On the Quantitative Reasoning over Incomplete Abstract Argumentation Frameworks

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Abstract

We recall percVer and percAcc, the problems asking for the percentages of the completions of an *incomplete Abstract Argumentation Framework* (iAAF) where a set S is an extension and an argument a is accepted, respectively, introduced in [1]. Specifically, we recall the relationship between the proposed framework and *probabilistic AAFs* (prAAFs) *under the constellations approach* and the complexity characterization of both problems.

1. Introduction

Incomplete AAFs (iAAFs) [2] are generalizations of the well known Dung's *Abstract Argumentation Framework* (AAF [3]) that model uncertainty that may affect arguments and attacks. Recent proposals [4, 5] further extend iAAFs with "dependencies", expressing, for instance, that (the presence of) an argument is alternative to other arguments, that an attack implies another attack, and so on. Therefore, an iAAF encodes multiple scenarios (called *completions*), corresponding to the different combinations of presence/absence of the uncertain arguments and uncertain attacks satisfying the dependencies.

The main reason for the popularity of iAAFs is their simplicity, mostly deriving from the fact that they do not require a quantitative modeling of the uncertainty. Nevertheless, some quantitative reasoning over iAAFs is likely to be useful when analyzing the dispute modeled by an iAAF. For instance, in the absence of quantitative measures implying a rank of the completions, it is reasonable to consider them "*alternative scenarios that may occur with the same probability*", and, in turn, to measure the closeness of a set (resp. an argument) to being an extension (resp., accepted) as the percentage of completions where this happens.

In this paper, we recall the reasoning paradigm over iAAFs proposed in [1], that is based on three problems, whose input includes an iAAF IF, a set of dependencies $\mathcal{D}$, a set of arguments S, an argument a:
1) percVer$^{\sigma}(IF,\mathcal{D},S)$ and percAcc$^{\sigma}(IF,\mathcal{D},a,X)$, asking for the percentages of completions of IF (satisfying $\mathcal{D}$) where S is an extension and a accepted, respectively;
2) cntCom$(IF,\mathcal{D})$, counting the completions of IF (satisfying $\mathcal{D}$): this problem supports a preliminary analysis, as its answer is a measure of the uncertainty encoded in the iAAF and helps interpret the answers of percVer and percAcc.

We first recall the relationship between this framework and *probabilistic AAFs in the constellations approach* (prAAFs) [6, 7, 8, 9], that are iAAFs where a probability distribution function is defined over the completions (called *possible worlds* in the context of prAAFs). In particular, we focus on the relationship with the problems probVer and probAcc over prAAFs, that ask for the overall probability of the possible worlds of a prAAF PF where a set is an extension and an argument accepted, respectively. We observe that, although in principle solving percVer and percAcc is the same as solving probVer and probAcc over a suitably constructed prAAF, constructing this "equivalent" prAAF can require a heavy computational cost (besides the fact that the so obtained prAAF is dramatically less compact and less user-friendly than the iAAF). We show that this issue holds also when no dependency is specified. Then, we report the investigation of the computational complexity of the quantitative-reasoning paradigm, showing that cntCom is #P-complete and percVer and percAcc are $FP^{\#P}$-complete, with islands of

8th Workshop on Advances in Argumentation in Artificial Intelligence

tractability depending on the semantics of extensions and the size of S or the structural properties of the iAAF.

2. Preliminaries

Incomplete AAFs (iAAFs). Assuming familiarity with Abstract Argumentation Frameworks (AAFs), we recall the Incomplete AAFs (iAAFs) [2].

Definition 1 (iAAF). *An* incomplete AAF *(iAAF) is a tuple $\langle A, A^?, D, D^? \rangle$, where A and $A^?$ are disjoint sets of arguments, and D and $D^?$ disjoint sets of attacks between arguments in $A \cup A^?$. The arguments and attacks in A and D (resp., $A^?$ and $D^?$) are said to be* certain *(resp.,* uncertain*), i.e. they are (resp., are not) guaranteed to occur.*

An iAAF compactly represents the alternative scenarios (*completions*) for the argumentation.

