Not all judgment aggregation should be neutral

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Abstract. Judgment aggregation is concerned with the problem of aggregating individual views on logically related issues. It offers a general framework in which several different types of aggregation problems can be represented and studied. Furthermore, judgment aggregation can be applied to problems that consider together issues that are typically considered as separate aggregation problems. *E.g.*, judgment aggregation can be used to determine group goals and collectively advanced supporting beliefs. While goals can be seen as preferences, typically studied by preference aggregation, beliefs cannot. Neutrality of a judgment aggregation rule is the property of aggregating the views on all issues in the same manner and it is uniformly considered to be a desirable property. This paper defends the position that multi issue type aggregation problems may require non-neutral judgment aggregation rules, and proposes a method for developing such rules.

Keywords: Judgment aggregation, neutrality, preference aggregation, merging beliefs and goals

1 Introduction

A committee of investors is considering whether or not to buy the stock of company X (proposition ϕ). It is known that company X has launched a new product. Some consider that a market success of X (proposition φ_1) and sufficient available committee funds (proposition φ_2) are reasons to purchase X shares. The committee thus needs to assign truth values to ϕ, φ_1, and $(\varphi_1 \wedge \varphi_2) \rightarrow \phi$. The committee is making these decisions around lunch time and they are feeling a bit peckish. Typically they order pizza, however some are contemplating ordering burgers instead. They have to yet find out where to order the burgers from, but there are really only two options for pizza in town: Napoli and Peppe's. If people are not worried about spending too much money for lunch (proposition sPv), then Napoli is far better than Peppe's, at least according to some members' opinion. After some discussion they realise they that what they have to decide is: whether pizza is better than burger (proposition pPb), whether Napoli is indeed better than Peppe's (proposition nPe) and whether $(pPb \wedge sPv) \rightarrow nPe$.

Judgment aggregation is a theory concerned with finding a combination of consistent collective opinions on logically related issues by aggregating individual views over those issues. It offers a general framework in which several different types of aggregation problems can be represented and studied. Judgment aggregation can be used to solve both the lunch and the investment problems of the committee. Furthermore, both committee problems have the same underlying structure. The same judgment aggregation rule can be applied to determine both whether to invest in X, which is a belief aggregation problem, and whether to order pizza from Napoli, which is a preference aggregation problem[1].

That same framework abstractness that allows for such versatility of problems representable in judgment aggregation also allows for more complex problems to be represented in it as well, problems that consider together issues that are typically considered as separate aggregation problems. Such framework flexibility can be useful, particularly in multi-agent system's settings.

Self governing cooperating agents would encounter the need to agree on which beliefs to uphold as a group and which goals to collectively pursue. Beliefs and goals influence one another. A rational agent is one that chooses her goals in accordance to the believes she holds about the world, but also that adjusts her goals as her beliefs change. If the same consideration is to be extended to groups of agents, a choice on whether to pursue a goal cannot be made separate from the decisions on what to believe as a group. Agreements can be reached by aggregating, or merging, the individual opinions of the agents; in the case of determining collectively held beliefs and goals, by aggregating the individual beliefs and goal choices of the agents [15, 3]. Judgment aggregation can be used to concurrently reach agreements regarding goals and beliefs.

Judgment aggregation was originally concerned with collective reasoning in collegiate courts of law [16, 17]. It was later shown that the same theory can be used to represent preference aggregation problems [24]. Further, judgment aggregation has been considered for the purpose of: identifying collective goals and collectively supported beliefs [3, 28], finding collective argument labelings in argumentation theory [4, 2], and most recently also to collective annotate linguistic resources [11].

A judgment aggregation problem is specified by an agenda and possibly a set of constraints. An agenda is the set of, possibly logically related, issues for which individual judgments are collected and collective judgments are requested. The issues are typically binary questions to which a positive (yes, true, agree, etc.) or a negative answer (no, false, disagree, etc) can be given. A judgment is "an answer" to an issue. The constraints are a set of formulas expressing additional relations among the agenda issues that must be observed when "answering" the issues. In the logical framework of judgment aggregation, in line with [6], an agenda issue is represented in an abstract manner with a pair of propositions $\varphi, \neg\varphi$. A positive "answer" or judgment is represented with φ, and a negative one with $\neg\varphi$.

