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# THE NEW VERSION OF LATVIAN CGE MODEL

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# The New Version of Latvian CGE Model

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**Abstract**

This paper describes the new version of Latvian CGE model, which is now an integral part of the joint CGE-EUROMOD modelling system. Special attention is devoted to the labour market and consumption blocks of CGE that are substantially improved compared with the previous version. We briefly describe the motivation to link Latvian CGE with Latvian EUROMOD and provide major technical details. We also provide an example of the policy simulation by the joint CGE-EUROMOD system, demonstrating how the introduction of the progressive personal income tax rate affected the Latvian economy at macro, industry and micro level.

**Keywords:** CGE model, Latvia, labour market, consumption, EUROMOD

**JEL codes:** D58, C68, H2, H6, D9

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# 1 Introduction

While the history of Computable General Equilibrium (CGE) models is not very long in Latvia, CGE models were already intensively used in various policy simulations, mostly related to the fiscal sector.[1] The previous version of Latvian CGE model was described in Benkovskis et al. (2016), but many important changes have been introduced into the model since then. This paper mostly serves as a technical document and describes the up-to-date state of the model. The new version of Latvian CGE model is an extension of Benkovskis et al. (2016) based on more recent and detailed Supply and Use tables (SUT) with more elaborate labour market and consumption blocks. Moreover, now the CGE model is linked with Latvian EUROMOD (see Sutherland and Figari 2013 for the general description of the EUROMOD, and Pluta 2021 for the latest version of the Latvian EUROMOD model) to introduce the analysis of income distribution and make use of EUROMOD's policy rules for labour taxes and household benefits.

One of the changes is related to the underlying data, since now the updated version of the model is based on more recent, detailed and reliable Supply and Use tables for 2015 provided by the Central Statistical Bureau of Latvia. As a result, the dimensions of the current model increased from 32 to 63 industries and from 55 to 63 commodities, including more than 30 000 variables.

Perhaps the most crucial changes are not related to the CGE model itself, but to the linkage of Latvian CGE model with Latvian EUROMOD. While the CGE model is an extremely rich tool that allows going beneath the aggregate macroeconomic surface, the model still assumes a representative agent at the industry level. The analysis of income distribution requires a much higher level of disaggregation, which is not always possible due to an exponential increase in the size of the model. In addition, the linearity of the model complicates the analysis of some policies, for example, progressive taxation or the minimum wage. An alternative approach is to link the CGE model with another model designed to simulate the issues related to the income distribution. Latvian EUROMOD is such a model.It is based on the EU-SILC data and allows simulating taxes that are applied to individual income: the personal income tax, social security contributions and the solidarity tax. On the benefit side, it allows to simulate all major child-related benefits, the unemployment benefit and two main means-tested benefits that are paid to the poorest population groups (for more information on the latest version of Latvian EUROMOD, see Pluta 2021).

Linking CGE with EUROMOD can address some of the abovementioned shortcomings of both

[1] See, for example, https://www.makroekonomika.lv/vienalga-kuru-nodokli-cel-sap-tapat.

individual models. The EUROMOD ability to simulate policy changes at individual and household level would compensate the lack of disaggregation and income distribution aspect in CGE, while the general equilibrium nature of CGE would overcome the absence of the indirect policy effect in EUROMOD. We use an iterative top-down bottom-up approach when the information from EUROMOD to CGE (the bottom-up link) is transmitted through the percentage change in disposable income, budget revenues from labour taxes and expenditures on benefits. The information from CGE to EUROMOD (the top-down link) is transmitted by the labour market variables: employment and activity rate changes, as well as wage rate changes by industry and skill. This paper provides some details regarding the linkage of the two models, although more technical details are provided in a companion paper by Benkovskis et al. (2023).

Several important adjustments were made to the CGE model as well. In particular, the labour market block is expanded substantially compared with the previous version of the model. First, the previous version of the model assumed a homogeneous labour, thus ignoring the difference in skills for different industries and occupations. The current version of the model contains three types of labour: high-, medium- and low-skilled labour. In addition to better reflection of economic realities and relaxation of the homogeneous wage rate changes assumption, the split of labour into three skill groups allows better linkage of CGE model with Latvian EUROMOD. Second, labour was perfectly mobile between industries, as a result the changes in nominal gross wage were uniform in all sectors of the economy. The current version of the model accounts for the imperfect mobility of labour both between industries and between skills, imposing a heterogeneity in the industry- and skills-specific wage. The third substantial innovation is related to labour supply that was assumed to be constant previously. The overlapping-generation demographics block together with the activity and natural unemployment rate provides a much richer description of labour supply in Latvia. It also allows for a detailed modelling of old-age pensions that are a very important fiscal expenditure category.

The second improvement to the CGE model is related to the consumption block: the Cobb-Douglas utility function was replaced by the Klein-Rubin function to allow for the different income elasticities by commodity. Another important change was implemented by splitting consumption into five income quintiles. This modification achieves two goals. First, it allows more granular modelling of consumption habits. Second, it provides the possibility for a better linkage with the microsimulation model EUROMOD.

The policy simulations presented in this and the companion paper (see also Benkovskis et al.

2023) demonstrate the advantages of the updated version of the model. In particular, we show that the capabilities of CGE model become wider, and one can now also observe the income distributional effects for various macroeconomic or industry specific shocks. Second, the outsourcing of the labour tax and household benefit block to EUROMOD allows for a broad set of policy simulations not available in the previous version of CGE model. Some of such policy simulations, like the introduction of the progressive personal income tax rate, are highly non-linear and cannot be performed without a rough approximations in the traditional CGE framework. In addition to the usual advantages of CGE and microsimulation models, our system includes the endogenised informal wage section, which allows more realistic modelling of policy measures related to the fiscal sector.

The paper is organized as follows. The next section provides a brief overview of CGE model. Section 3 describes the major improvements in the labour market block, while Section 4 – improvements in the consumption block. The new old-age pension block is covered in Section 5. Section 6 is devoted to a brief description of the CGE model link to Latvian EUROMOD. The data and the calibration of the model are documented in Section 7. Section 8 provides an example of the fiscal policy analysis, simulating how the introduction of the progressive personal income tax affected the Latvian economy at macro, industry and micro level. The last section concludes.

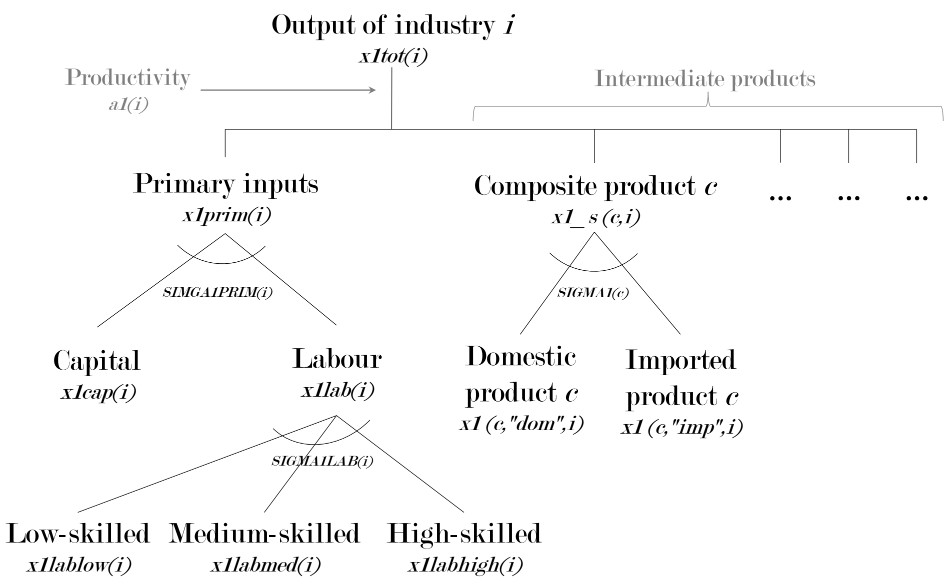
## 2 Short overview of Latvian CGE model

The new version of Latvian CGE model is an extension of Benkovskis et al. (2016) based on more recent and detailed Supply and Use tables (SUT) with more elaborate labour market and consumption blocks. Moreover, now the CGE model is linked with Latvian EUROMOD (see Sutherland and Figari 2013 for the general description of EUROMOD, and Pluta 2021 for the latest version of Latvian EUROMOD model) to introduce the analysis of income distribution and make use of EUROMOD's policy rules for labour taxes and household benefits. The general structure of Latvian CGE model follows, to a large extent, the seminal MONASH-style models, in particular a single-country ORANI-G model (see Horridge 2014, Dixon and Rimmer 2002, and Dixon et al. 2013). In this section we briefly overview the features that remain (almost) unchanged compared with the previous version of Latvian CGE model, while more explanations and details can be found in Benkovskis et al. (2016).

## 2.1 Structure of production

The new version of the model contains 63 industries and 63 commodities (compared with 32 industries and 55 commodities in the previous version). Each industry can produce several commodities reflected in the Supply table. The total demand faced by an industry equals the sum of demand for individual commodities produced by a given industry. It is assumed that each commodity supplied by a particular industry has the same production structure. Similar to Horridge (2014), the production of the commodity is substituted between different industries based on producer prices of the respective industries (see equation (A21) in Appendix A.2).

Figure 1: Production structure of an industry




All industries have the same structure of production, consisting of three nests. Figure 1 summarizes the production structure in the new model, which is a slightly simplified version of the production structure in ORANI-G (see Horridge 2014, p. 18). After acknowledging the total demand, an industry determines its need for an intermediate commodity and primary factor aggregates. The first nest assumes the Leontief production function – all the inputs are demanded proportionally to total output. The first nest also includes the exogenous production technology: the growth rate of total output equals the growth rate of aggregated (intermediate commodity and primary factor) inputs plus changes in production technology of the respective industry (see equations (A17) and (A18) in Appendix A.2).

At the second nest, all industries substitute between domestic and imported commodities according to the intuition of Armington (1969). Assuming a constant elasticity of substitution (CES) function, one arrives to equation (A6): the industry's choice between domestic and imported commodities depends on changes in relative producer prices of the respective commodity. The degree of substitution is determined by the elasticity of substitution ($SIGMA1(c)$).

Industries also substitute between primary factors – capital and labour. Similar to the substitution between domestic and imported commodities, the choice between labour and capital is driven by relative costs (see equations (A13)–(A14)). While labour inputs were assumed to be homogeneous in Benkovskis et al. (2016), the new version of the model also consists of the third nest, where industries substitute between three types of labour, and the choice between different skills of labour is driven by the relative wage (see Section 3.3 for more details).

The major reason behind the introduction of different types of labour into the production function is to relax the assumption of the proportional wage movements for all types of labour (see, e.g. Boeters and Savard 2013). The functional implementation of the production function and labour demand is uncertain, however, and the structure used by Horridge (2014) is not the only one possible. For example, Falk and Koebel (1997) use a translog production function. First, value added is split into the contributions of low-skilled labour and a bundle consisting of the rest of labour and capital. At a second level, capital is separated from non-low-skilled labour. High and medium-skilled labour are split at the lowest level. Such a production function accounts for the fact that non-low-skilled labour is more substitutable with capital, not with the low-skilled labour. On the other hand, such a production structure requires more elasticities of substitution for each industry, so we stick to the ORANI-G structure in the current version of the model.

### 2.2 Demand for commodities

The aggregate demand for domestic and foreign commodities comes from nine different categories of users. As described above, industries use domestic and foreign commodities as intermediate inputs in the production process. The other eight categories of users are the final users of commodities: domestic private consumption, domestic government consumption (we separate the value added tax (VAT) taxable and VAT exempt government consumption), domestic investments (that include private non-housing investments, private housing investments and government investments), exports as well as direct purchases abroad (see equation (A5) in Appendix A.2). Note that the last item was not present in Benkovskis et al. (2016) due to data constraints. It contains direct purchases made by non-residents in Latvia in case of demand for domestic commodities, while it consists of direct purchases by Latvian residents abroad in case of demand for foreign commodities.

Most of the final use categories are modelled by the two-nest structure. First, the final user decides on the amount of each composite commodity. This decision is modelled differently depending on the category of the final use. At the second stage, the choice between the domestic and foreign source for the particular commodity is made based on the relative prices and elasticities of substitution (see equations (A7)–(A12) in Appendix A.2). The definition of relative price may differ by the categories of the final use depending on the indirect taxes. The second nest is absent for private housing investments (as there is no foreign housing in the model) and exports.

Households decide on the amount of composite commodities by maximizing utility for a given level of total nominal consumption. Although the Cobb-Douglas household utility function was used in the previous version of the model, the new version switches to the Klein-Rubin utility function making it closer to Horridge (2014). Moreover, private consumption is now split by five income quintiles to capture income distribution effects and to improve the linkage with EUROMOD (see Sections 4 and 6).

The nominal government consumption (both VAT taxable and VAT exempt) and government investments are set exogenously for any composite commodity, although the government can still substitute between domestic and imported commodities. The nominal private housing investments are proportional to the disposable income of households (see equation (A81) in Appendix). Private non-housing investments are modelled differently: we assume that the total level of productive investments (i.e. private non-housing and government investments) keeps the aggregate real capital level constant in the long run. Moreover, similar to Horridge (2014) we assume that the real structure of private non-housing investments remains unchanged (see equations (A82)–(A84)).

Latvia's exports as well as direct purchases by non-residents in Latvia are driven by exogenous foreign demand for the respective commodity. In addition, from the cost minimization of non-residents, the export demand is driven by the relative price of domestic commodities to foreign prices. Finally, the current version of the model also contains re-exports (direct exports of imported commodities, constituting more than 30% of total Latvian merchandise exports, see Benkovskis et al. 2016), which only depends on exogenous foreign demand (see equations (A85)–(A87)).

## 2.3 Prices and costs

There are three commodity prices in the model: producer, basic and purchaser prices. We implicitly assume perfect competition and zero profits, thus the basic prices of domestic industry include only input costs of intermediate production, capital and labour. Once basic prices of industries are known, the basic price for a commodity is determined as a weighted industry basic price. Producer prices of domestic and foreign commodities equal basic prices of the respective commodity plus excise tax payments. The purchaser prices also include the VAT payments, which are commodity-specific following Giesecke and Nhi (2010) and Giesecke and Nhi (2012). In addition, as Latvia has a substantial size of informal activities, we keep assuming that only a fraction of agents make excise and VAT payments due to the shadow economy (see equations (A31)–(A34) in Appendix A.2).

We assume that capital is a homogeneous good used by all industries as a primary factor of production. Capital costs consist of two parts: the exogenous real interest rate that is similar to all industries and the industry-specific depreciation rate. Also, since capital is assumed to be a homogeneous good, we define the price of investments as a weighted producer price of private non-housing investments and government investments (see equations (A62) and (A63)).

While the modelling of the labour costs remains similar to Benkovskis et al. (2016), two major changes were introduced in the new version of the model. First, total labour costs are now an aggregate of high-, medium- and low-skilled labour costs. Second, the personal income tax and social security payments that enter labour costs are now modelled by Latvian EUROMOD (see sections 3.3 and 6 for more information).

## 2.4 Fiscal block and shadow economy

The main focus of the previous Latvian CGE model in Benkovskis et al. (2016) was the fiscal block. This focus remains in the new version of the model. The government revenues consist of five major parts: social security contributions, personal income tax (PIT) payments, VAT revenues, excise tax revenues, and other revenues. However, modelling revenues from the labour taxes – PIT as well as an employer's and employee's social contributions – is now outsourced to Latvian EUROMOD (see a more detailed description in Section 6), allowing much larger flexibility in simulating progressive tax rates and exemptions.

The modelling of VAT, excise tax, and other revenues remains unchanged compared with the previous version. Income from VAT depends on nominal private and government consumption

(VAT taxable), private housing investments, the commodity-specific VAT rate and the share of users paying commodity taxes (see equation (A104) in Appendix A.2). All users except exporters are subject to excise tax payments, where the tax rate is commodity-specific and is applied to the volume of commodity use. Excise tax revenue also depends on the share of users paying VAT and excise tax (equation (A105)). The remaining revenues are modelled as a fixed proportion to nominal GDP.

The government expenditures are now modelled in much more detail, mostly expanding the social expenditure block by introducing old-age pensions (described below in Section 5), and the availability of numerous private person benefits in Latvian EUROMOD, for instance, social expenditures on parental benefits, unemployment benefits, sickness benefits, disability pensions, and other social expenditures (see Section 6). Apart from that, government expenditures also include nominal government consumption (VAT taxable and VAT exempt), nominal government investments, interest payments on government debt, and other expenditure. Interest payment expenditure is determined by the current level of government debt and nominal interest rate, while the remaining components are treated as exogenous.

The issues of informal economy and tax evasion are still important in Latvia (see, for instance, Eurobarometer 2020 and Putnins and Sauka 2015). Modelling the changes in the informal economy is essential for an adequate analysis of fiscal policy. The shadow economy in our model refers to labour (PIT and social security contributions) and commodity (VAT and excise tax) payments. The share of tax evaders is partially endogenised by assuming that changes in tax rates and real activity affect the relative size of the shadow economy. The share of tax payers both with regard to labour tax and commodity tax payments is modelled as a logistic function (see equations (A88) and (A91) in Appendix A.2; more discussion in Benkovskis et al. 2016).

