

---

# Formal Concept Analysis for Modelling Students in a Technology-enhanced Learning Setting

Michael A. Bedek, Michael Kickmeier-Rust, and Dietrich Albert

Knowledge Technologies Institute, Graz University of Technology, Austria

{michael.bedek, michael.kickmeier-rust, dietrich.albert}@  
tugraz.at

**Abstract.** We suggest the Formal Concept Analysis (FCA) as theoretical backbone in technology-enhanced learning settings to support a students' learning process in two ways: i) by engaging with concept lattices, the structure of the knowledge domain and the interrelations of its concepts become explicit, and ii) by providing visual feedback in form of open learner modelling, the student's reflection on the own strengths and weaknesses is facilitated. For teachers, the FCA provides intuitive visualizations for a set of pedagogically relevant questions, concerning the performance of students on the individual- as well as on the class-level.

**Keywords:** Formal Concept Analysis, Learning Analytics, Visualizations, Learner Modelling

## 1 Introduction

The increasing availability of comprehensive technology-enhanced learning (TEL) environments or single educational tools and apps enables students to easily advance their knowledge without direct support from a teacher. However, teachers are challenged by the need to provide appropriate learning resources and to keep up with students' learning progress without reverting to exams or tests [1]. Learning analytics and educational data mining are two highly interrelated research fields that aim to help teachers and educators to make previously hidden insights explicit (e.g. [2]). When applying learning analytics and educational data mining in schools, it is of high importance to meet the requirements of teachers and students. Teachers usually want to have user-friendly tools that help them to reduce the time required for personalized assessment and tailored competence development of their students.

We suggest the Formal Concept Analysis (FCA) as a framework for addressing these requirements. A so-called FCA-tool has been developed in the course of the EU-funded project weSPOT (<http://wespot.net/home>), which provides a *Working Environment with Social and Personal Open Tools* to support students in developing their inquiry based learning skills. In the context of weSPOT, the FCA-tool is mainly used by students by guiding them through a knowledge domain, predefined and en-

riched with learning resources by their teacher. For a more technical description of the FCA-tool's features see [3]. In weSPOT, the FCA-tool supports learners by enabling domain and open learner modelling. The fields of application of the FCA in general, and the FCA-tool in particular, have been extended in the course of the LEA's BOX project (<http://leas-box.eu/>) which stands for Learning Analytics Toolbox. In the context of LEA's BOX, the FCA-tool is mainly used by teachers for student modelling and visualization of educational data. By applying the formal concept analysis with students' performance data, a set of pedagogically relevant questions for teachers can be addressed and visualized.

## 2 Formal Concept Analysis

The FCA describes concepts and concept hierarchies in mathematical terms, based on the application of order and lattice theory [4]. The starting point is the definition of the formal context  $K$  which can be described as a triple  $(G, M, I)$  consisting of a set of objects  $G$ , a set of attributes  $M$  and a binary relation  $I$  between the objects and the attributes (i.e. " $g I m$ " means "the object  $g$  has attribute  $m$ "). A formal context can be represented as a cross table, with objects in the rows, attributes in the columns and assigned relations as selected cells. An example of a formal context is shown in Fig. 1. This formal context has been created by the FCA-tools *Editor View*. Teachers use the *Editor View* to define the formal context and to add learning resources (URLs or files) which can be assigned to both objects and attributes, respectively.

	Is toxic	hatched from egg	is able to fly	lives in/on the water	is able to swim	live birth	performs photosynt...	bear fruits
Bumble-bee	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Bee	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Tree frog	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Goldfish	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Root vole	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

Fig. 1. FCA-tool's *Editor View* for creating a domain with objects, attributes, and relations.

In order to create a concept lattice, for each subset  $A \in G$  and  $B \in M$ , the following derivation operators need to be defined:

$$A \mapsto A' := \{m \in M \mid g I m \text{ for all } g \in A\}, \text{ which is the set of common attributes of the objects in } A, \text{ and}$$

$$B \mapsto B' := \{g \in G \mid g I m \text{ for all } m \in B\}, \text{ which is the set of objects which have all attributes of } B \text{ in common.}$$

A formal concept is a pair  $(A, B)$  which fulfils  $A' = B$  and  $B' = A$ . The set of objects  $A$  is called the extension of the formal concept; it is the set of objects that encompass the formal concept. The set  $B$  is called the concept's intension, i.e. the set of attributes, which apply to all objects of the extension. The ordered set of all formal concepts is called the concept lattice  $\mathfrak{B}(K)$  (see [5] for details), which can be represented as a labelled line diagram (see Fig. 2).

