Spike-Time Dependant Plasticity in a Spiking Neural Network for Robot Path Planning

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Abstract. This paper will present a path planning technique for autonomous mobile robot, based on the representation of the environment as a cognitive map through a spiking neural network (SNN) of O'Keefe place cells. The method is based on the concept of the travelling wave. For this purpose, we use a biologically plausible neural model (Izhikevitch model) which is the medium of a travelling wavefront stabilized by the Spike-Time Dependant Plasticity (STDP) process. The obstacles are represented by inhibited neurons and the robot by the unique externally excited place cell that initiates the wave. This method produces a gradient map that allows fast and reliable calculation of a feasible path.

Keywords: autonomous mobile robot, path planning, spiking neural network, travelling wave, Spike-Time Dependant Plasticity

1 Introduction

The path planning for autonomous mobile robots is a problem that has been studied extensively for over than 30 years. The aim of this research is to develop algorithms which allow robots to move in an environment, that may be static or dynamic, fully or partially known, in a secure manner; ie according to a path guaranteed without collisions.

The first attempts to solve this problem were based on global methods which can be considered as a search process for a path in a graph which represents the accessible paths along objects. The most widely known global methods are Roadmaps[1], visibility graph method[2][3], cell-decomposition[4], trapezoidal-decomposition[5] and voronoi general decomposition[6]. The main drawback of these methods is their increasing complexity when environment itself becomes complex.

In response to this problem, attempts based on local methods were proposed. The idea was to consider only the goal and the direct close surrounding of the robot, therefore reducing the complexity due to the global environment. The Potential Fields[7][8][9] that consider the environment, the robot and the goal as electric charges is a famous example. The drawback of these methods (and of any local method) is the local minima[10]. Although several suggestions have been presented to circumvent these problems[7][11][12], no algorithm which guarantees an exact solution was presented.

To solve the problems related to global method complexity and the local minima, algorithms based on a environment discretization had been designed. Firstly, these algorithms consider the environment as an occupancy grid, thus solving the problem of the environment complexity. Secondly, the underlying idea of these algorithms is to organise wave propagation in a way similar to waves in water spreading, for instance, around a dropped stone. This expansion wave carries information regarding the distance from it birth point allowing, later, the calculation of a path between the wave birth point and any other point of the environment. Several approaches had been proposed, such as the resistive grids[13][14] or neural networks[15][19][20][21][22]. The main challenge of these methods is to guarantee a wave that grows stably without evolving to chaotic patterns.

Proposed neural networks are effective, nonetheless, biologically implausible. Weidong et al.[22], Yang et al.[15][17][16][18], Glasius et al.[20] and Lebedev et al.[21] propose neural networks built on non-pulsating neurons (type Hopfield) completely unrealistic, the wave stability is guaranteed by parameters checking the Liapunov conditions. Qu et al.[19], propose a spiking neural network, however; to stabilize the expansion of the wave it uses the internal changes in the neurons (change in threshold) which were not observed in nature. Ponulak and Hopfield [28] propose a spiking neural network representing a cognitive map biologically plausible, despite that neurons are simulated with a very basic model (Integrate and fire).

Our work aims to propose a path planner based on a biologically plausible neural network (Izhikevich) and which the wave is stabilized according to a learning process actually implemented in the mammalian brain : the Spike-Time Dependant Plasticity (STDP). We will demonstrate the feasibility of this method through experiments and comparisons with other methods in the literature.

This paper is organized as follows: section 2 outlines our approach to the path-planning problem. Section 3 presents experimentation and results that show the efficiency of our proposal. In Section 4, we discuss the results and perform comparisons. Finally, we present our conclusions and perspectives in the last section.

2 The Navigation Model

The proposed model is based on a spiking neural network (SNN) in which each neuron is modeled according to the Izhikevich neuron model[23] and represents a place cell, an elementary portion of the discretized environment. Place cells are particular neurons in the hippocampus that respond to mostly unique spatial locations. Place cells were discovered by O'Keefe in 1976 [30]. Our SNN is a connected network of place cells that forms a cognitive map and is a medium for a traveling wave passing through the network with the successive activation of different neurons in a huge domino effect. The wave indirectly carries information about the distance by generating a gradient field potential (thanks to the potential of each neuron) around the starting point. This gradient fields allows the calculation of a feasible path by ascending the gradient from any point of the environment to the starting point.

