On the Affordance-Theoretic bases of the Landscape of Knowledge Paradigm

Francisco J. Valverde-Albacete[1,*], Carmen Peláez-Moreno[2], Inma P. Cabrera[3], Pablo Cordero[3] and Manuel Ojeda-Aciego[3]

[1]Depto. Teoría de Señal y Comunicaciones, Sistemas Telemáticos y Computación, Univ. Rey Juan Carlos, Madrid, Spain
[2]Depto. Teoría de Señal y Comunicaciones, Univ. Carlos III de Madrid, Madrid, Spain
[3]Depto. Matemática Aplicada, Univ. de Málaga, Málaga, Spain

Abstract

In this paper we set out to understand the cognitive basis of Formal Concept Analysis used as an Exploratory Data Analysis framework under the guise of the Landscapes of Knowledge metaphor introduced by Wille. We show that it can be re-interpreted and extended in the framework of the Theory of Affordances from Ecological Psychology to provide not only different affordances for different flavours of formal analysis of the information captured by a formal context, but also a theory that sheds light on how we learn to do it, Perceptual Learning. This raises the issue of what it is that a formal analysis of a formal context provides. We introduce the concept of formal qualia as basic, incomparable, privative items of information afforded by each possible analysis and illustrate these concepts by the formal qualia provided by Formal Concept, Independence and Equivalence Analysis.

Keywords

Closure system, Galois connection, Fuzzy lattice

1. Introduction and Motivation

Exploratory Data Analysis (EDA) was introduced by Tukey [17] as a complement to Confirmatory Data Analysis (CDA)—often called nowadays *Predictive Modelling*. EDA preconizes the analysis of data prior to issuing hypotheses, that may lead to better models for it, the premise being that by investigating the data and gaining intuitions about it, better informed hypotheses and models may arise for CDA. So in this original purpose, EDAs is subservient to CDA.

The theory of Ecological Psychology [10] would have otherwise: every inquisitive action carried out by an organism—indeed every instance of cognition—is of an interactive, exploratory nature. Predictive analysis can therefore only be a transient result in an endless cycle of interaction: as soon as we act on the prediction the cycle closes and we have again explored (just prior to a new prediction). The basis for this interaction is the notion of *affordance* in the

Published in Pablo Cordero, Ondrej Kridlo (Eds.): The 16th International Conference on Concept Lattices and Their Applications, CLA 2022, Tallinn, Estonia, June 20–22, 2022, Proceedings, pp. 93–104.
*Corresponding author.
francisco.valverde@urjc.es (F. J. Valverde-Albacete); carmen@tsc.uc3m.es (C. Peláez-Moreno); ipcabrera@uma.es (I. P. Cabrera); pcordero@uma.es (P. Cordero); aciego@uma.es (M. Ojeda-Aciego)
0000-0002-5874-7604 (F. J. Valverde-Albacete); 0000-0003-1425-6763 (C. Peláez-Moreno); 0000-0001-5129-0085 (I. P. Cabrera); 0000-0002-5506-6467 (P. Cordero); 0000-0002-6064-6984 (M. Ojeda-Aciego)

original sense coined by James J. Gibson—the father of Ecological Psychology—in the context of animal psychology, "The affordances of the environment are what it *offers* the animal, what it *provides* or *furnishes*, either for good or ill [10]."

Actually, Ecological Psychology can be taken as a philosophical foundation for Embodied, Embedded, Extended and Enacted (4E) Cognition [14]. These adjectives refer to qualities that the system organism-within-an-environment exhibits to support cognition:

- *embodiment*, refers to the fact of having a body to behave with,
- *embedding*, to the fact of being situated within an enveloping environment,
- *extension*, to the fact that organisms can supplement their bodies through tools to extend their effect onto the environment, and
- *enaction*, to the fact that the interaction with the environment, e.g. *carrying out behaviours*, mediates in developing meaning and goals for the organism.

The consideration of Mathematics as the study of "possible realities" as well as the consideration of the concept of a (mathematical) model, suggests that Maths as a whole is a mechanism for extending reality [15]. We adhere in this paper to the hypothesis that

Hypothesis 1: Mathematics as Extended Cognition

Mathematics is a systematic way to build Extended Cognition in "alternate realities": those that appear by the processing of abstracting models of the real world.

