A Set-membership Approach for Visible Light Positioning with Fluctuated RSS Measurements*

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Abstract. Visible Light Positioning (VLP) is considered as one of the most promising technologies for achieving low-cost and massive coverage indoor location-based service. However, traditional trilateration-based VLP methods suffer the Received Signal Strength (RSS) fluctuation problem which would significantly limit the positioning performance. This paper proposes an interval analysis-based set-membership approach to improve the positioning accuracy and stability of the VLP system in noisy environments. The proposed method utilizes a statistics method to construct confidence intervals from the fluctuated RSS measurements and casts the positioning process into a set-inversion problem which is then solved via an interval analysis-based algorithm in the framework of set-membership. Simulation results have been compared with the traditional least-square based positioning method, showing that the proposed method can provide more accurate and stable positioning results in different noisy interference environments.

Keywords: Visible light positioning · RSS · Confidence interval · Set-membership · Interval analysis.

1 Introduction

In the upcoming 5G Internet of Things (IoT) era, most of the mobile services will be generated in indoor environments and there would be an explosive growth of Location-Based Service (LBS). As the core technology of LBS, indoor positioning technology lays a technical foundation for housekeeping services, emergency security, smart warehousing, crowd monitoring, precision marketing, mobile health, cultural entertainment, etc. To meet the diversified and massive LBS demands in indoor environment, different technologies, namely UWB, WLAN, RFID, BLE and VLP, have been proposed and developed to tackle the positioning issues, committed to achieve accurate, reliable, and full coverage solution.

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Among the aforementioned technologies, VLP is gaining more and more attention nowadays due to numbers of inherent advantages: modest infrastructure cost, adequate coverage, free of magnetic interference and potential centimeter accuracy. Received Signal Strength (RSS) is perhaps the mostly used metric in VLP system due to its simplicity and low hardware requirement [6]. However, the main challenge of such method is the continuous signal fluctuations [3]. The measured RSS values usually have a high variability over time due to the fluctuating nature of wireless signals. Besides the thermal and shot noise, they could be significantly affected by shadowing, fading, and multipath propagation in indoor scenarios [10]. Such high variability will affect the ranging results and degrade the performance of the VLP system in terms of accuracy and reliability.

To deal with the RSS signal fluctuation problem while still maintaining the simplicity of VLP system, this paper proposes an interval analysis-based set-membership approach to improve the accuracy and stability of the RSS-based trilateration method. Interval analysis based methods have achieved promising results in parameter and state estimation tasks [4], as well as the mobile robotic localization and mapping area [5, 8, 9]. Our proposed method constructs confidence intervals from fluctuated RSS measurements by utilizing a statistics method, and then characterizes the confidence region of the receiver's position with the Set Inversion Via Interval Analysis (SIVIA) algorithm. Afterwards, the nominal position is characterized by a weight-coefficients method.

This paper is organized as follows: Section 2 details the framework of our proposed method and implementation. Section 3 presents the simulation results with a comparison to least-square method. Section 4 concludes the paper and proposes the perspective of future work.

2 Proposed Set-membership method for VLP system

2.1 Bootstrap-based RSS ranging

Denoting the RSS original data obtained during the ranging phase by $P_r = (\mathrm{RSS}_1, \mathrm{RSS}_2, \cdots, \mathrm{RSS}_{K_r})^T$. Traditional methods usually utilize a Gaussian filter to firstly remove the irregular values and then use the averaged RSS value is used for distance estimation. Our proposed method adopts the Bootstrap method to construct confidence intervals from the raw RSS measurement data. Firstly, we randomly sample with replacement from origin data P_r, which leads to a new series of measurement data

$$P_r^1 = (\mathrm{RSS}_1^{*(1)}, \mathrm{RSS}_2^{*(1)}, \cdots, \mathrm{RSS}_{K_r}^{*(1)})^T \tag{1}$$

P_r^1 is called the Bootstrap sample, where some original data may be drawn more than once and some others may be never drawn. We can repeat the resample procedure K_b times to generate a set of Bootstrap samples, denoted by $\{P_r^1, P_r^2, \cdots, P_r^{K_b}\}$.

