*, vol. 3, no. 2, 1999, pp. 124-141.

[28] S. C. Esquivel and C. A. Coello Coello. "On the use of particle swarm optimization with multimodal functions," *Proc. of the 2003 IEEE Congr. on Evol. Comput.*, 2003, pp. 1130-1136.

[29] H. Y. Fan and J. Lampinen, "A trigonometric mutation operation to differential evolution,"*J. Global Opt.*, vol. 27, no. 1, pp. 105-129, Sep. 2003.

[30] C. Ferreira, , "Gene Expression Programming: A New Adaptive Algorithm for Solving Problems," Complex Systems, vol. 13, 2001, pp. 87-129.

[31] L. J. Fogel, A. J. Owens and M. J. Walsh, "Artificial Intelligence through Simulated Evolution," New York: John Wiley, 1966.

[32] D. B. Fogel. "An introduction to simulated evolutionary optimization," *IEEE Trans. Neural Networks*, vol. 5, 1994, pp. 3-14.

[33] D. E. Goldberg. (1990, Oct.), Probability Matching, the Magnitude of Reinforcement, and Classifier System Bidding, *J. Mach. Learn.*, vol. 5, no. 4, pp. 407-425.

[34] J. J. Grefenstette, "Genetic algorithms for changing environments," in *Proc. 2nd Int. Conf. Parallel Problem Solving From Nature*, 1992, pp. 137-144.

[35] EA. Grimaldi, F. Grimacia, M. Mussetta, P. Pirinoli, and RE. Zich. "A new hybrid genetical particle swarm algorithm for electromagnetic optimization," *Proc. of Int. Conf. on Comput. Electromagnetics and its Applications*, 2004, pp. 157-160.

[36] N. Hansen and Ostermeier. "Convergence properties of evolution strategies with the derandomized covariance matrix adaptation: The $(\mu/\mu\_I, \lambda)$-ES ," in *Proc. 5th Europ. Congr. Intelligent Techniques and Soft Computing*, 1997, pp. 650-654.

[37] N. Higashi and H. Iba. "Particle swarm optimization with Gaussian mutation," *Proc. of the 2003 IEEE Swarm Intelligence Symphosium*, 2003, pp. 72-79.

[38] J. H. Holland, "Adaptation in natural and artificial systems," Ann Arbor: The University of Michigan Press, 1975.

[39] X. Hu and R. C. Eberhart, "Multiobjective optimization using dynamic neighborhood particle swarm optimization," in *Proc. Congr. Evol. Comput.*, 2002, pp. 1677-1681.

[40] S. Janson and M. Middendorf, "A hierachical particle swarm optimizer for dynamic optimization problems," in *Evo. Workshops 2004: Appl. Evol. Comput.*, LNCS, vol. 3005,2004 pp. 513-524.

[41] S. Janson and M. Middendorf, "A hierarchical particle swarm optimizer and its adaptive variant," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 35, no. 6, 2005, pp. 1272-1282.

[42] J.J. Liang, PN. Suganthan, and K. Deb."Novel composition test functions for numerical global optimization," in *Proc. 2005 Symp. Swarm Intelligence*, 2005, pp. 68-75.

[43] J. J. Liang, A. K. Qin, P. N. Suganthan, and S. Baskar. "Comprehensive learning particle swarm optimizer for global optimization of multimodal functions," *IEEE Trans. Evol. Comput.*, vol. 10, 2006, pp. 281-295.

[44] Y. Jin and J. Branke, "Evolutionary optimization in uncertain environments: a survey," *IEEE Trans. Evol. Comput.*, vol. 9, no. 3, 2005, pp. 303-317.

[45] S. C. Johnson, "Hierarchical Clustering Schemes", Psychometrika, vol. 2, pp. 241-254, 1967.

[46] D. Karaboga, "An Idea Based On Honey Bee Swarm for Numerical Optimization", Technical Report-TR06,Erciyes University, Engineering Faculty, Computer Engineering Department 2005.