### 2.5 Mathematical form of the model

Although many underlying relationships of the model are non-linear, the model is solved by representing it as a series of linear equations following Johansen (1960). Such linearized systems are easy to solve even for large dimensions, but are accurate only for small changes. Many policy simulations require large shocks. Therefore, a conventional iterative solution procedure is adopted. The idea behind the procedure is to break large changes in exogenous variables into smaller changes and reiterate the system while updating the coefficients at each step. The reader is referred to Dixon et al. (2013) for more information.

### 2.6 Dynamics of the model

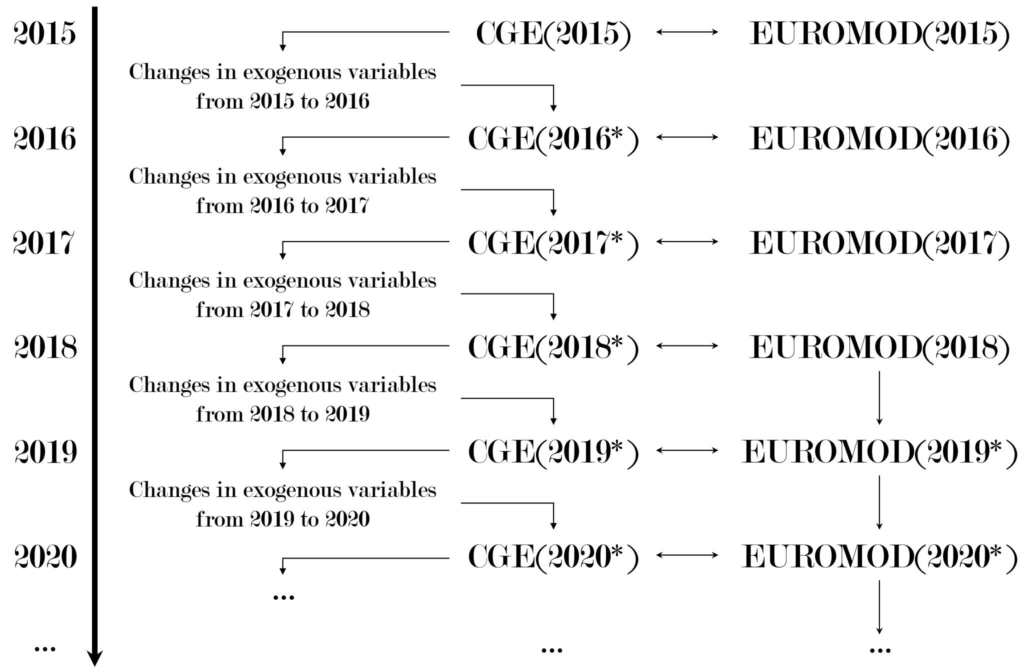
The Latvian CGE model is a recursive quasi-dynamic model. While the latest SUT is available for 2015 in Latvia, the model can be solved for any consecutive year. The CGE model starts with an initial database (2015 in our case). Then, assuming/observing the changes in exogenous variables between 2015 and 2016 the CGE model simulates the changes in endogenous variables. The SUT 2015 database is then updated to the 2016 SUT database and the process can be repeated recursively (see Dixon et al. 2013, pp. 58–60). The dynamic CGE models require links between years either through expectations about the consequtive years or through the memory of the lagged variables. The Latvian CGE model contains a limited set of dynamic equations (therefore, we refer to our model as being a quasi-dynamic). Primarily, it accounts for the dynamics in the employment gap (equation (A61) and (5)) by assuming that the deviation in real wage from its baseline increases proportionally to the deviation in aggregate employment (see section 3.4). This is the most important dynamic equation ensuring that real wage is sticky in the short run and flexible in the long run. In addition, the model captures the aging and death process by linking the population by age and gender in year $t$ and $t-1$ (see equations (A2)–(A3) in Appendix A.2), and the accumulation of pension capital (equations (A94)–(A97)).

The fact that Latvian CGE is linked with EUROMOD complicates the year-on-year sequence of solutions (see Figure 2). EUROMOD is based on the actual EU-SILC data in 2015–2018. Thus, the CGE-EUROMOD system is solved on the basis of actual SUT and EU-SILC data in 2015. Afterwards, in 2016–2018, the CGE-EUROMOD system uses the actual EU-SILC data, but relies on the predicted SUT data as described in the previous paragraph. Finally, EUROMOD also uses the predicted microeconomic data starting from 2019: it is updated using the EU-SILC from the previous year and taking into account changes in the labour market variables obtained from the CGE model. More details about the linkage between CGE and EUROMOD will follow in Section 6.

## 3 Labour market block

The labour market block is one of the blocks expanded substantially compared with the previous version of the model. There are three major improvements. First, the previous version of the model

Figure 2: Sequence of solutions for the joint CGE-EUROMOD system




Notes: * denotes that the model uses a predicted dataset: SUT in the case of CGE model, and EU-SILC in the case of EUROMOD (the year indicates the income data year in EU-SILC).

assumed a homogeneous labour, thus ignoring the difference in skills by industry and occupation. The new version of the model contains three groups of labour: high-, medium- and low-skilled labour. In addition to better reflection of economic realities, the split of labour into three skill groups allows better linkage of CGE model with Latvian EUROMOD (see Section 6). Second, labour was perfectly mobile between industries in Benkovskis et al. (2016), as a result the changes in nominal gross wage were uniform across all sectors of the economy. The new version of the model accounts for the imperfect mobility of labour both between industries and between skills, imposing a heterogeneity in industry- and skills-specific wage. The third substantial innovation is related to labour supply that was previously assumed to be constant. The overlapping-generation demographics block together with equations for activity and natural unemployment rates provide a much richer description of labour supply in Latvia.

### 3.1 Labour supply: demographics, migration and activity

Latvia has experienced a decline in population in recent decades both due to natural demographic changes and outward migration. The long-term demographics forecasts are not bright for Latvia (see, e.g. Vilerts et al. 2019), making the assumption of constant labour supply unrealistic. In order to account for this inconsistency, we introduce an overlapping generation (OLG) demographic block into our model. OLG-CGE models have a long history, widely used to analyse the effects of changes in various economic policies. The advantages of OLG-CGE models are not limited to labour supply, but they also include the possibility to model the life-cycle effects of policies (see Summers 1981, Zodrow and Diamond 2013, and Fehr et al. 2013). We limit the use of OLG to capture the effect of aging and model the old-age pensions (see Section 5) in the new CGE model, as our primary goal was to capture the impact of demographic changes on labour supply.[2] Thus, one should keep in mind that the policy analysis in our model does not account for the intertemporal decisions of economic agents.

We introduce demographics by adding age- and gender-specific mortality, fertility and net migration. For that we use a straightforward accounting approach and model birth, aging and death processes. The number of newborns (of zero age) depends on the fertility rate and the number of females by age. In addition, some newborn children die or migrate together with their parents during the first year of their life:

$$\forall g \in GDR,$$
$$POP0(g) = (1 - DRATE0(g)) \cdot (1 + MRATE0(g)) \cdot \sum_{a \in AGE} \left(POP(a, "w") \cdot BRATE(a)\right), \tag{1}$$

where $POP0(g)$ denotes the number of newborns of gender $g$, $POP(a, "w")$ – the number of females of age $a$, $BRATE(a)$ corresponds to the birth rate for women of age $a$, while $DRATE0(g)$ and $MRATE0(g)$ indicate the death rate and the migration rate[3] for newborns of gender $g$ respectively. The log-linearised version of equation (1) models the change in the number of newborn children by gender (see equation (A1) in Appendix A.2).

Aging and mortality are described by equation (2). The number of persons of age $a$ and gender $g$ ($POP(a, g)$) equals the number of persons of the same gender, but one year younger in the previous year ($POP_{-1}(a-1, g)$), excluding persons who did not survive (accounted by the age- and gender-specific death rate, $DRATE(a, g)$) and adding net migration (accounted by the age- and

[2] The CGE model uses 2015 SUT data, but we cannot ignore the decline in labour supply due to demographic factors since then.

[3] The positive number for the migration rate denotes immigration.

gender-specific migration rate, $MRATE(a,g)$):

$$\forall g \in GDR, \forall a \in AGE \backslash \{1\},$$

$$POP(a,g) = (1 - DRATE(a,g)) \cdot (1 + MRATE(a,g)) \cdot POP_{-1}(a-1,g), \quad (2)$$

Equations (A2) and (A3) are the log-linearised versions of the aging process for people of age 1 to 100 years and newborns separately. The birth, death and migration rates are calibrated using Latvian demographic data and Eurostat demographic projections.[4] The changes in migration rates ($mrate(a,g)$ and $mrate0(g)$) are treated as exogenous, allowing to simulate the effect of migration on the labour market and Latvian economy.

Finally, after modelling the natural demographic changes and migration, we also introduce the activity rate: the active population forms the labour supply in our model. The changes in the activity rate depend on the changes in the overall employment gap (reflecting higher activity rates in the periods of tight labour market), and the migration rate (since active males have a higher tendency to emigrate; no statistically significant effect was found for females). In addition, we also control for the effect of changes in pension age on the activity rate (see equations (A52) and (A53) in Appendix A.2). The coefficients were obtained from a regression based on the actual data for 2001–2015.

As Section 6 specifies, the existence of the OLG demographic block allows us to make an additional linkage to EUROMOD and adjust the EU-SILC database accordingly. This would partially endogenise expenditures on some of the family-related benefits. However, given the small effect of demographic changes in the short to medium run and the relatively low share of such benefits in budget expenditures, we leave this improvement for the future versions of the model.

### 3.2 Labour supply: skill transformations

In order to account for the imperfect mobility of labour between skills and industries, we relax the assumption about the homogeneity of labour supply and augment the Latvian CGE model with constant elasticity of transformation (CET) labour supply functions. We follow Dixon and Rimmer (2003) and Dixon and Rimmer (2006) assuming that individuals maximize a CES utility function containing earnings in each labour variety by choosing labour supply subject to the total

[4] See https://ec.europa.eu/eurostat/web/population-demography/population-projections

employment constraint. This approach is less intuitive compared with an alternative approach when individuals maximize earnings subject to the constraint where total labour is a CET function of labour varieties (see, e.g. Gaasland 2008). The latter approach leads to the problem of non-additive labour supply, however. See Boeters and Savard (2013) for more discussion.

In our setup individuals face a nested CES utility function, where the first level consists of utility obtained from high-, medium- and low-skilled labour income ($UHIGH$, $UMED$ and $ULOW$ respectively), while at the second level the utility for the respective skill is formed as a CES function of income from different industries (variables $NGWAGE*$ denote the nominal gross wage, while $EMP*$ – employment for the respective skill and industry). This utility function is maximized subject to standard labour constraints: total labour supply equals the sum of high-, medium-, and low-skilled labour supply, while the latter equals the sum of the respective labour supply by industries (see equation (3)).

$$\max_{\substack{EMPHIGH(i),\\ EMPMED(i),\\ EMPLOW(i)}} \left( UHIGH^{\frac{1}{1+SIGMALAB}} + ULOW^{\frac{1}{1+SIGMALAB}} + UMED^{\frac{1}{1+SIGMALAB}} \right)^{1+SIGMALAB}, \quad (3)$$

$$UHIGH = \left( \sum_{i \in IND} (NGWAGEHIGH(i) \cdot EMPHIGH(i))^{\frac{1}{1+SIGMALABHIGH}} \right)^{1+SIGMALABHIGH},$$

$$UMED = \left( \sum_{i \in IND} (NGWAGEMED(i) \cdot EMPMED(i))^{\frac{1}{1+SIGMALABMED}} \right)^{1+SIGMALABMED},$$

$$ULOW = \left( \sum_{i \in IND} (NGWAGELOW(i) \cdot EMPLOW(i))^{\frac{1}{1+SIGMALABLOW}} \right)^{1+SIGMALABLOW},$$

subject to

$$EMPHIGH = \sum_{i \in IND} EMPHIGH(i),$$

$$EMPMED = \sum_{i \in IND} EMPMED(i),$$

$$EMPLOW = \sum_{i \in IND} EMPLOW(i),$$

$$EMP = EMPHIGH + EMPMED + EMPLOW.$$

Equations (A42)–(A47) in Appendix A.2 are the solutions to this optimization problem in the log-linearized form. Equations (A45)–(A47) account for the imperfect mobility of high-, medium-and low-skilled labour across industries. More rapid increase in the industry gross wage compared to the average gross wage for the particular skill category is associated with a larger labour supply for this industry. The inverse elasticity of labour supply to the relative wage ($SIGMALABHIGH$, $SIGMALABMED$ and $SIGMALABLOW$)[5] reflects the degree of mobility between industries for a certain skill. Note that in the case of perfect mobility, the inverse elasticity tends to infinity, reducing to the specific case of homogeneous wage changes across all industries. In addition to the imperfect mobility between industries, labour is also imperfectly mobile between skills; this is captured by equations (A42)–(A44) in a similar fashion, where $SIGMALAB$ denotes the inverse elasticity of labour supply to the relative wage.

### 3.3 Labour demand: differentiating labour by skill

We follow Horridge (2014) and split the labour into several skills similarly to the ORANI-G model. The labour force splits into three parts in the current version of the model: the high-skilled, medium-skilled and low-skilled labour (see Figure 1).[6] All industries substitute between those three types of labour by minimizing the costs of aggregated labour, where the use of aggregated labour is defined as a CES function:

$$\min_{\substack{EMPHIGH(i),\\ EMPMED(i),\\ EMPLOW(i)}} \begin{pmatrix} NGWAGEHIGH(i) \cdot EMPHIGH(i) + \\ +NGWAGEMED(i) \cdot EMPMED(i) + \\ +NGWAGELOW(i) \cdot EMPLOW(i) \end{pmatrix}, \tag{4}$$

subject to

$$EMP(i) = \begin{pmatrix} BHIGH(i) \cdot (EMPHIGH(i))^{\frac{SIGMA1LAB(i)}{SIGMA1LAB(i)-1}} + \\ +BMED(i) \cdot (EMPMED(i))^{\frac{SIGMA1LAB(i)}{SIGMA1LAB(i)-1}} + \\ +BLOW(i) \cdot (EMPLOW(i))^{\frac{SIGMA1LAB(i)}{SIGMA1LAB(i)-1}} \end{pmatrix}^{\frac{SIGMA1LAB(i)-1}{SIGMA1LAB(i)}},$$

[5] Note that inverse elasticity of labour supply to the relative wage equals $\frac{1}{\sigma-1}$, where $\sigma$ denotes the elasticity of substitution of income in the utility function in equation (3).

[6] Although the data allowed for a more fine split of labour by occupation, we limited ourselves to three groups only. One of the reasons is the linkage with Latvian EUROMOD, where wages and the activity status by skill and broad industries are consistent between the two models (see Section 6). Splitting labour into more fine groups would create problems in the convergence of both models due to the small size of the Latvian economy and the EU-SILC database.

where $EMPHIGH(i)$, $EMPMED(i)$ and $EMPLOW(i)$ represent the high-, medium- and low-skilled labour in the industry $i$ respectively, $EMP(i)$ indicates the aggregate use of labour in industry $i$, $NGWAGEHIGH(i)$, $NGWAGEMED(i)$ and $NGWAGELOW(i)$ – the labour costs of the respective types of labour, but $SIGMA1LAB(i)$ stands for the elasticity of substitution between different types of labour. The $BHIGH(i) > 0$, $BMED(i)_i > 0$ and $BLOW(i)_i > 0$ are the industry-specific exogenously set parameters. The solution of the optimization problem (4) together with additional restriction of $EMP = EMPHIGH + EMPMED + EMPLOW$, and log-linearisation leads to equations (A15)–(A16) and (A19) in Appendix A.2, reflecting that producers substitute one type of labour for another if the relative price of labour changes, while the degree of the changes is driven by the elasticity of substitution.

### 3.4 Wages and labour market dynamics

Similarly to the previous version of the model, we assume that the unit costs of labour are comprised only of gross wage and an employer's social contributions, while the net wage equals gross wage net of social security payments of employees and PIT payments (see equations (A35)–(A38) in Appendix A.2). Moreover, we assume that some firms evade paying labour taxes and the share of enterprises paying labour taxes is industry-specific. Therefore, wages in a particular industry should be interpreted as effective wage.

The average wage rate in the economy is driven by the demand for supply of labour. The demand for labour is determined by industries, while the supply of labour is determined by the demographic factors and activity rate as explained in Section 3.1. However, the supply of labour is positively related to real wage growth in the short run, which introduces some dynamics into our model. Here we follow the conventional approach in CGE models (see Dixon and Rimmer 2002, p. 357) and assume that the real wage is sticky in the short run and flexible in the long run. In other words, we assume that the deviation in the real wage from its baseline increases proportionally to the deviation in aggregate employment (here we differ from Dixon and Rimmer 2002 who relate real wage to deviation in aggregate hours of employment; we also express the real wage equation in a different form):

$$\frac{RGWAGE_{t+1} - RGWAGE_t}{RGWAGE_t} = GAMMA \cdot \frac{EMP_{t+1} - EMPNAT_{t+1}}{EMPNAT_{t+1}}, \tag{5}$$

where $RGWAGE_t$ denotes real average gross wage at time $t$, $EMP$ is total employment, and $EMPNAT$ represents the natural level of employment (the log-linearized version can be found in equations (A60)–(A61) in Appendix A.2). The parameter $GAMMA > 0$ defines the wage flexibility to the employment gap. Equation (5) is very similar to the one in Benkovskis et al. (2016) except the use of the varying natural level of employment that depends on demographics, activity and the natural rate of unemployment:

$$EMPNAT = ATOT \cdot (1 - UNAIRU)\,, \tag{6}$$

where $UNAIRU$ stands for the exogenous natural rate of unemployment. Equation (A56) represents the log-linearized relationship between the employment gap, economically active population and natural unemployment.