The abstract representation of the judgments and agenda issues induces the versatility of the judgment aggregation framework. It also allows us to focus

[1] Remark that a preference aggregation problem would typically be represented as the problem of selecting one from the list of options: order pizza from Peppe's, order pizza from Napoli, order a burger, as specified in Section 4.

on the relations between individual and collective judgments and to study the properties of aggregation functions in an abstract way, with findings of such studies extending to all the domains of application. The abstract representation of the agenda issues is beyond reproach when all the agenda issues have the same meaning, *i.e., are of the same type*, for example when they are all beliefs, or all preferences. However when the agenda contains a mixture of different types of issues, as is the case when judgment aggregation is used to reach an agreement on a common goal, the "flat" representation is no longer adequate.

In [6] it was shown that the representation language of the agenda, in general, cannot be exploited to circumvent the impossibility results regarding judgment aggregation rules [24]. This result appears to have ended the interest in the representation language in judgement aggregation. However, the representation of the agenda issues is rather the lesser problem, more crucial is that different types of issues would require different treatment by the aggregation function, which is not the case at present.

The property of aggregation functions which stipulates that all issues should be aggregated in the same fashion is called *neutrality*. At present, neutrality is considered desirable to that extent, that non-neutral rules virtually have not been designed and a precise formulation of the neutrality property has not been given. When the agenda has the same type of issues, neutrality seems natural to require. However there are agendas where this is not the case. Consequently, neutrality should not be considered a *de facto* desirable condition in judgment aggregation and non-neutral aggregation methods must be explored. Furthermore, the type of issues being aggregated must be taken into consideration and desirable properties of aggregation should be associated with the distinct type.

This work is structured as follows. In Section 2 we elaborate on different types of issues that can be found in the agendas of various judgment aggregation problems. In Section 3 we give the definitions of the main concepts in the judgment aggregation framework. In Section 4 we discuss the property of neutrality and other assumed-desirable properties. In Sections 5 one method of non-neutral aggregation is proposed. In Section 6 we make a summary and outline directions for future work.

2 Types of issues and their aggregation

The representation of issues in the logic framework abstracts from the meaning of those issues. An issue $\varphi, \neg\varphi$ that arrises from a preference has the intuitive reading of "is it the case that you prefer option x to option y". Often special predicates $xPy, \neg xPy$ are used, *e.g.*, in [8, 20], for representing issues that are preferences, or as in the lunch committee decision example. An issue $\varphi, \neg\varphi$ that arises from beliefs has the intuitive reading of "is it the case that you believe φ is true", whereas if that same issue arises from possible agent goals it can be interpreted both as "should goal φ be pursued" and "do you want to pursue goal φ". An issue $\varphi, \neg\varphi$ may represent an, at present, unknown or unobservable fact about the world, *e.g.*, will the project of company X be a market success, thus having the intuitive reading of "are you of the conviction that φ is true".

In a preference aggregation problem all issues in the corresponding judgment aggregation agenda are of the same type – preferences. When a jury considers evidence, all the issues are of the same type – past-observable facts of the world. However, there is nothing preventing anyone from using judgment aggregation on problems with agendas that concurrently have different types of issues.

In fact, the original judgment aggregation problem, the "doctrinal paradox"[18], has an agenda in which two types of issues occur: premises and a conclusion. The agenda in the "doctrinal paradox" consists of three issues. The premises are: *'Is there a contract?'*, and *'Is there an estoppel?'*. The conclusion is the decision on the case – *'Does the judge(s) rule for the plaintiff?'* (or against). The judges must rule in favour of the plaintiff when there was both a contract and an estoppel. The truth of both the issue of the existence of contract and estoppel are facts of the world (which might not be observable), while the decision on the case is the (informed) opinion of the judge(s).

It is straightforward to observe that preferences differ from presently unobservable facts about the world. One agent can prefer pizza over burger (pPb), and another can prefer burger over pizza ($\neg pPb$) but both of these positions can be held at the same time by different agents because one does not invalidate the other; preferences are matters of subjective views and tastes. One agent might be convinced that the project of company X will be a market success (ϕ), while another might be convinced that the project will be failure ($\neg\phi$). However, only one of these judgments is correct, and some time after the project's release it would be known which one. Preferences give rise to *subjectively valued propositions*, while unknown facts give rise to *objectively valued propositions* in the agenda. The truth-value of subjectively valued propositions exists based solely on the bias, opinion, emotions, convictions, etc. of the entity that assigns this value. In contrast, the truth-value of an objectively valued proposition exists independently of the bias of the entity that assigns the value.