[47] H. Kellerer, U. Pferschy and D. Pisinger, Knapsack Problems.Springer, 2004.

[48] J. Kennedy, "Stereotyping: Improving particle swarm performance with cluster analysis," in *Proc. 2000 Congr. Evol. Comput.*, 2000, pp. 1507-1512.

[49] J. Kennedy and R. C. Eberhart. *Swarm Intelligence*. Morgan Kaufmann Publishers, 2001.

[50] J. Kennedy and R. Mendes, "Population structure and particle swarm performance", in *Proc. 2002 Congr. Evol. Comput.*, 2002, pp. 1671-1676.

[51] J. Kennedy and R. C. Eberhart. "Particle Swarm Optimization," in *Proc. 1995 IEEE Int. Conf. on Neural Networks*, 1995, pp. 1942-1948.

[52] J. Kennedy, "The particle swarm: social adaptation of knowledge," in *Proc. 1997 Congr. Evol. Comput.*, 1997, pp. 303-308.

[53] J. Kennedy, "Small worlds and mega-minds: Effects of neighborhood topology on particle swarm performance," in *Proc. 1999 Congr. Evol. Comput.*, 1999, pp. 1931-1938.

[54] J. R. Koza, "Genetic programming: a paradigm for genetically breeding populations of computer programs to solve problems," Stanford,CA, USA, Tech. Rep., 1990.

[55] R. A. Krohling. "Gaussian particle swarm with jumps," *Proc. of the 2005 IEEE Congr. on Evol. Comput.*, 2005, pp. 1226-1231.

[56] R. A. Krohling and L. dos Santos Coelho. "PSO-E: Particle swarm with exponential distribution," *Proc. of the 2006 IEEE Congr. on Evol. Comput.*, 2006, pp. 1428-1433.

[57] P. Larrañaga and J. A. Lozano. *Estimation of Distribution Algorithms: A New Tool for Evolutionary Computation*, Kluwer Academic Publishers, 2001.

[58] Y. Leung, Y. Wang, "An orthogonal genetic algorithm with quantization for global numerical optimization,"*IEEE Trans. Evol. Comput.*, vol. 5, no. 1, 2001, pp. 41-53.

[59] J. Lewis, E. Hart and G. Ritchie." A comparison of dominance mechanisms and simple mutation on non-stationary problems," *Proc. of the 5th Int. Conf. on Parallel Problem Solving from Nature*, 1998, pp. 139-148.

[60] D. G. Li and C. Smith, "A new global optimization algorithm based on Latin square theory," in *Proc. IEEE Int. Conf. Evol. Program. VII,* May 20-22, 1996, pp. 628-630.

[61] X. Li, "Adaptively choosing neighborhood bests using species in a particle swarm optimizer for multimodal function optimization," in *Proc. 2004 Genetic Evol. Comput. Conf.*, 2004, pp. 105-116.

[62] C. Li, M. Yang, and L. Kang. "A new approach to solving dynamic TSP, " *Proc of the 6th Int. Conf. on Simulated Evolution and Learning*, 2006,pp. 236-243.

[63] C. Li, Y. Liu, L. Kang, and A. Zhou. "A Fast Particle Swarm Optimization Algorithm with Cauchy Mutation and Natural Selection Strategy". *ISICA2007, LNCS4683*, 2007, pp. 334-343,

[64] C. Li and S. Yang, "A generalized approach to construct benchmark problems for dynamic optimization," in *Proc. 7th Int. Conf. Simulated Evolution and Learning*, 2008, pp. 391-400.

[65] C. Li, S. Yang, and I. A. Korejo. "An adaptive mutation operator for particle swarm optimization," in *Proc. 2008 UK Workshop Comput. Intell.*, 2008, pp. 165-170.

[66] C. Li and S. Yang, "Fast multi-swarm optimization for dynamic optimization problems," in *Proc. 4th Int. Conf. Natural Comput.*, 2008, vol. 7, pp. 624-628.