Recall that Wille introduced early on his "Landscapes of Knowledge" (LofK) metaphor [22] that Formal Concept Analysis (FCA) is eminently suited for the EDA of binary table data collected in the form of *formal contexts*. In this paper we try to elucidate the following question:

Question 1: 4E Cognition applied to Maths: the FCA case.
What are the affordances of applying hypothesis 1 to the particular instance of Mathematical thinking and modelling efforts encapsulated within FCA?

We make a case below that LofK was embodied, embedded and extended as a matter of principle, and, furthermore, we provide contrastive evidence in this paper that it was (implicitly) enacted, by providing examples of different, complementary enaction in the form of Formal Independence Analysis (FIA) [20], and Formal Equivalence Analysis (FEA) [21].

For that purpose, we will briefly review the Ecological Theory of Affordances (section 2.1), and in particular its operative interpretation of learning (section 2.2). To pave the way for our analysis of enactment we will briefly introduce the notion of Formal *Context* Analysis (FxA) as a generalisation of FCA (section 2.4), that will provide the support for our analysis of the enactment afforded by Formal Concept Analysis.

2. Preliminary notions and theoretical basis

In order to answer question 1 we should have an answer to the following ones:

1. What is a satisfactory explanation of cognition for the human animal, that is, up to a certain extent, a *Theory of Cognition*? We suggests that the Theory of Affordances is one such.
2. What is an appropriate *Theory of Learning* within the above constraints? We suggest the theory of Perceptual Learning within the Ecological Approach to Perception might be one such theory.
3. What is a *Theory of the praxis of EDA*? For the case of boolean tables cast as formal contexts, we want to propose an adequate extension of LofK.

2.1. Brief Introduction to the Theory of Affordances.

As mentioned above "... The affordances of the environment are what it *offers* the animal, what it *provides* or *furnishes*, either for good or ill" [10]. This basic operational definition has been explored, deepened and complemented in the works of a number of researchers. The theory here presented was first put forward in [2] and later completed in [3].

Consider E to be a set of abstract environments (yet to be fully determined), O a set of organisms, and consider the three sets of *ontological modes*: *abilities* (of cognitive states of organisms) A, *behaviours* (of organisms) B, and *features* (of situations of environments) C.

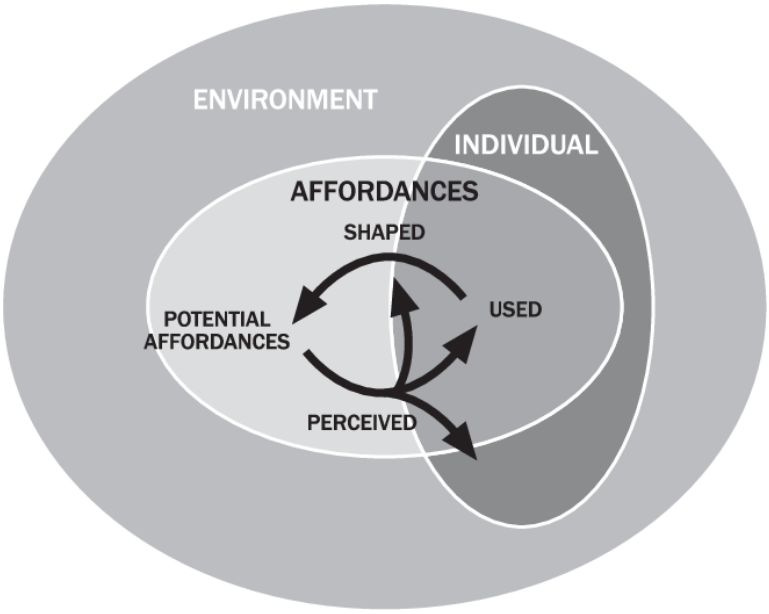

Figure 1: A conceptual view of the relational nature of affordances in the subject-environment interaction, from [12]

By virtue of its dynamics, the environment is in a *situation*—for all purposes a *state*—in which some of its *features* are salient. Note that situations are here understood as in Situation Theory [1]. In a first approach, describe the *situation of the environment* as a subset of features, that is, $s : E \to 2^C$.

On the other hand, by virtue of its evolutionary development and, possibly, cultural history, the organism finds itself in a *functional state* $r : O \to 2^A$ described by its abilities, since such abilities allow it to carry out a catalogue of behaviours that interact with the environment.