Regardless of whether the agenda issue is a subjectively or objectively valued proposition, there may exist only one entity, agent, or source, that is in the unique position to assign the judgment for this issue. For example, I am the only person in the position to decide whether for me Napoli pizza is affordable lunch-food. My affordability preference however might be included in an agenda with other issues that are related but over whose truth-value I do not have full control. Objectively valued propositions yield even better examples of full control issues. Consider the owner of the funds that the committee will invest. Whether there are available funds for investing (if φ_2 is true or false) can only be decided by the owner, as he or she fully controls this resource. An issue can also be fully controlled by a group of agents. For example, Mary and John can together decide to marry each other, thus determining the judgment on the issue of "Mary and John are married". Instances of objectively valued fully controlled judgments can be expected to occur when one agent coordinates with a group of agents, or a group of agents coordinates with another group.

The beliefs and goals give rise to different types of propositions as well. The beliefs of the agents represent the current information that the agent has about the world. The proposition $G\ x$ denotes the decision of the agent to have his future state be one in which x holds. We can view the agent's goals as giving rise to subjective valued propositions, because the truth value of the associated proposition is the prerogative of the agent. The agent however, may not be able to guarantee a future in which x holds, as much as a person can not always guarantee that pizza will be served to them instead of a burger, as per their preference. Beliefs, on the other hand can be viewed as giving rise to objectively valued propositions. Although an agent chooses which beliefs to adopt, the aim of beliefs is to as faithfully as possible represent the state of the world.

The type of the issues that represent labels of arguments in an argumentation context and annotations of linguistic resources would depend on the specific context in which these issues are created.

It is not our aim at this section to give an ultimate taxonomy of issue types that can occur in judgment aggregation but to emphasise that different types do exist and that there is a need for them to be aggregated accordingly. For the purpose of this article it is sufficient to distinguish between objectively valued and subjectively valued issues, and orthogonally, between fully controlled and partially controlled issues.

3 Preliminaries

We begin by introducing the definitions of the basic concepts that can be found in a logical framework of judgment aggregation before discussion the property of neutrality in greater detail.

An agenda $\mathcal{A}$ is a set of propositions, not necessarily atomic, from a set of well formed propositions $\mathcal{L}$. This set $\mathcal{L}$ is often taken to be the set of well formed formulas of the (classical) propositional logic. For each $\varphi \in \mathcal{A}$ it holds that $\neg\varphi \in \mathcal{A}$, and φ is neither tautology nor a contradiction. An agenda issue is a pair $\{\varphi, \neg\varphi\} \subseteq \mathcal{A}$ and a judgment is thus either φ or $\neg\varphi$, corresponding to taking the position that the issue in question is true or false respectively[2]. For example, the agenda of the investment problem is $\{\phi, \neg\phi, \varphi_1, \neg\varphi_1, (\varphi_1 \wedge \varphi_2) \rightarrow \psi, \neg((\varphi_1 \wedge \varphi_2) \rightarrow \psi)\}$, while the agenda of the lunch problem is $\{pPb, \neg pPb, nPe, \neg nPe, (pPb \wedge sPv) \rightarrow nPe, \neg((pPb \wedge sPv) \rightarrow nPe)\}$. In addition to the agenda, one can specify a set of constraints $\Gamma \subset \mathcal{L}$ which constrain further[3] the allowed combination of judgments that can be given for the specified agenda.

A judgment set J is a subset of the agenda that is *complete* when for each issue $\{\varphi, \neg\varphi\} \subset \mathcal{A}$ either $\varphi \in J$ or $\neg\varphi \in J$. For example, the opinions of a committee member that has no monetary concerns, thinks that pizza is preferable to a burger, and that pizza is better ordered from Peppe's are represented as the

[2] A proposition preceded by an odd number of consecutive negations is considered to be a negated proposition, while a proposition preceded by an even number of consecutive negations is considered to be a non negated proposition.

[3] The agenda issues are typically logically related among each other.

set of judgments $\{pPb, \neg((pPb \wedge sPv) \rightarrow nPe), \neg nPe\}$. The judgment set J is *consistent* if it is a consistent set and Γ-consistent if $J \cup \Gamma$ is a consistent set. In the rest of this article we use consistent to denote both consistent and Γ-consistent judgment sets. The set of all non-empty consistent judgment sets for a specified $\mathcal{A}$ and Γ is the set $\mathcal{D}(\mathcal{A}, \Gamma)$, while the set of all consistent judgments sets that are also complete is $\mathbb{D}(\mathcal{A}, \Gamma)$. Clearly $\mathbb{D} \subset \mathcal{D}$.

