Multi-Criteria Synthesis of the Software-Defined Network Structure

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Abstract. A software-defined networking is a complex system that consists of a set of end nodes and switching nodes, as well as communication channels that connect them to each other. In such networks, along with the information transmission processes in the communication channels, there are also processes of information sharing that take place and take place at the switching nodes. In a software-defined networking, the central controller manages these distribution processes. Therefore, in the construction of software-defined networking should take into account the quality indicators that characterize these processes. First of all, these indicators are related to delay and loss of messages, errors in the delivery of messages to specific recipients, time of delivery of the data stream, topology of the software-defined networking, etc. They can be described by numerical parametric characteristics, which may be restricted. To design and construct SDNs, developers try to find the values of the characteristics that would optimally satisfy the functioning of the network by minimizing or maximizing its quality. And the tasks of researching and finding such characteristics are inherently multi-criteria. For a complex solution of the general issue of construction a software-defined networking, the article discusses the basic requirements that apply to the criteria in multicriteria tasks. The process of normalization of the parameters of the criterion functions in the formation of the Pareto-optimal set of solutions is described and recommendations are given with their subsequent use.

Keywords: Software-Defined Networking, Data Transmission System, Optimization, Network Design.

1 Introduction

Since the creation of the concept of software-defined networking (SDN), their optimization has been conducted in three main areas:

- development of theoretical foundations of packet switching and network flows. In this direction, the following parameters of the SDN were investigated, such as network response time, its performance, capacity of resources, indicators of their use, etc.;
- development of mathematical methods for optimization of data flows in SDN by choosing the optimal route of their delivery;
- scientific and applied researches on development of modern SDN hardware and software, creation of routing, switching protocols, etc. [1].

This article deals with the theoretical aspects of the application of multicriteria optimization methods to software-defined networking and refers to the second direction.

2 Formulation of the problem

According to the performed tasks, a software-defined networking is considered as a complex object of research, which is characterized by many features. Describing it, take simultaneously into account many indivisible one of the another characteristics. In other words, for SDN research, a modern systematic approach requires the involvement of the full spectrum of its characteristics.

Since being in different conditions, the network exhibits different system properties, it is necessary to consider not one of its theoretical models, but a set of different, sometimes conceptually contradictory models. However, none of them can be neglected, because each characterizes some property of the SDN under study and none can be accepted as a single one because it does not express the complete set of all properties of the network.

Multiple SDN properties under different conditions can be quantified by appropriate partial criteria. However, contributing to the acquisition of different properties, the corresponding conditions are also evaluated by different partial criteria. Thus, it can be said that mutually exclusive theoretical network models are characterized by contradictory partial criteria, each of which is applied in its own, special conditions and in the values of its special characteristics. And only a complete set of partial criteria makes it possible to adequately evaluate the functioning of the software-switched network. Therefore, a multi-criteria approach with contradictory criteria must be applied to the holistic perception of SDNs in different working conditions. In the general case, the results of solutions of this approach will contain not one but several variants of SDN construction and be in the area of Pareto-optimal (effective) design solutions.

Content essence of many practical problems is the choice of operating conditions SDN that give it an opportunity to show their best qualities. If the conditions, on which these properties depend, are quantitatively expressed by some variables $x_1, x_2, ..., x_m$, which are defined on X, then they can be called optimization arguments. In turn, if the properties themselves are also quantified by variables f, then their values will characterize the quality of the network with respect to these properties.

In general, indicators $f_1,\ f_2, \ldots, f_s$ are called quality criteria and determine vector $f = \{f_k\}_{k=1}^{s}$. Its components quantify the properties of the software-defined networking with a given set of optimization arguments $x = \{x_i\}_{i=1}^{m} \in X$, and the quality of the solution is evaluated by the set of contradictory partial criteria that form s-dimensional vector $f(x) = \{f_k(x)\}_{k=1}^{s} \subset Y, x \in X$, which is defined on the set *X*. Here, a feasible region $X \subset E^m$ is defined and consists of vectors $x = \{x_i\}_{i=1}^{m}$ of *m*-dimensional Euclidean space, expression $f \subset Y$ means belonging to the vector *f* to the class *Y* of valid efficiency, and vector of partial criteria is limited by the codomain $f \in M$.

In addition to the specified conditions, the SDN may also be affected by external, random factors *r*. Although these factors are independent, they are known to take their values from a compact set *R*, and in calculations consider that this set is given and the vector of external influences is known [2, 3].