## 4 Private consumption block

The utility function of households in Benkovskis et al. (2016) was based on the traditional Cobb-Douglas functional form, implying the constant income shares allocated to the consumption of individual commodities. This is a significant shortcoming: keeping consumption shares of all goods constant contradicts economic reality, such that different commodities have different income elasticities. Some goods are income elastic, other goods, which we usually think of satisfying basic human needs, are inelastic, while some can even be inferior. By ignoring these differences, we underestimate possible unique reactions of industry output after the introduction of new economic policy.

To overcome this restriction, the model should allow setting different absolute and marginal shares of commodities in the consumption basket. This logic is the ground for the Linear Expenditure System introduced by Stone (1954). Based on his idea, the Klein-Rubin function was developed, which now becomes a conventional way of modelling private consumption in CGE models.

In addition to the use of Klein-Rubin function, another important change to the new version of Latvian CGE model was introduced by splitting consumption into five income quintiles. This modification achieves two goals. First, it allows more fine modelling of consumption habits. Second, it provides the possibility for a better linkage with the EUROMOD model described in section 6.

We improve the consumption block of the new CGE model by assigning different elasticities to commodities, thereby simulating the behavioural process of households maximizing their utility by choosing what to consume in a more realistic manner. This also brings our model closer to the one of Horridge (2014). We employ the Linear Expenditure System. The core idea of this system is the distinction between absolute and marginal budget shares of each commodity. As suggested by Stone (1954), an easy way to achieve it is to distinguish between subsistence quantities for each good and the auxiliary or luxury consumption. The latter is funded by what is left after the purchase of the subsistence quantities. Such an approach is based on the intuition that, regardless of income, households have some necessary amounts of most goods, and they will consume them in any case. There are several important limitations that we have to make by adopting this approach. First, this assumption rules out inferior goods and assures budget shares monotonous with income growth. Second, all commodities are treated equally, with no individual substitution or complementary effect occurring between certain types of commodities.

The Klein-Rubin utility function is the underlying functional form to design the instrument of integration of the Linear Expenditure System into the CGE model. Households face budget constraint, but in spite of that they meet their need for the minimum required amount of all commodities. Next, households, having satisfied their subsistence demand, are free to allocate their spare income to commodities in the proportion that brings them the highest utility. It is assumed that subsistence demand contains the amount of commodities vital for living only, and their consumption does not bring utility. Therefore, households from the respective quintile maximize utility according to the following utility function:

$$\max_{X2\_S(c,q)} \prod_{c \in COM} X2LUX\_S(c,q)^{S2LUX(c,q)}, \tag{7}$$

subject to

$$\sum_{c \in COM} (PPUR2\_S(c) \cdot X2\_S(c,q)) = W2TOT(q),$$

where $X2LUX\_S(c,q) = X2\_S(c,q) - X2SUB\_S(c,q)$ is luxury consumption of a composite commodity $c$ of households from income quintile $q$. The luxury consumption equals the difference between total consumption $X2\_S(c,q)$ and minimum required subsistence consumption $X2SUB\_S(c,q)$ of composite commodity $c$. The total nominal consumption is constraint by the total disposable income $W2TOT(q)$ of the respective quintile, ($PPUR2\_S(c)$ defines a purchaser price for commodity $c$). $S2LUX(c,q)$ defines the marginal share of consumption of commodity $c$ in income quintile $q$ and is the same parameter as in the generalized Cobb-Douglas function (must sum to unity for all commodities).

The solutions of the maximization problem in equation (7) lead to the following demand equation for the composite product $c$:

$$X2\_S(c,q) = X2SUB\_S(c,q) + \frac{S2LUX(c,q) \cdot W2TOTLUX(q)}{PPUR2\_S(c)}, \tag{8}$$

where

$$W2TOTLUX(q) = W2TOT(q) - \sum_{c \in COM} \left( PPUR2\_S(c) \cdot X2SUB\_S(c,q) \right).$$

From equation (8), the demand for each individual commodity depends on purchaser prices, disposable income remaining after the subsistence consumption, and subsistence quantities of all other commodities. The log-linearized version of equation (8) leads to the demand for luxury consumption equation (A65) in Appendix A.2. To relate changes in total and luxury consumption of the composite product $c$ we need to introduce the quintile-specific Frisch coefficient ($FRISCH(q)$). As described by Frisch (1959), the use of this coefficient eliminates the need to set cross-price elasticities of demand for different goods. The Frisch coefficient is the (negative) ratio of all disposable income to the part spent for purchasing the amount of the aggregate auxiliary demand for all goods (see, e.g. Jussila et al. 2012):

$$FRISCH(q) = -\frac{W2TOT(q)}{W2TOT(q) - \sum_{c \in COM} \left( PPUR2\_S(c) \cdot X2SUB\_S(c,q) \right)}. \tag{9}$$

In the log-linearized version, changes in total consumption relate to changes in luxury consumption in the following way (note that subsistence consumption is assumed to remain unchanged):

$$x2\_s(c,q) = \frac{X2LUX\_S(c,q)}{X2\_S(c,q)} \cdot x2lux\_s(c,q). \tag{10}$$

The use of the Frisch coefficient allows replacing $\frac{X2LUX\_S(c,q)}{X2\_S(c,q)}$ by $-\frac{EPSILON(c)}{FRISCH(q)}$, where $EPSILON(c)$ refers to the income elasticity of a composite product $c$ (see, e.g. Horridge 2014, pp. 68–69 for more details), thus obtaining equation (A66) in Appendix A.2. Finally, from the definition of the disposable income remaining after the subsistence consumption ($W2TOTLUX(q)$) and the definition of the Frisch coefficient, one can arrive at to the equation linking the changes in total disposable income remaining after the subsistence consumption with the changes in total disposable income (see equation (A67)).

## 5 Old-age pension block

The introduction of the demographic block to the Latvian CGE model (see Section 3.1) allows for a detailed modelling of the very important fiscal expenditure category – old-age pensions. Pension blocks become a usual part of the overlapping generations CGE models, see Fehr et al. (2013) and Zodrow and Diamond (2013), while the example of a more elaborated pension block can be found in Zuo et al. (2020). Although most of other social expenditures like unemployment or sickness benefits are outsourced to Latvian EUROMOD (see Section 6), the old-age pensions are retained in the CGE model since the size of pension depends on the social security payments during the employment period – the relationship that cannot me modelled in the static microsimulation model.

The pension block is designed to capture the main features of the current Latvian pension system. It includes two pension pillars. The first, or Pay-As-You-Go (PAYG) pension pillar is a mandatory pension scheme based on the principle of the intergenerational solidarity, when tax contributions are used to finance current pension expenditures. The second, or Fully Funded (FF) pension pillar is also a mandatory scheme, where part of the social insurance contributions are channelled into individual pension accounts and invested. There is also a third, voluntary pension pillar in Latvia that offers a possibility for private accumulations in pension funds. However, we do not include the third pillar into the current CGE model, so voluntary pension savings are indistinguishable from other private savings.[7]

Although the overall design of the pension system was mostly set in early 2000s, there were certain transition periods and changes in system parameters that should be taken into account while modelling. In particular, although the second pillar was introduced already in July 2001, only cohorts of 1952 and younger were allowed to participate (with mandatory participation for cohorts of 1971 or younger). The share of both pillars in the social contribution payments also changed, gradually increasing the importance of FF pillar since 2001 with a temporary decline during and after the global financial crisis. Also, the retirement ageis continuously rising and is planned to be set at 65 years in 2025.

The old-age pension depends on two factors in the Latvian CGE model: the size of the accumulated pension capital, coming from the first pillar notional capital and the second pillar capital, and the remaining life expectancy at the retirement age. The remaining life expectancy (which is not

[7]The contribution of the third pillar is relatively small compared with the second pillar (646.8 million euro and 5 999.4 million euro at the end of 2021 respectively).

gender-specific in pension calculations, so it equals the average remaining life expectancy for a given cohort) negatively depends on the death rates captured by quations (A92)–(A93) in Appendix A.2.

The first pillar notional capital is modelled by equation (A94) by gender and cohort: the accumulated notional capital depends on the level of capital in the previous year, augmented by the social security revenues contributing to the first pillar. The latter are determined by the social security contribution rate to the first pillar, employment and the average reported nominal wage. In addition, we take into account the possibility of changes in the retirement age that determines the cohorts contributing to the pension capital (we assume no early retirement in the model). The second pillar capital is modelled in a similar way by equation (A96) in Appendix A.2.

On top of social security contributions, the accumulated pension capital depends on capital returns. The notional return on the first pillar capital is determined by the annual changes in social security tax revenues for the old-age pensions[8], while the return on the second pillar capital is linked to the nominal interest rate (see equations (A98)–(A99) in Appendix A.2).

The old-age pension is received by all persons of retirement age or older. For all persons at retirement age, the overall benefits (for the respective gender) are derived as the accumulated pension capital (from the first and second pillars) divided by the remaining life expectancy (see equations (A95) and (A97)). Afterwards, old-age pension payments are indexed by the consumer inflation rate[9] in equation (A100).

# 6 Linking CGE with EUROMOD

While the CGE model is an extremely rich tool that allows going beneath the aggregate macroeconomic surface, the model still assumes a representative agent at the industry level. The analysis of income distribution requires a much higher level of disaggregation, which is not always possible due to an exponential increase in the size of the model. Thus, CGE models are not well suited for the analysis of individual-level and distributional effects (see Cockburn et al. 2014 and DeBacker

[8]The revenues between August of the previous year and July of the current year are taken into account, so we weight the changes of the previous and the current year accordingly.

[9]The actual indexation mechanism is rather complex and non-linear. In particular, the indexation applies to the par of pension that does not exceed 50% of the average wage subject to insurance contributions of the previous year. Pensions are reviewed once a year on 1 October, considering the actual consumer price index and 50% of the real increase in the aggregate wage subject to insurance contributions. In addition, pensions cannot be indexed downwards, but if the real increase in the wage subject to insurance contributions is higher than 15%, the value of 15% is used (see the law "On State Pensions", Section 26, https://likumi.lv/ta/en/en/id/38048-on-state-pensions). In the current version of the model, we index pensions only according to the inflation rate (between August of the previous year and July of the current year) for simplicity and take into account the fact that pensions are indexed on 1 October. Moreover, only positive inflation matters for the indexation, while pensions are not reduced in the case of deflation.

et al. 2019). In addition, the linearity of the model complicates the analysis of some policies, and CGE often lacks the degree of detail necessary to simulate effects taking into account all tax-benefit system complexities and interrelations (see Peichl 2009), for example, progressive taxation or the minimum wage.

One way to introduce a detailed tax and benefit block into the model is to use a Social Accounting Matrix (SAM) based CGE model (see Pyatt 1988 and Robinson and Roland-Holst 1988 for the general discussion about SAM-based CGE models; see Socci et al. 2021 for the recent empirical implementation). The use of SAM ensures a consistency of the tax and benefit data with the national accounts – the feature that makes this approach superior to the linkage between CGE and the microsimulation model. Also, SAM allows disaggregating the households by income into quintiles or deciles, thus capturing the effect of policy measures not only on the aggregate economic variables, but also on the income distribution. The level of household sector disaggregation may still not be granular enough compared with microsimulation models to capture some changes in the tax and benefit policy. Furthermore, SAM is not currently available for Latvia.

An alternative approach is to link the CGE model with another (microsimulation) model designed to simulate the issues related to the income distribution. The Latvian EUROMOD is such a model.

### 6.1 Short description of EUROMOD

EUROMOD is a tax-benefit microsimulation model covering all EU Member States built to simulate tax liabilities and benefit entitlement at individual and household level (see Sutherland and Figari 2013 for more details). The model uses two key ingredients: the input micro data at individual and household level (EU-SILC in models of most countries) and the model code describing the national tax and benefit policy rules. Simulations involve three major steps. First, the model creates tax-benefit assessment units (which may differ from households), using information about partners and children contained in the database. Second, the model assesses the eligibility for benefits and taxes for each individual within the assessment unit, depending on the composition of the assessment unit and the reported incomes from all sources. Finally, the model simulates taxes and benefits for each individual within the assessment unit based on the policy rules. EUROMOD is being widely used for policy analysis in EU Member States, both at the country and EU level. For instance, Bargain et al. (2014) use EUROMOD for euro area countries to simulate the effect of a fiscal union by imposing a homogeneous tax system in the Member States and introducing automatic stabilization and redistribution mechanisms across countries. Jara Tamayo and Simon (2021) use EUROMOD to simulate the effects of a common unemployment insurance system for the euro area and estimate the effect of such a system on income of part-time and temporary workers. Euromod (2021) estimates the distributional effects of all policy changes implemented in EU countries and the UK in 2019–2020.

Latvian EUROMOD is based on the EU-SILC data and allows simulating the taxes applied to individual income: the personal income tax, social security contributions and the solidarity tax. On the benefit side, it allows to simulate all major child-related benefits, the unemployment benefit and two main means-tested benefits paid to the poorest population groups (for more information on the latest version of Latvian EUROMOD, see Pluta 2021). Latvian EUROMOD was previously used to analyse the distributional impact of selected policy changes in Latvia (see Pluta and Zasova 2017 and Pluta and Zasova 2018).

We modified the current version of Latvian EUROMOD by adding non-reported wages to the reported gross wage available in the EU-SILC data. This was done to facilitate linkage with Latvian CGE that contains both the reported wage and total wage (that also includes non-reported income). The total wage at the individual level was imputed to the EU-SILC database. Here we follow the recently introduced approach by Benkovskis and Fadejeva (2022) who extended the Gavoille and Zasova (2021) evaluation of the probability of a firm to engage in labour tax evasion by estimating the size of the unreported payments at the employee level. A simple rule stating that the total gross wage cannot be lower than the reported gross wage was introduced to the set of rules in EUROMOD.

EUROMOD has several drawbacks: this is a static model, since it does not account for any possible behavioural responses that may be caused by reforms, and it also abstracts from any changes in population demographic composition that occur over time. Thus, the simulated effects of a reform should be interpreted as very short run – EUROMOD provides only the direct or the first-round effect of the policy on the income distribution, tax revenues and benefits, ignoring the indirect effects coming from changes in the disposable income and general economic activity. Such indirect effects can be quite substantial, however (see Barrios et al. 2019).

### 6.2 Linking the Latvian CGE model with Latvian EUROMOD

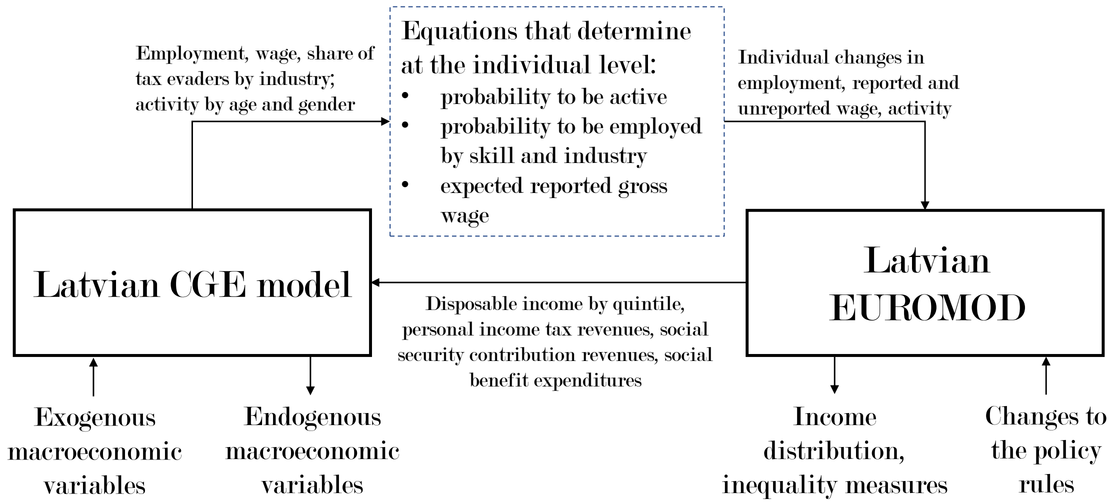
Linking CGE with EUROMOD can address some of the abovementioned shortcomings of both individual models. The EUROMOD ability to simulate policy changes at individual and household level would compensate the lack of disaggregation and income distribution aspect in CGE, while the general equilibrium nature of CGE would overcome the absence of the indirect policy effect in EUROMOD. We follow the approach when both models are linked in a "layered" way: the linkage is performed in a specially built interface using a set of linkage variables common to both models. This can be a "top-bottom" linkage when the shock is first modelled in CGE and then transmitted to the microsimulation model (see, e.g. Herault 2010 and Tiberti et al. 2018). The linkage can work "bottom-up", which involves using output of the microsimulation model as an input to the CGE model (see Benczur et al. 2018). Finally, the linkage can occur in an iterative manner with numerous rounds of simulations and exchanges of results between the two models (see Savard 2010).