To calculate a consistent path, the wave must be stable; ie to have a single activation front moving from the starting point to the outer limits of the environment. The necessary condition for this stability is that each neuron should fire (activate) a single time at the passage of the wave and should not fire in the future. This is impossible without a control process because an activated parent neuron will necessarily stimulate its neighbors which by emitting their pulses will reactivate the parent neuron, causing the wave instability.

The control process we are proposing in this model is the STDP (Spike-Time Dependant Plasticity). Its a temporally asymmetric form of Hebbian learning induced by tight temporal correlations between the spikes of pre- and postsynaptic neurons. As with other forms of synaptic plasticity, it is widely believed that it underlies learning and information storage in the brain.

We will demonstrate that this rule is sufficient to stabilize the traveling wave in the SNN.

2.1 Network Architecture

The network architecture consists in a grid X of $N \times N$ elements. Each element is an excitatory neuron which represents a portion of the environment (place cell). Each neuron x_i is connected to a maximum of eight immediate neighbors (neurons located in a Euclidean distance less than or equal to $\sqrt{2}$)(Fig. 1). The 8 neighbors form a neighborhood set s_i.

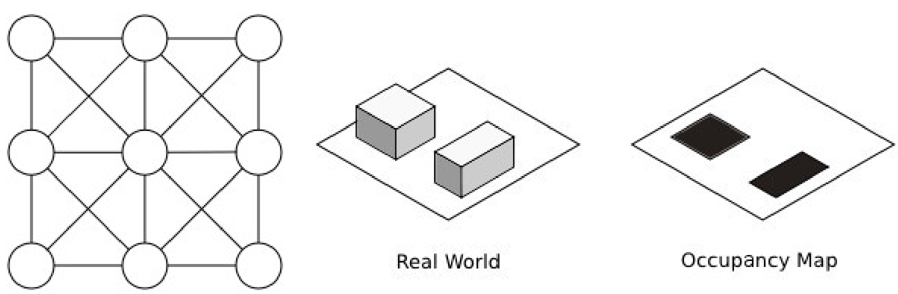

Fig. 1: At left, neuron neighbors in a 8-connected configuration. At center and right, real world and its equivalent occupancy grid.

The initial connections between neurons are synaptic weighted links which are inversely proportionals to the distance:

$$w_{ij} = \begin{cases} 1/d_{ij} & \text{if } d_{ij} \leq \sqrt{2} \\ 0 & \text{else} \end{cases} \quad (1)$$

d_{ij} is the distance between the neuron i and neuron j on the grid. The neuron of the grid has another synaptic link from the occupancy grid E. The occupancy map is a N^2 vector. It also represents a discretization of the environment. Each cell e_i of the map (represented as a vector) has 3 states: free, occupied by an obstacle or occupied by the robot.

$$e_i = \begin{cases} 0 & \text{if free space} \\ -\frac{3}{2} & \text{if occupied by obstacle} \\ +\frac{1}{4} & \text{if occupied by robot} \end{cases} \quad (2)$$

The synaptic connection, between a cell of the occupancy grid and a neuron of the grid, exist only if the cell and the neuron represent the same portion of space. Its weight is $w_{ext}^{k} = 80$ knowing that k is the index of e_k a cell of E and x_k a neuron of X. This connection may be inhibitory in the case of an obstacle or excitatory in the case of a robot.

2.2 Neuron Model

For this proposition, the Izhikevich neural model[23] is used. This model combines the biologically plausibility of Hodgkin-Huxley type [24] dynamics and the computational efficiency of integrate-and-fire neurons. Bifurcation methodologies [25] enable to reduce many biophysically accurate Hodgkin-Huxley type neuronal models to a two-dimensional (2-D) system of ordinary differential equations of the form

$$\frac{dv}{dt} = 0.04v^2 + 5v + 140 - u + I_{syn}(t) \quad (3)$$

$$\frac{du}{dt} = a(bv - u) \quad (4)$$

with the auxiliary after-spike reset:

$$\text{if } v \geq 30 \text{ mV, then} \begin{cases} v \leftarrow c \\ u \leftarrow u + d \end{cases} \quad (5)$$

where $v(t)$ represents the membrane potential, and $u(t)$ represents membrane recovery. The parameter a describes the time scale of the recovery variable u. Smaller values result in slower recovery. The parameter b describes the sensitivity of the recovery variable u to the subthreshold fluctuations of the membrane potential v. The parameter c describes the after-spike reset value of the membrane potential v. The parameter d describes after-spike reset of the recovery variable u. Synaptic currents or injected dc-currents are delivered via the variable I_{syn}. In this model, the synaptic current I_{syn} is the sum of two currents (Eq.

