Towards Novel Inference Relations based on Logical Argumentation*

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Abstract

Dealing with inconsistencies is a long time studied topic. The current paper introduces a mechanism for reasoning with conflicts in knowledge bases. The conflicts are addressed through a form of logic-based argumentation. We explore argument structures to define various inference relations that can rationally draw meaningful answers to queries even from inconsistent knowledge bases. We investigate the productivity of these entailments and their relationships with other well-known existing relations.

Keywords

Logical argumentation, Inference relations, Inconsistency handling

1. Introduction

Significant research efforts have focused on reasoning under conflicts, with two main strategies to deal with inconsistency: A traditional approach is to analyse and repair it when we encounter contradiction [1]. The second approach is simply to accept the inconsistency and to apply a non-standard reasoning method to draw meaningful answers from inconsistent knowledge bases. In this paper, we focus on the latter and consider the argumentative reasoning methods [2]. Argumentation has emerged as one of the most important fields in Artificial Intelligence. The basic idea of argumentation is that each plausible conclusion inferred from a knowledge base is justified by some reasons, called *arguments*, for believing in it. Due to inconsistency, those arguments may be attacked by other *counterarguments*. The problem is thus to evaluate the arguments in order to select the most *acceptable* ones. Using argumentation-based reasoning instead of classical approaches has several interesting features [3]: (1) the *maxcon* approach (selecting a maximal consistent subset) results in a loss of useful information, as it may not be certain which subset to choose, leading to an arbitrary choice; (2) the *oracle* approach (constructing a consistent knowledge base by getting extra information to help resolve the conflicts) involves a lot of work that may not be necessary. For example a query can be answered from a small part of the agents' knowledge, and furthermore, that knowledge may not even be in conflict; and (3) argumentation-based approaches can be used for explanatory purposes. For example, if one wants to know why a particular conclusion is accepted, an argument having that conclusion can be presented. That argument can be attacked by other arguments and so on [4].

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Additionally, it might be possible to construct only a part of the tree related to the conclusion in question, thus having a better representation.

After briefly introducing the necessary notions in Section 2, we study in Section 3 an approach that can be employed to define multiple conflict-tolerant inference relations in order to draw meaningful answers, while preserving the non-conflicts of the derived conclusions. We also study the productivity landscape of these new entailment relations.

2. Propositional Logic & Logical Argumentation

In this section, we briefly sketch some basics in propositional logic and deductive argumentation. We assume an arbitrary finite set of propositional variables $\mathcal{V}$. We use the set $\mathcal{V}$ with the classical connectives ($\neg$, $\vee$, $\wedge$, $\rightarrow$) as well as the two constants $\top$ (true) and $\bot$ (false) to build in the usual way the propositional language $\mathcal{L}(\mathcal{V})$. Well-formed formulas from $\mathcal{L}(\mathcal{V})$ are denoted by Greek letters α, β, γ, etc. We also denote by $\vdash$ the classical consequence relation. Two formulas $\alpha, \beta \in \mathcal{L}(\mathcal{V})$ are called equivalent, denoted as usual by $\alpha \equiv \beta$, if $\{\alpha\} \vdash \beta$ and $\{\beta\} \vdash \alpha$. A knowledge base is a finite set of propositional formulas. We write $\mathbb{K}_{\mathcal{V}}$ to denote the set of all knowledge bases built over $\mathcal{V}$. Also, K is said to be **inconsistent** if there exists a formula α such that $K \vdash \alpha$ and $K \vdash \neg\alpha$. Otherwise, K is **consistent**.

Given a knowledge base $K \in \mathbb{K}_{\mathcal{V}}$, a subset $M \subseteq K$ is a ***maximal consistent set*** (MC, in short) of K iff M is consistent and $\forall \alpha \in K \setminus M$, $M \cup \{\alpha\}$ is inconsistent.