Then an affordance $f(b) \in F$ for a behaviour $b \in B$ is an instance of a binary relation $f(b) = (\alpha, \gamma) \in 2^A \times 2^C$, a tuple between a subset of abilities $\alpha \in 2^A$ and a subset of features of the environment $\gamma \in 2^C$, paraphrased as "(In situation γ the environment for an organism having the ability set α) the behaviour b is afforded."

Then Perceives$[o, e, f(b) \equiv (\alpha, \gamma)]$ is the act of perception studied by ecological psychology. But typically the organism perceives the behaviour as being afforded, not the relation itself Perceives$[o, f(b)]$ and the environment is probably *perceptually transparent*.

For Chemero [2, 3], any change in the set of available features or abilities is an *event*. We want to distinguish the dynamics of the environment, "embodied" in the physical world—that we refer to as *happening*—and *the dynamics of the* changes in abilities embodied in the organism's body—that we call *learning*. For notating dynamics we need to index the environment and the organism in a time flow t.

2.2. Perceptual Learning in the Ecological Approach.

Affordances may fail to be detected. This classical observation is due to [8] in the context of technological affordances, but applies equally to any other type. Figure 2 presents the phenomenology of affordances in terms of classical detection theory.

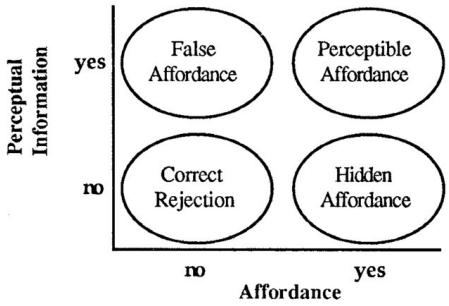

Figure 2: The phenomenology of affordances under detection theory, from [8]

One of the tenets of E. Gibson's Ecological Theory of perception [9] is that detection can be improved through *perceptual learning* that involves long-term changes to perception due to practice or experience [16]. In this view perceptual learning is "any relatively permanent and consistent change in the perception of a stimulus array, following practice or experience with this array" [9]. Confirmatory and exploratory are both modes of operation in Perceptual Learning, hence both CDA and EDA are supported by this framework.

A further development of this demands that this change must:

1. involve a change in perception, e.g. must be *truly perceptual*, that is, only if it affects how the stimulus appears to the subject in her perceptual experience.
2. be *long term*, e.g. short term adaptations will not count.

Metaphor number	Statement
1	Undiscovered knowledge is uncharted territory.
2	The intricacies of knowledge are features of the landscape.
3	Building a formal context is setting yourself on a vantage point on the knowledge landscape.
4	A concept lattice is a map of the knowledge landscape.
5	Knowledge is an exhibit.
6	Knowledge can be visualized.
7	Knowledge can be (bodily) manipulated.

Table 1
Summary of the basic metaphors in LofK, from [18]. Sub-metaphors are further indented.

3. result *from practice or experience*, not contingent on whether the perceptual organ changes (e.g. in blindness or correction).

The ecological theory of perception asserts that developmental perceptual learning—in infants—is carried out seamlessly into adulthood, where it is properly conceived as perceptual learning. In their first year of life infants learn (a) to communicate and interact with other people, (b) to reach, grasp and interact with objects, and (c) to locomote and ambulate in the real world. This process of perceptual learning is always on as "Potential new affordances never stop becoming available, nor do people of any age stop learning to perceive them (...) as reaching, handling and locomotion develop, encounters with the environment broaden and potential affordances multiply (...). Increasing experience provides a backlog of knowledge that breeds new opportunities." [9, Ch.10].

Critically, since it tries to understand human behaviour, Ecological Perception Theory considers Perceptual Learning the means to achieve the four distinctive emergent hallmarks of human behaviour [9, Ch.9]:

1. *agency*, the control over your own behaviour.
2. *prospectivity*, the forward-looking[1] character of behaviour that results in plans.
3. *perceiving order*, because it is all around and we need to detect and use it.
4. *flexibility*, that is, accommodating to new circumstances with alternatives that are not random, but appropriate to the task.

2.3. FCA as a Type of Exploratory Data Analysis: Landscapes of Knowledge

Landscapes of Knowledge (LofK, [22]) is a collection of metaphors on how to use FCA [7] for EDA. A partial list of the metaphors elicited in [18] can be found in Table 1. More metaphors, in the context of other frameworks for EDA, data mining or Knowledge Discovery in Databases can be found in that reference.