6): external (I_{ext}) and internal (I_{int}) current relative to the fields of neurons. I_{ext} is the synaptic current from a cell in the occupancy map, I_{int} is the sum of synaptic currents from the neighborhood in the neuron field.

$$I^i_{syn}(t) = I^i_{ext}(t) + I^i_{int}(t) = e_i(t)w^i_{ext} + \sum_{x_j \in a_i(t)} w_{ji} \quad (6)$$

Using of Izhikevich model was motivated by the lack of complexity required (13 operations) to simulate a wide variety of neuronal behaviors (22 behaviors) with a very satisfactory accuracy compared to natural counterparts.

2.3 Spike-Time Dependant Plasticity

The STDP is a Hebbian learning rule related to SNNs that is believed to be responsible for learning and storage information in the brain. Schematically, let a neuron x_i connected to a neuron x_j by a synapse going from x_i to x_j with a weight w_{ij}: if the neuron x_i fires before x_j ($t_i^{fire} < t_j^{fire}$ or $\Delta t_{ij} < 0$), the synapse is strengthened (w_{ij} increases) else if the neuron x_i fires after x_j ($t_i^{fire} > t_j^{fire}$ or $\Delta t_{ij} > 0$), the synapse weakens (w_{ij} decreases).

Let $F(.)$ the STDP function, the synapse's weight is changed according to the Delta rule $w_{ij} = w_{ij} + \beta F(\Delta t_{ij})$, β being a weighting of STDP function, Δt_{ij} the difference between t_i^{fire} and t_j^{fire} and $F(.)$ such as:

$$F(\Delta t) = \begin{cases} +A_+ exp(\Delta t/\tau_+) & if \Delta t < 0 \\ -A_- exp(-\Delta t/\tau_-) & if \Delta t \geq 0 \end{cases} \quad (7)$$

The parameters τ_+ and τ_- determine the ranges of pre-to-postsynaptic interspike intervals over which synaptic strengthening and weakening occur. A_+ and A_-, which are both positive, determine the maximum amounts of synaptic modification, which occur when Δt is close to zero[26].

In our proposal, we use the STDP rule to strengthen the synapses between parent neuron and child neurons and especially to significantly weaken the synapses between child neurons and parent neuron. This helps to prevent the reactivation of the parent neuron by child neurons and avoid wave instability.

2.4 Wave Expansion Algorithm

Before introducing the algorithm, some matrices and vectors will be defined. Let V be a vector representing the membrane potential of all the neurons (N^2) of the network and U a vector representing the membrane recovery. The SNN is supplied with information by the occupancy grid which is actually a vector E whose values are determined by Eq. 2. The vector E allows the calculation of the external input current vector $I_{ext} = 80 \times E$ applied to the neurons. Let A an amplified activation vector which is such as:

$$A_i = \begin{cases} \alpha & \text{if } V_i \geq 30 \\ 0 & else \end{cases} \quad (8)$$

The $(N^2 \times N^2)$ ΔT matrix is an matrix expressing the activation time differential between two neurons of the network : $\Delta T_{ij} = t_i^{fire} - t_j^{fire}$. W is the $(N^2 \times N^2)$ synapses weight matrix and C the network connexion matrix.

The algorithm 1 illustrates the process giving birth to the wavefront and its expansion. During the first $5ms$, a positive current of $20mV$ is applied as an external input to the neuron representing the portion of space including the robot. This fast excitation is sufficient to initiate the wave expansion. In order to stabilize the wave travel, an STDP control process is applied on the weights of synapses connecting activated neurons at time t. This process will strengthen the synapses that had the same connecting direction as the wave expansion and will greatly weaking synapses on the opposite direction. This will prevent the reactivation of previously activated neurons and guarantee a single side of expansion. In this algorithm, $(\times)$ express the matrix product, $(\circ)$ the Hadamard (element-wise) product.