There are not many examples of CGE models linked to a microsimulation model. The only example known to us is Peichl (2009) who linked a microsimulation model for Germany and the CGE model by a top-down bottom-up approach. However, the CGE model can be linked with the microsimulation model similarly to other macroeconomic models. Moreover, the CGE model has the advantage of a detailed disaggregation by industry, allowing for a more fine linkage through the labour market variables.

We use an iterative top-down bottom-up approach, when the information from EUROMOD to CGE (the bottom-up link) is transmitted through the percentage change in disposable income, budget revenues from labour taxes and expenditures on benefits. The information from CGE to EUROMOD (the top-down link) is transmitted by the labour market variables: employment and activity rate changes, as well as the wage rate changes by industry and skill. To translate industry-level changes in employment and wages into the microsimulation model, we follow a regression-based approach similar to Marx et al. (2012).

Figure 3 schematically summarizes how the new version of the Latvian CGE model is linked with Latvian EUROMOD. This paper only briefly describes the way both models are linked. More detailed technical information, together with various simulations and illustrations of the advantages provided by the linkage can be found in a companion paper by Benkovskis et al. (2023).

Figure 3: Latvian CGE model linked with Latvian EUROMOD




Note: More details about the linkage of the two models can be found in Benkovskis et al. (2023).

### 6.3 EUROMOD output as input to the CGE model

The general idea of the linkage is to use the strength of each model. EUROMOD is a microsimulation model based on the household data (EU-SILC) designed to simulate tax liabilities and benefit entitlement at individual and household level. Thus, the variables related to household disposable income, benefits and labour taxation are not modelled in the new version of the CGE model, but treated as exogenous variables obtained from Latvian EUROMOD (see Figure 3).

Changes to nominal disposable income by income quintile ($w2totnetADD(q)$) is the most important input to CGE obtained from EUROMOD. Any changes in Latvian policy rules related to labour taxation or benefits directly change the simulated disposable income of each household in EUROMOD. Aggregating those changes using the respective household sample weights provides the overall changes to nominal disposable income that is used as an exogenous input in equation (A68) in Appendix A.2.[10] Changes in the disposable income directly transmit to changes in private consumption, affecting the overall economic activity and consequently all variables in the CGE model. Thus, any change in the EUROMOD policy rules has a crucial effect on the CGE output.

Apart from the disposable income, the CGE model uses two groups of fiscal inputs originating

[10]The EU-SILC database captures capital income of households on a weekly basis. Thus, we use EUROMOD for obtaining the nominal disposable income net of capital income. This coincides with total disposable income for the first four quintiles, while the capital income is added to obtain the total disposable income of the fifth quintile (see equation (A70) in Appendix A.2).

from EUROMOD. One group of inputs consists of labour tax revenues. While the previous version of the CGE model took into account some features of labour taxes like the non-taxable minimum, it was way behind the possibilities offered by EUROMOD. Thus, modelling of the overall labour tax revenues is "outsourced" to EUROMOD now. The CGE model uses personal income tax payments as well as social security contributions of employers and employees as exogenous inputs obtained from EUROMOD by weighted aggregation (see equations (A106), (A108)–(A109) in Appendix A.2). This allows simulating virtually any changes in labour taxation.

Another important group of fiscal inputs obtained from EUROMOD is related to benefits. EUROMOD has a large number of different household and individual benefits described in its policy rules. We limit the number of exogenous variables from EUROMOD to five: unemployment benefits, sickness benefits, parental benefits, invalidity pension benefits, and all other benefits (summed together) available in Latvian EUROMOD (see equations (A116)–(A120) in Appendix A.2). All benefits are aggregated at the national level using household weights. Old-age pensions are the only important social benefit that is not taken from EUROMOD, but still modelled within CGE (see Section 5). This is related to the static nature of EUROMOD that does not account for demographic processes and life-time accumulation of pension capital, which makes the CGE modelling framework more suitable.

### 6.4 CGE output adjusting the EUROMOD micro dataset

There are two major blocks of CGE output used in EUROMOD simulations.[11] First, the CGE model provides employment and activity changes by industry. The changes in employment are further disaggregated by three skill groups (see Section 3.3). This allows adjusting the employment status of individuals in the EUROMOD database. There are four possible status transitions: unemployed to employed, employed to unemployed, inactive to unemployed, and unemployed to inactive (two consecutive transitions are possible, e.g. an inactive person may become employed through an intermediate unemployed status).

The second block of CGE outputs consists of changes in the gross wage by industry and skill, as well as the change in the share of unreported wage payments by industry (note that the gross wage includes unreported payments in the Latvian CGE model). Thus, we can alter the wages (both

[11] In principle, it is also possible to use the demographic output of CGE to adjust the EUROMOD database (accounting for the newborn children, deceased persons and migration). Such an additional link would not much affect the output of the simulations in the short to medium run therefore we limited the linkage to the labour market for simplicity.

reported and unreported) for individuals that remain employed, and assign a wage rate for a newly employed person in EUROMOD.

Transferring the output of a macroeconomic model to a microsimulation model is not as straightforward as the reverse process: the output of the CGE model is available at the industry level[12], while EUROMOD requires the individual level for the wage rate and employment status. We address this issue similarly to Marx et al. (2012) by including the additional block of equations that transforms the industry-level output of CGE to individual changes in the EUROMOD database (see Figure 3). First, the probit regressions determine the probability of an individual to be employed (at the high-, medium- or low-skilled position), unemployed or inactive. Second, the income regressions explain the reported gross wage (for the high-, medium- or low-skilled position) using the Heckman two-stage procedure. All regressions are estimated using the EU-SILC data for 2011–2019 (corresponding to the income reference years 2010–2018), which contains various individual characteristics like gender, age, and education. More information on the above regressions can be found in Benkovskis et al. (2023).

The results from these additional regressions allow creating an algorithm that transforms industry level shocks to changes in labour variables at the individual level. Each individual in the EUROMOD database has an estimated probability to be employed, unemployed or inactive. If, for example, the CGE model output suggests that 25 employees in Agriculture should become unemployed, the algorithm finds 25 individuals[13] in the EUROMOD database currently employed in Agriculture with the highest probability to be unemployed[14], and change their status to unemployed. Other changes in the individual employment status are simulated in a similar way. As to changes in the wage rate, the newly employed persons receive the wage predicted by the income regression. Afterwards we similarly adjust the wages of all employees of the respective industry to obtain the numbers suggested by the CGE model.

[12] Although CGE works with 63 2-digit industries, EUROMOD only consists of 12 broad sectors, so the output of CGE was aggregated respectively.

[13] Weights are assigned to all households and individuals in EUROMOD, so each individual in the dataset represents several individuals in Latvia. To avoid the situation when all similar individuals change their employment status simultaneously, the EUROMOD database was transformed. Every household (and every individual within this household) with weight $w_i$ (household $i$ in the database representing $w$ households in Latvia) was replaced by $k_i$ similar households with weight $n = w_i \div k_i$, so that any household in the new database represents $n$ households in Latvia. See Benkovskis et al. (2023) for more details.

[14] A random number is added to the estimated probability of being unemployed to avoid determinism in EUROMOD solution. See Benkovskis et al. (2023) for more details.

### 6.5 Convergence of both models

Figure 3 shows the joint system of the two models. Shocks can be applied to both models either by changing exogenous variables in the CGE model or altering the policy rules in EUROMOD. Any of those shocks induce an iterative process, when changes in the CGE labour market output make adjustments to the employment status and labour income at the individual level in EUROMOD, while the changes in disposable income, tax payments and benefits create a new exogenous shock for CGE. We continue the iterative process until the joint system of the two models converge: both models report similar[15] changes in the variables present in both models. Namely, both models should have a similar output in terms of disposable income growth (by income quintile), labour tax payments and social benefits.[16] In other words, we ensure that changes of the abovementioned variables are consistent in both models (we do not ensure consistency in levels since the CGE model operates with changes in economic variables). Although there is no theoretical proof that the system of both models should converge, in practice the convergence is usually achieved after 4–6 iterations.[17]

## 7 Data and calibration of parameters

### 7.1 Data

The major difference compared with the previous version of the model in Benkovskis et al. (2016) is the use of a more recent, detailed and reliable Latvian SUT for 2015 provided by the Central Statistical Bureau of Latvia (CSB). As a result, the dimensions of the current model increased from 32 to 63 industries and from 55 to 63 commodities. Industries are classified according to NACE Rev. 2, while commodities – according to the CPA 2008 classification. The Supply table was used to obtain the information on the supply of commodity by industries ($V1SUP$) at basic price. Similarly, the Use table provides the data on the use of commodities by different industries ($V1BAS$) or final users ($V2BAS$-$V9BAS$) at basic prices, as well as labour and capital costs ($V1CAP$) by industries.

Data on employment and labour costs by different skill categories ($EMPHIGH$, $EMPLMED$, $EMPLOW$, $V1LABHIGH$, $V1LABMED$, $V1LABLOW$) were imputed using the additional

[15] In practical terms, the difference should not exceed $2.5 \cdot 10^{-4}$ or 0.025 percentage points. This difference is negligible from the economic point of view.

[16] In total there are 13 variables that count for convergence.

[17] It can require more iterations for large shocks.

information provided by the CSB.[18] The same data source was used for the demographics-related information: number of persons by age and gender ($POP$), birth rates by age ($BRATE$), death, net migration and activity rates by age and gender ($DRATE$, $MRATE$, $ARATE$). Information related to the design of the Latvian pension system: remaining life expectancy ($LIFEEXP$), pensions age ($PENSAGE$), contributions to the 1st and 2nd pillars ($TPAYG$, $TFF$), gender and age-specific pension capital ($KPAYG$, $KFF$) mostly comes from the State Social Insurance Agency with some additional imputations. The rest of the data sources are mostly related to the fiscal sector, including the State Revenue Service data on the excise tax and VAT revenues, Ministry of Finance budgetary documents on government consumption and investments (see Benkovskis et al. 2016 for more detailed description).

## 7.2 Calibration of parameters

Results of the CGE model largely depend on the values of model parameters, in particular elasticities of substitution that define the substitutability, e.g. between domestic and imported products or labour and capital. This version of the model continues using the calibrated elasticities and other parameters. Moreover, most of the parameters remain unchanged from Benkovskis et al. (2016), relying on expert judgements (see Tables A3 and A4 for commodity- and industry-specific elasticities of substitutions).

Several important parameters were added compared with the previous version, though. The introduction of several groups of labour into the production structure requires the elasticity of substitution between different skills of labour ($SIGMA1LAB(i)$) that was calibrated to 0.5 for all industries. We also assume that low-skilled labour is more substitutable and therefore also more mobile between industries compared with middle- and especially high-skilled labour. This is reflected by lower inverse elasticity of relative supply by industries to the relative wage ($SIGMALABLOW = 0.10$, $SIGMALABMED = 0.25$, $SIGMALABHIGH = 0.40$). Labour is also mobile between skills as $SIGMALAB = 0.40$.

Determining the level of the Frisch coefficient is important for the consumption function since it reflects sensitivity of consumers to the price changes. In the current version of the model the Frish coefficient varies by income quintile and is calibrated to be between 5 for the poorest and 1.25 for the richest income group ($FRISCH(1) = 5.0$, $FRISCH(2) = 3.0$, $FRISCH(3) = 1.85$,

[18] The data are available by major occupation group, further aggregated into high- (ISCO-08 major groups 1, 2, 3), medium- (0, 4, 6, 7), and low-skilled labour (5, 8, 9).

$FRISCH(4) = 1.4$, and $FRISCH(5) = 1.25$). The value of the Frisch coefficient in the middle-income group is set close to the value used by Powell et al. (2002). Finally, the consumption block also requires the product-specific income elasticity of consumption that was calibrated using expert judgements and can be found in Table A3.

# 8 Simulation: introduction of the progressive personal income tax rate

This section provides an example of the simulation that can be produced by the joint CGE-EUROMOD system. In order to demonstrate the advantage of the new model compared with Benkovskis et al. (2016), we choose the recent personal income tax reform in Latvia, when a progressive tax rate was introduced in 2018. First, this is not a shock that is easy to implement in a macroeconomic model, since the overall effect depends on the actual income distribution; therefore the link with EUROMOD helps capturing the initial shock more precisely. Second, the progressive taxation influences income distribution in addition to the aggregate macroeconomic changes – the new model captures these distributional effects. More simulations using the CGE-EUROMOD system can be found in Benkovskis et al. (2023).

## 8.1 Description of the reform

The tax reform covered many areas in 2018, including changes in the minimum wage, the increase in social security contribution rates, adjustments in the microenterprise and enterprise income tax. The most notable changes, perhaps, were made to the personal income tax. Before 2018 the PIT rate was flat at the level of 23%, subject to the non-taxable minimum and reliefs for dependant persons. Starting from 2018 Latvia introduced a progressive PIT system, where the rate is based on the level of annual income:[19]

- A PIT rate of 20% applies to the annual income up to 20 004 EUR (corresponding to the average monthly income of 1 667 EUR).
- The portion of annual income between 20 004 EUR and 55 000 EUR is subject to a 23% PIT rate.

[19]See the amendments to the law "On Personal Income Tax" of 22.11.2017, https://www.vestnesis.lv/op/2017/242.15 (in Latvian). The annual income corresponds to the gross wage that includes an employee's social security contributions, but excludes an employer's social security contributions.

- Any annual income over 55 000 EUR is subject to a 31.4% PIT rate.

In other words, the annual income below 20 004 EUR saw a reduction of PIT rate by 3 percentage points, while income above 55 000 EUR – a substantial increase in the tax burden by 8.1 percentage points. However, according to the data of CSB, the average gross monthly wage in 2018 was 1 004 EUR, while the median – 774 EUR (corresponding to the annual wage of 12 048 EUR and 9 288 EUR respectively). The threshold of 20 004 EUR falls into the ninth decile of gross wage distribution, while the threshold of 55 000 EUR – to the tenth decile. Therefore, the abovementioned changes were a substantial tax relief for the vast majority of employers and only a few workers with very high wages saw an increase in their overall personal income tax payments.

There were changes in the non-taxable minimum and reliefs for dependant persons introduced in addition to the progressive PIT rate.[20] Although these changes were substantial and affected the income distribution and macroeconomic situation, we limit our simulation to the changes in the PIT rate only to isolate the effect of the introduced progressivity.

### 8.2 General overview of the results

Changes in the PIT rate had a substantial effect on the nominal and real disposable income of households, stimulating real consumption and GDP. This was a direct channel transmitting the fiscal shock to changes in activity. We can quantify the direct effect to some extent by simulating EUROMOD separately from CGE. In particular, the introduction of the progressive PIT rate reduced the PIT revenues by 10.5% directly, increasing the budget deficit by approximately 200 million euro or 0.7% to GDP. The nominal disposable income of households went up by 2.0% as a result of the fiscal stimulus in the very short run.

The separate EUROMOD simulation lacks the indirect effects, which can be accounted for by our newly developed CGE-EUROMOD system. There are several channels that should be mentioned. First, the increase in nominal (and real) disposable income stimulated the real consumption, boosted the aggregate activity and improved budget revenues (primarily from the VAT). Another important channel accounted for by the endogenised informal wage section was the reduction of the informal

[20] In particular, the monthly relief for dependant persons was increased from 175 EUR to 200 EUR. As to the non-taxable minimum, it depends on the level of gross wage. In 2017, the highest monthly non-taxable minimum equalled 115 EUR, applied to the monthly gross wage below 400 EUR. The lowest monthly non-taxable minimum equalled 5 EUR, applied to the monthly gross wage above 1 100 EUR. In 2018, the highest monthly non-taxable minimum increased to 200 EUR, applied to the wage below 440 EUR. Employers with the monthly wage above 1 000 EUR had no non-taxable minimum in 2018.

economy, stemming both from the higher economic activity and the lower labour tax burden. The third important channel included the increasing price level that reduced the external competitiveness and dampened the real exports (and overall activity).

Table 1: Changes in selected macroeconomic, fiscal and income distribution variables
(Deviation of the actual situation from the counterfactual scenario with flat 23% PIT rate; %)

| Variable | The effect of progressive PIT tax (2018) |
|---|---|
| Macroeconomic variables | |
| Real GDP | 0.71 |
| Real private consumption | 2.10 |
| Real investments | 0.73 |
| Real exports | -0.31 |
| Real imports | 0.71 |
| Real disposable income | 2.37 |
| Nominal disposable income | 2.63 |
| GDP deflator | 0.36 |
| Consumption deflator | 0.25 |
| Export deflator | 0.18 |
| Import deflator | 0.00 |
| Gross real wage | 0.23 |
| Gross nominal wage | 0.48 |
| Gross nominal wage (high-skilled) | 0.47 |
| Gross nominal wage (medium-skilled) | 0.51 |
| Gross nominal wage (low-skilled) | 0.48 |
| Total employment | 0.27 |
| Unemployment (percentage points) | -0.18 |
| Fiscal variables | |
| Budget balance (% to GDP) | -0.26 |
| Budget expenditures | -0.01 |
| Social expenditures | -0.19 |
| Unemployment benefits | -3.46 |
| Budget revenues | -0.78 |
| Personal income tax revenues | -9.34 |
| Social security contribution revenues | 0.80 |
| Value added tax revenues | 2.43 |
| Excise tax revenues | 1.22 |
| Income distribution | |
| GINI (equalised disposable income) | 0.63 |
| S80/20 ratio | 1.61 |
| At risk of poverty (percentage points) | 0.84 |

Notes: Own calculations.