A profile of judgments $P \in \mathbb{D}(\mathcal{A}, \Gamma)^n$ is a collection of n judgment sets, each associated with an agent, or source, $1 \leq i \leq n$. For convenience, we use $J_i \in P$ to denote that J_i is the i'th element of P. The number of all judgment sets, or agents, in P that contain a particular $\varphi \in \mathcal{A}$ is $N_P(\varphi)$, $N_P(\varphi) = |\{i \mid \varphi \in J_i, \text{ and } J_i \text{is the } i\text{th element of } P\}|$.

A judgment aggregation rule R, also called an *irresolute rule*, is a function of type $R : \mathbb{D}(\mathcal{A}, \Gamma)^n \rightarrow 2^{\mathbb{D}(\mathcal{A}, \Gamma)}$. A judgment aggregation rule maps a profile of judgments to a set of complete and consistent judgment sets. A judgment aggregation function F, also called a *resolute rule*, is defined as a function of type $F : \mathbb{D}(\mathcal{A}, \Gamma)^n \rightarrow \mathbb{D}(\mathcal{A}, \Gamma)$. A resolute rule maps a profile of judgments to a single complete and consistent judgment set. The judgment sets in the set $R(P)$ are called collective judgment sets for P. The elements of a collective judgment set are called collective judgments.

Typically the impossibility characterisation results, namely studies showing which properties of aggregation cannot be satisfied at the same time, have been done for the resolute rules, see for example [24] for an overview. Almost all specific proposed judgment aggregation operators are irresolute rules [25, 19, 26, 5, 10, 7, 12]. The possibly best known judgment aggregation rule is issue-majority defined as $m(P) = \{\varphi \mid \varphi \in \mathcal{A}, N_P(\varphi) > \frac{1}{2}\}$. A profile $P \in \mathbb{D}(\mathcal{A}, \Gamma)^n$ is majority-consistent if $m(P)$ is a consistent set, *i.e.*, $m(P) \in \mathcal{D}(\mathcal{A}, \Gamma)$.

The following properties have been defined for irresolute rules, however the definitions extend in a straightforward way to resolute rules as well.

A judgment aggregation rule R is *anonymous* if for every $P \in \mathbb{D}(\mathcal{A}, \Gamma)^n$, and every $\mathcal{A}$ and Γ, for every permutation σ on the order of the judgment sets in P it holds that $R(P) = R(\sigma(P))$.

A judgment aggregation rule R is *majority-preserving* if for every majority-consistent profile $P \in \mathbb{D}(\mathcal{A}, \Gamma)^n$, and every $\mathcal{A}$ and Γ, it holds that $R(P) = \{J \mid m(P) \in \mathcal{D}(\mathcal{A}, \Gamma) \text{ and } m(P) \subseteq J\}$. Intuitively, whenever a profile P is majority-consistent and $m(P)$ is a complete judgment set, then a majority-preserving function R would select only the $m(P)$ as a collective judgment set. For those majority-consistent profiles for which $m(P)$ is not a complete judgment set, only all the complete and consistent judgment sets that are supersets of $m(P)$ are selected as collective judgment sets.

Systematicity is one of the first properties studied, together with anonymity [23, 24], and defined for resolute judgment aggregation rules. It is composed of two properties: *neutrality* and *independence of irrelevant alternatives* (IIA). A judgment aggregation function F satisfies IIA if for all all $\varphi, \phi \in \mathcal{A}$, and all $\mathcal{A}$, Γ, for every $P, P' \in \mathbb{D}(\mathcal{A}, \Gamma)^n$, where $P = \langle J_1, \ldots J_n\rangle$ and $P' = \langle J'_1, \ldots J'_n\rangle$, if [$\varphi \in J_i$ iff $\phi \in J'_i$], then [$\varphi \in F(P)$ iff $\phi \in F(P')$]. Intuitively, a function satisfies

IIA if the collective judgment on each issue depends only on the individual judgments for that issue in the profile[4].

The *neutrality property* of a judgment aggregation rule R is informally the property: the collective judgment on an issue depends only on the number of positive and negative judgments given for that issue in the profile.

A version of neutrality called *issue-neutrality*, has been formally defined for resolute rules in [13] in the following way. A judgment aggregation function F is issue-neutral if for all $\varphi, \phi \in \mathcal{A}$, and all $\mathcal{A}$, Γ, $P \in \mathbb{D}(\mathcal{A}, \Gamma)^n$, where $P = \langle J_1, \dots J_n \rangle$, if [for all $J_i \in P$, $\varphi \in J_i$ iff $\phi \in J_i$], then [$\varphi \in F(P)$ iff $\phi \in F(P)$].