Algorithm 1 Wave expansion algorithm

```
t ← 0 ms
T_th ← 10000 ms
while t < T_th do
    if t > 5 ms then
        E_robot ← 0
    end if
    Calculate ΔT
    Calculate A (according to Eq. 8)
    if A_target = α then
        T_th ← t + 10 ms
    end if
    I_ext ← 80.E
    W ← W + β.F(ΔT) (according to Eq. 7)
    I_int ← (W ∘ C) × A
    I_syn ← I_ext + I_int
    Calculate V according to Eq. 3
    Calculate U according to Eq. 4
    t ← t + 1 ms
end while
```

The parameter T_{th} is the time-threashold. Actually, the target is known as a reward cell which is a theoretical neuron representing the reward value associated with a place cell. Each place cell is bijectively connected to a reward cell. It's the simplified version of a pre-frontal cortex column presented in Erdem and Hasselmo [29]. The algorithm does not stop until the target neuron is activated.

Although the target neuron is activated, we must add an extra-time to allow removal of the wave because the path computation request a target neuron that is in a rest state. If the activation time of the target neuron is T_a and the extra-time is a constant parameter T_{et} then $T_{th} = T_a + T_{et}$.

2.5 Gradient map and path calculation

For computing a feasible path, our method is based on the classic case of gradient ascent[15][20][22][21]. However, unlike the gradient ascent in the literature, we propagate the wave from the robot and the ascent is made from the target to the robot. In order to obtain a gradient map, we take advantage of the asymptotic potientiel raising of the activated neurons to the resting potential. This asymptotic expansion can temporally differentiating neurons according to their activation time. A neuron x_i which is activated earlier than a neuron x_j has a higher potential at a future time t after the wave expansion phase. This property ensures that, in the neighborhood of a neuron x_j, the neighbor neuron with the highest potential is the one who had activated first and is therefore the parent of x_j. This allows, from children to parents, to reconstruct a feasible path from the target to the robot.

3 Experiments

The experiments were performed on a 30×30 spiking-neurons grid. The selected parameters are the same for all experiments. The neuron is a 'Regular Spiking' type with parameters $(a, b, c, d) = (0.02, 0.2, -65, 8)$. The membrane potential is initialized at $-65mV$ and the membrane recovery at -13. The amplification value of A is $\alpha = 100$. The values of the parameters that regulate synaptic plasticity are $A_- = 4$, $A_+ = 4.28$, $\tau = 10ms$ and $\beta = 0.25$, all obtained empirically. The neurons of the grid are connected according to the equation Eq.1.

Three types of environment were tested. Firstly, a simple environment with two perpendicular crossed walls with the aim of illustrating the wave expansion as shown in Fig.2 (a.-h.). The waveform rises at the right of the grid (a.), propagates in the medium and bypassing the walls (b.-g.) before stamping and revealing a gradient map (h.).

Secondly, an environment where barriers are walls forming two U-shapes (Fig.2 i.). This environment is particularly problematic for local methods because it has two local minimas, one in the center of each U-shape. Our method is insensitive to these local minima and allows the calculation of a feasible path.

Thirdly, a maze (Fig.2 j.). This experiment prove the algorithm's ability to overcome the complexity of obstacles. The execution time depends entirely on the length of the feasible path between the robot and the target, not the complexity of the environment.

The simulations were performed on a computer with a 2GHz Core I3 processor and 8GB of RAM and implemented with the Python language. The execution speed is an average of 1.6 seconds per millisecond of neuronal simulation. This