[67] C. Li and S. Yang. "An island based hybrid evolutionary algorithm for optimization," in *Proc. 7th Int. Conf. Simulated Evolution and Learning*, 2008, pp. 180-189.

[68] C. Li, S. Yang, T. T. Nguyen, E. L. Yu, X. Yao, Y. Jin, H.-G. Beyer, and P. N. Suganthan, "Benchmark Generator for CEC'2009 Competition on Dynamic Optimization," *Technical Report 2008*, Department of Computer Science, University of Leicester, U.K., 2008.

[69] C. Li and S. Yang, "A clustering particle swarm optimizer for dynamic optimization," in *Proc. 2009 Congr. Evol. Comput.*, 2009, pp. 439-446.

[70] C. Li and S. Yang. "An adaptive learning particle swarm optimizer for function optimization," in *Proc. 2009 Conf. Evol. Comput.,* 2009, pp. 381-388.

[71] C. Li and S. Yang. "Adaptive learning particle swarm optimizer-II for function optimization," in *proc. 2010 Congr. Evol. Comput.,* 2010, pp. 1-8.

[72] J. J. Liang, P. N. Suganthan, and K. Deb. "Novel composition test functions for numerical global optimization," *Proc. of the 2005 IEEE Congr. on Evol. Comput.*, 2005, pp. 68-75.

[73] M. Lovbjerg, and T. Krink, "Extending particle swarm optimisers with self-organized criticality," in *Proc. Evol. Comput.*, 2002, pp. 1588-1593.

[74] R. I. Lung and D. Dumitrescu, "A collaborative model for tracking optima in dynamic environments," in *Proc. 2007 Congr. Evol. Comput.*, 2007, pp. 564-567.

[75] R. Mendes, "Population topology and their influence in particle swarm performance," Ph.D. dissertaion, Departmento de informaticam Escola de Engenharia, Univerdade do Minho, 2004.

[76] R. Mendes, J. Kennedy, and J. Neves, "The fully informed particle swarm: simple, maybe better," *IEEE Trans. Evol. Comput.*, vol. 8, 2004, pp. 204-210.

[77] V. Miranda and N. Fonseca. "New evolutionary particle swarm algorithm (EPSO) applied to voltage/var control," in *Proc. 14th Power Systems Comput. Conf.*, Jun, 2002.

[78] R. W. Morrison and K. A. De Jong, "Triggered hypermutation revisited," in *Proc. 2000 Congr. Evol. Comput.*, 2000, pp. 1025-1032.

[79] R. W. Morrison and K. A. De Jong. "A test problem generator for non-stationary environments," *Proc. of the 1999 Congr. on Evol. Comput.*, 1999, pp. 2047-2053.

[80] G. Nicolis, I. Prigogine. (1977) *Self-organization in nonequilibrium systems: from dissipative systems to order through fluctuations*. John Wiley, NY.

[81] K. P. Ng and K. C. Wong."A new diploid scheme and dominance change mechanism for non-stationary function optimisation," in *Proc. of the 6th Int. Conf. on Genetic Algorithms*, pp. 159–166, 1995.

[82] C. K. Oei, D. E. Goldberg, and S. Chang, "Tournament selection, niching, and the preservation of diversity", IlliGAl Report 91011, Illinois Genetic Algorithms Laboratory (IlliGAL), Department of Computer Science, Department of General Engineering, University of Illinois at Urbana-Champaign, December 1991.

[83] D. Parrott and X. Li, "A particle swarm model for tracking multiple peaks in a dynamic environment using speciation," in *Proc. 2004 Congr. Evol. Comput.*, 2004, pp. 98-103.

[84] D. Parrott and X. Li, "Locating and tracking multiple dynamic optima by a particle swarm model using speciation," *IEEE Trans. Evol. Comput.*, vol. 10, no. 4, 2006, pp. 440-458.