As a software-defined networking is a multifunctional system, and limitations are imposed on many of its characteristics, the task of research is inherently multicriterion.

In the most general case, to solve such tasks means to find the optimal decision $x^* \in X$, which optimizes the efficiency vector under given conditions, relations and constraints *f*(*x*) [4]. However, this solution formulation is so general that it can difficult be used in practice. Therefore, summarizing the above and describing mathematical model for searching of optimal solutions of SDN, it is necessary to consider a set of basic conditions for its criteria and for its parameters, namely:

- network parameters $x_1, x_2, \ldots, x_m$, on the one hand, should sufficiently and reliably characterize it, on the other hand, allow to reduce the dimension of the solving task;
- to consider the possibility of using the simplest methods of analysis of these parameters;
- to take into account the possibility of normalization of selected parameters, i.e. reducing them to a single dimension or to a dimensionless form;
- a set of partial criteria $f = f(x_1, x_2, \ldots, x_m)$, by which the quality of the network is evaluated, should be a function of the selected parameters;
- in turn, the criterion should be considered sensitive to analyzed variants of SDN, in particular to the selected parameters $x_1, x_2, \ldots, x_m$;
- to specify the domain of existence of parameters for unambiguous determination of their constraints;
- to provide specified level of network quality and efficiency of its operation;
- criteria should take into account the assessment of the efficiency performance of the network main tasks;
- the criterion should be sufficiently simple and visual, have a clear physical meaning, so that there are no difficulties in the physical interpretation of the research results;
- choosing criteria it is necessary also to take into account the existence of close links and contradictions between them [5].

3 Analysis of Publication and Problem Statement

Today, the issue of finding optimal decisions in the problems of criterion research has been given considerable attention. This is confirmed by the large number of publications, and is caused by the fact that this topic covers a very wide range of tasks in a variety of fields.

There are many publications that classify and uncover the various ways and methods used in multicriteria optimization tasks. Also, many publications are devoted to disclosing the basic conditions for describing mathematical models of finding optimal decisions [1-5, 8, 9]. Although different authors invest in the concept of optimality, different understanding of most rules for choosing the best project decisions has a common feature: the selection is made on the basis of information on the comparison of systems variants. Of course, such comparisons can be made on many permissible variants of systems, but more often it is convenient to perform in the criterion space, since here the design decisions are compared by means of a set of quality indicators having a numerical character [6, 12, 14].

Development of a method of finding the optimal variant of construction of the software-defined networking, which would be economically justified and at the same time would ensure the fulfillment of the tasks assigned to it and is the purpose of this article. And as can be seen from the goal, this method will be related to the problem of multicriteria optimization of complex systems [11, 13-16].

4 Formation of the set of Pareto-optimal decisions

As described above, the results of multi-criteria decisions will be in the domain of Pareto-optimal decisions. However, before forming the Pareto-optimal set of variants of SDN construction, it is necessary to clearly define the basic requirements for it. In addition, it is necessary to set clearly the requirements for the criteria functions, their arguments, and also to take into account all the relations between the criteria, their physics and nature. All of these requirements should be an integral part of the process of proving the choice of the optimal SDN construction option.

Characterizing a software-defined networking, its parametric characteristics (PC), in fact, are said to be coordinates of a point in a multidimensional space. In other words, any complex system can be formally described by point coordinates in a multidimensional space of parametric network characteristics. The equation of existence distinguishes in this space the area of technically implemented variants of SDN construction, and the use of models of functioning of this network and methods of evaluating its efficiency provides an opportunity for each point of this area to match the numerical values of the specified criteria.

As is known, there is a method that allows to construct a Pareto-optimal set of decisions using a quadratic approximation of criterion functions, with given constraints on parameters and two contradictory criteria. [3].

For this purpose, each objective function is given as a quadratic polynomial

$$f_1(x)=\sum_{i=0}^{m}\sum_{j=0}^{m}a_{ij}x_i x_j\ ,\ \ f_2(x)=\sum_{i=0}^{m}\sum_{j=0}^{m}a_{ij}x_i x_j\ ,$$

or in matrix form

$$\begin{aligned} f_1(x) &= A_0 + 2A_1 x + x^T A_2 x, \\ f_2(x) &= A_0 + 2A_1 x + x^T A_2 x. \end{aligned} \tag{1}$$

In this case x is a column vector of SDN parametric characteristics, x^T is a transposed column vector of x, A_0 is a free term of approximating polynomial, A_1 is a vector of approximating coefficients at the first degrees of the parametric characteristics of SDN, which do not multiply with each other, A_2 is a symmetric square matrix for the second degrees of parametric characteristics, as well as for parameters that multiply with each other. To find matrices A_0, A_1 and A_2 it is necessary to find the coefficients of the polynomial a_{ij}.