Secondly, for each Bootstrap sample P_r^i, we can compute the Bootstrap statistics $\overline{P_r^i}$ by

$$\overline{P_r^i} = \frac{1}{K_r} \sum_{j=1}^{K_r} \mathrm{RSS}_j^{*(i)} \tag{2}$$

where $i = 1, 2, \cdots, K_b$. Then the distance between the VLC receiver and transmitter can be estimated by using the Optical Wireless Channel (OWC) model described in [2]. At the receiver side, the RSS value can be expressed as

$$P_r = (H_{\mathrm{LOS}} + H_{\mathrm{NLOS}}) \cdot P_t + w_n \tag{3}$$

where P_t and P_r are the transmitted and received signal power. H_{LOS} and H_{NLOS} represent the VLC channel gain of light-of-sight (LOS) and non-light-of-sight (NLOS) channel respectively. w_n denotes the noise power at the receiver, i.e. the shot noise and thermal noise power. Since only the LOS channel signals are useful for RSS-based positioning algorithm, Eq. 3 can be rewritten as:

$$P_r = H_{\mathrm{LOS}} \cdot P_t + P_n \tag{4}$$

where $P_n = H_{\mathrm{NLOS}} \cdot P_t + w_n$ represents the total noise power that affects the RSS value of the LOS channel. According to Lambertian radiation model, the typical VLC channel gain H_{LOS} can be expressed as:

$$H_{\mathrm{LOS}} = \begin{cases} \frac{(m+1)A_r}{2\pi d^2} \cos^m(\varphi)\cos(\theta) & 0 \leq \theta \leq \phi_{\mathrm{FOV}} \\ 0 & \theta > \phi_{\mathrm{FOV}} \end{cases} \tag{5}$$

where φ and d are respectively the radiation angle and distance between the receiver and transmitter. A_r is the effective area of the receiver, and θ is the angle of light incident to the receiving surface of the detector. m represents the order of Lambertian emission and ϕ_{FOV} is the field-of-view of the receiver. The distance between the transmitter and receiver is thus given by

$$d = \sqrt[m+3]{\frac{(m+1)A_r H^{m+1}}{2\pi} \frac{P_t}{P_r}} \tag{6}$$

For K_b Bootstrap statistics, we can obtain K_b estimated distance results which can be sorted from small to large as follows:

$$d_1^* \leq d_2^* \leq \cdots \leq d_{K_b}^* \tag{7}$$

$(d_1^*, d_2^*, \cdots, d_{K_b}^*)$ is called the Bootstrap distribution. From this stage, we can utilize the Bootstrap percentile formula to construct the confidence interval of the distance estimation with a $(1-\alpha)\cdot 100\%$ confidence probability by :

$$[d] = [d_{u_1}^*, d_{u_2}^*] \tag{8}$$

where the lower and upper bound of $[d]$ are defined by the subscripts u_1 and u_2, with $u_1 = floor(K_b \cdot \alpha/2)$ and $u_2 = K_b - u_1 + 1$ [7]. On this way, the confidence intervals of the distances between the receiver and different VLC transmitters can be obtained.

2.2 Confidence region configuration with SIVIA

To deal with the random RSS fluctuation problem, we propose to define the trilateration problem as a set inversion problem and utilize the SIVIA algorithm to compute the confidence region where the receiver is assumed to be located.

Let's consider three deployed VLC transmitters with fixed coordinates, denoted by (t_{x_i}, t_{y_i}) $(i = 1, 2, 3)$. The distances between the transmitters and receiver are respectively d_1, d_2, d_3. According to the trilateration positioning formulation, the feasible values of the receiver's position (r_x, r_y) can be configured via the equations:

$$\begin{cases} (r_x - t_{x_1})^2 + (r_y - t_{y_1})^2 + h^2 = d_1^2 \\ (r_x - t_{x_2})^2 + (r_y - t_{y_2})^2 + h^2 = d_2^2 \\ (r_x - t_{x_3})^2 + (r_y - t_{y_3})^2 + h^2 = d_3^2 \end{cases} \tag{9}$$

where h is a constant, denoting the vertical distance between the receiver and VLC transmitters. By using the Bootstrap method, the estimated confidence interval of the distances $[d_1], [d_2]$ and $[d_3]$ between the receiver and each VLC transmitter can be calculated. The confidence region $\mathbb{X}$ is thus defined as a set of all the feasible values which satisfy the constraints (18) and can be characterized by solving the set inversion problem:

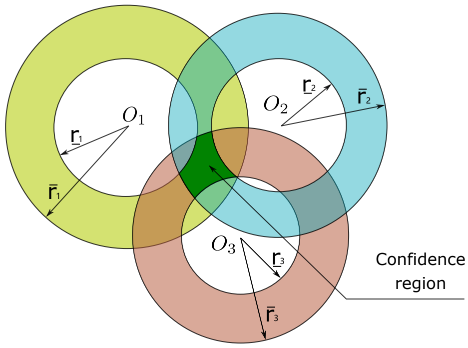
$$\mathbb{X} = \{(x, y) \in \mathbb{R}^2 \mid g(x, y) \in [\mathbf{D}]\} = g^{-1}([\mathbf{D}]) \tag{10}$$

where $[\mathbf{D}]$ is a three dimensional interval box $[\mathbf{D}] = [d_1] \times [d_2] \times [d_3]$ and $g(\cdot) : \mathbb{R}^2 \to \mathbb{R}^3$ is a vector function defined as:

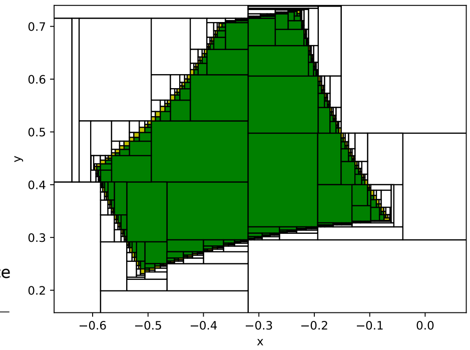
$$g(x, y) = \begin{cases} \sqrt{(x - t_{x_1})^2 + (y - t_{y_1})^2 + h^2} \\ \sqrt{(x - t_{x_2})^2 + (y - t_{y_2})^2 + h^2} \\ \sqrt{(x - t_{x_3})^2 + (y - t_{y_3})^2 + h^2} \end{cases} \tag{11}$$

The confidence region is usually an irregularly shaped area. It is equivalent to find the intersection area of three rings whose radius range is defined by $[r_i] = [\underline{r_i}, \overline{r_i}] = \sqrt{[d_i]^2 - [h]^2}$ $(i = 1, 2, 3)$, as shown in Fig. 1a. This problem can be consistently solved by the SIVIA algorithm. Assume that $[\mathbf{x}] = [x] \times [y]$ is the initial solution space, $[\mathbf{g}](\cdot)$ is the inclusion function of $g(x, y)$, the main steps of the solving process are carried out as follows:

- If $[\mathbf{g}]([\mathbf{x}]) \subset [\mathbf{D}]$, then any $(x, y) \in [\mathbf{x}]$ is consistent with the VLC ranging measurements and noise bounds. $[\mathbf{x}]$ is proved to be in $\mathbb{X}$ and is kept in the solution list.
- If $[\mathbf{g}]([\mathbf{x}]) \cap [\mathbf{D}] = \emptyset$, then the whole box is inconsistent with the VLC ranging measurements and noise bounds, $[\mathbf{x}]$ is eliminated from the solution list.
- If $[\mathbf{g}]([\mathbf{x}]) \cap [\mathbf{D}] \neq \emptyset$, then at least one configuration in $[\mathbf{x}]$ is consistent with the VLC ranging measurements and noise bounds, $[\mathbf{x}]$ is said to be undetermined. If its size conforms $w([\mathbf{x}_k]) \geq \varepsilon$ (ϵ is the prespecified precision), then it will be bisected and the same test should be applied to each of newly generated sub-boxes. Otherwise, $[\mathbf{x}]$ will be kept in the solution list due to its small size $(w([\mathbf{x}]) \leq \varepsilon)$.



(a) Illustration of receiver's position



(b) Confidence region obtained with SIVIA

Fig. 1: Receiver's confidence region configuration with SIVIA algorithm

The SIVIA algorithm performs the inclusion test and bisection process recursively to verify that all the boxes in the solution list belong to $\mathbb{X}$. As a result, it yields a list of non-overlapping boxes described in Fig. 1b. The green boxes are those which are validated and the yellows are the undetermined ones. The union of these non-overlapping boxes thus denotes the confidence region where the receiver is deemed to be located.

2.3 Nominal position determination

The confidence region obtained via the SIVIA algorithm is a list of interval boxes, from which we can calculate the receiver's final position estimation (we call it the nominal position). In our work, we propose to use the weighted arithmetic average method based on interval box dimensions to calculate the nominal position. Denote the list of solution boxes in the confidence region by $\mathcal{L} = \{[\mathbf{x}_1], [\mathbf{x}_2], \cdots, [\mathbf{x}_p]\}$, where p is the number of boxes in the confidence region. Then the nominal position is determined through

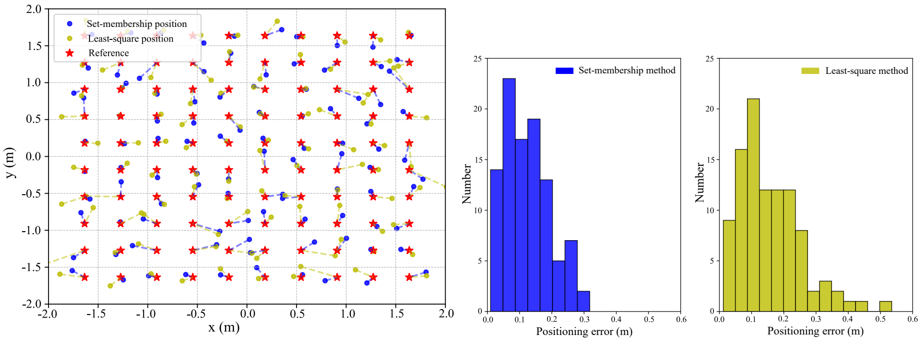
$$(t_x, t_y) \leftarrow \sum_{k=1}^{p} \Psi_k \cdot mid([\mathbf{x}_k]) \tag{12}$$

where Ψ_i is the weight-coefficient, calculated by $\Psi_k = \frac{vol([\mathbf{x}_k])}{\sum_{i=1}^{p} vol([\mathbf{x}_i])}$, $vol([\mathbf{x}_i])$ and $mid([\mathbf{x}_i])$ represent respectively the size and the center point of the i^{th} interval box.