The metaphors are supported on the "canonical behaviours" or *activities* to carry out when examining boolean data with FCA. Valverde-Albacete et. al. [18] collected and extended Wille's original proposal for activities [22] as listed on Table 2.

Note that, although these activities are aligned with the purposes of EDA, from the point of

[1]Sometimes called "predictive"

Activity	Description	Aliases
Focusing	Selecting what one wants to look into.	*contextualizing*
Exploring	"...looking for something of which one only has a vague idea..."	*browsing*
Searching	"...looking for something which one can more or less specify but not localize..."	
Recognizing	"...perceiving clearly circumstances and relationships..."	*Clarifying*
Identifying	"...determining the taxonomic position of an object within a given classification..."	*categorizing, placing in a taxonomy*
Analyzing	"...examining data in their relationships while guided by the theoretical views and declared purposes..."	
Investigating	"...study by close examination and systematic enquiry..."	
Deciding	"...resolving a situation of uncertainty by an order..."[2]	
Improving	"...enhancement in quality and value. ..."	*distilling, valorizing*
Restructuring	"...to reshape a given structure, which, within the scope of our discussion, is conceptual in its nature..."	
Memorizing	"...a process of committing and reproducing what has been learned and retained. ..."	*committing to memory*
Hypothesizing	Abducing facts and relationships based on data.	*forming an opinion*
Indexing	Indexing other knowledge resources using concepts, objects or attributes	*interfacing resources*

Table 2
Extended list of activities supporting the Landscape of Knowledge metaphor, their descriptions (from [18]) and some aliases that we use for them.

view of Perceptual Learning, of which EDA is certainly an instance, CDA is just an incomplete exploration, so these activities should also be relevant to it, despite the fact that FCA is seldom used for CDA.

2.4. Alternative Analyses for Formal Contexts

Ever since Wille himself cautioned against *only* reading hierarchical knowledge from FCA, there have been attempts at "other readings" from the information collected in a formal context, e.g. [5, 4]. More recently, Formal Independence Analysis (FIA) [20, 19] and Formal Equivalence Analysis (FEA) [21] have tried to cast representation theorems for formal contexts as lattices of anti-chains of a poset and partitions of a base set, respectively, in the guise of FCA-like theorems.

In previous works we have been calling collectively these three kinds of analyses emanating from a formal context Formal *Context* Analysis (FxA):

- FCA, the analysis in terms of upper and lower bounds of the order imposed by the polars of the context on the object extents and attribute intents,

- FIA, the analysis in terms of maximal antichains of that order, and
- FEA, the analysis in terms of the equivalence relations which are refinements of the standard congruences on objects and attributes imposed by the polars of the context.

3. Results: A Theory of Affordances for the Perceptual Learning of F*x*A

Our results adopt the form of an argument clarifying the affordance-theoretic bases of the LofK paradigm as an EDA technique. For that purpose, with respect to the questions stated in the previous section we hypothesize that:

1. 4E cognition (Embodied, Embedded, Extended and Enacted) is an adequate frame theory for cognition.
2. Perceptual Learning as interpreted by the Ecological Approach to Psychology is an adequate theory to approach/describe 4E learning, and in particular the EDA process.
3. Affordances also stem from Extended cognition environments and abilities, e.g. Mathematical "worlds". By increasing/extending/expanding the inventory of features of the environment, we expand its describable situations. For instance, we can extend the affordances of FCA to the new features and abilities provided by F*x*A.

In the following subsections we further argue these points.

3.1. LofK is Embodied, Embedded, Extended and Enacted

A strong case of why LofK complies with 4E cognition are the metaphors made evident in the analysis of Table 1. Specifically, recall that metaphorical interpretation eases the comprehension of an unfamiliar domain of knowledge (concept lattices) in terms of an already-known domain (terrain navigation, object grasping): the entities and issues in the new domain are mapped onto the well-known domain to make them more intellectually apprehensible [13].

First, this and our previous positing Mathematics as a type of Extended Cognition would entail that LofK is also of that type since it is based on the Mathematical concepts and abstractions of FCA.