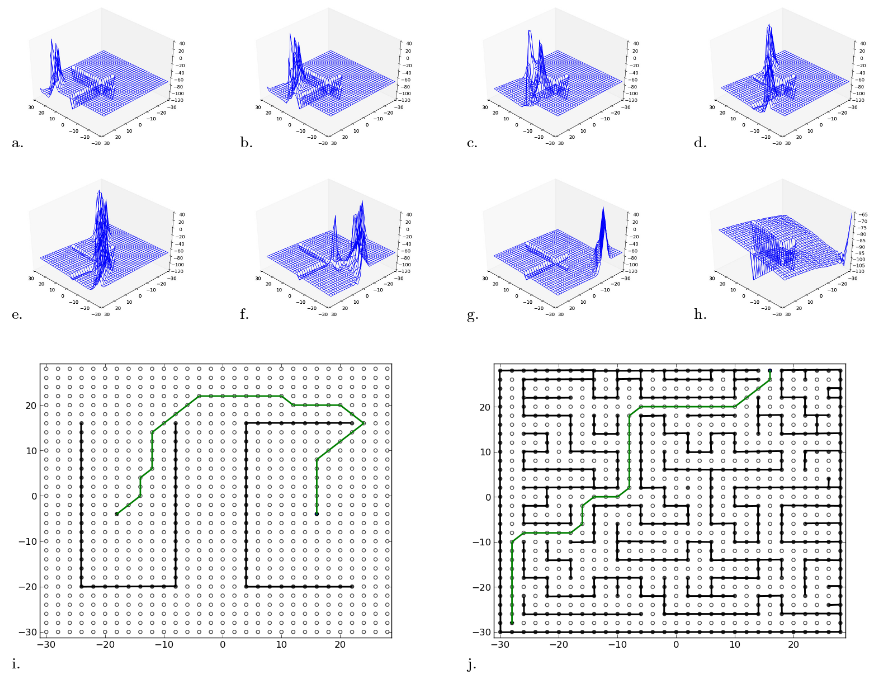

Fig. 2: Different stages of the wave expansion (a.-h.). Path planning in the 'double U' environment (i.). Path planning in the maze(j.).

slowness is mainly due to the matrix exponentials and products involving very large matrices (W and ΔT have dimensions of 900×900). This slowness prevents the implementation of this algorithm on the Von Neumann architectures for use under real-time conditions. However, the simulation has shown that biological and parallel neural architecture allow the calculation of a path in less than 60 ms, hence, about 16 updated paths per second. Theoretically, this speed allow a real-time use.

4 Discussion

Our work is primarily aiming to verify the possibility to build a neural path planner based exclusively on a biologically plausible neural model. The neural path planner models seen in the literature are based on a reducing or very free neuron modeling. Yang et al.[15], Chen et al.[22], Glasius et al.[12] use a non-pulsating potential variable neural model based on the Grossberg equation[27]:

$$\epsilon \frac{dn(t)}{dt} = -n(t) + (b^{+} - n(t))p^{+} - (n(t) + b^{-})p^{-} \tag{9}$$

where the parameters b^{+} and b^{-} are the passive decay rate, the upper and lower bounds of the neural activity. p^{+} is the positive input (0 otherwise), p^{-} is the negative input (0 otherwise). These non-pulsating models have the advantage to provide stable and uniform gradient maps after a certain time. This comes from the fact that these models fulfill the Lyapunov stability criteria, but, a neuron remained stable at a very high potential is biologically implausible.

Qu et al.[19] proposed a PCNN model which is closer to the Hodgkin-Huxley model (pulsating neuron). However, the inputs of the neurons are voluntarily standardized in order to obtain constant wave velocity everywhere. Furthermore, Qu et al. artificially change (against biological premise) the neuron activation threshold after the passage of the wave in order to avoid the reactivation of parent neuron and keep a coherent wave front.

In our proposal, the wave is stabilized by taking advantage of the natural plasticity process of biological synapses that regulates the synapse weight according to pre- and post-neuron spiking time. We used the natural input in the neuron without any modification. This produces a drawback: the neuron output (peak and frequency) is directly related to current input intensity. Since all neurons don't receive the same amount of current in the input (the number of active neighbors varies) that generates waves that does not move at a uniform velocity throughout the network. The difference is minimal but sufficient to generate, in some cases, non-optimal paths.

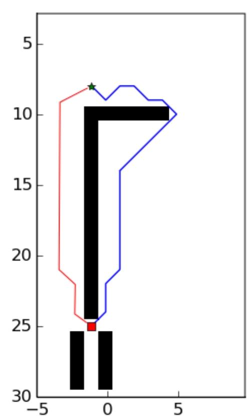

Fig. 3: Impact of propagation speed uniformity on paths optimality.

This scenario is illustrated in figure Fig. 3. The red path is the one obtained by Qu et al. whose wave has a uniform velocity whatever the space configuration. The blue path is the one obtained by our algorithm. After initialization, the wave splits into two sub-waves. The first goes through the left side of the vertical wall, the second through the right side. In our case, and since the left side is narrower than the right side, the number of neurons is reduced. The average intensity of the input current of the left side neurons is slightly lower than the right side ones, leading to pulses of lower frequencies. Accordingly, the right sub-wave will be faster than the left one and will reach the goal first: generating a non-optimal path. This phenomenon is also encountered by Yang et al. and Chen et al.[19]. Qu et al. has standardized the input current of neurons and avoided the problem.