All in all, the real economic activity increased by 0.71%. The magnitude of the indirect effects vis-a-vis the direct impact of the PIT rate reduction due to progressivity can be highlighted by the increase in nominal disposable income in the CGE-EUROMOD system (by 2.63%, see Table 1) compared with the increase of the same variable in the EUROMOD simulation (2.0%). Another way to observe the role of the indirect effects is to look at fiscal sector variables: the revenue from all major taxes (except the PIT) increased in the range of 0.8–2.4%, while the overall reduction of the budget balance was approximately 2.5 times lower compared with the direct effect (−0.26% to GDP). More details on the simulation results follow in the subsequent sections.

### 8.3 Macroeconomic effects

The overall effect of progressive PIT rate on the main macroeconomic indicators can be found in Table 1. Note that the table reflects the simulated effect of the reform, so the actual outcome is compared to the counterfactual scenario when the progressive tax rate was not introduced and the rate remained flat at 23%.

As mentioned above, the majority of employees experienced a substantial decrease in the effective personal income tax rate from 23% to 20%, which positively affected real disposable income (2.37%) and private consumption (2.10%). The fiscal stimulus to the private consumption affected the overall activity, so real GDP increased by 0.71% also driving up real investments and imports. Higher economic activity put pressure on the labour market by higher labour demand, boosting employment (0.27%) and increasing gross nominal wage in all skill groups (by 0.48% on average). Growing labour costs push up prices for domestic products causing higher consumer inflation (by 0.25 percentage points) and loss in price competitiveness due to higher export deflator. Given the unchanged foreign demand, this materialized in lower real exports (by 0.31%).

### 8.4 Income distribution effects

The income distribution changes after the introduction of the progressive PIT rate can be observed in Table 2. It shows the effect on nominal disposable income and consumption by disposable income quintiles.[21] In addition, it also reports the original change in the nominal disposable income by quintile produced by EUROMOD. Since the introduction of progressive taxation was initially modelled with EUROMOD, the last column of Table 2 reports the direct effect of the reform that does not account for any indirect effects coming through changes in the macroeconomic environment. By comparing the changes of nominal disposable income reported by the joint CGE-EUROMOD system with direct effects reported by EUROMOD we can see the magnitude of indirect effects following the initial fiscal shock.

[21] Note that we use equalised disposable income quintiles, when the average disposable income per household member is adjusted for the composition of the household. Also, the numbers in Table 2 refer to the changes of income and consumption for households that actually belonged to the respective quintile in 2018, so it does not include the structural effect of households moving from one income quintile to another.

Table 2: Changes in nominal income and consumption by household disposable income quintile (Deviation of the actual situation from the counterfactual scenario witha flat 23% PIT rate; %)

| Equalised disposable income quintiles | Nominal consumption | Nominal disposable income | Direct EUROMOD shock: nominal disposable income |
|---|---|---|---|
| First quintile | 0.51 | 0.58 | 0.46 |
| Second quintile | 1.31 | 1.39 | 1.35 |
| Third quintile | 2.33 | 2.45 | 2.07 |
| Fourth quintile | 3.00 | 3.14 | 2.57 |
| Fifth quintile | 2.65 | 2.75 | 2.06 |

Notes: Own calculations.

The last column reveals which households directly gained most in terms of the after-tax income: the largest direct increase in nominal disposable income was observed for the fourth quintile (around 2.5%), followed by the third and fifth quintile (around 2%). The fourth quintile contains a lot of households whose employed members have monthly gross wage close to the 1 667 EUR threshold, so the introduction of the progressive tax effectively means reduction of the PIT rate by three percentage points for them. In addition, these employees typically have no non-taxable minimum that maximizes their marginal gains from the reform. The fifth quintile contains employees with a higher gross wage whose gains were lower or tax payments even higher (in case of a very high gross wage). As to the third quintile, the direct disposable income gains are lower due to the positive non-taxable minimum for lower gross wages (or old-age pensions). The lowest direct effect was observed for the poorest quintile containing households with a low share of employees and pensioners. These households contain a substantial number of unemployed adults or children, therefore any changes in the PIT rate do not affect their disposable income.

The response of the nominal disposable income obtained from the joint CGE-EUROMOD system demonstrates that the indirect impact of PIT rate changes account for about a third of the total effect. As described above, it mainly comes from the growth in overall activity that follows higher private consumption, stimulating employment and wages. This growth in economic activity provides less benefits to the first quintile due to low numbers of employed persons. Higher disposable income goes hand in hand with a larger nominal consumption, which grows the most for the fourth quintile (3.14%) and the least – for the first quintile (0.58%).

The last three rows of Table 1 contain the response of various income inequality and poverty measures to the progressive PIT rate (obtained from the EUROMOD part of the CGE-EUROMOD system). It flags the growing inequality and poverty as a result of the changes in the PIT rate. As Table 2 shows, the largest gains (both direct and overall) were obtained by the fourth quintile, but the poorest groups remained almost unaffected. Also, the threshold for the 31.4% tax was too high, so the majority of the households belonging to the richest group did not see the increase in tax payments. All these factors resulted in an increased inequality. As to the increase in being at the risk of poverty – this comes from the increasing median income that outpaced the growth of income for the poorest group. In general, these results signal that reducing poverty and income inequality by a progressive tax is not a straight-forward task if there are no adjustments to the benefit system.

### 8.5 Fiscal effects

Table 1 also contains the response of the main fiscal variables. The adjustments to the PIT rate produce a substantial decline in PIT revenues (by 9.34%, which is lower than the direct decline of 10.5% flagged by EUROMOD), which were partially compensated by the growing economic activity and prices, also materializing in higher revenues from other major taxes (primarily VAT – by 2.43%). The overall decline in budget revenues is simulated to be modest (0.78%).

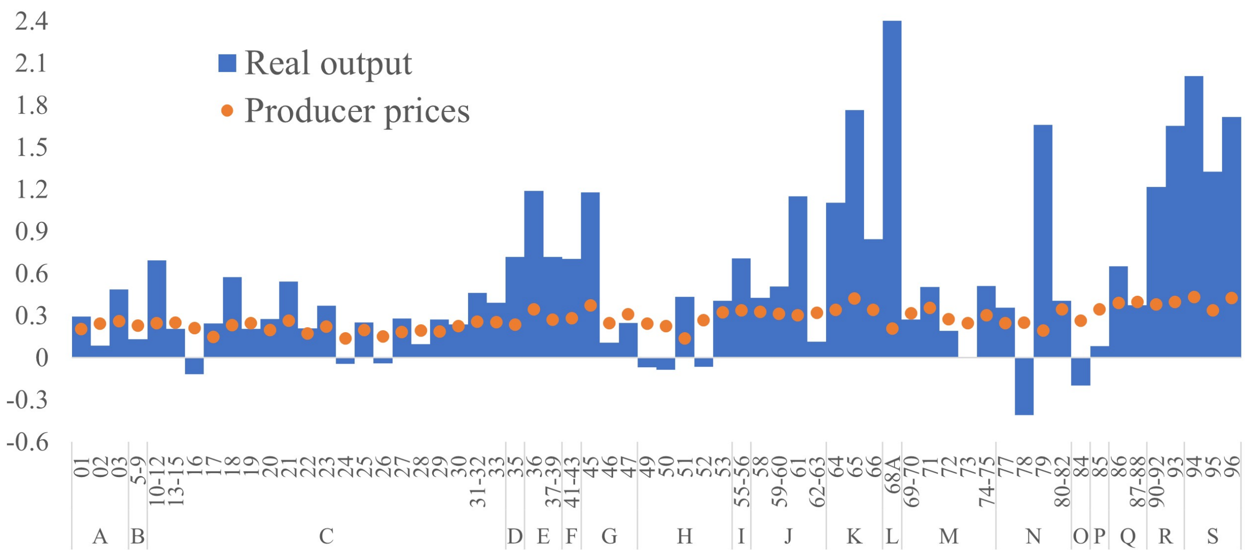
One of the advantages from linking the CGE model with EUROMOD is the availability of various benefits in the latter model. In particular, our simulations now account for the changes in the unemployment benefits (decline by 3.46%) following the higher economic activity and lower unemployment. Changes in other social benefits were found to be negligible. Since we assume the exogenous fiscal policy with no changes in government consumption and investments, the overall decline in budget expenditures was only 0.01%, and the overall budget balance declined by 0.26% to GDP.

### 8.6 Industry effects

Finally, the CGE model allows for going into the sectoral details and uncovering the industries gaining and losing the most from changes in the PIT rate. The reaction of two most important indicators – the real output and producer prices – is reflected in Figure 4.

While the impact on producer prices remains comparatively homogeneous (around 0.35%, with larger price pressure for services due to the high share of labour costs), the response of real output differs substantially. In general, the industries can be split into two groups. The first group includes export-oriented industries that face a decline in price competitiveness that negatively affects their output. These are mostly the manufacturing and transportation industries, while some services providing inputs to export-oriented industries were also hurt (e.g. trade, advertising and market

Figure 4: Changes in real output and producer prices by industry
(Deviation of the actual situation from the counterfactual scenario with a flat 23% PIT rate; %)




Notes: Own calculations. The horizontal axis contains broad (letters) and two-digit NACE Rev.2 industries.

research, rental and leasing activities, employment activities). In some cases the negative effect of price competitiveness losses even outweigh the growing domestic activity. The second group consists of domestically-oriented industries, primarily services (financial services, other services, as well as energy and construction). The impact of fiscal stimulus from introducing the progressive PIT rate is clearly positive for this group of industries.

# 9 Conclusions

This paper provides a technical description of the new version of the Latvian CGE model. This model is based upon the previous version described in Benkovskis et al. (2016) and largely follows the MONASH-style models, in particular a single-country ORANI-G model (Horridge 2014; Dixon et al. 2013).

Several important improvements were made in comparison with the previous version. First, the model is now based on the most recent and more reliable SUT for 2015; the model also has a more disaggregated industry and commodity structure. Second, the labour market block was expanded to account for the substantial changes in labour supply due to the demographic process and migration, as well as to introduce an imperfect mobility of labour between skills and industries. Third, the consumption is modelled by disposable income quinties to account for the distributional effects, while the Klein-Rubin utility function allows non-unity income elasticity of consumption for individual commodities. Fourth, the availability of overlapping generations made it possible to build a block devoted to the Latvian old-age pension system (the first and the second pillar), which is one of the major expenses of the government budget. Finally, this paper also briefly describes the link of the Latvian CGE model with Latvian EUROMOD (more details can be found in Benkovskis et al. 2023). This linkage, as demonstrated in the last section of the paper, allows overcoming several drawbacks of the CGE model, in particular introducing the possibility to analyse the effects on income distribution.

The CGE-EUROMOD system is one of the workhorse models at Latvijas Banka used for various policy simulations therefore the improvement of this model is a never-ending process. One of the directions of future developments is related to the energy sector transformation and green economy in Latvia.

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# A Appendices

## A.1 List of variables and coefficients

Table A1: List of variables

| Variable | Type | Description |
|---|---|---|
| $a1(i)$ | % | Productivity shift in industry $i$ |
| $arate(a,g)$ | Δ | Activity rate for person of age $a$ and gender $g$ |
| $arate\_exog(a,g)$ | Δ | Exogenous shock to the activity rate for person of age $a$ and gender $g$ |
| $atot$ | Δ | Active population |
| $brate(a)$ | Δ | Birth rate for women of age $a$ |
| $drate(a,g)$ | Δ | Death rate for persons of age $a$ and gender $g$ |
| $drate0(a)$ | Δ | Death rate for newborns of gender $g$ |
| $f8q(c)$ | % | Real foreign demand for commodity $c$ |
| $govbb$ | Δ | Government budget balance |
| $govbbcum$ | Δ | Cumulative government budget balance |
| $govbbcum_{-1}$ | Δ | Cumulative government budget balance in the previous period |
| $govbbgdp$ | Δ | Cumulative government budget balance to GDP |
| $govdebt$ | % | Government debt |
| $govrev$ | % | Total government expenditures |
| $govexpcons$ | % | Aggregate government consumption |
| $govexpcons1(c)$ | % | Government consumption (VAT taxable) of commodity $c$ |
| $govexpcons2(c)$ | % | Government consumption (VAT exempt) of commodity $c$ |
| $govexpint$ | % | Government interest payments |
| $govexpinv$ | % | Aggregate government investments |
| $govexpoth$ | % | Other government expenditures |
| $govexpsocill$ | % | Government social expenditures on sickness benefits |
| $govexpsocillADD$ | % | EUROMOD shock to the government social expenditures on sickness benefits |
| $govexpsocinv$ | % | Government social expenditures on disability pensions |
| $govexpsocinvADD$ | % | EUROMOD shock to the government social expenditures on disability pensions |
| $govexpsocoth$ | % | Other government social expenditures |
| $govexpsocothADD$ | % | EUROMOD shock to other government social expenditures |
| $govexpsocotheur$ | % | Other government social expenditures present in EUROMOD |
| $govexpsocpar$ | % | Government social expenditures on parental benefits |
| $govexpsocparADD$ | % | EUROMOD shock to the government social expenditures on parental benefits |
| $govexpsocpens$ | % | Government social expenditures on age pensions |
| $govexpsocunempl$ | % | Government social expenditures on unemployment benefits |
| $govexpsocunemplADD$ | % | EUROMOD shock to the government social expenditures on unemployment benefits |
| $govirate$ | Δ | Interest rate for government debt |
| $govrev$ | % | Total government revenues |
| $govrevexc$ | % | Total government revenues from excise tax |
| $govrevoth$ | % | Other government revenues |
| $govrevssc$ | % | Total government revenues from social security contributions |
| $govrevsscee$ | % | Total government revenues from employee social security contributions |
| $govrevsscer$ | % | Total government revenues from employer social security contributions |
| $govrevvat$ | % | Total government revenues from VAT |
| $irate$ | Δ | Nominal interest rate |
| $kff(a,g)$ | Δ | FF pension capital at age $a$ for gender $g$ |
| $kff_{-1}(a,g)$ | Δ | FF pension capital at age $a$ for gender $g$ in the previous period |
| $kpayg(a,g)$ | Δ | PAYG pension capital at age $a$ for gender $g$ |
| $kpayg_{-1}(a,g)$ | Δ | PAYG pension capital at age $a$ for gender $g$ in the previous period |
| $lifeexp(a)$ | Δ | Remaining life expectancy at age $a$ |
| $mrate(a,g)$ | Δ | Migration rate for person of age $a$ and gender $g$ |
| $mrate0(a)$ | Δ | Migration rate for newborns of gender $g$ |
| $ncon$ | % | Aggregate nominal private consumption |
| $ncon(q)$ | % | Nominal private consumption of households from $q$th income quintile |
| $nexp$ | % | Nominal exports |
| $ngdp$ | % | Nominal GDP |
| $ngov$ | % | Aggregate nominal government consumption |
| $ngwagehigh(i)$ | % | Nominal gross wage of high-skilled labour in industry $i$ |
| $ngwagehigh$ | % | Nominal average gross wage of high-skilled labour |
| $ngwagehighADD(i)$ | % | EUROMOD shock to gross wage of high-skilled labour in industry $i$ |
| $ngwageleg(i)$ | % | Nominal legal gross wage in industry $i$ |
| $ngwageleg$ | % | Nominal average legal gross wage |