3 Simulation results

3.1 Experiment set-up

To test our proposed method, we consider a typical VLP scenario in simulation, i.e., a $4\,\mathrm{m} \times 4\,\mathrm{m} \times 3\,\mathrm{m}$ area. Three LEDs are deployed on the ceiling downward



(a) Illustration of positioning results (b) Histograms of the positioning errors

Fig. 2: Positioning results for 100 points

vertically. with coordinates (-1, 1, 0), (1, 1, 0) and (0, -1, 0). To setup the simulation in Matlab, the total noise power P_n considered in the simulation is generated based on the time-variant deviation model presented in [1]. The time-variant noise interference $P_n(t)$ is defined as

$$\begin{cases} \chi_n(t) = \lambda \cdot \chi_n(t-1) + \mathcal{N}(0, \sigma_n^2) \\ P_n(t) = \chi_n(t) \cdot H_{\mathrm{LOS}} \cdot P_t \end{cases} \tag{13}$$

where λ is an arbitrary number between $0 \sim 1$, $\mathcal{N}(0, \sigma_n^2)$ is Gaussian white noise, and $\chi_n(t)$ denote the LOS channel noise factor. The noise vibration level depends on the σ_n value: the bigger the σ_n is, the larger noise fluctuation will be.

3.2 Simulation result

Firstly, 100 unknown points are evenly distributed on the plane, their positions are estimated through the two positioning methods with the noise level $\sigma_n = 0.3$. The results are described in Fig. 2a, the red stars are the reference positions, the blue circles are the nominal positions estimated by our proposed method and the yellow circles are the results obtained by least-square method. We utilize the Euclidean distance between estimated position and reference position to calculate the positioning error. Fig. 2b gives the statistics of the positioning error for the two methods. Our proposed method could achieve more stable positioning results when dealing with fluctuated RSS measurements, as it can be seen from Fig. 2b, the position errors of our method are all below 0.3 m, while for the least-square method, the largest error reaches 0.54 m. Calculating an average value gives 0.11 m accuracy for our method and 0.16 m for the least-square method, showing that our proposed method expresses better performance in terms of accuracy.

A robust positioning scheme should accommodate interference in different noisy environments. In order to get a quantitative evaluation of our proposed

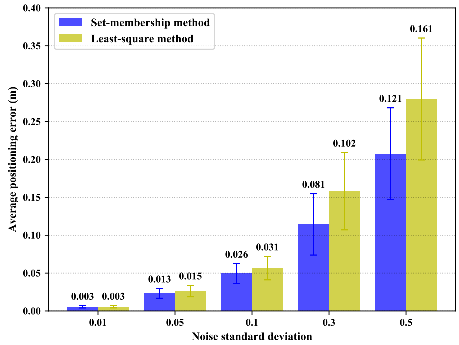


Fig. 3: Positioning error with different noise level

method, we perform the experiments with different level of noise fluctuation by changing the standard deviation σ of white noise in Eq. 13. The average positioning error and the variance of the positioning accuracy are computed for different values of σ_n ranging from 0.01 to 0.5. Fig. 3 presents the results obtained over 1000 randomly positioned points. The black error bars denote the standard deviations of the positioning errors. As we can see from the figure, when the noise fluctuation is very small ($\sigma_n = 0.01$), both methods obtain almost the same results, the positioning error is about 0.5 cm. When the noise fluctuation increases, the positioning errors of both methods increase as well. But the positioning error of least-square method increases more rapidly which means it is more vulnerable to the noise fluctuation than ours. The standard deviation of positioning error (the black error bar on the figure) of our proposed method is also smaller than the least-square method at all noise fluctuation levels, which demonstrates the positioning results obtained through our method are more stable than the least-square method.

4 Conclusion

This paper presents an interval analysis-based trilateration positioning approach in the scheme of set-membership. The proposed approach takes advantage and combines Bootstrap and SIVIA algorithm to compute a confidence region of the feasible positions from fluctuated RSS measurements and gives a nominal position estimation with a weighting method. Simulation results demonstrate that our method expresses better performance in terms of accuracy and stability in comparison with least-square method, which indicates that our method is more tolerable to RSS signal fluctuation. Future work will focus on validating the performance of the proposed method in real indoor environment.

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