[85] K. E. Parsopoulos and M. N. Vrahatis, "Recent approaches to global optimization problems through particle swarm optimization," *Natural Comput.*, vol. 1, no. 2-3, 2002, pp. 235-306.

[86] K. E. Parsopoulos and M. N. Vrahatis, "UPSO A unified particle swarm optimization scheme", in *Proc. 2004 Int. Conf. Computat. Methods Sci. and Eng.*, 2004, vol. 1, pp. 868-873.

[87] A. Passaro and A. Starita, "Particle swarm optimization for multimodal functions: A clustering approach," *J. of Artif. Evol. and Appl.*, vol. 2008, Article ID 482032, 2008.

[88] R. Poli , C. D. Chio, W. B. Langdon," Exploring extended particle swarms: a genetic programming approach," in *Proc. 2005 Conf. on Genetic and Evol. Comput.*, 2005, pp. 33-57.

[89] R. Poli, W. B. Langdon and O. Holland, "Extending particle swarm optimization via genetic programming," in *Proc. 8th European Conf. Genetic Programming*, 2005, pp. 291-300.

[90] R. Poli, J. Kennedy, and T. Blackwell, "Particle swarm optimization: An overview," *Swarm Intell.*, vol. 1, no. 1, 2007, pp. 33-58.

[91] A. K. Qin, V. L. Huang, P. N. Suganthan, "Differential Evolution Algorithm With Strategy Adaptation for Global Numerical Optimization," *IEEE Trans. Evol. Comput.*, vol. 13, no. 2, April 2009, pp. 398-417.

[92] A. Ratnaweera, S. K. Halgamuge, and H. C. Watson, "Self-organizing hierarchical particle swarm optimizer with time-varying acceleration coefficients," *IEEE Trans. Evol. Comput.*, vol. 8, no. 3, 2004, pp. 240-255.

[93] I. Rechenberg, "Evolutions strategie - Optimierung technischer Systeme nach Prinzipien der biologischen Evolution," Stuttgart: Frommann-Holzboog, 1973.

[94] H. Richter, "Detecting change in dynamic fitness landscapes," in *Proc. 2009 Congr. Evol. Comput.*, 2009, pp. 1613-1620.

[95] S. Ronald. Preventing diversity loss in a routing genetic algorithm with hash tagging. Complexity International, 2, April 1995.

[96] S. Ronald. "Genetic Algorithms and Permutation-Encoded Problems. Diversity Preservation and a Study of Multimodality". PhD thesis, University Of South Aus- tralia. Department of Computer and Information Science, 1996.

[97] R. Salomon. "Reevaluating genetic algorithm performance undercoordinate rotation of benchmark functions: A survey of sometheoretical and practical aspects of genetic algorithms". *BioSystems*, vol. 3, 1996, pp. 263-278.

[98] Y. Shi and R. C. Eberhart,"Fuzzy adaptive particle swarm optimization," *in Proc. 2001 Congr. Evol. Comput.*, 2001, vol. 1, pp. 101-106.

[99] XH. Shi, YC. Liang, HP. Lee, C. Lu, and LM. Wang, "An improved GA and a novel PSO-GA-based hybrid algorithm," *Information ProcessingLetters*, vol. 93(5), 2005, pp. 255-261.

[100] Y. Shi, R. Eberhart, "A modified particle swarm optimizer," In *Proc. 1998 IEEE Conf. Evol. Comput.*, 1998, pp. 69-73.

[101] J. E. Smith, "Credit assignment in adaptive memetic algorithms," in *Proc. 9th Annu. Conf. Genetic and Evolut. Comput.*, 2007, pp. 1412-1419.

[102] J. E. Smith and T. C. Fogarty, "Operator and parameter adaptation in genetic algorithms," *Soft Computing*, 1997, pp. 81-87.

[103] A. Stacey, M. Jancic, and I. Grundy."Particle swarm optimization with mutation," *Proc. of the 2003 IEEE Congr. on Evol. Comput.*, 2003, pp. 1425-1430.