| | | |
|---|---|---|
| $ngwageleg_{-1}$ | % | Nominal average legal gross wage in the previous period |
| $ngwageleg_{-2}$ | % | Nominal average legal gross wage two periods ago |
| $ngwageleg_{-3}$ | % | Nominal average legal gross wage three period ago |
| $ngwagelow(i)$ | % | Nominal gross wage of low-skilled labour in industry $i$ |
| $ngwagelow$ | % | Nominal average gross wage of low-skilled labour |
| $ngwagelowADD(i)$ | % | EUROMOD shock to gross wage of low-skilled labour in industry $i$ |
| $ngwagemed(i)$ | % | Nominal gross wage of medium-skilled labour in industry $i$ |
| $ngwagemed$ | % | Nominal average gross wage of medium-skilled labour |
| $ngwagemedADD(i)$ | % | EUROMOD shock to gross wage of medium-skilled labour in industry $i$ |
| $ngwage$ | % | Nominal average gross wage |
| $ngwage(i)$ | % | Nominal gross wage in industry $i$ |
| $nimp$ | % | Nominal imports |
| $ninv$ | % | Nominal investments |
| $nnwage(i)$ | % | Nominal net wage in industry $i$ |
| $p1cap(i)$ | % | Rental price of capital for industry $i$ |
| $p1lab(i)$ | % | Price of labour for industry $i$ |
| $p1labhigh(i)$ | % | Price of high-skilled labour for industry $i$ |
| $p1lablow(i)$ | % | Price of medium-skilled labour for industry $i$ |
| $p1labmed(i)$ | % | Price of low-skilled labour for industry $i$ |
| $p1prim(i)$ | % | Price of primary factors for industry $i$ |
| $p1tot(i)$ | % | Output price in industry $i$ |
| $p2tot$ | % | Private consumption deflator |
| $p2tot_{-1}$ | % | Private consumption deflator in the previous period |
| $p2tot_{-2}$ | % | Private consumption deflator two periods ago |
| $p5tot$ | % | Unit cost of capital |
| $pbas(c, s)$ | % | Basic price of commodity $c$ from source $s$ |
| $pensage(a, g)$ | $\Delta$ | Proportion of population of age $a$ and gender $g$ subject to pension age |
| $pensff(a, g)$ | $\Delta$ | FF pension for age $a$ and gender $g$ |
| $pensff_{-1}(a, g)$ | $\Delta$ | FF pension for age $a$ and gender $g$ in the previous period |
| $pensfftot$ | % | Total FF pensions |
| $pensind$ | % | Indexation of the age pension |
| $pensind_{-1}$ | % | Indexation of the age pension in the previous period |
| $penspayg(a, g)$ | $\Delta$ | PAYG pension for age $a$ and gender $g$ |
| $penspayg_{-1}(a, g)$ | $\Delta$ | PAYG pension for age $a$ and gender $g$ in the previous period |
| $penspaygtot$ | % | Total PAYG pensions |
| $pensretff$ | % | Return of the FF pension capital |
| $pensretpayg$ | % | Notional return of the PAYG pension capital |
| $penstotADD$ | % | EUROMOD shock to total age pensions |
| $pgdp$ | % | GDP deflator |
| $pop(a, g)$ | % | Number of persons of age $a$ and gender $g$ |
| $pop_{-1}(a, g)$ | % | Number of persons of age $a$ and gender $g$ in the previous period |
| $poptot$ | % | Total population |
| $pop0(g)$ | % | Number of newborns of gender $g$ |
| $pop0_{-1}(g)$ | % | Number of newborns of gender $g$ in the previous period |
| $ppur(c, s)$ | % | Purchaser price of commodity $c$ from source $s$ |
| $ppur2\_s(c)$ | % | Purchaser price of composite commodity $c$ for "Private consumption" |
| $ppur2\_s(c, q)$ | % | Purchaser price of composite commodity $c$ for $q$th income quintile "Private consumption" |
| $ppur3\_s(c)$ | % | Purchaser price of composite commodity $c$ for "Government consumption (VAT taxable)" |
| $pprod(c, s)$ | % | Producer price of commodity $c$ from source $s$ |
| $pprod1\_s(c, i)$ | % | Producer price of composite commodity $c$ for industry $i$ |
| $pprod4\_s(c)$ | % | Producer price of composite commodity $c$ for "Government consumption (VAT exempt)" |
| $pprod5\_s(c)$ | % | Producer price of composite commodity $c$ for "Private non-housing investments" |
| $pprod7\_s(c)$ | % | Producer price of composite commodity $c$ for "Government investments" |
| $rcon$ | % | Aggregate real private consumption |
| $revpens$ | % | Actual payments of to PAYG and FF pension capital |
| $revpens_{-1}$ | % | Actual payments of to PAYG and FF pension capital in the previous period |
| $revpens_{-2}$ | % | Actual payments of to PAYG and FF pension capital two periods ago |
| $revpit(i)$ | % | Actual payments of personal income tax by industry $i$ |
| $revpitADD$ | % | EUROMOD shock to the actual payments of personal income tax by all industries |
| $revsscee(i)$ | % | Actual payments of employee social security contributions by industry $i$ |
| $revssceeADD$ | % | EUROMOD shock to the actual payments of employee social security contributions by all industries |
| $revsscer(i)$ | % | Actual payments of employer social security contributions by industry $i$ |
| $revsscerADD$ | % | EUROMOD shock to the actual payments of employer social security contributions by all industries |
| $rexp$ | % | Real exports |
| $rgdp$ | % | Real GDP |

| | | |
|---|---|---|
| $rgov$ | % | Aggregate real government consumption |
| $rgwage$ | % | Real average gross wage |
| $rimp$ | % | Real imports |
| $rinv$ | % | Real investments |
| $sav$ | % | Aggregate nominal savings of households |
| $sav(q)$ | % | Aggregate nominal savings of households from $q$th income quintile |
| $savrate$ | $\Delta$ | Aggregate saving rate of households |
| $savrate(q)$ | $\Delta$ | Aggregate saving rate of households from $q$th income quintile |
| $taxlabsh(i)$ | $\Delta$ | Share of enterprises in industry $i$ paying labour taxes |
| $taxlabshADD(i)$ | $\Delta$ | EUROMOD shock to the share of enterprises in industry $i$ paying labour taxes |
| $taxlab\_exog(i)$ | $\Delta$ | Exogenous shock to the share of enterprises in industry $i$ paying labour taxes |
| $taxprodsh(c)$ | $\Delta$ | Share of users paying VAT and excise tax for commodity $c$ |
| $taxprod\_exog(c)$ | $\Delta$ | Exogenous shock to the share of users paying VAT and excise tax for commodity $c$ |
| $tff(a)$ | $\Delta$ | Contribution to FF pension capital for age $a$ |
| $tpayg(a)$ | $\Delta$ | Contribution to PAYG pension capital for age $a$ |
| $texc(c)$ | % | Ad valorem equivalent of excise tax for commodity $c$ |
| $tvat(c)$ | $\Delta$ | Effective value added tax rate for commodity $c$ |
| $unairu$ | $\Delta$ | Natural unemployment (NAIRU) |
| $x0tot(c,s)$ | % | Total use of commodity $c$ from source $s$ |
| $x1(c,s,i)$ | % | Use of commodity $c$ from source $s$ by industry $i$ |
| $x1\_s(c,i)$ | % | Use of composite commodity $c$ by industry $i$ |
| $x1cap(i)$ | % | Use of capital by industry $i$ |
| $x1lab(i)$ | % | Use of labour by industry $i$ |
| $x1lab(a,g)$ | % | Number of employees by age $a$ and gender $g$ |
| $x1labhigh(i)$ | % | Use of high-skilled labour by industry $i$ |
| $x1labmed(i)$ | % | Use of medium-skilled labour by industry $i$ |
| $x1lablow(i)$ | % | Use of low-skilled labour by industry $i$ |
| $x1prim(i)$ | % | Use of primary factors by industry $i$ |
| $x1sup(c,i)$ | % | Supply of commodity $c$ by industry $i$ |
| $x1tot(i)$ | % | Total output by industry $i$ |
| $x2(c,s)$ | % | Use of commodity $c$ from source $s$ by "Private consumption" |
| $x2(c,s,q)$ | % | Use of commodity $c$ from source $s$ by $q$th income quintile "Private consumption" |
| $x2\_s(c)$ | % | Use of composite commodity $c$ by "Private consumption" |
| $x2\_s(c,q)$ | % | Use of composite commodity $c$ by $q$th income quintile "Private consumption" |
| $x2lux\_s(c,q)$ | % | Luxury use (consumption) of composite commodity $c$ by $q$th income quintile "Private consumption" |
| $x3(c,s)$ | % | Use of commodity $c$ from source $s$ by "Government consumption (VAT taxable)" |
| $x3\_s(c)$ | % | Use of composite commodity $c$ by "Government consumption (VAT taxable)" |
| $x4(c,s)$ | % | Use of commodity $c$ from source $s$ by "Government consumption (VAT exempt)" |
| $x4\_s(c)$ | % | Use of composite commodity $c$ by "Government consumption (VAT exempt)" |
| $x5(c,s)$ | % | Use of commodity $c$ from source $s$ by "Private non-housing investments" |
| $x5\_exog(c)$ | % | Exogenous shock to the use commodity $c$ from source $s$ by "Private non-housing investments" |
| $x5\_s(c)$ | % | Use of composite commodity $c$ by "Private non-housing investments" |
| $x5tot(i)$ | % | Aggregate real private non-housing investments |
| $x6(c,s)$ | % | Use of commodity $c$ from source $s$ by "Private housing investments" (defined only for domestic source, s="dom") |
| $x6\_s(c)$ | % | Use of composite commodity $c$ by "Private housing investments" |
| $x7(c,s)$ | % | Use of commodity $c$ from source $s$ by "Government investments" |
| $x7\_s(c)$ | % | Use of composite commodity $c$ by "Government investments" |
| $x8(c,s)$ | % | Use of commodity $c$ from source $s$ by "Exports" (for s="imp" $x8(c,s)$ represents re-exports) |
| $x9(c,s)$ | % | Use of commodity $c$ from source $s$ by "Direct purchases abroad" (for s="dom" $x9(c,s)$ corresponds to direct purchases of non-residents in Latvia, for s="imp" $x9(c,s)$ corresponds to direct purchases of residents abroad) |
| $x9(c,s,q)$ | % | Use of commodity $c$ from source $s$ by $q$th income quintile "Direct purchases abroad" (defined only for imported source, s="imp", direct purchases of residents abroad) |
| $xlabgap$ | % | Unemployment gap |
| $xlabgap_{-1}$ | % | Unemployment gap in the previous period |
| $xtotcap$ | % | Aggregate use of capital |
| $xtotcap_{-1}$ | % | Aggregate use of capital in the previous period |
| $xtotinv$ | % | Aggregate real nonhousing investments |
| $xtotlab$ | % | Aggregate use of labour |
| $xtotlabhigh$ | % | Aggregate use of high-skilled labour |
| $xtotlablow$ | % | Aggregate use of low-skilled labour |
| $xtotlabmed$ | % | Aggregate use of medium-skilled labour |
| $xtotnat$ | % | Natural employment |
| $utot$ | % | Number of unemployed |
| $urate$ | $\Delta$ | Unemployment rate |

| | | |
|---|---|---|
| $w2tot$ | % | Aggregate nominal disposable income of households |
| $w2tot(q)$ | % | Nominal disposable income of households from $q$th income quintile |
| $w2totnet(q)$ | % | Nominal disposable income excluding capital income of households from $q$th income quintile |
| $w2totnetADD(q)$ | % | EUROMOD shock to nominal disposable income excluding capital income of households from $q$th income quintile |
| $w2totlux(q)$ | % | Nominal disposable income of households from $q$th income quintile after subsistant consumption |

Table A2: List of coefficients

| Parameter | Description | Update |
|---|---|---|
| $ARATE(a,g)$ | Activity rate for a person of age $a$ and gender $g$ | $ARATE(a,g)$+$arate(a,g)$ |
| $BRATE(a)$ | Birth rate for women of age $a$ | $BRATE(a)$+$brate(a)$ |
| $DELTA(i)$ | Depreciation rate in industry $i$ | |
| $DRATE(a,g)$ | Death rate for a person of age $a$ and gender $g$ | $DRATE(a,g)$+$drate(a,g)$ |
| $DRATE0(g)$ | Death rate for newborns of gender $g$ | $DRATE0(g)$+$drate0(g)$ |
| $EMPHIGH(i)$ | Employment of high-skilled labour in industry $i$ | $EMPHIGH(i)$ *exp($x1labhigh(i)$ |
| $EMPLOW(i)$ | Employment of low-skilled labour in industry $i$ | $EMPLOW(i)$ *exp($x1lablow(i)$ |
| $EMPMED(i)$ | Employment of medium-skilled labour in industry $i$ | $EMPMED(i)$ *exp($x1labmed(i)$ |
| $EPSILON(c)$ | Income elasticity of commodity $c$ consumption | |
| $FRISCH(q)$ | Frisch coefficient for $q$th income quintile | $FRISCH(q)$ *exp($w2tot(q)$-$w2totlux(q)$) |
| $GAMMA$ | Adjustment rate of real wage | - |
| $GOVDEBT$ | Government debt | $GOVDEBT$+$govdebt$ |
| $GOVEXPINT$ | Government interest payments | $GOVEXPINT$*exp($govexpint$) |
| $GOVEXPOTH$ | Other government expenditures | $GOVEXPOTH$*exp($govexpoth$) |
| $GOVEXPSOCPENS$ | Government social expenditures on age pensions | $GOVEXPSOCPENS$* exp($govexpsocpens$) |
| $GOVEXPSOCUNEMPL$ | Government social expenditures on unemployment benefits | $GOVEXPSOCUNEMPL$* exp($govexpsocunempl$) |
| $GOVEXPSOCINV$ | Government social expenditures on disability pensions | $GOVEXPSOCINV$* exp($govexpsocinv$) |
| $GOVEXPSOCILL$ | Government social expenditures on sickness benefits | $GOVEXPSOCILL$* exp($govexpsocill$) |
| $GOVEXPSOCPAR$ | Government social expenditures on parental benefits | $GOVEXPSOCPAR$* exp($govexpsocpar$) |
| $GOVEXPSOCOTH$ | Government social expenditures on other benefits | $GOVEXPSOCOTH$* exp($govexpsocpoth$) |
| $GOVEXPSOCOTHEUR$ | Government social expenditures on other benefits present in EUROMOD | $GOVEXPSOCOTHEUR$* exp($govexpsocotheur$) |
| $GOVREVOTH$ | Other government revenues | $GOVREVOTH$* exp($ngdp$) |
| $KAPPA$ | Share of capital income retained domestically | - |
| $KFF_{-1}(a,g)$ | FF pension capital at age $a$ for gender $g$ in the previous period | $KFF_{-1}(a,g)$+$kff_{-1}(a,g)$ |
| $KPAYG_{-1}(a,g)$ | PAYG pension capital at age $a$ for gender $g$ in the previous period | $KPAYG_{-1}(a,g)$+$kpayg_{-1}(a,g)$ |
| $LIFEEXP(a)$ | Remaining life expectancy at age $a$ | $LIFEEXP(a)$+$lifeexp(a)$ |
| $MRATE(a,g)$ | Migration rate for a person of age $a$ and gender $g$ | $MRATE(a,g)$+$mrate(a,g)$ |
| $MRATE0(g)$ | Migration rate for newborns of gender $g$ | $MRATE0(g)$+$mrate0(g)$ |
| $PENSAGE(a,g)$ | Proportion of population of age $a$ and gender $g$ subject to pension age | $PENSAGE(a,g)$+$pensage(a,g)$ |
| $PENSFF(a,g)$ | FF pension for age $a$ and gender $g$ | $PENSFF(a,g)$+$pensff(a,g)$ |
| $PENSFF_{-1}(a,g)$ | FF pension for age $a$ and gender in the previous period $g$ | $PENSFF_{-1}(a,g)$+$pensff_{-1}(a,g)$ |
| $PENSIND$ | Indexation of pensions | $PENSIND$* exp($pensid$) |
| $PENSIND_{-1}$ | Indexation of pensions in the previous period | $PENSIND_{-1}$* exp($pensid_{-1}$) |
| $PENSINDALL$ | Indexation of pensions including negative indexation | 1+ 7/12 $p2tot$ + 5/12 $p2tot_{-1}$ |
| $PENSINDALL_{-1}$ | Indexation of pensions including negative indexation in the previous period | 1+7/12 $p2tot_{-1}$ + 5/12 $p2tot_{-2}$ |
| $PENSPAYG(a,g)$ | PAYG pension for age $a$ and gender $g$ | $PENSPAYG(a,g)$+$penspayg(a,g)$ |
| $PENSPAYG_{-1}(a,g)$ | PAYG pension for age $a$ and gender in the previous period $g$ | $PENSPAYG_{-1}(a,g)$+$penspayg_{-1}(a,g)$ |
| $PENSRETFF$ | Return of the FF pension capital | $PENSRETFF$* exp($pensretff$) |
| $PENSRETPAYG$ | Notional return of the PAYG pension capital | $PENSRETPAYG$* exp($pensretpayg$) |
| $POP(a,g)$ | Number of persons of age $a$ and gender $g$ | $POP(a,g)$*exp($pop(a,g)$) |