Second, note how the Extension forcibly makes approaching concept lattices as embodied (graspable) and embedded (navigable). Therefore, this metaphorical transfer essentially pretends that LofK is an embodied and embedded approach to the EDA of formal contexts, mainly using (bodily) navigation and manipulation.

Third, seeing LofK through the lens of Perceptual Learning further supports this extension hypothesis: the metaphors in Table 1—the extension mechanism—suggest that what is being exported towards the extended environment are the capabilities for "navigation" around a lattice, "manipulating" it and communicating its description, which—we found in Sec. 2.2—are the basic abilities developed and accrued by humans on the physical world.

Finally, to argue that LofK is enacted, we use Affordance Theory. After Chemero's proposal [2], to evince affordances, actions have to be available on the adequate features of the environment. Recall that affordances are possibilities for action, therefore each of the activities, essentially

behaviours, of Table 2 are the result of one affordance at least. Since Enaction refers to the con-formation of information through action [14], it follows to reason that LofK is enacted.

Note that an affordance may fail to be evinced by an organism in a particular environment and that not all affordances are *effective*, e.g. despite the fact that a drought may induce me to dance to the Rain God, this is not going to bring about any rain. Instead of using the heavily-connoted concept of *superstition* we have been using that of *mirage* to refer to these non-effective affordances in the context of FCA. For instance, the presence of a top—most abstract—and bottom—most specific—concepts in a concept lattice is a mirage demanded by the technique of analysis.

3.2. Interpreting LofK by means of Affordance Theory

In the case of FCA, the information provided by standard concept lattices refers to the two processes of *concept formation* and *concept hierarchisation.* It is not immediate, or indeed possible, to act jointly on extents and intents based on the information provided by the formal context. Neither are the hierarchical relationships between concepts readily available as features of the (extended) environment.

Note that an affordance may *fail to be evinced* by an organism in a particular environment, e.g. because of a failure in its senses, and that not all affordances are *effective*, e.g. despite the fact that a drought may induce me to dance to the Rain God, this is not going to bring about any rain.

Instead of using the heavily-connoted concept of *superstition* we have been using that of *mirage* to refer to these non-effective affordances in the context of FCA. For instance, the presence of a top—most abstract—and bottom—most specific—concepts in a concept lattice is a mirage demanded by the technique of analysis. Likewise, FCA fails to distinguish between objects that have identical intensions—an example of a non-evinced affordance from the formal context—in which case we have elsewhere asserted that is has a *blind spot* for such phenomenon. Indeed, the technique of Conceptual Exploration was invented to address this limitation [6].

3.3. Extending the Enaction in LofK: Formal Qualia

However, we know that FCA is not the only possible analysis that can be carried out for a particular formal context: both FIA and FEA shed light on what *type of information* is carried by it (Section 2.4). Can we formalize such *types of information*?

1. In the previous section we saw that LofK is Enacted through the conformation of information provided by FCA based on formal concept formation and hierarchisation.
2. It stands to reason that FIA will provide a different conformation of information to LofK based on *tomoi*—the analogue of formal concepts but based on pairs of antichains—and independence [20].
3. Likewise, FEA will provide a different conformation based on partitions of objects and attributes and the indistinguishability relations between objects or attributes implied by them [21].

Note that these different *conformations of information* are incomparable—formal concepts vs. tomoi vs. pairs of partitions, or hierarchisation vs. independence vs. indistinguishability—

and we apprehend intuitions on what knowledge they convey based on our mathematical pre-conceptions of (order-theory) bounds, anti-chains and partitions, and their typical order relation.

We introduce the following definition of a *formal quale (*plural *qualia)* as the basic, incomparable, privative conformation of information afforded by each possible analysis on a formal context. Here we suggest that *it is productive to think of FCA as providing formal qualia on hierarchisation, FIA on independence and FEA on indistinguishability, and this is consistent with the Ecological approach to defining information Enactively.*

Note that the term "quale" comes from the Philosophy of the Mind, and is both heavily defended and contested even in the context of Neuroscience [14]. Formal qualia transcend this debate by adhering to the interpretation afforded by mathematical training about the basic units of analysis: that is, what an order relation is, equivalently a partition, an anti-chain, an so on. Of course the basic challenge next is to find whether there are any more means of obtaining formal qualia from a context. This is left for further work.