The following consequence relations are typically defined on the basis of maximal consistent subsets of a given knowledge base $K \in \mathbb{K}_{\mathcal{V}}$:

Definition 1. *$K \vdash^{\forall}_{MC} \alpha$ iff for all $M \in MC(K)$, $M \vdash \alpha$. $K \vdash^{\exists}_{MC} \alpha$ iff for some $M \in MC(K)$, $M \vdash \alpha$.*

Now, we introduce the logical argumentation based on propositional logic studied by Besnard and Hunter [5].

Definition 2. *For $K \in \mathbb{K}_{\mathcal{V}}$, an argument is a pair $\langle \Phi, \alpha \rangle$ where $\Phi \nvdash \bot$, $\Phi \subseteq K$, $\Phi \vdash \alpha$ and $\nexists \Phi' \subset \Phi$ s.t. $\Phi' \vdash \alpha$.*

Definition 3. *$K \vdash_A \alpha$ iff there exists an argument for α in K, and there is no argument for $\neg\alpha$.*

Arguments are not necessarily independent in the sense that an argument can be contained within another one. Formally, $\langle \Phi, \alpha \rangle$ is more conservative than an argument $\langle \Psi, \beta \rangle$ iff $\Phi \subseteq \Psi$ and $\beta \vdash \alpha$.

Since information may be conflicting, arguments may also be conflicting among them. Various attack relations based on inconsistency have been studied (see [6] for an overview). The common idea for these relations is that $\langle \Phi, \alpha \rangle$ attacks $\langle \Psi, \beta \rangle$ if $\Phi \cup \Psi \vdash \bot$. Following [2], we consider the undercut relation where an *undercut* of an argument $\langle \Phi, \alpha \rangle$ is an argument $\langle \Psi, \neg(\beta_1 \wedge \ldots \wedge \beta_n) \rangle$ s.t. $\{\beta_1, \ldots, \beta_n\} \subseteq \Phi$.

Next, we assume that there exists a *canonical* enumeration of all maximally conservative undercuts for an argument $\langle \Phi, \alpha \rangle$. Such canonical enumeration allows us to avoid some amount of redundancy among counter-arguments.

Definition 4. *$\langle\Psi, \neg(\beta_1 \wedge \ldots \wedge \beta_n)\rangle$ is a canonical undercut of $\langle\Phi, \alpha\rangle$ iff $\langle\Psi, \neg(\beta_1 \wedge \ldots \wedge \beta_n)\rangle$ is a maximal conservative undercut of $\langle\Phi, \alpha\rangle$ and $\langle\beta_1, \ldots, \beta_n\rangle$ is the canonical enumeration of Φ.*

To merge arguments and counter-arguments for/against a given conclusion, the authors of [2] define the so-called argument tree for a formula α as a tree whose nodes are arguments such that: (1) the root is an argument for α, (2) for every node $\langle\Psi, \beta\rangle$ whose ancestor nodes are $\langle\Psi_1, \beta_1\rangle, \ldots, \langle\Psi_n, \beta_n\rangle$, there exists $\gamma \in \Psi$ such that for $1 \leq i \leq n$, $\gamma \notin \Psi_i$, and (3) each child node is a canonical undercut of its parent node.

As many different argument trees for a given claim can exist, one can represent all these trees in a global manner by using the argument structure defined as follows:
Given a formula α, the *argument structure* for α is a pair of sets $\langle\mathcal{P}, \mathcal{S}\rangle$ s.t. $\mathcal{P}$ (resp. $\mathcal{S}$) is the set of argument trees for α (resp. $\neg\alpha$).

3. Novel Argumentation based Inference Relations

Given an argument tree, we want to determine whether the root argument (i.e., goal) wins (i.e., it is undefeated) or whether it loses (i.e., it is defeated), or undecided. This is called argument labelling. Let T be an argumentation tree for a conclusion α and $\mathbb{A} = \langle\Phi, \alpha\rangle$ be an argument in T. An **argument labelling** is a total function $Lab : \mathbb{A} \rightarrow \{\mathcal{A}', \mathcal{R}, \mathcal{U}\}$.

