Can FCA Provide a Framework for AGI?

Francisco J. Valverde-Albacete[1,*,†], Carmen Peláez-Moreno[2,†], Inma P. Cabrera[3,†], Pablo Cordero[3,†] and Manuel Ojeda-Aciego[3,†]

[1] *Depto. Teoría de Señal y Comunicaciones, Sistemas Telemáticos y Computación, Univ. Rey Juan Carlos, Madrid, Spain*
[2] *Depto. Teoría de Señal y Comunicaciones, Univ. Carlos III de Madrid, Madrid, Spain*
[3] *Depto. Matemática Aplicada, Univ. de Málaga, Málaga, Spain*

Abstract
This paper is an attempt at bridging two strains of research being developed by the authors: a theory of information flows to subserve intelligence and a theory of affordances for the modelling of Embodied, Embedded, Extended and Enacted Computational Intelligence as provided by FCA. We list previous successes, present challenges, and future avenues of research that suggest themselves.

Keywords
Formal Concept Analysis. Embodied, Embedded, Extended and Enacted Intelligence. Affordances. Qualitative Semantics.

1. Towards a theory of 4E Intelligence

One of the inceptors of present-day's state-of-the-art in AI, blames the failure of old-time, symbol-based incarnation of AI on its refusal to take Biological Neural Networks (BNN) as a source of inspiration [1]. They cite three "generations" of NN in Machine Learning (ML) and suggest how each succeeded, once the hurdles of modelling specific behaviour of neural tissue and devising computational techniques to implement them were overcome.

In the Ecological Theory of Perception [2] intelligence amounts to performant cognition. At least in one stream of Cognitive Theory, cognition is *embodied, embedded, extended and enacted (4E)* [3]. These adjectives refer to qualities that the system organism-within-an-environment should have to demonstrate cognition and ultimately intelligence:

- *embodiment*, refers to having a body to behave with,
- *embedding*, to being situated within an enveloping environment,
- *extension*, to the fact that organisms can supplement their bodies through tools to extend their effect onto the environment, and

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*Corresponding author.
†These authors contributed equally.
francisco.valverde@urjc.es (Francisco J. Valverde-Albacete); cpm@tsc.uc3m.es (C. Peláez-Moreno); ipcabrera@uma.es (I. P. Cabrera); pcordero@uma.es (P. Cordero); aciego@uma.es (M. Ojeda-Aciego)
0000-0002-5874-7604 (Francisco J. Valverde-Albacete); 0000-0003-1425-6763 (C. Peláez-Moreno); 0000-0001-5129-0085 (I. P. Cabrera); 0000-0002-5506-6467 (P. Cordero); 0000-0002-6064-6984 (M. Ojeda-Aciego)

- *enaction*, to the fact that the interaction with the environment, e.g. *carrying out behaviours*, mediates in developing meaning and goals for the organism.

In this paper we speculate about machines attaining *(4E) intelligence*—as a synonym of (Natural) General Intelligence—using the techniques and tools stemming from Formal Concept Analysis (FCA), as the general framework to deal with Galois connections induced by formal contexts [4].

From the *description* of the phenomenology of cognition and intelligence above, we move naturally onto the *design* of 4E intelligence: one way forward is to invoke the Predictive Coding or Processing hypothesis [5]. We have elsewhere summarized how this leads to a series of hypotheses about the forward and backward flows of information in BNN [6], to wit:

- BNN implement *communications at multiple spatial and temporal scales* with the goal of maintaining enaction through the *sensory-motor loop* closed by the environment. In this view, rhythms at different temporal scales subserve both the information flow along dual forwards/backwards streams as well as the prediction of the timing of events at different time-scales adapted to environment dynamics [7].
- BNNs are instances of biological *Complex Dynamical Systems (CDS)*, whose model [8] is tuple (X, S, M, A, Ω) describing an agent (that defines the "internal" space) embedded within and environment (defining the "external" space) with explicit dependences on the tuple of states $b = (s, a)$, and implicit dependence on a set of statistical parameters θ, where we have
  a) *hidden external states* $\eta \in X \subset \mathbb{R}^n$, governed by a nonlinear funtion $f_\eta : X \times S \times A \rightarrow R^n$, modeling the outer environment,
  b) *sensory states* $s \in S \subset \mathbb{R}^p$, governed by another nonlinear function $f_s : S \times S \times A \rightarrow S$, capturing the evolution of perception,
  c) *hidden internal states* $\mu \in M \subset \mathbb{R}^q$, governed by another nonlinear function $f_\mu : M \times S \times A \rightarrow M$ capturing its evolution, and
  d) *(control) actions* $a \in A \subset \mathbb{R}^m$, governed by another nonlinear function $f_a : A \times S \times A \rightarrow A$ designed to drive the dynamics,

$$\dot{\eta} = f(\eta, b; \theta) + \omega^\eta \qquad \dot{s} = g(s, b; \theta) + \omega^s \tag{1}$$
$$\dot{\mu} = f(\mu, b; \theta) + \omega^\mu \qquad \dot{a} = g(a, b; \theta) + \omega^a$$

  e) Ω is a sample space from which *random fluctuations* $\omega^\eta \in \mathbb{R}^n$, $\omega^s \in \mathbb{R}^p$, $\omega^\mu \in \mathbb{R}^q$, and $\omega^a \in \mathbb{R}^m$ are drawn.
- Free-Energy (FE) models are specializations of said CDS, Bayesian models developed to reconcile the predictive coding hypothesis with neural architectures in a sensory-motor loop [5]. A basic FE model is a scale-agnostic model of the behaviour of situated agents for perception and learning [9].
- Neural processing is the communication of *entropy flows* though BNN. However, entropy being a *quantitative* property of probability measure values, it is "blind" to any particular events or "bins" of distributions, so
- BNN require *topical maps* to keep track of the *qualities* quantified by entropy flows from their information sources to their destinations.

For lack of a better name, let's call the model we envision a *4E Artificial Neural Network (4E-ANN)*. Then these should be machines (embodied) that route entropy—the quantity of information—flows between sensors, actuators and the environment (embodied and embedded) and provide

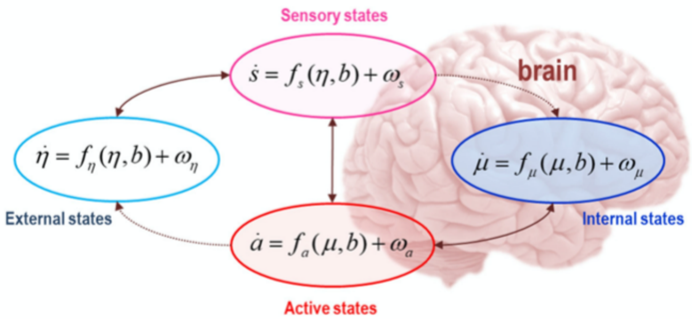

Figure 1: Situated Complex Dynamical System in the FE framework, with implicit dependence $b = (s, a)$ (from [8]).

interpretation of these—the quality of information—in the form of connectivity or topic maps, to subserve their own goals (so as to attain extension and enaction).

2. The Affordances of FCA for a theory of 4E Intelligence: Quanta and Qualia

2.1. The Formal Qualia in Binary Formal Contexts

Ever since Wille himself cautioned against *only* reading hierarchical knowledge from FCA, there have been attempts at "other readings" from the information collected in a formal context, e.g. [10, 11].

Recently, we have been calling collectively these three kinds of analyses emanating from a formal context Formal *Context* Analysis [12, 13, 14] but we now think about these in terms of *Formal Context Transform (FCT)*, using an analogy based on Fourier Analysis:

- In the *analysis phase* of any type of FCT the information contained in a formal context $\mathbb{K}$ is described in terms of the *formal qualia (sing. quale)*, basic abstractions Q that capture some essence of mathematical modelling, e.g. chains, antichains, partitions, etc. endowed with a partial order $\langle Q, \leq_Q \rangle$. The sets of formal qualia result in a pair of (possibly dual) complete lattices join- or meet-embedded within their order.
- In the *synthesis phase* using the join- and meet-irreducibles of some complete lattice $\langle L, \leq_L \rangle$ we synthesise a context that would return an isomorphic lattice $' \equiv L$.