3.4. Modelling the Theory of Affordances by means of F*x*A

What are the affordances of the environment e for an organism o at a definite time point t? Clearly, at least, those affordances for any possible behaviour $b \in B$ whose abilities α are encompassed by the present cognitive state of o and whose situation γ is "compatible" with the present state of the environment:

$$F(o,e) = \{(\alpha,\gamma) \mid \exists b \in B, f(b) = (\alpha,\gamma), \alpha \subseteq r(o), \gamma \subseteq s(e)\} \tag{1}$$

Note that we have posited here two different order relations between sets of abilities of the organism and sets of features of the environment.

After Gibson [10], a *niche* is a set of affordances for a particular organism, and this can be found by considering all possible situations of the environment, that is, as it evolves over time $t \in T$:

$$n(o) = \bigcup_{t \in T} F(o, e(t)) \tag{2}$$

If we want to model *learning* we should extend this to every possible cognitive state too, so we rather provide the time variable to the affordance functions:

$$F(o,e,t) = \{(\alpha,\gamma) \mid \exists b \in B, f(b) = (\alpha,\gamma), \alpha \subseteq r(o,t), \gamma \subseteq s(e,t)\} \tag{3}$$

where the new $r(o,t)$ and $s(e,t)$ have the obvious nuance of fixing the time instant where the states are being observed.

4. Summary, Discussion and Further Work

In this paper we have argued how not only standard FCA but our long-sustained effort to extend its modelling capabilities that—called F*x*A—can be better understood in the framework

of Embodied, Embedded, Extended and Enacted (4E) Cognition. For which purpose we must presuppose that Mathematics is a form of Extended cognition.

We started from previous metaphorical analyses of the flavour of EDA provided by standard FCA, that Wille himself called the "Landscapes of Knowledge" approach. These lend support to the hypothesis that LofK is Embodied and Embedded, but also argue for the Extendedness of those types of model-based EDA.

In order to make Enaction apparent, we recollected the extensive list of data-exploring activities licenced by the LofK approach. Such data-exploring behaviours make abilities appear in the functional state (for EDA) of humans which are, in the Perceptual Learning tradition, the basic components for affordances to be available for behaviour. The "features of the environment" that support such abilities are data to be analysed and encapsulated in the formal context. Finally we argued that each "flavour" of formal analysis in FxA brings a different kind of qualitative information to the EDA: FCA brings hierarchy, FIA independence and FEA indistinguishability. In providing different types of qualitative information through the behaviour afforded by the data in the context, FxA brings in a rich Enaction.

To start the discussion, note that the affordances that emerge from mathematical constructs like FCA are beyond those offered by the LofK paradigm or metaphor theory. A previous but more extensive analysis of the "affordances" of LofK vis-à-vis other metaphors in Data Mining can be found in [18, § 4.1]. But the "affordances" there are more those of interface design [8] than those of the ecological approach to perception [10, 11]. We leave for future work, a more extensive elicitation and formalization of the ecological affordances provided by FxA techniques.

Also, the embodiment and embedding of lattices in the extended environment seems to be a matter of physical complexity scale: when the lattice has few nodes we prefer to consider it a thing to be grasped and manipulated, while if the lattice is moderately to really complex the preferred metaphor is that of navigation. Further investigation of this issue should be carried out.

It may be construed that formal concepts and their counterparts, concept lattices, are equally "physical" and identically informative—of so the Basic Theorem of FCA would have us believe—so that lattices may not offer more affordances than contexts. These dual domains of information are readily seen in science and engineering: signals in time are often *transformed* into a dual domain—think of their frequency description. Despite signals having unique representations in either domain—up to a set of points of measure zero—the time and frequency representations enable different e.g. perceptual affordances that the human auditory system is only too eager to capitalise on. The investigation of what this concept of a *(domain) transform* brings to the consideration of concept lattices vs. formal contexts in the Enaction phenomenon is also left for future work.

To drop formal qualia in the midst of the (perceptual, philosophical) qualia debate, are the former more like glimpsing at the information in the formal context with glasses of differently-coloured lenses, or rather like using different senses on it? This latter metaphor makes more sense, for the time being, since we do not know if there is an analogue of the concept of "spectrum of light" for the gamut of possible conformations of the information in a formal context. Indeed, the theory of affordances assures that as long as we find new opportunities for operating on formal concepts, new information will be born out of the Enaction of such actions. This seems an open question for the time being.

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