Various studies in literature rely on some given qualitative or quantitative information to determine, or give more refined accounts of, the justification status of arguments. Other works refine the notion of acceptability based on the number of (counter)-attacks on arguments (see [7] for an overview). Our approach of defining the new inference relations is based on a given argument labelling without any particular requirements, so we do not instantiate such a function in this paper. For a given argument structure $\langle\mathcal{P}, \mathcal{S}\rangle$ for α w.r.t. a knowledge base K, let us introduce the following logical conditions:

- **C1.** $\mathcal{P} \neq \emptyset$ and $\mathcal{S} = \emptyset$.
- **C2.** $\exists\, T \in \mathcal{P}$, $Judge(T) = Warranted.$
- **C3.** $\forall\, T \in \mathcal{P}$, $Judge(T) = Warranted$, and $\mathcal{P} \neq \emptyset$.
- **C4.** $\forall\, T \in \mathcal{S}$, $Judge(T) = Unwarranted.$
- **C5.** $\forall\, T \in \mathcal{P}$, $Judge(T) \neq Unwarranted.$
- **C6.** $\bigcap_{i=1}^{n} \Phi_i \models_\pi \alpha$, where $\mathcal{P} = \{T_1, \ldots, T_n\}$ with $\mathbb{A}_r(T_i) = \langle\Phi_i, \alpha\rangle$, and $Judge(T_i) = Warranted$ for $1 \leq i \leq n$.
- **C7.** $\exists\, T \in \mathcal{P}$ with $\mathbb{A}_r(T) = \langle\Phi, \alpha\rangle$, and $\Phi \subseteq \bigcap_{M \in \mathrm{MC}(K)} M$, $Judge(T) = Warranted.$
- **C8.** $\forall\, T \in \mathcal{P}$ with $\mathbb{A}_r(T) = \langle\Phi, \alpha\rangle$ and $\Phi \subseteq \bigcap_{M \in \mathrm{MC}(K)} M \neq \emptyset$, $Judge(T) = Warranted.$

These logical constraints give rise to various entailment relations based on argumentation which are all conflict-tolerant. Next, we discuss two sorts of semantics: the family of classical semantics and the family of IAR semantics. Other inference relation that approximates the IAR argumentation semantics, the cardinality based semantics, and the agent preferences based semantics can be further defined as in [8] and omitted in this paper due to space limit.

Now, we revisit various classical inference relations in light of the previous constraints over argument structures. For the first type of reasoning, we suggest that a conclusion follows from a knowledge base if the latter has an argument structure that supports this conclusion but no

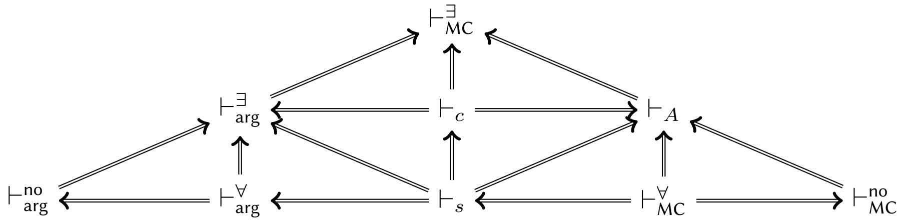

Figure 1: Productivity comparison of inference relations, where $A \Rightarrow B$ means that the entailment relation A is less productive than B, i.e., each conclusion of A is also a conclusion of B

argument structure against that conclusion. Let K be a knowledge base and α is a formula. We say α is **credulously** (resp. **skeptically**) entailed from K, denoted $K \vdash_c \alpha$ (resp. $K \vdash_s \alpha$), if the argument structure $\langle \mathcal{P}, \mathcal{S} \rangle$ for α satisfies C1 and C2 (resp. C1 and C3).

Note that the above inference relations $\vdash_A, \vdash_c$, and $\vdash_s$ for a conclusion α are conservative due to the requirement that there must be no argument against α, hence rather unproductive. To relax such constraint, we propose in the following another reasoning type via four logical consequence relations, namely $\vdash^{\forall}_{\text{arg}}$, $\vdash^{\exists}_{\text{arg}}$, $\vdash^{\text{no}}_{\text{arg}}$, and $\vdash^{\text{IAR}}_{\text{arg}}$.