Definition 2 (Completion). *Given an iAAF $IF = \langle A, A^?, D, D^? \rangle$, a completion of IF is an AAF $F = \langle A', D' \rangle$ where $A \subseteq A' \subseteq (A \cup A^?)$ and $D \cap (A' \times A') \subseteq D' \subseteq (D \cup D^?) \cap (A' \times A')$.*

The possible and necessary perspectives are natural ways to deal with multiple completions when adapting the notions of *extension* (renamed "i*-*extension*") and *accepted argument* to iAAFs:

Definition 3. *Let IF be an iAAF, S a set of arguments and a an argument. Under a semantics σ, S is an i*-extension and a an X-accepted argument (with $X \in \{Cr, Sk\}$) under the possible (resp., necessary) perspective if, for at least one (resp., every) completion F of IF, S is an extension of F and a an X-accepted argument of F, respectively.*

In [5, 4], the possibility of restricting the set of completions (in order to make it better fit the alternative scenarios) via OR, NAND, CHOICE, IMPLY dependencies involving arguments/attacks was studied (this set was proved to be sufficient to encode any propositional constraint over the completions, expressing which combinations of arguments or of attacks can/cannot occur). Given two sets of arguments Y, Z, the semantics is: **OR**(Y)**:** (the completions to be considered are all and only those where) at least one argument in Y occurs; **NAND**(Y)**:** at least one argument in Y does not occur; **CHOICE**(Y)**:** exactly one argument in Y occurs; **IMPLY**(Y, Z)**:** if every argument in Y occurs, then all the arguments in Z occur. If Y and Z are sets of attacks, the semantics is analogous, but, when checking if a completion satisfies a dependency, only the attacks in Y and Z between arguments that both belong to the completion are considered (this means conditioning the dependencies to the presence of the arguments involved in the attacks). In the presence of a set of dependencies $\mathcal{D}$, the set of completions of an iAAF IF satisfying $\mathcal{D}$ will be denoted as $\mathcal{C}(IF, \mathcal{D})$, and the problems of checking, under the perspective $P \in$ {Possible, Necessary}, if S is an i*-extension and a X-accepted (with $X \in$ {Cr, Sk}) as iVER$^\sigma(IF, \mathcal{D}, S, P)$ and iAcc$^\sigma(IF, \mathcal{D}, a, P, X)$, respectively.

Probabilistic Abstract Argumentation Frameworks (prAAFs). We consider prAAFs following the "*constellations approach*", where probabilities quantitatively model the uncertainty affecting the knowledge of which "possible world" (i.e. combination of the arguments and attacks) actually occurs. In the context of prAAFs, "*possible world*" has the same meaning as "*completion*" in the context of iAAFs, so prAAFs can be viewed as iAAFs where a pdf (*probability distribution function*) is defined over the completions.

Definition 4. *A prAAF PF is a tuple $\langle A, D, \mathcal{PW}, p \rangle$, where A is a set of arguments, $D \subseteq A \times A$ a set of attacks, $\mathcal{PW}$ a set of possible worlds over A and D, and p is a pdf over $\mathcal{PW}$.*

When independence between arguments/attacks is assumed, the pdf p can be encoded compactly, by specifying the marginal probabilities μ of the arguments/attacks, so that the probability of a possible world $\omega = \langle A', D' \rangle$ is

$$p(\omega) = \Pi_{a \in A'} \mu(a) \times \Pi_{a \in A \setminus A'} \big(1 - \mu(a)\big) \times \Pi_{(a,b) \in D'} \mu\big((a, b)\big) \times \Pi_{(a,b) \in \left(D \cap (A' \times A')\right) \setminus D'} \Big(1 - \mu\big((a, b)\big)\Big)$$

which means that the marginal probability of an attack (a, b) is interpreted as the probability that the attack occurs assuming that a and b occur. Thus, under the independence assumption, $\mathcal{PW}$ and p can be replaced by a *marginal probability function* $\mu : A \cup D \to [0, 1]$. In this case, the prAAF will be called IND-prAAF and denoted as a tuple $PF = \langle A, D, \mu \rangle$.