It is not straightforward to extend issue-neutrality to irresolute rules because there are several ways to do this, each varying in strength. The strongest corresponding property can be expressed as in Definition 1.

Definition 1. *An irresolute judgment aggregation rule R is strongly issue-neutral when for all $\varphi, \phi \in \mathcal{A}$, and all $\mathcal{A}$, Γ, $P \in \mathbb{D}(\mathcal{A}, \Gamma)^n$, where $P = \langle J_1, \dots J_n \rangle$, if [$\varphi \in J_i$ iff $\phi \in J_i$, for $1 \leq i \leq n$], then [for all $J \in R(P)$, $\varphi \in J$ iff $\phi \in J$].*

4 Desirable properties in judgment aggregation

The initial inspiration for a minimal set of desirable properties in judgment aggregation was the desirable properties for preference aggregation according to Arrow [1]. A preference aggregation problem is specified as follows. Given is a set of options (or candidates, or alternatives) $\mathcal{O}$ a preference profile is a collection of n preference orders $\succsim_i$, $1 \leq i \leq n$, over $\mathcal{O}$. A preference aggregation function maps a preference profile to a preference order over $\mathcal{O}$. A preference aggregation function is neutral if the options are treated the same, regardless of what the option is. It is reasonable to expect that all alternatives are treated the same by the aggregation function.

Non-neutral aggregation functions that satisfy IIA are still *dictatorial*. A resolute judgment aggregation function F is dictatorial when for every profile P the same collective judgment J is selected. Note that in the original formulation of the framework and the definition of the judgment aggregation function, functions were defined as $F(J_1, \dots J_n) = J$, where $J_1, \dots, J_n, J$ are subsets of the agenda but not required to be complete nor consistent. Two additional properties for F have been additionally explicitly defined: *universal domain* stipulates that function F has to be defined for each possible profile; *collective rationality* stipulates that the collective judgment set J is a consistent and complete set. The premise based rule [9], that satisfies IIA , and is anonymous and neutral, does not satisfy universal domain.

Judgment aggregation irresolute rules that do not satisfy IIA have been considered [25, 19, 26, 5, 10, 7, 12], but all of them consider only the support for a judgment on an issue and not the issue itself, *i.e.*, they are neutral in the informal sense. Their neutrality is not explicitly considered, testifying to the universal understanding of neutrality as a desirable property for rules beyond discussion.

[4] Observe that the co-domain of F(P) is the set of consistent and complete judgment sets, thus $\phi \notin F(P')$ implies $\neg\phi \in F(P')$

A property that also appears to be universally accepted as desirable, possibly also as a legacy from the close connections between judgment aggregation and preference aggregation, is majority-preservation. A rule that is majority-preserving, when applied to a preference aggregation problem represented as a judgment aggregation problem, produces collective judgment sets that contain the Condorcet winner when such an alternative exists.

The motivation for desiring to maximise the number of collective judgments supported by a majority in the profile, which is the typical method for constructing majority-preserving rules, is clear when the agenda issues are preferences. Since every agent is the ultimate authority on what they want, and since not everyone can get what they individually want, the aim of the aggregation is to maximise the satisfaction of as many agents as possible. However, if the issues are objectively valued, it does not matter how many agents are "happy" with the collective judgments. What matters is to increase the likelihood that the issues' objective truth-values coincide with the assigned collective judgments.

The Condorcet jury theorem [14] states that if the agents are more likely than not to individually find the correct valuation for a proposition, than the valuation supported by the majority is most likely to be correct. However the propositions in question are independent, whereas the issues in an agenda are logically related [22]. When the conditions of the Condorcet jury theorem do not apply, it is no longer clear why a judgment aggregation rule should be majority-preserving. Furthermore, in the case of an agenda with objectively valued issues, rational agents can be expected to be more "satisfied", or have an increase in utility, with the increase of the likelihood of truth of the collective judgments, not with the increase of their likelihood to their own opinions. In conclusion, majority-preservation can be clearly seen as desirable only in the case of subjectively valued propositions.

Lastly let us briefly concern ourself with the desirability of the anonymity property. Clearly, when there is no way of establishing the relevance or reliability of a judgment's source, it is prudent to seek anonymous judgment aggregation rules. However, when the value of an agenda issue is fully controlled by some, this agent, or these agents, should have veto over the issues they control. To aggregate an agenda that consists both of fully controlled issues and partially controlled issues, one needs a rule that is non-neutral and non-anonymous with respect to the fully controlled issues.