Methods of finding polynomial coefficients, as well as arguments for choosing a quadratic approximation of a criterion function, as described in the second degree polynomial class, are described in [5].

After finding the coefficients a_{ij}, the matrices A_0, A_1 and A_2 are formed for the criterion functions, and using the rules of differentiation of the matrix expressions by the scalar argument, the values $\frac{f_1(x)}{dx}$ and $\frac{f_2(x)}{dx}$ equals to zero. The values of the coordinates of the unconditional minima of the criteria functions are also found. It is a point $A(\text{opt}\,x_1^1, \text{opt}\,x_2^1)$ for $f_1(x)$, and point $B(\text{opt}\,x_1^2, \text{opt}\,x_2^2)$ for $f_2(x)$ (fig. 1).

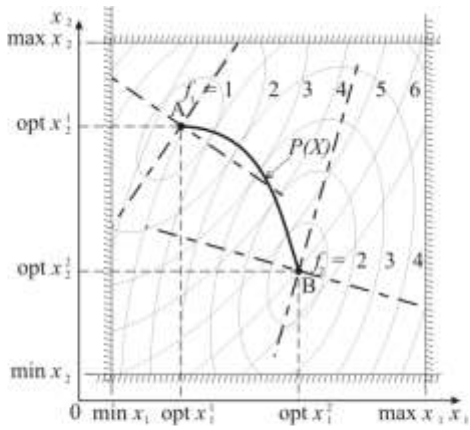

Fig. 1. Families of initial ellipses

If these points belong to the feasible region, the solutions, that meet these points, are said to be optimal by the criteria $f_1(x)$ and $f_2(x)$ respectively. If the points do not belong to feasible region, then significant constraints and additional researches, based on the Lagrange multiplier method, should be introduced [2].

In the first case, when the criterion surfaces $f_1(x)$ and $f_2(x)$ = const are multidimensional ellipsoids, and points *A* and *B* belong to the feasible region, in the future this case will be considered, we can say that the set of points of the spatial curve *AB*, which is a locus of adherent points of the second-order lines of the family f_1 with lines belonging to the family f_2, corresponds to set of Pareto-optimal decisions.

However, for further analysis and comparison of the components of the criteria $f_1(x)$ and $f_2(x)$, these components should have the same dimension, that is, be normalized.

5 Rotation of the coordinate system

To reduce the initial families of second order lines f_1 and f_2 to the same dimension, it is necessary with criterion functions $f_1(x)$ and $f_2(x)$ to do such orthogonal transformations that would allow to move to the new coordinate system using the replacement of variables, i.e. to move to a new basis, but in the same criterion space.

However, this must be done so that the properties of the initial second-order line families f_1 and f_2 do not change according to the moving from the initial coordinate system to the new system and vice versa.

In other words, it is necessary to rotate the coordinate axes of both families of the second-order lines and to move the origin of the coordinates of one of the families.

For further clearness, let's present a quadratic polynomial in the form (1) if m = 2,

$$\begin{aligned} f_1(x_1, x_2) &= a_{11}^1 x_1^2 + 2a_{12}^1 x_1 x_2 + a_{22}^1 x_2^2 + 2a_1^1 x_1 + 2a_2^1 x_2 + a_{00}^1, \\ f_2(x_1, x_2) &= a_{11}^2 x_1^2 + 2a_{12}^2 x_1 x_2 + a_{22}^2 x_2^2 + 2a_1^2 x_1 + 2a_2^2 x_2 + a_{00}^2. \end{aligned} \tag{2}$$

where the upper indices of the coefficients a_{ij} correspond to the criterion number, and rotate the coordinate system counterclockwise at an angle ϕ. Then the basis vectors e_1, e_2 move to the new basis vectors, respectively:

$$\tilde{e}_1 = \cos\phi e_1 + \sin\phi e_2,$$

$$\tilde{e}_2 = -\sin\phi e_1 + \cos\phi e_2.$$

The old coordinates x_1, x_2 is expressed by the new coordinates $\tilde{x}_1, \tilde{x}_2$ as:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \tilde{x}_1\tilde{e}_1 + \tilde{x}_2\tilde{e}_2 = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix}.$$