3. Quantitative Reasoning Over iAAFs Vs. Probabilistic Reasoning Over prAAFs

We report the definitions of the problems cntCom, percVer and percAcc, and their relationship with probVer and probAcc (the probabilistic counterparts of the verification and acceptance problems over prAAFs), provided in [1]. In the rest of the paper, an iAAF $IF = \langle A, A^?, D, D^? \rangle$, a set of dependencies $\mathcal{D}$, a set of arguments S of IF, and a semantics $\sigma \in \{\mathtt{ad}, \mathtt{st}, \mathtt{co}, \mathtt{gr}, \mathtt{pr}\}$, are assumed to be given.

Definition 5. *cntCom$(IF, \mathcal{D})$ is the problem of computing $|\mathcal{C}(IF, \mathcal{D})|$, and percAcc$^\sigma(IF,\mathcal{D},a,X)$ (with $X \in \{Cr, Sk\}$) and percVer$^\sigma(IF,\mathcal{D},S)$ the problems of computing the percentage of completions in $\mathcal{C}(IF, \mathcal{D})$ where, under σ, a is X-accepted and S is an extension, respectively. Conventionally, the answer of percVer and percAcc is 0 if $\mathcal{C}(IF, \mathcal{D}) = \emptyset$.*

Observe that percAcc$^\sigma(IF,\mathcal{D},a,X)$ trivially returns 0 if $\sigma = \mathtt{ad}$ and $X = Sk$, as no argument can be skeptically accepted in any completion (as $\emptyset$ is always an admissible extension). Hence, in most of the results regarding percAcc, the combination $\sigma = \mathtt{ad}$ and $X = Sk$ will not be considered.

The following proposition states that percVer and percAcc can be solved by viewing IF as a prAAF whose possible worlds are the completions of IF and whose pdf is the uniform distribution, and then reasoning on this prAAF via probVer and probAcc. Obviously, translating iAAFs to prAAFs makes sense if $\mathcal{D}$ is satisfiable, as prAAFs are not defined if there is no possible world.

Proposition 1. *Assume $\mathcal{C}(IF, \mathcal{D}) \neq \emptyset$. Let $PF = \langle A \cup A^?, D \cup D^?, \mathcal{PW}, p \rangle$ be the prAAF where $\mathcal{PW} = \mathcal{C}(IF, \mathcal{D})$ and p is the pdf that assigns $1/|\mathcal{PW}|$ to every possible world of PW. Then percVer$^\sigma(IF,\mathcal{D},S)$=probVer$^\sigma(PF, S)$ and percAcc$^\sigma(IF,\mathcal{D},a,X)$=probAcc$^\sigma(PF, a, X)$.*

The theorem below states that cntCom$(IF, \mathcal{D})$ (which returns $1/p$) is #P-complete.

Theorem 1. *cntCom$(IF, \mathcal{D})$ is #P-complete, even if $\mathcal{D} = \emptyset$ and IF contains no certain argument and no certain attack.*

The following theorem states that, under the hypothesis #P$\not\subseteq$ FP, there is no polynomial time transformation of an iAAF into a prAAF over the same arguments that allows for solving any instance of percVer via an instance of probVer.

Theorem 2. *Under the hypothesis #P$\not\subseteq$ FP, for every $\sigma \in \{\mathtt{ad}, \mathtt{st}, \mathtt{co}, \mathtt{gr}, \mathtt{pr}\}$, there is no algorithm taking as input an iAAF IF that runs in polynomial time and translates IF into a prAAF over the same arguments as IF such that, for every set of arguments S, probVer$^\sigma(PF, S)$= percVer$^\sigma(IF,\mathcal{D},S)$, or, for each argument a, percAcc$^\sigma(IF,\mathcal{D},a,X)$= probAcc$^\sigma(PF, a, X)$ (except for the case $\sigma = \mathtt{ad}$ and $X = Sk$).*

It is natural to wonder whether there is some general easy-to-compute translation from IF into an equivalent IND-prAAF over the same arguments. Unfortunately, even if $\mathcal{D}= \emptyset$, computing such a translation (assuming that it exists) would require a huge computational effort, as formally stated below.