5 Sequential judgment aggregation

Having argued that a judgment aggregation rule should only be neutral if the agenda issues are type-homogenous, we need to consider methods for constructing non-neutral judgment aggregation rules. A judgment aggregation rule that satisfies IIA can be easily made non-neutral: since each issue is aggregated separately, one simply needs to use different, type-adequate, rules, to aggregate every issue. However, since IIA rules will necessarily violate either universal domain or anonymity, using a IIA rule may not be possible.

One option to treat issues differently is to apply, possibly different, judgment aggregation rules sequentially to the profile. This method is what is used when the premise based rule [9] is applied, and also in the complete conclusion based procedure of [27]. The method of [27], applicable to agendas that are split into premises and conclusions, is to use a resolute judgement aggregation rule to one issue of the agenda, the conclusion issue, and add the resulting collective judgment to the constraints before aggregating the full agenda with an irresolute rule. An extreme example of a sequential rule is the one considered in [21], where each issue is aggregated one by one. A similar approach can be used to aggregate profiles for mixed issue type agendas: to aggregate the agenda sequentially in several steps, at least one for each different type of issue. Thus, all of the completely controlled issues would form one issue type, that can possibly be further divided to a subset of objectively valued issues and a subset of subjectively valued issues.

In [27], issue-majority over one issue and in the context of an odd number of agents, which is a resolute, is used as the first step. In general a resolute rule might not be available, meaning that several collective judgment sets might be the result of a first aggregation step. Since the collective judgments from different collective judgment sets can be mutually inconsistent, they cannot all be included in the constraints of the next step. There are several options on how to select the constraints. For example, one can: a) add the disjunction of the collective judgment sets; b) add the intersection of the collective judgment set; c) select one of the collective judgment sets using a tie-breaking mechanism; etc.

The definitions of the logic framework now need to be extended.

Definition 2.

Partial agendas. *An agenda $\mathcal{A}$ is partitioned into k partitions when $\mathcal{A} = \mathcal{A}_{p_1} \cup \mathcal{A}_{p_2} \cup \cdots \cup \mathcal{A}_{p_k}$, and all the partitions $\mathcal{A}_{p_1}, \mathcal{A}_{p_2}, \cdots, \mathcal{A}_{p_k}$ are mutually disjoint. Given an aggregation order, namely some permutation of the partitions of the agenda $\sigma(\langle \mathcal{A}_{p_1}, \ldots, \mathcal{A}_{p_k} \rangle)$, the step agendas are obtained as $\mathcal{A}_1 = \sigma(\mathcal{A}_{p_1})$, $\mathcal{A}_2 = \sigma(\mathcal{A}_{p_2}) \cup \mathcal{A}_1$, $\ldots$, $\mathcal{A}_k = \sigma(\mathcal{A}_{p_k}) \cup \mathcal{A}_{k-1} = \mathcal{A}$.*

Partial profile. *A partial profile P_i, $1 \leq i \leq k$ for profile $P =< J_1, \ldots, J_n >$ of the agenda $\mathcal{A}_i$ is defined as $P_i = \langle J_1 \cap \mathcal{A}_i, \ldots, J_n \cap \mathcal{A}_i \rangle$.*

Collective selector. *A collective selector function $\varsigma : 2^{\mathbb{D}(\mathcal{A},\Gamma)} \to 2^{\mathcal{L}}$ maps a set of collective judgment sets into a consistent set of formulas. It holds that $\varsigma(R(P)) = S$, $S \subset 2^{\mathcal{L}}$, $S \not\models \bot$ and if $R(P) = J$, then $\varsigma(R(P)) = J$.*

Partial aggregator. *A partial aggregator function $R_{i,\Gamma_i} : \mathbb{D}(\mathcal{A}_i, \Gamma)^n \to 2^{\mathbb{D}(\mathcal{A}_i,\Gamma_i)}$, where $\Gamma, \Gamma_i \in \mathcal{L}$, $1 \leq i \leq k$ are consistent and mutually consistent constraints, is an irresolute judgement aggregation rule that aggregates a profile of judgment sets consistent with respect to a set of constraints Γ into a set of judgment sets consistent with respect to the set of constraints Γ_i.*

Partial constraints. *An aggregation constraint at step i, $1 \leq i \leq k$, is $\Gamma_i \supseteq \Gamma$, such that $\Gamma_1 = \Gamma$, and $\Gamma_i = \varsigma(R_{i-1,\Gamma_{i-1}}(P_{i-1})) \cup \Gamma_{i-1}$.*

Sequential judgment aggregation rule. *Given a k-partitioned agenda $\mathcal{A}$, and an aggregation order σ, a judgment aggregation rule R is sequential when*

$R(P) = R_{k,\Gamma_k}(P)$, and $R_{i,\Gamma_i}(P_i)$, for $1 \leq i \leq k$, is recursively defined as: $\Gamma_1 = \Gamma$, and $\Gamma_i = \varsigma(R_{i-1}, \Gamma_{i-1}(P_{i-1})) \cup \Gamma_{i-1}$.