In comparison with Ponulak and Hopfield model [28], our neural model (Izhikevich model) is a biologically more accurate model in comparison with the "integrate and fire" model used by the authors. Ponulak model considers the cognitive map as a network of excitatory neurons that does not take into consideration obstacles and dangerous places. Obstacles are not part of cognitive map so they are unreachables. The authors do not mention the case of a dynamic environment where obstacles are in motion. In our model, obstacles and dangerous places are place cells which are inhibited from the pre-frontal cortex. An inhibited neuron in the SNN can not be a medium to the wavefront, consequently, any future paths will avoid (bypass) the inhibited neurons (dangerous places). To calculate the next movement of the robot, Ponulak and Hopfield propose an elegant solution base on the synaptic vector field in order to obtain motor commands. They also explore the biological revelance of the travelling waves. The authors claim that recent electrophysiological results suggest existence of expanding waves of neural activity in the hippocampus during, so called, sharp wave ripple (SWR) episodes [31]. Sharp wave ripples are brief highfrequency bursts of neural activity observed during sleep or at awake rest.

5 Conclusion

This paper presents a model for collision free path-planning organized on a bidimensional grid of 8-connected pulsing neurons. This pulsing neurons are place cells which form a cognitive map. The proposed method uses the concept of travelling waves (or wave expansion) among the spiking neural network (cognitive map) in order to find a faisible (not necessarily shortest) path. The computational complexity is only related to the lenght of the path. The originality of this method is that the considered neuron is very close to the biological neuron. In addition, in order to stabilize the wave expansion, a natural phenomenon present in the mammalian brain had been used : the Spike-Time Dependant Plasticity. This approach distinguishes our method from that of the literature using algorithmic corrections or non-plausible neurons to stabilize the wave. The main weakness of the proposed method is that the generated waves does not move at a uniform velocity through the network which does not guarantee the path optimality. The second weakness is that this method is not suited to the Von Neumann architectures and must be implemented on massively parallel machines or biological neural tissues to hope use in condition of dynamic environments. Thus, the challenge is to implement the method on parallel machine and improve the quality of the resulting paths.

References

1. Latombe, J.C: Robot motion planning, Springer (1990)

2. Lozano-Prez, T. and Wesley, M.A.: An algorithm for planning collision-free paths among polyhedral obstacles. Communications of the ACM, Vol. 22, No. 10, pp. 560–570 (1979)
3. Lozano-Prez, T.: Spatial Planning: A configuration space approach. IEEE Transaction on Computers, Vol. 100, No. 2, pp. 108–120 (1983)
4. Schwartz, J.T., Sharir, M.: On the piano moving problem: general techniques for computing topological properties of real algerbric manifolds. Advances in Applied Mathematics, Vol. 4, No. 3, pp. 298–351 (1983)
5. Bhattacharya, B.K., Zorbas, J.: Solving the two-dimensional find path problem using a line-triangle representation of the robot. Journal of Algorithms, Vol. 9, No. 4, pp. 449–469 (1988)
6. Takahashi, O., Schilling, R.J.: Motion planning in a plane using generalized Voronoi diagrams. IEEE Robotics and Automation, Vol. 5, No. 2, pp. 143–150 (1989)
7. Barraquand, J., Langlois, B., Latombe, J. C.: Numerical potential field techniques for robot path planning. Systems, Man and Cybernetics, IEEE Transactions, Vol. 22, No. 2, pp. 224–241 (1992)
8. Hwang, Y. K., Ahuja, N.: A potential field approach to path planning. IEEE Robotics and Automation, Vol. 8, No. 1, pp. 23–32 (1992)
9. Wang, Y., Chirikjian, G. S.: A new potential field method for robot path planning. Robotics and Automation, 2000. Proceedings. ICRA'00. IEEE International Conference, Vol. 2, pp. 977–982 (2000)
10. Koren, Y., Borenstein, J.: Potential field methods and their inherent limitations for mobile robot navigation. Robotics and Automation, IEEE Transactions, pp. 1398–1404. (1991)
11. Warren, C. W.: Global path planning using artificial potential fields. In Proceedings of the IEEE International Conference on Robotics and Automation (Scottsdale, AZ) , Los Angeles. Computer Society Press of the IEEE, pp. 316–321 (1989)
12. Glasius, R., Komoda, A., Gielen, S. C. A. M.: Population coding in a neural net for trajectory formation. Network: Computation in Neural Systems, 5(4), pp. 549–563 (1994)
13. Connolly, C., Burns, J. B., Weiss, R.: Path planning using Laplaces equation. In Proceedings of IEEE International Conference on Robotics and Automation. Cincinnati, OH: IEEE. 1990, pp. 2102–2106 (1990)
14. Bugmann, G., Taylor, J. G., Denham, M.: Route finding by neural nets. In J. G. Taylor (Ed.), Neural networks, Alfred Waller Ltd., pp. 217–230 (1995)
15. Yang, S. X., Meng, M.: An efficient neural network approach to dynamic robot motion planning. Neural Networks, Vol. 13, No. 2, pp. 143–148 (2000)
16. D. Zhu, Y. Huang, S. X. Yang: Dynamic task assignment and path planning of multi-AUV system based on an improved self-organizing map and velocity synthesis method in three-dimensional underwater workspace. IEEE Transactions on Cybernetics. Vol. 43, No. 2, pp. 504–514 (2013)
17. J. Ni, C. Zhang, L. Ren, S. X. Yang: Abrupt event monitoring for eater environmental system based on KPCA and SVM. IEEE Transactions on Instrumentation and Measurement. Vol. 61, No. 4, pp. 980-989 (2012)
18. S. X. Yang, A. Zhu, G. Yuan, M. Q.-H. Meng: A bioinspired neurodynamics based approach to tracking control of mobile robots. IEEE Transactions on Industrial Electronics. Vol. 59, No. 8, pp. 3211-3220 (2012)
19. Qu, H., Yang, S. X., Willms, A. R., Yi, Z.: Real-time robot path planning based on a modified pulse-coupled neural network model. Neural Networks, Vol. 20, No. 11, pp. 1724–1739 (2009)