Definition 5 ($\vdash^{\forall}_{\text{arg}}$). *We say $K \vdash^{\forall}_{arg} \alpha$ if the argument structure $\langle \mathcal{P}, \mathcal{S} \rangle$ for α satisfies C3 and C4.*

Definition 6 ($\vdash^{\exists}_{\text{arg}}$). *We say $K \vdash^{\exists}_{arg} \alpha$ if the argument structure $\langle \mathcal{P}, \mathcal{S} \rangle$ for α satisfies C2 and C4.*

The intuition behind $\vdash^{\exists}_{\text{arg}}$ is that there exists *at least* one warranted argumentation tree for α. Both $\vdash^{\exists}_{\text{arg}}$ and $\vdash^{\forall}_{\text{arg}}$ require that the arguments against α should be labelled as rejected.

Next, we investigate a new argumentative inference relation based on the notion of non-objection [9]. The intuition behind is that no argumentation tree in the argument structure has an objection to the acceptance of the conclusion.

Definition 7 ($\vdash^{\text{no}}_{\text{arg}}$). *$K \vdash^{no}_{arg} \alpha$ if the argument structure $\langle \mathcal{P}, \mathcal{S} \rangle$ for α satisfies C2, C4 and C5.*

Other types of entailment relations may be defined by imposing further constraints on the argument structure. The following three conflict-tolerant entailments evaluate queries over the supports in the root nodes of the argumentation trees for the considered query. This gives rise to a family of IAR argumentation semantics defined as follows:

The first inference relation, called $\vdash^{\text{IAR}}_{\text{arg}}$, is based on the intuition that a valid conclusion has to be a consequence of the intersection of the supports of the root nodes of all accepted argumentation trees.

Definition 8 ($\vdash^{\text{IAR}}_{\text{arg}}$). *$K \vdash^{IAR}_{arg} \alpha$ if the argument structure $\langle \mathcal{P}, \mathcal{S} \rangle$ for α satisfies C4 and C6.*

To relax the quasi-equivalence requirement among the root nodes of warranted argument trees, we focus on two variants of IAR argumentation semantics defined through the selection of warranted argument trees, for a formula, that belong to the intersection of maximal consistent sets of K. Formally:

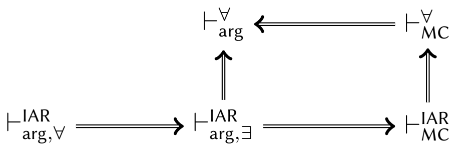

Figure 2: Productivity comparison of the inference relations from the IAR family semantics

Definition 9 **($\vdash^{\mathbf{IAR}}_{\mathbf{arg},\exists}$).** *$K \vdash^{IAR}_{arg,\exists} \alpha$ if the argument structure $\langle \mathcal{P}, \mathcal{S} \rangle$ for α satisfies C4 and C7.*

Definition 10 **($\vdash^{\mathbf{IAR}}_{\mathbf{arg},\forall}$).** *$K \vdash^{IAR}_{arg,\forall} \alpha$ if the argument structure $\langle \mathcal{P}, \mathcal{S} \rangle$ for α satisfies C4 and C8.*

Intuitively, the $\vdash^{\mathsf{IAR}}_{\mathsf{arg},\exists}$ semantics requires that the intersection of all the maximal conflict-free sub-bases has to contain a warranted argument for α. The $\vdash^{\mathsf{IAR}}_{\mathsf{arg},\forall}$ semantics is based on the idea that a given conclusion α can be considered as valid if the support of each root node for α is involved in the intersection of all the maximal conflict-free sub-bases.

Properties. Figure 1 and 2 show the productivity comparison among the different inference relations presented above. The proofs are omitted due to space limitations and can be found in [8]. It can be seen that the result strengthens the previous results for the IAR standard semantics showed by Bienvenu and Rosati [10] in the context of classical DL reasoning.

Discussion. This paper presents several fine-grained inference relations based on argument structure and labelling functions and their comparison. We will explore more aspects of argument semantics, e.g. ranking semantics or extensions, to investigate new inference relations.

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