Some clarifying observations are due before an illustrative example. The sequential judgment aggregation rule is not neutral even when all partial aggregators used are the same functions, because the issues at each step are considered under different constraints.

The collective selector selects $R(P)$ iff $R(P)$ is a singleton and employs some selection method when $R(P)$ is not a singleton, such as the disjunction of all the judgment sets produced by $R(P)$, their intersection, etc.

At the first step of aggregation, all the judgment sets in the partial profile are consistent with the aggregation constraint Γ. At each subsequent step, Γ is made stronger, namely more selective, by the outcome of the previous aggregation step. The partial judgment sets in the subsequent partial profiles may not be consistent with the new constraints Γ_t, but the result of the aggregation must be. Since at each step the constraints are obtained by adding the results of the aggregation in the previous constraints, it is not possible, at any point for Γ_t to be inconsistent with Γ_{t+1}. In the last step, the full profile $\mathcal{A}$ is aggregated with the strongest constraint Γ_k.

The judgment aggregation functions so far defined are not from the desired type as partial aggregator function R_t since they require that the same constraint Γ be imposed both on the judgment sets in the profile and on the collective judgment sets obtained by aggregation. However, the aggregation rules from [19, 10, 7], can be extended in a straightforward fashion, into partial aggregators.

Consider for example the $MCSA_\Gamma$ from[20], also known as the Slater rule for judgments, defined as $MCSA_\Gamma(P) = \mathit{MaxCardCons}(m(P), \Gamma)$, where $\mathit{MaxCardCons}(m(P), \Gamma))$ denotes the maximum cardinality set of Γ-consistent subsets of $m(P)$. The $MCSA_{i,\Gamma_i}$ partial aggregator can be defined as $MCSA_{i,\Gamma_i}(P_i) = \mathit{MaxCardCons}(m(P_i), \Gamma_i)$. Another example is the MWA_Γ from [20], also known as the median rule, defined as $MWA_\Gamma(P) = \underset{J \in \mathbb{D}(\mathcal{A},\Gamma)}{\text{argmax}}\, W_P(J)$, where $W_P(J) = \sum_{\varphi \in J} N_P \varphi$. The MWA_{i,Γ_i} partial aggregator can be defined as $MWA_{i,\Gamma_i}(P_i) = \underset{J \in \mathbb{D}(\mathcal{A}_i,\Gamma_i)}{\text{argmax}}\, W_P(J)$, where $W_P(J) = \sum_{\varphi \in J} N_{P_i} \varphi$.

Example 1. Let $\mathcal{A} = \mathcal{A}_{p_1} \cup \mathcal{A}_{p_2} \cup \mathcal{A}_{p_3}$, with $\mathcal{A}_{p_1} = \{p, \neg p, q \neg q,\}$, $\mathcal{A}_{p_2} = \{r, \neg r\}$, and $\mathcal{A}_{p_3} = \{s, \neg s\}$. Let $\Gamma = \{(p \wedge q) \leftrightarrow r, r \rightarrow s\}$. Consider the profile given in Table 1. Assume we aggregate first $\mathcal{A}_1 = \mathcal{A}_{p_2}$ using the issue-majority rule, then $\mathcal{A}_2 = \mathcal{A}_{p_1} \cup \mathcal{A}_1$ using $MCSA_{2,\Gamma_2}$ and last $\mathcal{A}_3 = \mathcal{A}$ using the MWA_{3,Γ_3}. Let the selector function be $\varsigma(R_i(P_i)) = \bigvee R_i(P_i)$, $1 \leq i \leq 3$. The collective judgments of, and constraints applied in, each aggregation step are also given in in Table 1.

The agenda can be split into as many partitions as there are issue types, with the issues in one partition being all of the same type. The question is then which type of issues should be given precedence in the order of aggregation. It is questionable whether a uniform rule for type order in sequential aggregation can be established in this regard.

