

Einstein-Rosen Bridge (ER), Einstein-Podolsky-Rosen Experiment (EPR) and Zero Measure Rindler-KAM Cantorian Spacetime Geometry (ZMG) Are Conceptually Equivalent

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Abstract

By viewing spacetime as a transfinite Turing computer, the present work is aimed at a generalization and geometrical-topological reinterpretation of a relatively old conjecture that the wormholes of general relativity are behind the physics and mathematics of quantum entanglement theory. To do this we base ourselves on the comprehensive set theoretical and topological machinery of the Cantorian-fractal E-infinity spacetime theory. Going all the way in this direction we even go beyond a quantum gravity theory to a precise set theoretical understanding of what a quantum particle, a quantum wave and quantum spacetime are. As a consequence of all these results and insights we can reason that the local Casimir pressure is the difference between the zero set quantum particle topological pressure and the empty set quantum wave topological pressure which acts as a wormhole “connecting” two different quantum particles with varying degrees of entanglement corresponding to varying degrees of emptiness of the empty set (wormhole). Our final result generalizes the recent conceptual equation of Susskind and Maldacena $ER = EPR$ to become

$$ZMG = ER = EPR$$

where ZMG stands for zero measure Rindler-KAM geometry (of spacetime). These results were only possible because of the ultimate simplicity of our exact model based on Mauldin-Williams random Cantor sets and the corresponding exact Hardy’s quantum entanglement probability $P(H) = \phi^5$ where ϕ is the Hausdorff dimension of the topologically zero dimensional random Cantor thin set, *i.e.* a zero measure set and $\phi = (\sqrt{5} - 1)/2$. On the other hand the positive measure spatial

separation between the zero sets is a fat Cantor empty set possessing a Hausdorff dimension equal ϕ^2 while its Menger-Urysohn topological dimension is a negative value equal minus one. This is the mathematical quintessence of a wormhole paralleling multiple connectivity in classical topology. It is both physically there because of the positive measure and not there because of the negative topological dimension.

Keywords

Zero Measure Thin Cantor Set, Fat Cantor Set, Cantorian Fractal KAM Spacetime, Quantum Gravity, Casimir Pressure, E-Infinity Theory

1. Introduction

The present paper had its roots around six years ago in our work on the geometrical-topological interpretation of Hardy's famous quantum entanglement and transfinite golden mean Turing computers [1] as well as its connection to a fractal version of Rindler spacetime combined with KAM theorem of nonlinear dynamics [2]. This work led to an exact determination of the ordinary and the dark energy density of the cosmos [2]-[6]. Shortly after that we started exploring the possibility that zero measure geometry could be interpreted as a wormhole [2]. This work was carried out admittedly in total ignorance of classical contributions on the subject [7]-[31]. In fact it was only the remarkable papers of Profs. L. Susskind and J. Maldacena which made us aware that we are not working in a vacuum [27].

One idea led to another and the logic of the situation set the course at attempting not only to validate the remarkable suggestions of Susskind and Maldacena but also to reduce their results to a natural consequence of a Cantorian Rindler-KAM zero measure and empty set geometry. Actually this is a grossly simplified label because the required E-infinity-Rindler-KAM spacetime is a multi fractal containing sets with positive and zero measure, u thin and fat fractals as well as positive and negative topological Menger-Urysohn dimensions as explained in the extensive literature on E-infinity theory and its application [32]-[118], particularly to spacetime Casimir-dark energy reactors [63]-[71] and transfinite Turing computers [120] [121].

2. Transfinite Set Theoretical Conception of Quantum Spacetime

Our general theory of quantum spacetime follows in a rigorous way from the von Neumann-Connes by now famous dimensional group function [119]

$$D = a + b\phi \quad (1)$$

where $a, b, \in Z$ and $\phi = (\sqrt{5} - 1)/2$. The above may be written in a simple compact notation known as the bijection formula which states that [1]-[5]

$$d_c^{(n)} = (1/\phi)^{n-1} \quad (2)$$

where $d_c = D_H$ is the Hausdorff dimension corresponding to the Menger-Urysohn topological dimension n . For instance if we take $n = 0$ which is a zero set modelling the quantum (pre)particle, we see that [1]-[5] [119]

$$\begin{aligned} d_c^{(0)} &= (1/\phi)^{-1} \\ &= \phi \end{aligned} \quad (3)$$