20. Glasius, R., Komoda, A., Gielen, S. C.: A biologically inspired neural net for trajectory formation and obstacle avoidance. Biological Cybernetics, Vol. 74, No. 6, pp. 511–520. (1996)
21. Lebedev, D. V., Steil, J. J., Ritter, H. J.: The dynamic wave expansion neural network model for robot motion planning in time-varying environments. Neural Networks, Vol. 18, No. 3, pp. 267–285 (2005)
22. Weidong, C., Changhong, F., Yugeng, X.: On-line safe path planning in unknown environments. In Robotics and Automation. Proceedings. ICRA'03. IEEE International Conference, Vol. 3, pp. 4191–4196 (2003)
23. Izhikevich, E. M.: Simple model of spiking neurons. Neural Networks, IEEE Transactions, Vol. 14, No. 6, pp. 1569–1572 (2003)
24. Hodgkin, A. L., Huxley, A. F.: A quantitative description of membrane current and its application to conduction and excitation in nerve. The Journal of physiology, Vol. 117, No. 4, pp. 500–544 (1952)
25. Izhikevich, E. M., Moehlis, J.: Dynamical Systems in Neuroscience: The geometry of excitability and bursting. SIAM review, Vol. 50, No. 2. (2008)
26. Song, S., Miller, K. D., Abbott., L. F.: Competitive Hebbian learning through spike-timing-dependent synaptic plasticity. Nature neuroscience, 3(9), pp. 919–926 (2000)
27. Grossberg, S.: Nonlinear neural networks: Principles, mechanisms, and architectures. Neural networks, Vol. 1, No. 1, pp. 17–61 (1988)
28. Ponulak, F., Hopfield, J. J. : Rapid, parallel path planning by propagating wavefronts of spiking neural activity. Frontiers in computational neuroscience, 7. (2013)
29. Erdem, U.M., Hasselmo, M.E. : A goal-directed spatial navigation model using forward trajectory planning based on grid cells. The European Journal of Neuroscience 35 (6), pp. 916–931 (2012)
30. OKeefe, J. : Place units in the hippocampus of the freely moving rat. Experimental Neurology 51 (1), pp. 78–109 (1976)
31. Ellender, T. J., Nissen, W., Colgin, L. L., Mann, E. O., and Paulsen, O. : Priming of hippocampal population bursts by individual perisomatic-targeting interneurons. J. Neurosci. 30, pp. 5979–5991 (2010)