	Agenda				Constraints
	$\mathcal{A}_{p_1}$		$\mathcal{A}_{p_2}$	$\mathcal{A}_{p_3}$	
	$\{p, \neg p,$	$q, \neg q$	$r, \neg r,$	$s, \neg s\}$	$\Gamma = \{(p \wedge q) \leftrightarrow r, r \rightarrow s\}$
$J_1 =$	$\{p$	q	r	$s\}$	
$J_2 =$	$\{p$	$\neg q$	$\neg r$	$s\}$	
$J_3 =$	$\{\neg p$	q	$\neg r$	$\neg s\}$	
$m(P_1) =$			$\{\neg r\}$		$\Gamma_1 = \Gamma$
$MCSA_{2,\Gamma_2}(P_2) =$	$\{\{$ p,		$\neg r\},$		$\Gamma_2 = \Gamma_1 \cup \{\neg r\}$
		$\{$ q,	$\neg r\}\}$		$\Gamma_2 = \Gamma_1 \cup \{\neg r\}$
$MWA_{3,\Gamma_3}(P) =$	$\{\{p,$	$\neg q,$	$\neg r,$	$s\},$	$\Gamma_3 = \Gamma_2 \cup \{(\neg p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge \neg r)\}$
	$\{\neg p,$	$q,$	$\neg r,$	$s\}\}$	$\Gamma_3 = \Gamma_2 \cup \{(\neg p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge \neg r)\}$

Table 1. An example of sequential aggregation.

It can be consistently assumed that the assigned values of the fully controlled issues, by their respective controllers, are mutually consistent. If these judgments are not mutually consistent, then either they are not really fully controlled, or there is a problem with setting the agenda. This assumption also imposes that the first aggregation step is allowing each of the controllers to establish the judgments for these issues, regardless of whether they are objectively, or subjectively valued, and adding these judgments to the aggregation constraints, forming the constraints for the next step. The partially controlled judgments can be considered in two separate agenda partitions with respect to whether they are subjectively or objectively valued. It is less clear how to aggregate the latter.

On one hand, a rational group of agents would be expected to base its decisions on what to desire based on what it determines is most likely to be the true state of the world. Thus the decisions on which goals should be pursued, should be made after making the decisions on what beliefs to support as a collective. This would imply that the subjectively valued issues should be aggregate after the objectively valued ones. However, an agent might choose what to adopt as the true state of the world based on what goals it has chosen to proceed, particularly if the nature of the objectively valued issues is such that their truthfulness cannot be expected to be established in the forceable future. Consider a family considering to join a religious denomination. It is unforeseeable when it would be established whether god X exists or not, however this can still be considered an objectively valued issue. The members of the congregation of believers in X are offered free housing. The family might first decide to pursue the goal of obtaining free housing and then decide that god X exits.

In conclusion, the decision of order of aggregation should be further specified by a more detailed consideration of the context of the aggregation problem.

6 Summary and Future work

The judgment aggregation framework is sufficiently abstract to allow a variety of aggregation problems to be represented as judgment aggregation problem. Thus studying the properties of judgment aggregation we obtain results that extend to all related aggregation problems. However, what the literature has failed to observe, or at least visibly emphasise, is that the abstractness of the framework also allows for more complex aggregation problems to be represented as well. The goal of this paper is to put a spotlight on this issue and advocate for more research attention to be directed towards aggregating for agendas that contain more than one type of issues.

A judgment aggregation rule is neutral if all the agenda issues are given equal treatment when aggregated. Neutrality has been uniformly accepted as a desirable property for rules, however this desirability is unfounded in the case of complex agendas. There are several issues that need to be pursued when developing non-neutral rules. Primarily, there exists no formal definition for neutrality for irresolute rules, although these are the only type of rules that exists which are anonymous and satisfy universal domain. One definition was offered here, however there are several approaches that can be taken and the best one, if it exists, should be established. Although some discussion is included here regarding the type of issues that can be considered in an agenda, the presented issue taxonomy is neither exhaustive nor precise, and needs to be improved in these regards. Further, adequate aggregation properties need to be associated for each issue types. For instance, majority-preserving rules are desirable for subjectively valued, but maybe not for objectively valued issues.

The here-defined sequential judgment aggregation rules and framework allow for neutral rules to be used in combination that yields non-neutral aggregation. However, the approach of sequential judgment aggregation opens its own set of unexplored problems. One is the issue of the selector function that determines the formation of constraints for each aggregation step. Three examples of selectors are given, but this is not an exhaustive list. The advantages and disadvantages of each need to be explored. Another big open issue is the question of property inheritance, for example, if all the partial aggregators satisfy property x is it the case that the resulting sequential aggregation rule will also satisfy x; which properties are inherited and which are not, etc. Lastly there possibly are other methods for non-neutral judgment aggregation beyond the sequential judgment aggregation rules.

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