[104] R. Storn and K. Price, "Differential evolution - A simpleand efficent adaptive scheme for global optimization overcontinuous spaces", Technical Report TR-95-012, Berkeley:International Computer Science Institute, 1995.

[105] R. Storn and K. Price.(Dec, 1997), "Differential Evolution - A Simple and Efficient Heuristic for Global Optimization over Continuous Spaces," *J. Global Optimization,* vol. 11, pp. 341-359.

[106] P. N. Suganthan, N. Hansen, J. J. Liang, K. Deb, Y.-P. Chen, A. Auger, and S. Tiwari. "Problem definitions and evaluation criteria for the CEC 2005 special session on real-parameter optimization," *Technical Report*, Nanyang Technological University, Singapore, 2005.

[107] P. N. Suganthan, "Particle swarm optimizer with neighborhood operator," in *Proc. 1999 Congr. Evol. Comput.*, 1999, pp. 1958-1962.

[108] T. Guo and L. S. Kang, "A new evolutionary algorithm for function optimization," *Wuhan University Journal of Natural Science*, vol. 4, 1999, pp. 409-414.

[109] D. Thierens. "An adaptive pursuit strategy for allocating operator probabilities," in *Proc. 2005 Congr. Evol. Comput.*, 2005, pp. 1539-1546.

[110] D. Thierens. *Parameter Setting in Evolutionary Algorithms,* vol. 54. Availale: http://www.springer.com/engineering/book/978-3-540-69431-1, April, 2007.

[111] R. Thomsen, "Multimodal optimization using crowding-based differential evolution," in *Proc. 2004 Congr. Evol. Comput.*, 2004, vol. 2, pp. 1382-1389.

[112] J.-T. Tsai, T.-K. Liu, and J.-H. Chou, "Hybrid Taguchi-genetic algorithm for global numerical optimization," *IEEE Trans. Evol. Comput.,* vol. 8, no. 4, pp. 365-377, Aug. 2004.

[113] Z. Tu, Y. Lu; , "A robust stochastic genetic algorithm (StGA) for global numerical optimization," *Evolutionary Computation, IEEE Transactions on* , vol. 8, no. 5, 2004, pp. 456-470.

[114] F. van den Bergh, "An analysis of particle swarm optimizers," *Ph.D. dissertation*, Dept. Comput. Sci., Univ. Pretoria, South Africa, 2002.

[115] F. van den Bergh, A. P. Engelbrecht,"A Cooperative approach to particle swarm optimization," *IEEE Trans. Evol. Comput.*, vol. 8, 2004, pp. 225-239.

[116] L. Wang, "A hybrid genetic algorithm-neural network strategy for simulation optimization, " *Applied Mathematics and Computation*, vol. 170(2), 2005, pp. 1329-1343.

[117] H. Wang, D. Wang, and S. Yang, "Triggered memory-based swarm optimization in dynamic environments," in *EvoWorkshops 2007: Appl. Evol. Comput.*, LNCS, vol. 4448, 2007, pp. 637-646.

[118] H. Wang, Y. Liu, C. Li, and S. Zeng, "A Hybrid Particle SwarmAlgorithm with Cauchy Mutation," *Proc.of the 2007 IEEE Swarm Intelligence Symposium*, 2007.

[119] H. Wang, Y. Liu, S. Zeng, and C. Li."Opposition-based Particle Swarm Algorithm with Cauchy Mutation," in *proc. of the 2007 IEEE Congr. on Evol. Comput.*, 2007, pp. 4750-4756.

[120] C. Wei, Z. He, Y. Zhang and W. Pei, "Swarm directions embedded in fast evolutionary programming," in *Proc. Congr. Evol. Comput.*, vol. 2, 2002, pp. 1278-1283.

[121] K. Weicker and N. Weicker. "Dynamic rotation and partial visibility, "*Proc. of the IEEE 2003 Congr. on Evol. Comput.*, 2003, pp. 1125-1131.