Theorem 3. *Under the hypothesis #P$\not\subseteq$ FP, even if $\mathcal{D}= \emptyset$, under $\sigma \in \{\mathtt{ad}, \mathtt{st}, \mathtt{co}, \mathtt{gr}, \mathtt{pr}\}$, there is no polynomial-time algorithm translating any iAAF IF into an IND-prAAF PF over the arguments of IF such that, for any set of arguments S, probVer$^\sigma(PF, S)$=percVer$^\sigma(IF,\mathcal{D},S)$, or, for any argument a, percAcc$^\sigma(IF,\mathcal{D},a,X)$= probAcc$^\sigma(PF, a, X)$ (except for the case $\sigma = \mathtt{ad}$,$X = Sk$).*

The following theorem describes a form of iAAF for which the polynomial-time translation mentioned in Theorem 3 exists.

Theorem 4. *Assume that $\mathcal{D}=\emptyset$ and every uncertain attack in IF involves at least one certain argument. Let $PF = \langle A \cup A^?,\, D \cup D^?, \mu\rangle$ be the IND-prAAF where $\forall a \in A\ \mu(a) = 1$, $\forall \delta \in D\ \mu(\delta) = 1$, $\forall \delta \in D^?\ \mu(\delta) = \frac{1}{2}$, and $\forall a \in A^?\ \mu(a) = \frac{2^{|D^?(a)|}}{1+2^{|D^?(a)|}}$, where $D^?(a)$ is the set of uncertain attacks involving a. Then, for any $S \subseteq A \cup A^?$, percVer$(IF, \mathcal{D}, S)$ = probVer$^\sigma(PF, S)$, and, for each $a \in A \cup A^?$, percAcc$^\sigma(IF,\mathcal{D},a,X)$= probAcc$^\sigma(PF, a, X)$.*

Finally, Theorem 5 below gives an insight into the sufficient condition of Theorem 4, as it states that this condition makes cntCom tractable too.

Theorem 5. *cntCom$(IF, \mathcal{D})$ is in P if $\mathcal{D}=\emptyset$ and every uncertain attack of IF involves at least one certain argument.*

4. Computational Complexity of Quantitative Reasoning Over iAAFs

We start by recalling the general characterization of percVer and percAcc done in [1].

Theorem 6. *percAcc$^\sigma(IF,\mathcal{D},a,X)$ (except for the case $\sigma = \texttt{ad}$ and X =Sk) and percVer$^\sigma(IF,\mathcal{D},S)$ are FP$^{\#P}$-complete, even if $\mathcal{D}=\emptyset$.*

Starting from this general result, we recall the sensitivity analysis of the computational complexity to the form of uncertainty encoded in the iAAF and to the semantics of extensions done in [1]. A first result straightforwardly follows from Theorem 4, which states that, when $\mathcal{D}=\emptyset$ and the uncertain attacks involve at least one certain argument, percVer$^\sigma(IF,\mathcal{D},S)$ is equivalent to probVer$^\sigma(PF, S)$, and percAcc$^\sigma(IF,\mathcal{D},a,X)$ to probAcc$^\sigma(PF, a, X)$, where PF is an IND-prAAFs obtained via a polynomial-time transformation from IF. Since, under $\sigma \in \{\texttt{ad}, \texttt{st}\}$, probVer$^\sigma(PF, S)$ is in FP over IND-prAAFs (as shown in [10]), we obtain:

Corollary 1. *Assume that $\mathcal{D}=\emptyset$ and IF is such that every uncertain attack involves at least one certain argument. Under $\sigma \in \{\texttt{ad}, \texttt{st}\}$, percVer$^\sigma(IF,\mathcal{D},S)$ is in FP.*

Under the Dungean semantics other than ad and st, probVer over IND-prAAFs is not known to be in FP but is FP$^{\#P}$-complete [10], and, under every Dungean semantics, probAcc is FP$^{\#P}$-complete. Thus, Theorem 4 implies that percVer is in FP$^{\#P}$ under $\sigma \in \{\texttt{gr}, \texttt{co}, \texttt{pr}\}$ and percAcc is in FP$^{\#P}$ under $\sigma \in \{\texttt{ad}, \texttt{st}, \texttt{gr}, \texttt{co}, \texttt{pr}\}$. The following theorem states that these are also lower bounds, even if only the arguments or only the attacks are uncertain.