| | | |
|---|---|---|
| $POP_{-1}(a,g)$ | Number of persons of age $a$ and gender $g$ in the previous period | $POP_{-1}(a,g)$*exp($pop_{-1}(a,g)$) |
| $POP0(g)$ | Number of newborns of gender $g$ | $POP0(g)$*exp($pop0(g)$) |
| $POP0_{-1}(g)$ | Number of newborns of gender $g$ in the previous period | $POP0_{-1}(g)$*exp($pop0_{-1}(g)$) |
| $R$ | Real interest rate | $R$+$irate$ |
| $SIGMA1(c)$ | Elasticity of substitution of commodity $c$ between domestic and imported source for industries | - |
| $SIGMA1LAB(i)$ | Elasticity of substitution between high-, medium- and low-skilled labour in industry $i$ | - |
| $SIGMA1PRIM(i)$ | Elasticity of substitution between labour and capital in industry $i$ | - |
| $SIGMA1SUP(c)$ | Elasticity of substitution between producers of commodity $c$ | - |
| $SIGMA2(c)$ | Elasticity of substitution of commodity $c$ between domestic and imported source for "Private consumption" | - |
| $SIGMA3(c)$ | Elasticity of substitution of commodity $c$ between domestic and imported source for "Government consumption (both VAT taxable and VAT exempt)" | - |
| $SIGMA5(c)$ | Elasticity of substitution of commodity $c$ between domestic and imported source for "Private non-housing investments" | - |
| $SIGMA7(c)$ | Elasticity of substitution of commodity $c$ between domestic and imported source for "Government investments" | - |
| $SIGMA8(c)$ | Elasticity of substitution of commodity $c$ between domestic and foreign source for "Exports" | - |
| $SIGMALAB$ | Inverse elasticity of labour supply to the relative wage by skill | - |
| $SIGMALABHIGH$ | Inverse elasticity of labour supply of high-skilled labour to the relative wage by industry | - |
| $SIGMALABLOW$ | Inverse elasticity of labour supply of low-skilled labour to the relative wage by industry | - |
| $SIGMALABMED$ | Inverse elasticity of labour supply of medium-skilled labour to the relative wage by industry | - |
| $TAXLABSH(i)$ | Share of enterprises in industry $i$ paying labour taxes | $TAXLABSH(i)$+$taxlabsh(i)$ |
| $TAXPRODSH(c)$ | Share of commodity taxes paying users by commodity $c$ | $TAXPRODSH(i)$+$taxprodsh(i)$ |
| $TEXC(c,s)$ | Ad valorem equivalent of the excise tax rate on the use of commodity $c$ from source $s$ | $TEXC(c,s)$*exp($texc(c,s)$-$pbase(c,s)$) |
| $TFF(a)$ | Contribution to the FF pension capital for age $a$ | $TFF(a)$+$tff(a)$ |
| $TPAYG(a)$ | Contribution to the PAYG pension capital for age $a$ | $TPAYG(a)$+$tpayg(a)$ |
| $TPROD(c)$ | Rate of other product taxes and subsidies on commodity $c$ | - |
| $TVAT(c)$ | Value added tax rate by commodity $c$ | $TVAT(c)$+$tvat(c)$ |
| $UNAIRU$ | Natural unemployment (NAIRU) | $UNAIRU$+$unairu$ |
| $V1BAS(c,s,i)$ | Use of commodity $c$ from source $s$ by industry $i$; value at basic prices | $V1BAS(c,s,i)$* exp($x1(c,s,i)$+$pbas(c,s)$) |
| $V1CAP(i)$ | Capital costs in industry $i$ | $V1CAP(i)$* exp($x1cap(i)$+$p1cap(i)$ |
| $V1K$ | Aggregate use of capital | $V1K$* exp($xtotcap$ |
| $V1LABHIGH(i)$ | Compensation of high-skilled employees in industry $i$ | $V1LABHIGH(i)$* exp($x1labhigh(i)$+$p1labhigh(i)$ |
| $V1LABLOW(i)$ | Compensation of low-skilled employees in industry $i$ | $V1LABLOW(i)$* exp($x1lablow(i)$+$p1lablow(i)$ |
| $V1LABMED(i)$ | Compensation of medium-skilled employees in industry $i$ | $V1LABMED(i)$* exp($x1labmed(i)$+$p1labmed(i)$ |
| $V1NNWAGE(i)$ | Total nominal net wage payments by industry $i$ | $V1NNWAGE(i)$* exp($x1lab(i)$+$nnwage(i)$ |
| $V1PIT(i)$ | Total actual payments of personal income tax by industry $i$ | $V1PIT(i)$* exp($revpit(i)$) |
| $V1SSCEE(i)$ | Total actual payments of employee social security contributions by industry $i$ | $V1SSCEE(i)$* exp($revsscee(i)$) |
| $V1SSCER(i)$ | Total actual payments of employer social security contributions by industry $i$ | $V1SSCER(i)$* exp($revsscer(i)$) |
| $V1SUP(c,i)$ | Supply of commodity $c$ by industry $i$; value at basic prices | $V1SUP(c,i)$* exp($x1(c,i)$+$pbas(c,\text{“}dom''$) |
| $V2BAS(c,s)$ | Use of commodity $c$ from source $s$ by "Private consumption"; value at basic prices | $V2BAS(c,s)$* exp($x2(c,s)$+$pbas(c,s)$ |
| $V2BAS(c,s,q)$ | Use of commodity $c$ from source $s$ by $q$th income quintile "Private consumption"; value at basic prices | $V2BAS(c,s,q)$* exp($x2(c,s,q)$+$pbas(c,s)$ |

| | | |
|---|---|---|
| $V3BAS(c,s)$ | Use of commodity $c$ from source $s$ by "Government consumption (VAT taxable)"; value at basic prices | $V3BAS(c,s)$* $\exp(x3(c,s)$+$pbas(c,s)$ |
| $V4BAS(c,s)$ | Use of commodity $c$ from source $s$ by ""Government consumption (VAT exempt)"; value at basic prices | $V4BAS(c,s)$* $\exp(x4(c,s)$+$pbas(c,s)$ |
| $V5BAS(c,s)$ | Use of commodity $c$ from source $s$ by "Private non-housing investments"; value at basic prices | $V5BAS(c,s)$* $\exp(x5(c,s)$+$pbas(c,s)$ |
| $V6BAS(c,s)$ | Use of commodity $c$ from source $s$ by "Private housing investments" (defined only for domestic source, s="dom"); value at basic prices | $V6BAS(c,s)$* $\exp(x6(c,s)$+$pbas(c,s)$ |
| $V7BAS(c,s)$ | Use of commodity $c$ from source $s$ by "Government investments"; value at basic prices | $V7BAS(c,s)$* $\exp(x7(c,s)$+$pbas(c,s)$ |
| $V8BAS(c,s)$ | Use of commodity $c$ from source $s$ by "Exports"; value at basic prices | $V8BAS(c,s)$* $\exp(x8(c,s)$+$pbas(c,s)$ |
| $V9BAS(c,s)$ | Use of commodity $c$ from source $s$ by "Direct purchases abroad"; value at basic prices | $V9BAS(c,s)$* $\exp(x9(c,s)$+$pbas(c,s)$ |
| $V9BAS(c,s,q)$ | Use of commodity $c$ from source $s$ by $q$th income quintile "Direct purchases abroad" (defined only for imported source, s="imp"); value at basic prices | $V9BAS(c,s,q)$* $\exp(x9(c,s,q)$+$pbas(c,s)$ |
| $YCRATIO(q)$ | Ratio of disposable income to consumption for households from $q$th income quintile | $YCRATIO(q)$* $\exp(w2tot(q)$-$ncon(q))$ |
| $W2SHNET(q)$ | The share of $q$th income quintile's disposable income in total households' disposable income (net of capital income) | $W2SHNET(q)$* $\exp(w2tot(q))$ |

### A.2 The list of equations

The following section defines all of the equations present in the model.

#### A.2.1 Demographics

**Birth**

$$\forall g \in GDR,$$
$$POP0(g) \cdot pop0(g) = (1 - DRATE0(g)) \cdot (1 + MRATE0(g)) \cdot \tag{A1}$$
$$\cdot \sum_{a \in AGE} \left( \frac{POP0(g)}{\sum_{g \in GDR} POP0(g)} \cdot POP(a, "w") \cdot (BRATE(a) \cdot pop(a, "w") + brate(a)) \right) -$$
$$- (1 + MRATE0(g)) \cdot \sum_{a \in AGE} \left( \frac{POP0(g)}{\sum_{g \in GDR} POP0(g)} \cdot POP(a, "w") \cdot BRATE(a) \right) \cdot drate0(g) +$$
$$- (1 - DRATE0(g)) \cdot \sum_{a \in AGE} \left( \frac{POP0(g)}{\sum_{g \in GDR} POP0(g)} \cdot POP(a, "w") \cdot BRATE(a) \right) \cdot mrate0(g)$$

**Aging and death**

$$\forall g \in GDR, \forall a \in AGE \setminus \{1\},$$
$$POP(a,g) \cdot pop(a,g) = (1 - DRATE(a,g)) \cdot (1 + MRATE(a,g)) \cdot \tag{A2}$$
$$\cdot POP_{-1}(a-1,g) \cdot pop_{-1}(a-1,g) - (1 + MRATE(a,g)) \, POP(a,g) \cdot drate(a,g) +$$
$$+ (1 - DRATE(a,g)) \, POP(a,g) \cdot mrate(a,g)$$

$$\forall g \in GDR, a \in \{1\},$$
$$POP(a,g) \cdot pop(a,g) = (1 - DRATE0(g)) \cdot (1 + MRATE0(g)) \cdot \tag{A3}$$
$$\cdot POP0_{-1}(g) \cdot pop0_{-1}(g) - (1 + MRATE(a,g)) \, POP(a,g) \cdot drate(a,g) +$$
$$+ (1 - DRATE(a,g)) \, POP(a,g) \cdot mrate(a,g)$$

**Total population**

$$POPTOT \cdot poptot = \sum_{g \in GDR} \left( \sum_{a \in AGE} (POP(a,g) \cdot pop(a,g)) + POP0(g) \cdot pop0(g) \right) \tag{A4}$$

where

$$POPTOT = \sum_{g \in GDR} \left( \sum_{a \in AGE} POP(a,g) + POP0(g) \right)$$

### A.2.2 Total demand for commodities

$$\forall c \in COM, \forall s \in SRC,$$
$$V0BAS(c,s) \cdot x0tot(c,s) = \sum_{i \in IND} (V1BAS(c,s,i) \cdot x1(c,s,i)) + \tag{A5}$$
$$+ V2BAS(c,s) \cdot x2(c,s) + V3BAS(c,s) \cdot x3(c,s) + V4BAS(c,s) \cdot x4(c,s) +$$
$$+ V5BAS(c,s) \cdot x5(c,s) + V6BAS(c,s) \cdot x6(c,s) + V7BAS(c,s) \cdot x7(c,s) +$$
$$+ V8BAS(c,s) \cdot x8(c,s) + V9BAS(c,s) \cdot x9(c,s)$$

where

$$V0BAS(c,s) = \sum_{i \in IND} V1BAS(c,s,i) + V2BAS(c,s) + V3BAS(c,s) + V4BAS(c,s) +$$
$$+ V5BAS(c,s) + V6BAS(c,s) + V7BAS(c,s) + V8BAS(c,s) + V9BAS(c,s)$$

### A.2.3 Substitution between imported and domestic commodities, substitution between primary factors

**Substitution between domestic and imported commodities**

$$\forall c \in COM, \forall s \in SRC, \forall i \in IND,$$
$$x1(c,s,i) = x1\_s(c,i) - SIGMA1(c) \cdot (pprod(c,s) - pprod1\_s(c,i)) \tag{A6}$$

$$\forall c \in COM, \forall s \in SRC, \forall q \in QNT,$$
$$x2(c,s,q) = x2\_s(c,q) - SIGMA2(c) \cdot (ppur(c,s) - ppur2\_s(c)) \tag{A7}$$
$$x9(c,"imp",q) = x2\_s(c,q) - SIGMA2(c) \cdot (pbas(c,"imp") - ppur(c,"imp")) \tag{A8}$$

$$\forall c \in COM, \forall s \in SRC,$$
$$x3(c,s) = x3\_s(c) - SIGMA3(c) \cdot (ppur(c,s) - ppur3\_s(c)) \tag{A9}$$
$$x4(c,s) = x4\_s(c) - SIGMA3(c) \cdot (pprod(c,s) - pprod4\_s(c)) \tag{A10}$$
$$x5(c,s) = x5\_s(c) - SIGMA5(c) \cdot (pprod(c,s) - pprod5\_s(c)) \tag{A11}$$
$$x7(c,s) = x7\_s(c) - SIGMA7(c) \cdot (pprod(c,s) - pprod7\_s(c)) \tag{A12}$$

**Substitution between primary production factors**

$$\forall i \in IND,$$
$$x1cap(i) = x1prim(i) - SIGMA1PRIM(i) \cdot (p1cap(i) - p1prim(i)) \tag{A13}$$
$$x1lab(i) = x1prim(i) - SIGMA1PRIM(i) \cdot (p1lab(i) - p1prim(i)) \tag{A14}$$
$$x1labhigh(i) = x1lab(i) - SIGMA1LAB(i) \cdot (p1labhigh(i) - p1lab(i)) \tag{A15}$$
$$x1labmed(i) = x1lab(i) - SIGMA1LAB(i) \cdot (p1labmed(i) - p1lab(i)) \tag{A16}$$

### A.2.4 Structure of production

**Demand for intermediate goods**

$$\forall c \in COM, \forall i \in IND,$$
$$x1\_s(c,i) = x1tot(i) - a1(i) \tag{A17}$$

**Demand for primary-factor composites**

$$\forall i \in IND,$$
$$x1prim(i) = x1tot(i) - a1(i) \tag{A18}$$
$$EMP(i) \cdot x1lab(i) = EMPHIGH(i) \cdot x1labhigh(i) + EMPMED(i) \cdot x1labmed(i) + \tag{A19}$$
$$+ EMPLOW(i) \cdot x1lablow(i)$$

where

$$EMP(i) = EMPHIGH(i) + EMPMED(i) + EMPLOW(i)$$

**Total activity by industry**

$$\forall i \in IND,$$
$$\left(\sum_{c \in COM} V1SUP(c,i)\right) \cdot x1tot(i) = \sum_{c \in COM} (V1SUP(c,i) \cdot x1sup(c,i)) \tag{A20}$$

**Substitution between domestic producers of the same commodity**

$$\forall c \in COM, \forall i \in IND,$$
$$x1sup(c,i) = x0tot(c,"dom") - SIGMA1SUP(c) \cdot (p1tot(i) - pbas(c,"dom")). \tag{A21}$$

### A.2.5 Prices of primary-factors composite and commodity composite

$$\forall i \in IND,$$
$$V1PRIM(i) \cdot p1prim(i) = V1LAB(i) \cdot p1lab(i) + V1CAP(i) \cdot p1cap(i) \tag{A22}$$
$$V1LAB(i) \cdot p1prim(i) = V1LABHIGH(i) \cdot p1labhigh(i) + V1LABMED(i) \cdot p1labmed(i) + \tag{A23}$$
$$+ V1LABLOW(i) \cdot p1lablow(i)$$

$$\forall c \in COM, \forall i \in IND,$$
$$\left(\sum_{s \in SRC} V1PROD(c,s,i)\right) \cdot pprod1\_s(c,i) = \sum_{s \in SRC} (V1PROD(c,s,i) \cdot pprod(c,s)) \tag{A24}$$

$\forall c \in COM, \forall q \in QNT,$

$$\left(\sum_{s \in SRC} V2PUR(c,s,q) + V9BAS(c,\text{"}imp\text{"},q)\right) \cdot ppur2\_s(c,q) = \tag{A25}$$
$$= \sum_{s \in SRC} \left(V2PUR(c,s,q) \cdot ppur(c,s)\right) + V9BAS(c,\text{"}imp\text{"},q) \cdot pbas(c,\text{"}imp\text{"})$$

$\forall c \in COM,$

$$\left(\sum_{s \in SRC} V2PUR(c,s) + V9BAS(c,\text{"}imp\text{"})\right) \cdot ppur2\_s(c) = \tag{A26}$$
$$= \sum_{s \in SRC} \left(V2PUR(c,s) \cdot ppur(c,s)\right) + V9BAS(c,\text{"}imp\text{"}) \cdot pbas(c,\text{"}imp\text{"})$$

$$\left(\sum_{s \in SRC} V3PUR(c,s)\right) \cdot ppur3\_s(c) = \sum_{s \in SRC} \left(V3PUR(c,s) \cdot ppur(c,s)\right) \tag{A27}$$

$$\left(\sum_{s \in SRC} V4PROD(c,s)\right) \cdot pprod4\_s(c) = \sum_{s \in SRC} \left(V4PROD(c,s) \cdot pprod(c,s)\right) \tag{A28}$$

$$\left(\sum_{s \in SRC} V5PROD(c,s)\right) \cdot pprod5\_s(c) = \sum_{s \in SRC} \left(V5PROD(c,s) \cdot pprod(c,s)\right) \tag{A29}$$

$$\left(\sum_{s \in SRC} V7PROD(c,s)\right) \cdot pprod7\_s(c) = \sum_{s \in SRC} \left(V7PROD(c,s) \cdot pprod(c,s)\right) \tag{A30}$$

where

$$V1LAB(i) = V1LABHIGH(i) + V1LABMED(i) + V1LABLOW(i)$$
$$V1PRIM(i) = V1LAB(i) + V1CAP(i)$$
$$V1PROD(c,s,i) = V1BAS(c,s,i) \cdot (1 + TPROD(c) + TEXC(c,s) \cdot TAXPRODSH(c))$$
$$V2PUR(c,s,q) = V2BAS(c,s,q) \cdot (1 + TPROD(c) + TEXC(c,s) \cdot TAXPRODSH(c)) \cdot$$
$$\cdot (1 + TVAT(c) \cdot TAXPRODSH(c))$$
$$V2PUR(c,s) = \sum_{q \in QNT} V2PUR(c,s,q)$$
$$V3PUR(c,s) = V3BAS(c,s) \cdot (1 + TPROD(c) + TEXC(c,s) \cdot TAXPRODSH(c)) \cdot$$
$$\cdot (1 + TVAT(c) \cdot TAXPRODSH(c))$$
$$V4PROD(c,s) = V4BAS(c,s) \cdot (1 + TPROD(c) + TEXC(c,s) \cdot TAXPRODSH(c))$$
$$V5PROD(c,s) = V5BAS(c,s) \cdot (1 + TPROD(c) + TEXC(c,s) \cdot TAXPRODSH(c))$$
$$V7PROD(c,s) = V7BAS(c,s) \cdot (1 + TPROD(c) + TEXC(c,s) \cdot TAXPRODSH(c))$$