In the new coordinates the quadratic part (1) $f_2(x_1, x_2)$ looks like:

$$f_2 = [\tilde{x}_1 \quad \tilde{x}_2]\left(\begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \times \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix}\begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}\right)\begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix}.$$

The matrix in parentheses is the product of three matrices $\tilde{A} = Q^T AQ$, moreover A is a symmetric matrix. Hence

$$\tilde{A}^T = \left(Q^T AQ\right)^T = Q^T A^T \left(Q^T\right)^T = Q^T AQ = \tilde{A}.$$

So, $\tilde{A} = [\tilde{a}_{ij}]$, $1 \le i, j \le 2$, is a symmetric matrix.

Let's try to choose the angle ϕ so that the matrix $\tilde{A}$ becomes diagonal matrix:

$$\begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix}\begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix}\begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

So, let’s equate to zero element

$$\tilde{a}_{12} = \tilde{a}_{21} = \left(\cos^2\phi - \sin^2\phi\right)a_{12} - \sin\phi\cos\phi\left(a_{11} - a_{22}\right) =$$

$$= \cos(2\phi)a_{12} - \sin(2\phi)\frac{a_{11} - a_{22}}{2} = 0.$$

If $a_{12} = 0$, so $\phi = 0$ can be taken. If $a_{12} \ne 0$, the following equation must be solved:

$$\operatorname{ctg}(2\phi) = \frac{a_{11} - a_{22}}{2a_{12}}.$$

Obviously, the solution of this equation exists. Therefore, there is always such ϕ that at which $\tilde{A}$ becomes diagonal matrix. In addition, with any choice of ϕ, we get:

$$\lambda_1 = \cos^2\phi a_{11} + 2\cos\phi\sin\phi a_{12} + \sin^2\phi a_{22},$$
$$\lambda_2 = \sin^2\phi a_{11} - 2\cos\phi\sin\phi a_{12} + \cos^2\phi a_{22}.$$

Hence $\lambda_1 + \lambda_2 = a_{11} + a_{22}$. At the same time, using equality $\begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix}\begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix}\begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$, and that the determinant of the product of matrices is equal to the product of the determinants, find $\lambda_1\lambda_2 = a_{11}a_{22} - a_{12}^2$. Thus, λ_1 and λ_2 are roots of the quadratic equation

$$\lambda^2 - \left(a_{11} + a_{22}\right)\lambda + \left(a_{11}a_{22} - a_{12}^2\right) = 0.$$

Note that the left-hand side of this equation is true $\det\begin{bmatrix} a_{11}-\lambda & a_{12} \\ a_{12} & a_{22}-\lambda \end{bmatrix}$. This polynomial of λ is a characteristic polynomial of a matrix $\begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix}$, so, λ_1 and λ_2 are the eigenvalues of this matrix.

It is easy to see that using turning the initial coordinate system by some angle ϕ equation $f(x_1, x_2) = 0$ is transformed into new coordinates to:

$$\lambda_1 \tilde{x}_1^2 + \lambda_2 \tilde{x}_2^2 + 2b_{13}\tilde{x}_1 + 2b_{23}\tilde{x}_2 + b_{33} = 0,$$

where $[b_{13} \quad b_{23}] = [a_{13} \quad a_{23}]\begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$, $b_{33} = a_{33}$, λ_1 and λ_2 are the eigenvalues of this matrix A_2 of a quadratic polynomial (1).

6 Movement of the coordinate system

It is natural to assume that the quadratic part f_2 in the rotated coordinate system is not identical to zero. Thus, λ_1 and λ_2 are not equal to zero at the same time.