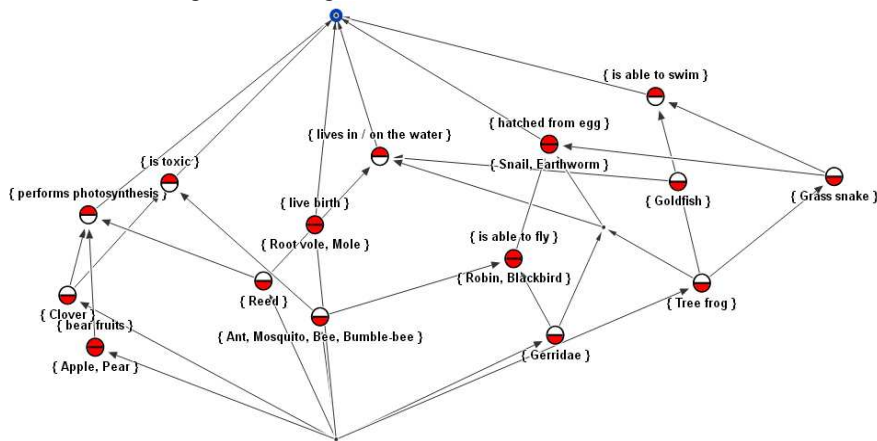


Fig. 2. Concept lattice

The concept lattice shown in Fig. 2 has been created by the FCA-tool's *Lattice View*. Every node represents a formal concept. The extension  $A$  of a particular formal concept is constituted by the objects that can be reached by descending paths from that node. As an example, the node with the label "Goldfish" has the extension  $\{ \text{Goldfish, Tree frog} \}$ . The intension  $B$  is represented by all attributes that can be reached by an ascending path from that node. In the example above, the intension consists of  $\{ \text{is able to swim, lives in / on the water} \}$ .

### 3 Domain Learning and Open Learner Modelling

Once the teacher has created the formal context, students can explore the resulting concept lattice by engaging in interactive graph visualizations (see Fig. 2). By selecting a node, the corresponding concept's extension and intension are illustrated in a highlighted manner. The concept lattice makes the structure of the knowledge domain and the interrelations of its concepts explicit. Similar as for concept maps, this kind of graphic organizer aims to facilitate meaningful learning by activating prior knowledge and illustrating its relationship with new concepts [6].

In case the teacher also assigned learning resources to the objects and attributes in the FCA-tools *Editor View* open learner modelling can be supported (see Fig. 3). Visualizations of open learner models (for an overview see [7]) are aiming to facilitate reflection on the side of the students and to support teachers to better understand strengths and weaknesses of their students.

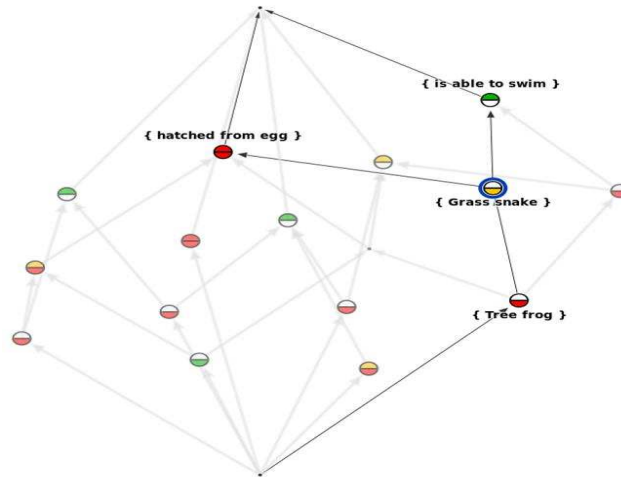


Fig. 3. FCA-tools *Lattice View* for visualizing domain- and learner models.

The FCA-tool's *Lattice View* applies the often-used traffic-light analogy (see e.g. [8]) to show the student the extent to which he or she already consumed learning resources.

#### 4 Applying the FCA(-tool) as a teacher

Similar as [9] who were the first who applied the FCA with students and their performance data we suggest formal contexts with student as “attributes” and problems or test-items as “objects”. The relation between these two sets means “student *m* has solved test item *g*”.