The composition of both steps only allows us to maintain structure up to an isomorphism that aligns with the restrictions that the formal qualia allow, that is, they provide a focus or lens on *some* type of information included in the context, missing others:

- FCA, the analysis in terms of upper and lower bounds of the order imposed by the polars of the context on the object extents and attribute intents, producing quale *dependence or hierarchy*.
- FIA, the analysis in terms of maximal antichains of that order, evincing quale *independence* and
- FEA, the analysis in terms of the equivalence relations which are refinements of the

standard congruences on objects and attributes imposed by the polars of the context, evincing quale *undistinguishability*.

But whether there might be other types of FCT is still an open issue.

2.2. The Need for Quantification in Modelling AI with FCA

A simplified (no control $u(t)$, only considers η and s states), linearized, discrete-time form of (1) can be Z-transformed as:

$$\eta_{k+1} = A_k \cdot \eta_k + \omega_k^\eta \qquad\qquad s_k = C_k \cdot \eta_k + \omega_k^s \tag{2}$$

where the instantaneous *transition and observation matrices A_k and C_k*, respectively, are to be learned[1].

Considering a set of state and observation dimensions I_η and I_s, respectively, and by virtue of the cryptomorphism between matrices with entries in an algebra, bipartite digraphs with weights in an algebra and relationships with strength in an algebra, we may consider the formal contexts $\mathbb{K}_k^\eta = \langle I_\eta, I_\eta, A_k \rangle$, describing the instantaneous dynamics of the CDS, and $\mathbb{K}_k^s = \langle I_s, I_\eta, C_k \rangle$, describing its instantaneous observation process.

We know that when the underlying algebra is an idempotent semifield $\mathcal{K}$ these formal contexts allow the definition of $\mathcal{K}$-FCA, that provides the FCA-flavour of analysis in a quantitative setting [15]. Indeed, the linear spaces associated with the input and output spaces of A_k and C_k are dually-isomorphic lattices that describe hierarchies of (quantitative) vectors that prove that standard FCA is the analogue of the Singular Value Decomposition for linear forms over vector spaces over idempotent semifields [16].

In fact, there are also different "conceptualizations" of the spaces associated with a matrix that resembles the structures needed for FIA and FEA [17, 18], and also the concept of "disparity" or "discord" [19] can be modelled in the fuzzy setting—a sister theory to $\mathcal{K}$-FCA. This is a different quale to those seen above, and a certain sign that more formal qualia can be discovered.

At least all of these (and more) strains of research would need to be fused to be able to provide instantaneous lattice-like pictures of what (2) represents. Note that, after our argument, important concepts in the study of CDS, e.g. phase space, evolve in a (constrained) lattice and therefore trajectories and cycles may take strange forms, unseen so far.

3. Conclusions: Towards a Quanta-and-Qualia Theory of 4E Intelligence

In summary, we believe the generic FCA framework shows promise to help with modelling 4E intelligence, e.g. as encapsulated in a model like Friston's [8]. But in order to do so, it has to be extended:

- Qualitatively, by enabling the inference of new formal qualia that cater for a better foundation of enaction and modelling the information-distinctions effectively carried out by BNNs. This is the purpose of our efforts towards better understanding FCT [20].

[1]Note that this is a qualitatively different model to that of (1).

- Quantitatively, by enabling such qualitative inferences in the presence of quantitatively rich data, e.g. as provided by transition and observation matrices with entries in an information semifield. This is the purpose of our efforts towards understanding information semifields [21], their linear-algebraic constructions [22, 17], and their relationship with Galois connections [23, 18].

This Quantitative-and-Qualitative, or *Quanta-and-Qualia (QaQ)* theory of 4E-intelligences seems a promising point for 4E-NN intelligent machines, whose learning theory we have not even broached in this paper. This is left for future work.

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