Then let's distinguish in the quadratic part the perfect squares

$$\lambda_1 \tilde{x}_1^2 + 2b_{13}\tilde{x}_1 = \lambda_1\left(\tilde{x}_1^2 + 2\frac{b_{13}}{\lambda_1}\tilde{x}_1 + \frac{b_{13}^2}{\lambda_1^2}\right) - \frac{b_{13}^2}{\lambda_1^2} = \lambda_1\left(\tilde{x}_1 + \frac{b_{13}}{\lambda_1}\right)^2 - \frac{b_{13}^2}{\lambda_1^2},$$

$$\lambda_2 \tilde{x}_2^2 + 2b_{23}\tilde{x}_2 = \lambda_2\left(\tilde{x}_2^2 + 2\frac{b_{23}}{\lambda_2}\tilde{x}_2 + \frac{b_{23}^2}{\lambda_2^2}\right) - \frac{b_{23}^2}{\lambda_2^2} = \lambda_2\left(\tilde{x}_2 + \frac{b_{23}}{\lambda_2}\right)^2 - \frac{b_{23}^2}{\lambda_2^2}.$$

Let's move the Cartesian coordinate system $\tilde{x}_1$ and $\tilde{x}_2$, and move its origin at the point

$$O' = \left(-\frac{b_{13}}{\lambda_1}, \quad -\frac{b_{23}}{\lambda_2}\right).$$

Then new coordinates z_1 and z_2 will be expressed through $\tilde{x}_1$ and $\tilde{x}_2$ as follows:

$$z_1 = \tilde{x}_1 + \frac{b_{13}}{\lambda_1}, \quad z_2 = \tilde{x}_2 + \frac{b_{23}}{\lambda_2}.$$

In the new coordinates, the equation $f(x_1, x_2) = 0$ loses its linear part and looks as

$$\lambda_1 z_1^2 + \lambda_2 z_2^2 + k = 0,$$

where $k = b_{33} - \frac{b_{13}^2}{\lambda_1^2} - \frac{b_{23}^2}{\lambda_2^2}$.

So, by rotating and moving the initial coordinate system, the equations $f(x_1, x_2) = 0$ is reduced in new coordinates to kind $\lambda_1 z_1^2 + \lambda_2 z_2^2 + k = 0$. Moreover, if an equation $f(x_1, x_2) = 0$ in any Cartesian coordinate system has this form, then it can have another form in no other Cartesian system.

7 Criteria Normalization

Returning to the issue of normalization, note that a new basis can be constructed by reducing the quadratic form (1) to the canonical form. As is known, any real symmetric matrix is orthogonally similar to a real diagonal matrix [7]

$$\Lambda = T^T A T, \; T^T = T^{-1},$$

where A is a matrix of coefficients a_{ij}, Λ is a diagonal matrix of eigenvalues of the matrix A, T is a matrix of transition or transposition from one basis to another.

In the new variables, the quadratic form (1) is an algebraic sum

$$f = \lambda_1 z_1^2 + \lambda_2 z_2^2 + \ldots + \lambda_m z_m^2.$$

In general case, for T only condition of invertibility is required. Finding the appropriate replacement of variables for a given quadratic form is called a canonical form. If T is an orthogonal matrix, then we are talking about bringing (1) to the principal axes, and the transition is made by expressions

$$z = Tx \text{, and accordingly } x = T^{-1} z.$$

But in our case we have two quadratic forms at once $f_1(x)$ and $f_2(x)$, which are second order curves on the plane. It will be wise to try to simplify their equations in a common coordinate system for both. In the general case, this coordinate system will be affine. Note that the points in the affine space are equal, they cannot be stacked with each other. In the affine space, there is also no concept of zero point or start of reference.

Suppose one of the curves is an ellipse. Then move to a Cartesian system, in which the equation is obtained for it $x_1^2 / a^2 + x_2^2 / b^2 = 1$. The equation of the second curve in this system may have the most general form. Changing the scales along the axes, move to the affine space, in which the equation of the ellipse is the equation of the circle $\tilde{x}_1^2 + \tilde{x}_2^2 = 1$. The equation of the second curve in the new (affine) system still has the general form. But using rotation, a form $\lambda_1 z_1^2 + \lambda_2 z_2^2$ can be obtained for its quadratic part. However, the rotation of the coordinate system can not change the form of the first equation.

So, let's have two real symmetric matrices A_2^1 and A_2^2, which are matrices of coefficients a_{ij} of quadratic parts of forms $f_1(x)$ and $f_2(x)$ accordingly, and thus A_2^1 positively defined. In the general case, the positivity of the matrices can be verified using the Sylvester's criterion.

Then there is the real invertible matrix T such that both matrices $T^T A_2^1 T$ and $T^T A_2^2 T$ are diagonal. That is, using the transposition of matrix *T*, forms $f_1(x)$ and $f_2(x)$ can be simultaneously reduced to a canonical form.