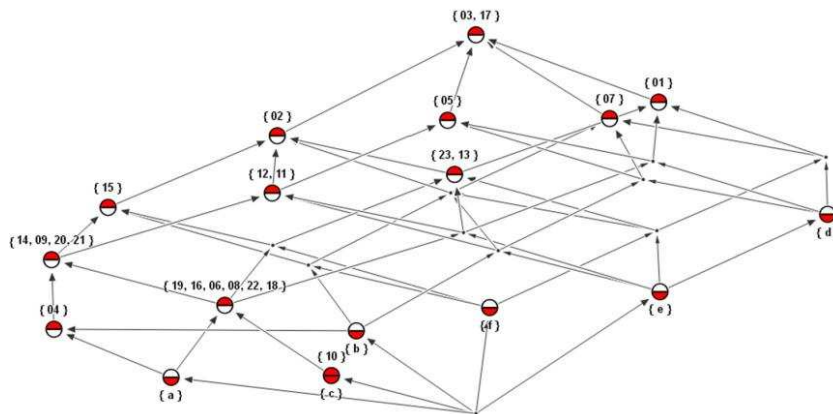
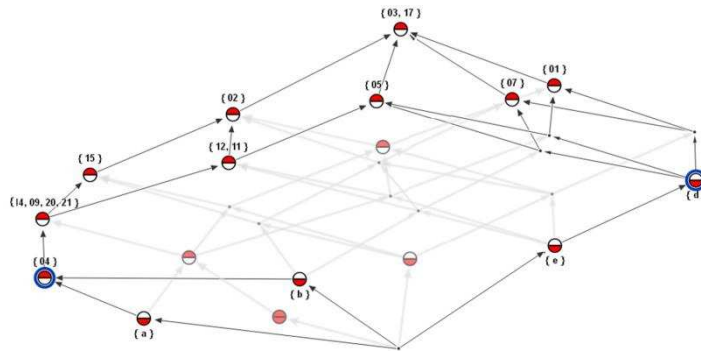


Fig. 4. Concept lattice with students as attributes (numbers from 01 to 23) and test items (letters *a*, *b*, *c*, *d*, *e*, and *f*) as objects (data reported by [10]).

An example of a concept lattice which results from such a formal context is shown in Fig. 4 (the data has been reported by [10]). As briefly outlined in the following sections, such a concept lattice visualizes answers to a set of pedagogical questions which are of high interest for teachers.

#### 4.1 Depicting information from the formal concepts extensions and intensions

As mentioned above, the set of test items which have been solved by a particular student can be directly depicted from the extension of the formal concept with the students' label assigned to it. As an example in Fig. 4, student 10 is the only one who solved only a single test item, *c*, and students 03 and 17 (assigned to the top element of the concept lattice) mastered all problems. When clicking on a particular node the formal concept's extension and intension is highlighted. As an example shown in Fig. 5 (left side), the student 04 has successfully mastered the test items *a* and *b*.



**Fig. 5.** The extension represents the set of test items solved by a student (see student 04) and the intension represents the set of students who solved the particular test item (see test item *d*)

The intension of a formal concept which has an object-label assigned to it indicates the set of students which have successfully mastered the according test item. As an example, the problem *d* in Fig. 5 (right side) has been solved by the students 01, 03, 05, 07 and 17. As it can be also seen, this formal concept located above the formal concept with the object-label *e* assigned to it. This means, that all students who solved item *d* were also able to solve item *e*.

#### 4.2 Highlighting overlaps and differences of students performances

The performances of two or more students can be compared when examining the intensions of the formal concepts with the according attribute-labels. As an example, the students 07 and 15 mastered different subsets of problems (see Fig. 4): Student 07 mastered the items *b*, *d*, *e* and *f* while student 15 mastered the items *a*, *b*, *c*, and *f*. Both students mastered items *b* and *f* (which is the set closure of their intensions) and together they mastered all problems (which is the set union of their intensions).

As a teacher, such kind of information might be of great interest since it helps to effectively arrange groups of students when aiming for collaborative, peer-learning (where students learn together in groups). In the example above, the students 07 and 15 together could be tutors for other students.

### 4.3 Visualizing a classrooms' learning progress over time

The concept lattices shown in Fig. 4 and Fig. 5 are the result of a formal context which is an evaluation of the students' performances at a certain point in time. However, in some cases it might be of great interest for a teacher to observe the learning progress over a longer period of time. Ideally, all students might be able to master all items at the end of course or the semester. In such a case, all cells in the formal context would be filled with crosses. This would result in a concept lattice with only a single formal concept. Fig. 6 exemplifies such a learning progress over time. The concept lattice in the middle results from adding one solved item to the students' performance states (except for the students 03 and 17). The concept lattice on the right results from adding another item to the student's performance states.