Theorem 7. *If $\mathcal{D}=\emptyset$ and IF contains no uncertain argument or no uncertain attacks, then: 1) under $\sigma \in \{\texttt{gr}, \texttt{co}, \texttt{pr}\}$, percVer$^\sigma(IF,\mathcal{D},S)$ is FP$^{\#P}$-complete, and 2) under $\sigma \in \{\texttt{ad}, \texttt{st}, \texttt{gr}, \texttt{co}, \texttt{pr}\}$, percAcc$^\sigma(IF,\mathcal{D},a,X)$ is FP$^{\#P}$-complete (except for the case $\sigma = \texttt{ad}$ and X = Sk).*

From what shown so far, it is natural to wonder whether the tractability islands of Theorem 5 for cntCom and of Corollary 1 for percVer can be extended to the case $\mathcal{D} \neq \emptyset$. The following theorem gives a negative answer: even in the simpler case where the uncertainty involves only arguments or only attacks, specifying any form of dependency makes cntCom and percVer under $\sigma \in \{\texttt{ad}, \texttt{st}\}$ intractable.

Theorem 8. *If IF contains no uncertain arguments or no uncertain attacks, and $\mathcal{D}$ contains dependencies of only one of the forms or, nand, choice, imply, then cntCom$(IF, \mathcal{D})$ is #P-hard and percVer$^\sigma(IF,\mathcal{D},S)$ is FP$^{\#P}$-hard.*

Theorem 8 does not mention percAcc, as it is already FP$^{\#P}$-hard with $\mathcal{D}=\emptyset$ (Theorem 7). As for percVer, Theorem 8 interestingly states that percVer is FP$^{\#P}$-hard whatever the form of dependencies, even when its decision counterpart (iVer) is tractable. We conclude by locating an island of tractability of PERCVER under $\sigma = \texttt{ad}$, that depends on the size of S and how S is connected to the rest of the iAAF. The tractability holds for a form of iAAF for which CNTCOM is not tractable (as no restriction is imposed on the portion of IF outside S). To state the result, we denote as *fr*(S) the *frontier* of S, i.e. the set of arguments outside S attacking or attacked by S. Moreover, we denote as $IF \setminus S$ the iAAF obtained from IF by removing every argument in S and every attack to/from S.

Theorem 9. *Assume that the size of S is logarithmic w.r.t. size of IF, that the arguments in $\mathrm{fr}(S)$ are certain, that, for each dependency $d \in \mathcal{D}$, d involves only arguments/attacks in S or only arguments/attacks in $IF \setminus S$, and that $|\mathcal{C}(IF, \mathcal{D})| > 0$. Then, under $\sigma = \texttt{ad}$, PERCVER$^{\sigma}(IF,\mathcal{D},S)$ is in FP.*

It is worth noting that this tractability result cannot be extended to the other semantics. In fact: 1) under $\sigma \in \{\texttt{co}, \texttt{gr}, \texttt{pr}\}$, the proof of Theorem 7 shows that PERCVER is $FP^{\#P}$-hard even in the restricted case where $\mathcal{D} = \emptyset$, *fr*$(S) = \emptyset$, and S is of constant size; 2) under $\sigma = \texttt{st}$, a minor change of the reduction used in the proof of Theorem 6 (consisting in adding no attack involving the fresh argument a) shows that PERCVER is $FP^{\#P}$-hard even when $\mathcal{D} = \emptyset$, *fr*$(S) = \emptyset$, and S is of constant size.

The above theorem is the only statement regarding the computational complexity where the satisfiability of the dependencies (i.e. $|\mathcal{C}(IF, \mathcal{D})| > 0$) is a prerequisite. Observe that the general hardness results of Theorem 8 hold even when $\mathcal{D}$ is known to be satisfiable, as they can be proved via parsimonious reductions to CNTCOM from counting problems that always return a value greater than 0. This means that CNTCOM and PERCVER have sources of complexity other than the uncertainty on the existence of at least one completion. In turn, this means that, in the general case, assuming $\mathcal{D}$ satisfiable cannot imply that PERCVER becomes tractable, and makes the result of Theorem 9 relevant.

5. Conclusions

We have recalled the new quantitative reasoning paradigm supporting the analysis of iAAFs and its computational complexity characterization, provided in [1].

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