Indeed, because of the positive defining, all the eigenvalues of the matrix A_2^1, $\lambda_i > 0$ for all *i*. Then A_2^1 is orthogonally similar or congruent to a diagonal matrix Λ

$$\Lambda = \begin{bmatrix} \lambda_1 & \\ & \lambda_2 \end{bmatrix} = Q^T A_2^1 Q, \quad Q^T = Q^{-1} ,$$

where *Q* is a matrix of eigenvectors of the matrix A_2^1.

Further note that A_2^1 is also congruent to the identity matrix *I*

$$I = \Lambda^{-1/2} Q^T A_2^1 Q \Lambda^{-1/2} = \left(Q\Lambda^{-1/2}\right)^T A_2^1 \left(Q\Lambda^{-1/2}\right).$$

Let congruence transformation in application to A_2^2 gives a matrix

$$C = \left(Q\Lambda^{-1/2}\right)^T A_2^2 \left(Q\Lambda^{-1/2}\right).$$

It is easy to verify that *C* remains a real symmetric matrix. Therefore, using the orthogonal matrix *V*, which is the matrix of eigenvectors of matrix *C*, a diagonal matrix $D = V^T C V$ is obtained. In this case, *D* is a diagonal matrix of eigenvalues of matrix *C*

$$D = \begin{bmatrix} \lambda_1^c & \\ & \lambda_2^c \end{bmatrix}.$$

At the same time, $V^T I V = I$. Finally $I = T^T A_2^1 T, \quad D = T^T A_2^2 T$, where $T = Q\Lambda^{-1/2} V$.

So, in order to normalize the criteria $f_1(x)$ and $f_2(x)$, that is, to reduce them to the canonical form, it is necessary:

– to select at least one positive definite quadratic part of these criteria, for example, let in our case be a positive definite quadratic part of the first criterion, i.e. A_2^1;

– find eigenvalues and eigenvectors of a positive definite matrix A_2^1 and add matrices Λ and *Q*;

– using the quadratic part of the second criterion A_2^2 and matrices Λ and Q calculate matrix C;
– find eigenvalues and eigenvectors of matrix C and add matrices D and V;
– using matrices Λ, Q and V calculate the transpose matrix T, with a help of which transition to a new basis is executed.

After diagonalization, the criterion functions (2) look like

$$f_1(z_1, z_2) = (z_1 - z_{01})^2 + (z_2 - z_{02})^2 + k_1,$$
$$f_2(z_1, z_2) = \lambda_1 z_1{}^2 + \lambda_2 z_2{}^2 + k_2,$$

or in general case

$$f_1(z_1, z_2, \ldots, z_m) = (z_1 - z_{01})^2 + (z_2 - z_{02})^2 + \ldots + (z_m - z_{0m})^2 + k_1,$$
$$f_2(z_1, z_2, \ldots, z_m) = \lambda_1 z_1{}^2 + \lambda_2 z_2{}^2 + \ldots + \lambda_m z_m{}^2 + k_2.$$

The transition from the old coordinate system to the new one and vice versa is done using expressions $z = Tx$ and $x = T^{-1}z$.

Therefore, using scale variation on the axes, rotation, and moving the coordinate system, the old ellipse family (Fig. 1) can be transformed into a family of circles centered at a point $(z_{01}, \quad z_{02})$ and into a family of deformed ellipses (Fig. 2).

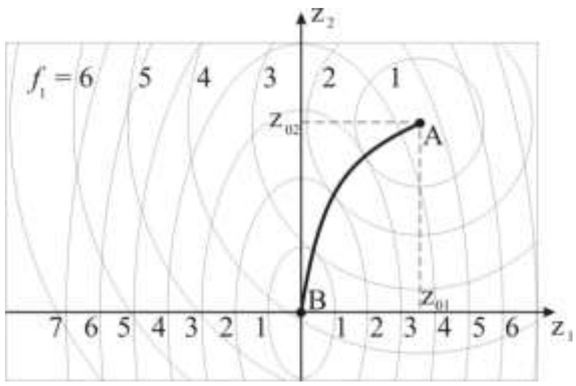

Fig. 2. Families of deformed ellipses

8 Finding Pareto-optimal decisions for SDN construction

In content terms, the idea of solving the issue of finding Pareto-optimal software-defined networking decisions is to determine the equation of the curve AB in the multidimensional space of the investigated parameters. It was noted above that the Pareto line, or curve AB, passes through the adherent point of isoquants. Moreover, moving to a new coordinate system, this property is not changed, it is still the locus of adherent points of the second order lines of a new family f_1 with lines of family f_2 (Fig. 2).