[122] T. Weise, "Global Optimization Algorithms - Theory and Application". Thomas Weise, 2008, Online available at http://www.it-weise.de/

[123] D. Whitley, D. Rana, J. Dzubera, and E. Mathias, "Evaluating evolutionary algorithms," *Artif. Intell.,* vol. 85, 1996, pp. 24-276.

[124] D. H. Wolpert and W. G. Macready." No free lunch theorems foroptimization. "*IEEE Trans. Evol. Comput.*, vol. 1(1), 1997, pp. 67-82.

[125] X. Xie,W. Zhang and Z. Yang, "Dissipative particle swarm optimization," in *Proc. Congr. Evol. Comput.*, 2002, vol. 2, pp. 1456-1461.

[126] S. Yang, Y. S. Ong, and Y. Jin (Eds), *Evolutionary Computation in Dynamic and Uncertain Environments*, Springer, 2007.

[127] S. Yang and H. Richter, "Hyper-learning for population-based incremental learning in dynamic environments," in *Proc. 2009 Congr. Evol. Comput.*, 2009, pp. 682-689.

[128] S. Yang and R. Tinos, "Hyper-selection in dynamic environments," in *Proc. 2008 Congr. Evol. Comput.*, 2008, pp. 3185-3192.

[129] S. Yang. "Non-stationary problem optimization using the primal-dualgenetic algorithm, " *Proc. of IEEE Congr. on Evol. Comput.*, 2003, pp. 2246-2253.

[130] S. Yang, "Associative memory scheme for genetic algorithms in dynamic environments," in *EvoWorkshops 2006: Appl. Evol.Comput.*, 2006, LNCS, vol. 3907, pp. 788-799.

[131] S. Yang, "Genetic algorithms with memory- and elitism-based immigrants in dynamic environments," *Evol. Comput.*, vol. 16, no. 3, 2008, pp. 385-416.

[132] S. Yang and X. Yao, "Experimental study on population-based incremental learning algorithms for dynamic optimization problems," *Soft Comput.*, vol. 9, no. 11, 2005, pp. 815-834.

[133] S. Yang and X. Yao, "Population-based incremental learning with associative memory for dynamic environments," *IEEE Trans. Evol. Comput.*, vol. 12, no. 5, 2008, pp. 542-561.

[134] S. Yang and C. Li. "A clustering particle swarm optimizer for locating and tracking multiple optima in dynamic environments," *IEEE Trans. Evol. Comput.*, published online first, 26 August 2010.

[135] X. Yao and Y. Liu. "Fast evolutionary programming," *Proc. of the Fifth Annual Conference on Evolutionary Programming (EP'96)*, 1996, pp. 451-460.

[136] X. Yao, Y. Liu and G. Lin. "Evolutionary programming made faster," *IEEE Trans. Evol. Comput.*, vol. 3, 1999, pp. 82-102.

[137] S. Zeng, H. de Garis, J. He, and L. Kang. "A novel evolutionary algorithm based on an orthogonal design for dynamic optimization problems. "*Proc. of the 2005 IEEE Congress on Evol. Comput.*,vol. 2, 2005, pp. 1188-1195.

[138] Z. Zhan,J. Zhang, Y. Li and H. S. Chung, "Adaptive Particle Swarm Optimization ", *IEEE Trans. Syst., Man, Cybern. B*, vol. 39, 2009, pp. 1362-1381.

[139] B.-T. Zhang. "A Bayesian framework for evolutionary computation," in *Proc. 1999 Congr. on Evol. Comput.*, 1999, pp. 722-728.

[140] Q. Zhang and Y. W. Leung, "An orthogonal genetic algorithm for multimedia multicast routing," *IEEE Trans. Evol. Comput.,* vol. 3, 1999, pp. 53-62.

[141] MA. Zmuda, MM. Rizki, and LA. Tamburino. "Hybrid evolutionary learning for synthesizing multi-class pattern recognition systems. "*Applied Soft Computing*, vol. 2(4), 2003, pp. 269-282.