On the other hand, for $n = -1$ which is the classical empty set and in our theory models the (pre)quantum wave, we see that

$$\begin{aligned} d_c^{(-1)} &= (1/\phi)^{-1-1} \\ &= \phi^2 \end{aligned} \quad (4)$$

Two further particular dimensions are of relevance to the present work, namely $n = -\infty$ for which we have clearly

$$d_c^{(-\infty)} = \phi^\infty = \text{zero} \quad (5)$$

and

$$\begin{aligned} d_c^{(4)} &= (1/\phi)^{4-1} \\ &= (1/\phi)^3 \\ &= 4 + \phi^3 \\ &= 4 + \frac{1}{4 + \frac{1}{4 + \dots}} \\ &= 4.2360679 \end{aligned} \quad (6)$$

Adding to the above the realization gained from the elementary theory of co-bordism [74] we see that the surface of the zero set $D \equiv (O, \phi)$ is the empty set $D \equiv (-1, \phi^2)$ [69] [70] [74] [86]. In turn the surface of the empty set is the emptier set $D \equiv (-2, \phi^3)$. On the other hand the average set of all sets is given by ϕ^3 so that it must be spacetime itself. To see this subtle point we look at the inversion of ϕ^3 which is [32]

$$1/\phi^3 = 4 + \phi^3. \quad (7)$$

In other words it gives us the average spacetime dimension [32]

$$\begin{aligned} \langle dc \rangle &= \frac{\sum_0^{\infty} n^2 \phi^n}{\sum_0^{\infty} n \phi^n} \\ &= \sum_{n=0}^{\infty} n \phi^n \\ &= 4 + \phi^3 \end{aligned} \quad (8)$$

That way we can construct a very simple and beautiful mental picture of our universe [73]-[76]. First we see a quantum pre-particle resembling a micro black hole fixed by $D = (O, \phi)$ which is the zero set. The surface surrounding the zero set is the empty set (pre)quantum wave $D = (-1, \phi^2)$. Then we see that the surface of the wave is a multi fractal with average bi dimension $\langle D \rangle = \langle (-2, \phi^3) \rangle$ and this is nothing else but our quantum spacetime itself where the particle resides “inside” a wave floating in spacetime, which is its fractal surface. Now we ask ourselves the natural questions about the surface of spacetime. This is clearly $\langle D \rangle = \langle (-\infty, O) \rangle$ which means nothingness, *i.e.* where there are no answers because there are no questions [74]. Next we contemplate the place of a wormhole in the above using the terminology of set theory and measure theory.

3. What Is a Wormhole from the View Point of Measure Theory?

It is almost a trivial conclusion that a wormhole [7]-[31] is the quintessential empty set with varying degrees of emptiness corresponding to varying degrees of entanglement [1] [37] [45]. Our spacetime consists exclusively of zero sets quantum particles and empty sets quantum waves. Consequently the infinitely many empty sets with positive measure are of negative dimensions and thus somewhat esoteric although they have a positive measure but in negative dimensions as a wormhole connecting two zero sets quantum particles. One could envisage the situation by looking at a single Cantor set. Imagine removing the middle third “randomly” as we do in the conventional construction of a one dimensional random Cantor set. Now the Hausdorff dimension of the “gaps” is $1 - \phi = \phi^2$. That means it is an empty set, in fact in this case, total nothingness [74]. Now at end “points” we have a Hausdorff dimension of a zero set particle, namely ϕ . Thus we have a topological pressure gradient acting inside the “gap”, *i.e.* the empty set which we can now call the wormhole and the gradient is obviously equal $\phi - \phi^2$ which is ϕ^3 . Again ϕ^3 is not only the universal fluctuation of spacetime [32] [74] but also equal to the local Casimir effect which we should call henceforth, the Casimir pressure. When all is said and done, mathe-

matically a wormhole is best modelled by a totally empty set given by the bi-dimension $(-\infty, O)$ or at most an empty set $(-1, \phi^2)$ and with all fat empty Cantor sets in between these two limits [74]. The important notions of fat Cantor sets and wild topology is explained in detail in Ref [55].