So, at these points the tangent line and normal to the isoquants of the family f_1 coincide with the tangent line and normal to the isoquants of the family f_2, which allow to form a system of equations of the type

$$\left.\begin{aligned} \frac{x_1 - x_1^O}{\left.\frac{df_1}{dx_1}\right|_{M_O}} = \frac{x_2 - x_2^O}{\left.\frac{df_1}{dx_2}\right|_{M_O}} \\ \frac{x_1 - x_1^O}{\left.\frac{df_2}{dx_1}\right|_{M_O}} = \frac{x_2 - x_2^O}{\left.\frac{df_2}{dx_2}\right|_{M_O}} \end{aligned}\right\} \Rightarrow \left.\begin{aligned} \frac{x_1 - x_1^O}{x_2 - x_2^O} = \frac{\left.\frac{df_1}{dx_1}\right|_{M_O}}{\left.\frac{df_1}{dx_2}\right|_{M_O}} \\ \frac{x_1 - x_1^O}{x_2 - x_2^O} = \frac{\left.\frac{df_2}{dx_1}\right|_{M_O}}{\left.\frac{df_2}{dx_2}\right|_{M_O}} \end{aligned}\right\},$$

where MO is an adherent point, x_1^O, x_2^O are coordinates of adherent point.

In order for the adherent point to be Pareto-optimal, it must belong to the curve AB, that is $M_O\left(x_1^O, x_2^O\right) \in AB$.

In the transition to a new coordinate system value $M_O\left(x_1^O, x_2^O\right)$ is identical both for family f_1 and family f_2, therefore, in the new coordinate system, the initial equation system looks like this:

$$\left.\begin{aligned} \frac{df_1}{dz_1} = 2(z_1 - z_{01}) \\ \frac{df_2}{dz_1} = 2\lambda_1 z_1 \end{aligned}\right\} \qquad \left.\begin{aligned} \frac{df_1}{dz_2} = 2(z_2 - z_{02}) \\ \frac{df_2}{dz_2} = 2\lambda_2 z_2 \end{aligned}\right\}$$

So, knowing one coordinate of point, the value of another point from a new system of equations can be determined by the formula

$$z_2 = \frac{z_{02}}{1 - \frac{\lambda_2}{\lambda_1}\left(1 - \frac{z_{01}}{z_1}\right)}.$$

It can be shown that, in the multidimensional case, the coordinates of the points belonging to the Pareto line, can be determined by a similar formula

$$z_m = \frac{z_{0m}}{1 - \frac{\lambda_m}{\lambda_1}\left(1 - \frac{z_{01}}{z_1}\right)},$$

where z_{0m} is m-th coordinate of center of family f_1 in new coordinate system; λ_m is eigenvalue of the quadratic form f_1.

So, knowing the coordinates of the family center f_1, that is $(z_{01}, z_{02}, .. z_{0m})$, eigenvalues of the quadratic form f_1, that is $(\lambda_1, \lambda_2, ..., \lambda_m)$, and changing the coordinate value z_1 in the range $[0, z_{01}]$, it is possible to calculate other coordinates of points belonging to the Pareto line. Application of inverse transformations of the coordinate system allows to obtain Pareto-optimal solutions in the initial coordinate system.

9 Conclusions

Thus, the proposed method allows to obtain from the Pareto-optimal range of criteria by the value of one of any given parameter the whole set of optimal network parameters, as well as the value of the criterion functions in the multidimensional criterion space. The values thus found on the set of several criteria will be the Pareto-optimal values of the software-defined networking construction.

In addition, applying the approaches proposed in the method described in [8], we can find the unique optimal solution. Therefore, it must be remembered that the fundamental complexity of the selection task under many criteria is the inability to determine a priori what is called the best solution. Therefore, in decision-making, researchers should not only rely on their experience and intuition, but also turn to well-developed mathematical models of decision support, which allow correct selection of the best alternatives from available ones [9, 10]. After all, the success of further development of the whole research, design and development as a whole depends on how competent and competent the decision support is.

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