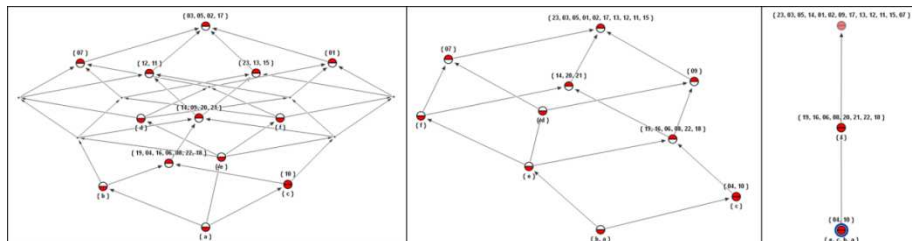


Fig. 6. Concept lattices changing over time reflect the learning progress of the class of students

In general, the visual appearance of the concept lattice gives a first impression of the student's coherence with respects to their performance: A concept lattice which looks "complex" due to a large amount of formal concepts is an indication for a high diversity among the students' performances.

## 5 Discussion and Outlook

In the previous sections, we suggested to apply the FCA to support students and teachers. Students apply the FCA, respectively the FCA-tool, to learn a knowledge domain by interacting with the concept lattice which makes previously hidden interrelationships between the domain's concepts explicit. In addition to that, a student's reflection upon his or her learned and still-to-learn concepts is supported by an open learning modelling approach. Summative evaluation studies on the effect of these pedagogical principles are still ongoing in the course of the weSPOT project.

In the context of LEA's BOX, also teachers apply the FCA(-tool) to visualize the answers to a set of pedagogical questions which are of high interest for them. These

pedagogical questions described in this paper are the result of small focus groups and interviews with teachers in the early phase of the LEA's BOX project. The resulting visualizations as shown above are currently in the spotlight of formative, qualitative evaluation studies with small focused groups of teachers. Current work on the technical side of the project focuses on the development of interactive visualizations which can be easily used by teachers in the classroom. Early feedback of teachers concerns the complexity of the concept lattices, in particular when dealing with a great amount of problems (respectively competences and skills). Conceptual research and the elaboration of ideas on how to reduce this complexity without reducing the amount of information which can be extracted and deduced from the visualizations will be the main focus of our work in the near future.

**Acknowledgement.** The research leading to these results has received funding from the European Community's Seventh Framework Program (FP7) under grant agreements no 318499 (weSPOT project) and no. 619762 (LEA's BOX). This document does not represent the opinion of the EC and the EC is not responsible for any use that might be made of its content.

## References

1. Shute, V.J.: Simply Assessment. *International Journal of Learning, and Media*. 1, 1-11 (2009)
2. Steiner, C., Kickmeier-Rust, M., Albert, D.: Learning Analytics and Educational Data Mining: An Overview of Recent Techniques. In: *Proceedings of the EC-TEL 2014 workshop 'Learning analytics for and in serious games'*, pp. 6 – 15, (2014)
3. Bedek, M., Kopeinik, S., Pruenster, B., Albert, D.: Applying the Formal Concept Analysis to introduce guidance in an inquiry-based learning environment. In: *15th IEEE International Conference on Advanced Learning*. IEEE Press, New York (2015, in press)
4. Wille, R.: Restructuring lattice theory: an approach based on hierarchies of concepts. In: Rival, I. (ed.) *Ordered sets*, pp. 445-470. Reidel, Dordrecht-Boston (1982)
5. Wille, R.: Formal Concept Analysis as Mathematical Theory of Concepts and Concept Hierarchies. In: Ganter, B., Stumme, G., Wille, R. (eds) *Formal Concept Analysis*, pp 1-34. Springer, Berlin (2005)
6. Nesbit, J.C., Adesope, O.O.: Learning With Concept and Knowledge Maps: A Meta-Analysis. *Review of Educational Research*. 76, 413–448 (2006)
7. Bull, S., Kay, J.: Open Learner Models. Nkambou, R., Bordeau, J., Miziguchi, R. (eds.) *Advances in Intelligent Tutoring Systems*, pp. 318-388. Springer, Heidelberg: Springer (2010)
8. Arnold, K.E., Pistilly, M.D.: Course Signals at Purdue: Using learning analytics to increase student success, In: *Proceedings of the 2nd International Conference on Learning Analytics and Knowledge*, pp. 267-270. ACM, New York (2012)
9. Rusch, A., Wille, R.: Knowledge spaces and formal concept analysis. In: Bock, H.H., Polasek, W. (eds.) *Data analysis and information systems: Statistical and conceptual approaches*, pp. 427-436. Springer, Berlin (1996)
10. Korossy, K.: Modeling Knowledge as Competence and Performance. In: Albert, D., Lukas, J. (eds.) *Knowledge Spaces: Theories, Empirical Research, and Applications*, pp. 103-132. Springer, Mahwah, NJ (1999)