4. From Local Casimir Pressure to Globally Concentrated Dark Energy

The Casimir topological pressure ϕ^3 is clearly a spacetime latent pressure which is the difference between the average Hausdorff dimension of spacetime and the Menger-Urysohn dimension of the same. That means it is $(4 + \phi^3) - 4 = \phi^3$. Locally it is explained as the difference between the zero set ϕ and the empty set ϕ^3 which means again $\phi - \phi^2 = \phi^3$. However, at the “end” of the universe we have a one sided Möbius like boundary separating ϕ^3 from nothingness. Taking the measure concentration theorem of Dvoretzky into account [100] [101] we have one sided topological pressure causing expansion. It then turned out that almost 96% of energy is at the boundary of the holographic boundary and is given exactly by the dark energy density $E(D) = 5\phi^2/2(mc^2)$ while the rest is inside the “universe” and is given by the ordinary energy density $E(O) = (\phi^5/2)(mc^2)$ as discussed in great detail in [10] [88] and the references therein.

5. The Dimensionality of Spacetime from a Fractal Strings Interpretation of E-Infinity Theory

As mentioned before, there are two basic sets in E-infinity theory, the zero set $D \equiv (O, \phi)$ and the empty set $D_{-1} \equiv (-1, \phi^2)$. By the usual inversion one moves to positive “observable” space dimensions, namely to a fractal string given Hausdorffly by $d_c^{(2)} = 1/\phi = 1 + \phi$ and $d_c^{(3)} = 1/\phi^2 = 2 + \phi$. In other words the zero pre-quantum particle transmutes to a fractal string with a dimension $1 + \phi = 1.618033989\dots$ and the empty pre-quantum wave transmutes to a fractal world sheet $2 + \phi = 2.618033989\dots$. Our E-infinity spacetime expectation dimension, *i.e.* average multi fractal dimension is then found either as the union or the intersection of the fractal string “particle” with the fractal world sheet “wave” [32] [74]. That means

$$\begin{aligned} D &= (1 + \phi) \oplus (2 + \phi) \\ &= 4 + \phi^3 \end{aligned} \tag{9}$$

or

$$\begin{aligned} D &= (1 + \phi) \otimes (2 + \phi) \\ &= 4 + \phi^3 \end{aligned} \tag{10}$$

This is the remarkable indistinguishability condition of E-infinity spacetime [32] which stresses the intrinsic fuzziness of this space where intersection and union give exactly the same result and leads to the famous outcome of all “which way” quantum experiment. Nature is not paradoxical but only irreducibly fuzzy on the fundamental quantum scale. It is both chaotic and deterministic. In the terminology of G. ‘tHooft, we are simply dealing with a deterministic quantum mechanics and the conceptual equality [16]

$$EP = EPR = ZMG \tag{11}$$

is just another manifestation of this geometrical-topological fractal-Cantorian fuzziness.

6. Conclusion

We have known since a long time that orthodox quantum mechanics has no place for our human intuitive need for a concept related to spatial separation. However, even general relativity evades this concept and leaves the possibility open by contemplating a multiply connected topology and consequently the possible existence of wormholes [7]-[37]. It did not take long before many researchers started to suspect that quantum entanglement of wormholes may be two sides of the same coin. String theory offered a theoretical possibility which could not be overlooked by pioneers like Susskind and Maldacena. In fact speculation on such a connection leading to a theory of quantum gravity was discussed in the excellent text book of Prof. P. Holland [114] on the quantum theory of motion which is an account of the de Broglie-Bohm causal interpretation of quantum mechanics and therefore in many respects, related to the deterministic quantum mechanics of Nobel Laureate G. ‘tHooft [115] and the fractal-Cantorian indeterministic classical mechanics proposed by the present author [116] as well as G.

Ord and L. Nottale [32]. However, in the Cantorian version of this theory exposed here we go one step further. We are not just satisfied by showing that quantum entanglement is a form of a wormhole or vice versa. We want to show that both quantum entanglement and wormholes are manifestations of something fundamental to mathematical logic as expressed in set theory and one of its main off springs, namely measure theory and the Menger-Urysohn dimensional theory [74]. It is time and time again the same problem which only a few gifted mathematicians of the calibre of von Neumann and Alain Connes [119] noticed and could deal with as they are interested in physics but are also well aware of the definite difference between zero, empty and nothingness [74]. It is as simple and as difficult as that and nothing more but also nothing less than taking pure mathematics very seriously when working on foundational deep problems in physics. Let us conclude this paper by stressing again that mathematically a wormhole is an empty set with varying degrees of emptiness. It is not zero measure because it is a fat fractal [55]. However, it provides no classical spatial separation because it is a positive measure of an empty set converging to nothing and living in a somewhat difficult to classically imagine negative topological dimension. It is really there and not there at the same time. It is like life itself: one moment we are there and in a split of a second we are not there although the universe as a whole still goes on unperturbed by us ceasing to exist.

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