### A.2.6 Basic producer's and purchaser's prices

**Industry unit costs**

$$\forall i \in IND,$$
$$V1TOT(i) \cdot (p1tot(i) + x1tot(i)) = V1LAB(i) \cdot (p1lab(i) + x1lab(i)) +$$
$$+ V1CAP(i) \cdot (p1cap(i) + x1cap(i)) +$$
$$+ \sum_{c \in COM} \sum_{s \in SRC} (V1PROD(c,s,i) \cdot (pprod(c,s) + x1(c,s,i))) \tag{A31}$$

where

$$V1TOT(i) = V1LAB(i) + V1CAP(i) + \sum_{c \in COM} \sum_{s \in SRC} V1PROD(c,s,i)$$

**Basic prices for commodities**

$$\forall c \in COM,$$
$$\left( \sum_{i \in IND} V1SUP(c,i) \right) \cdot pbas(c, "dom") = \sum_{i \in IND} (V1SUP(c,i) \cdot p1tot(i)) \tag{A32}$$

**Producer's prices for commodities**

$$\forall c \in COM, \forall s \in SRC$$
$$C0PROD(c,s) \cdot pprod(c,s) = C0PROD(c,s) \cdot pbas(c,s) + TEXC(c,s) \cdot taxprodsh(c)$$
$$+ TEXC(c,s) \cdot TAXPRODSH(c) \cdot texc(c) \tag{A33}$$

where

$$C0PROD(c,s) = (1 + TPROD(c) + TEXC(c,s) \cdot TAXPRODSH(c))$$

**Purchaser's prices**

$$\forall c \in COM, \forall s \in SRC$$
$$C0PUR(c) \cdot ppur(c,s) = C0PUR(c) \cdot pprod(c,s) + TVAT(c) \cdot taxprodsh(c) +$$
$$+ TAXPRODSH(c) \cdot tvat(c) \tag{A34}$$

where

$$C0PUR(c) = 1 + TVAT(c) \cdot TAXPRODSH(c)$$

### A.2.7 Labour market

**Labour costs**

$\forall i \in IND,$

$$V1LAB(i) \cdot (p1lab(i) + x1lab(i)) = V1NNWAGE(i) \cdot (nnwage(i) + x1lab(i)) + \tag{A35}$$
$$+ V1PIT(i) \cdot revpit(i) + V1SSCER(i) \cdot revsscer(i) + V1SSCEE(i) \cdot revsscee(i)$$

$$V1LAB(i) \cdot (p1labhigh(i) + x1labhigh(i)) = \tag{A36}$$
$$= V1NGWAGE(i) \cdot (ngwagehigh(i) + x1labhigh(i)) + V1SSCER(i) \cdot revsscer(i)$$

$$V1LAB(i) \cdot (p1labmed(i) + x1labmed(i)) = \tag{A37}$$
$$= V1NGWAGE(i) \cdot (ngwagemed(i) + x1labmed(i)) + V1SSCER(i) \cdot revsscer(i)$$

$$V1LAB(i) \cdot (p1lablow(i) + x1lablow(i)) = \tag{A38}$$
$$= V1NGWAGE(i) \cdot (ngwagelow(i) + x1lablow(i)) + V1SSCER(i) \cdot revsscer(i)$$

where

$$V1NGWAGE(i) = V1NNWAGE(i) + V1PIT(i) + V1SSCEE(i)$$

**Nominal and real effective gross wage**

$\forall i \in IND, \setminus\{1\}$

$$V1LAB(i) \cdot ngwage(i) = V1LABHIGH(i) \cdot ngwagehigh(i) + \tag{A39}$$
$$+ V1LABMED(i) \cdot ngwagemed(i) + V1LABLOW(i) \cdot ngwagelow(i)$$

$$\left(\sum_{i \in IND} V1NGWAGE(i)\right) \cdot ngwage = \sum_{i \in IND} (V1NGWAGE(i) \cdot ngwage(i)) \tag{A40}$$

$$rgwage = ngwage - p2tot \tag{A41}$$

**Substitution of labour between skills and industries**

$$ngwagehigh = ngwage + SIGMALAB \cdot (xtotlabhigh - xtotlab) \tag{A42}$$

$$ngwagemed = ngwage + SIGMALAB \cdot (xtotlabmed - xtotlab) \tag{A43}$$

$$ngwagelow = ngwage + SIGMALAB \cdot (xtotlablow - xtotlab) \tag{A44}$$

$\forall i \in IND,$

$$ngwagehigh(i) = ngwagehigh + SIGMALABHIGH \cdot (x1labhigh(i) - xtotlabhigh) + \tag{A45}$$
$$+ ngwagehighADD(i)$$

$$ngwagemed(i) = ngwagemed + SIGMALABMED \cdot (x1labmed(i) - xtotlabmed) + \tag{A46}$$
$$+ ngwagemedADD(i)$$

$$ngwagelow(i) = ngwagelow + SIGMALABLOW \cdot (x1lablow(i) - xtotlablow) + \tag{A47}$$
$$+ ngwagelowADD(i)$$

**Total employment**

$$\left(\sum_{i\in IND} EMPHIGH(i)\right)\cdot xtotlabhigh = \sum_{i\in IND}(EMPHIGH(i)\cdot x1labhigh(i)) \quad \text{(A48)}$$

$$\left(\sum_{i\in IND} EMPMED(i)\right)\cdot xtotlabmed = \sum_{i\in IND}(EMPMED(i)\cdot x1labmed(i)) \quad \text{(A49)}$$

$$\left(\sum_{i\in IND} EMPLOW(i)\right)\cdot xtotlablow = \sum_{i\in IND}(EMPLOW(i)\cdot x1lablow(i)) \quad \text{(A50)}$$

$$\left(\sum_{i\in IND} EMP(i)\right)\cdot xtotlab = \sum_{i\in IND}(EMP(i)\cdot x1lab(i)) \quad \text{(A51)}$$

**Activity rate and natural unemployment**

$\forall g\in GDR, a\in\{15:74\}$,

$$arate(a,"m") = 0.230980\cdot xlabgap - 0.491142\cdot pensage(a,"m")+ \quad \text{(A52)}$$

$$+0.752628\cdot mrate(a,"m") + arate\_exog(a,"m")$$

$$arate(a,"w") = 0.180456\cdot xlabgap - 0.329546\cdot pensage(a,"w") + arate\_exog(a,"w") \quad \text{(A53)}$$

$$ATOT\cdot atot = \sum_{g\in GDR}\sum_{a\in AGE}(ARATE(a,g)\cdot pop(a,g) + POP(a,g)\cdot arate(a,g)) \quad \text{(A54)}$$

$$ATOT\cdot atot = ATOT\cdot poptot + POPTOT\cdot arate \quad \text{(A55)}$$

$$xtotnat = atot - \frac{1}{1-UNAIRU}\cdot unairu \quad \text{(A56)}$$

where

$$ATOT\cdot atot = \sum_{g\in GDR}\sum_{a\in AGE}(ARATE(a,g)\cdot POP(a,g))$$

**Employment by age and gender**

$\forall g\in GDR, a\in\{15:74\}$,

$$ARATE(a,g)\cdot x1lab(a,g) = arate(a,g) + ARATE(a,g)\cdot(pop(a,g) + xtotlab - atot) \quad \text{(A57)}$$

**Unemployment**

$$(ATOT-EMP)\cdot utot = ATOT\cdot atot - EMP\cdot xtotlab \quad \text{(A58)}$$

$$ATOT\cdot urate = (ATOT-EMP)\cdot utot - (ATOT-EMP)\cdot atot \quad \text{(A59)}$$

where

$$EMP = \sum_{i\in IND}(EMPHIGH(i) + EMPMED(i) + EMPLOW(i))$$

**Dynamic link between the employment gap and real wage**

$$rgwage = GAMMA \cdot xlabgap \tag{A60}$$

$$EMPNAT \cdot xlabgap = EMPNAT \cdot xlabgap_{-1} - EMP \cdot xtotnat + EMP \cdot xtotlab \tag{A61}$$

### A.2.8 Capital costs and aggregate capital

**Capital costs**

$\forall i \in IND$,

$$(R + DELTA(i)) \cdot p1cap = (R + DELTA(i)) \cdot p5tot + irate \tag{A62}$$

$$\left(\sum_{c \in COM} V5PROD\_S(c) + \sum_{c \in COM} V7PROD\_S(c)\right) \cdot p5tot = \tag{A63}$$
$$\sum_{c \in COM} (V5PROD\_S(c) \cdot pprod5\_s(c)) + \sum_{c \in COM} (V7PROD\_S(c) \cdot pprod7\_s(c))$$

where

$$V5PROD\_S(c) = \sum_{s \in SRC} (V5BAS(c,s) \cdot (1 + TPROD(c) + TEXC(c,s) \cdot TAXPRODSH(c)))$$
$$V7PROD\_S(c) = \sum_{s \in SRC} (V7BAS(c,s) \cdot (1 + TPROD(c) + TEXC(c,s) \cdot TAXPRODSH(c)))$$

**Aggregate capital**

$$V1K \cdot xtotcap = V1K \cdot \left(1 - \sum_{i \in IND} \frac{DELTA(i) \cdot V1CAP(i)}{\sum_{i \in IND} V1CAP(i)}\right) \cdot xtotcap_{-1} + \tag{A64}$$
$$+ V1INV \cdot xtotinv$$

where

$$V1INV = \sum_{c \in COM} \sum_{s \in SRC} V5PROD(c,s) + \sum_{c \in COM} \sum_{s \in SRC} V7PROD(c,s)$$

### A.2.9 Private consumption

**Luxury and total consumption by quintile**

$\forall c \in COM, \forall q \in QNT$,

$$x2lux\_s(c,q) + ppur2\_s(c,q) = w2totlux(q) \tag{A65}$$

$$x2\_s(c,q) = (1 - GAMMAC(c,q)) \cdot x2lux\_s(c,q) \tag{A66}$$

where

$$GAMMAC(c,q) = 1 + \frac{EPSILON(c)}{FRISCH(q)}$$

**Disposable income**

$\forall q \in QNT,$

$$w2totlux(q) = -FRISCH(q) \cdot w2tot(q) + \sum_{c \in COM} FRISCH(q) \cdot WGAMMAC(c,q) \cdot ppur2\_s(c,q) \tag{A67}$$

$$W2TOT \cdot w2totnet(q) = W2TOT \cdot w2totnetADD(q) + \tag{A68}$$
$$+ \sum_{i \in IND} (V1NNWAGE(i) \cdot (x1lab(i) + nnwage(i))) + GOVEXPSOC \cdot govexpsoc$$

$\forall q \in QNT \backslash \{5\},$

$$w2tot(q) = w2totnet(q) \tag{A69}$$

$q \in \{5\},$

$$W2TOT(q) \cdot w2tot(q) = \left( W2TOT(q) - KAPPA \cdot \sum_{i \in IND} V1CAP(i) \right) \cdot w2totnet(q) + \tag{A70}$$
$$+ KAPPA \cdot \sum_{i \in IND} (V1CAP(i) \cdot (x1cap(i) + p1cap(i)))$$

$$\left( \sum_{q \in QNT} W2TOT(q) \right) w2tot = \sum_{q \in QNT} (W2TOT(q) \cdot w2tot(q)) \tag{A71}$$

where

$$WGAMMAC(c,q) = \frac{GAMMAC(c)}{YCRATIO(q)} \cdot \frac{\sum_{s \in SRC} V2PUR(c,s,q)}{\sum_{s \in SRC} \sum_{c \in COM} V2PUR(c,s,q)}$$

$$GOVSOCEXP = GOVSOCEXPSPENS + GOVSOCEXPUNEMPL +$$
$$+ GOVSOCEXPINV + GOVSOCEXPILL + GOVSOCEXPPAR +$$
$$+ GOVSOCEXPOTH$$

$$W2TOT(q) = \left( \sum_{i \in IND} V1NNWAGE(i) + GOVTRANS \right) \cdot W2SHNET(q), \qquad \forall q \in QNT \backslash \{1\}$$

$$W2TOT(q) = \left( \sum_{i \in IND} V1NNWAGE(i) + GOVTRANS \right) \cdot W2SHNET(q) +$$
$$+ KAPPA \cdot \sum_{i \in IND} V1CAP(i), \qquad q \in \{5\}$$

$$W2TOT = \sum_{q \in QNT} W2TOT(q)$$

$$KAPPA = 0.7$$

**Consumption aggregates**

$\forall c \in COM, \forall s \in SRC,$

$$\left(\sum_{q\in QNT} V2PUR(c,s,q)\right)\cdot x2(c,s) = \sum_{q\in QNT}\left(V2PUR(c,s,q)\cdot x2(c,s,q)\right) \tag{A72}$$

$\forall c \in COM,$

$$\left(\sum_{q\in QNT} V9BAS(c,\text{"}imp\text{"},q)\right)\cdot x9(c,\text{"}imp\text{"}) = \sum_{q\in QNT}\left(V9BAS(c,\text{"}imp\text{"},q)\cdot x9(c,s,\text{"}imp\text{"})\right) \tag{A73}$$

$$\left(\sum_{q\in QNT}\sum_{s\in SRC} V2PUR(c,s,q) + \sum_{q\in QNT} V9BAS(c,\text{"}imp\text{"},q)\right)\cdot x2\_s(c) = \tag{A74}$$

$$= \sum_{q\in QNT}\left(\left(\sum_{s\in SRC} V2PUR(c,s,q) + V9BAS(c,\text{"}imp\text{"},q)\right)\cdot x2\_s(c,q)\right)$$

$\forall q \in QNT,$

$$NCON(q)\cdot ncon(q) = \sum_{c\in COM}\sum_{s\in SRC}\left(V2PUR(c,s,q)\cdot\left(x2(c,s,q)+ppur2(c,s)\right)\right) + \tag{A75}$$

$$+\sum_{c\in COM}\left(V9BAS(c,\text{"}imp\text{"},q)\cdot\left(x9(c,\text{"}imp\text{"},q)+pbas(c,\text{"}imp\text{"})\right)\right)$$

$$NCON\cdot ncon = \sum_{c\in COM}\sum_{s\in SRC}\left(V2PUR(c,s)\cdot\left(x2(c,s)+ppur2(c)\right)\right) + \tag{A76}$$

$$+\sum_{c\in COM}\left(V9BAS(c,\text{"}imp\text{"})\cdot\left(x9(c,\text{"}imp\text{"})+pbas(c,\text{"}imp\text{"})\right)\right)$$

where

$$NCON(q) = \sum_{c\in COM}\left(\sum_{s\in SRC} V2PUR(c,s,q) + V9BAS(c,\text{"}imp\text{"},q)\right)$$

$$NCON = \sum_{c\in COM}\left(\sum_{s\in SRC} V2PUR(c,s) + V9BAS(c,\text{"}imp\text{"})\right)$$

**Private savings**

$$\left(\sum_{q\in QNT} W2TOT(q) - NCON\right)\cdot sav = \sum_{q\in QNT} W2TOT(q)\cdot w2tot - NCON\cdot ncon \tag{A77}$$

$$W2TOT\cdot savrate = \tag{A78}$$

$$= (W2TOT - NCON)\cdot sav - (W2TOT - NCON)\cdot w2tot$$

$\forall q \in QNT,$

$$(W2TOT(q) - NCON(q)) \cdot sav(q) = W2TOT(q) \cdot w2tot(q) - NCON(q) \cdot ncon(q) \quad \text{(A79)}$$

$$W2TOT(q) \cdot savrate(q) = \quad \text{(A80)}$$

$$= (W2TOT(q) - NCON(q)) \cdot sav(q) - (W2TOT(q) - NCON(q)) \cdot w2tot(q)$$

#### A.2.10 Private housing and non-housing investments

$\forall c \in COM,$

$$x6\_s(c) + ppur(c, "dom") = w2tot \quad \text{(A81)}$$

$$x5\_s(c) = x5tot + x5_e xog(c) \quad \text{(A82)}$$

$$xtotinv = \sum_{i \in IND} \left( \frac{DELTA(i)}{R + DELTA(i)} \cdot x1cap(i) \right) \quad \text{(A83)}$$

$$V1INV \cdot xtotinv = \sum_{c \in COM} (V5PROD\_S(c) + V7PROD\_S(c)) \cdot x5tot + \quad \text{(A84)}$$

$$+ \sum_{c \in COM} (V5PROD\_S(c) \cdot x7\_s(c))$$

#### A.2.11 Exports

$\forall c \in COM,$

$$x8(c, "dom") = f8q(c) - SIGMA8EXP(c) \cdot (pbas(c, "dom") - pbas(c, "imp")) \quad \text{(A85)}$$

$$x8(c, "imp") = f8q(c) \quad \text{(A86)}$$

$$x9(c, "dom") = f8q(c) - SIGMA8EXP(c) \cdot (ppur(c, "dom") - ppur(c, "imp")) \quad \text{(A87)}$$

#### A.2.12 Shadow economy

**Share of enterprises paying labour taxes**

$\forall i \in IND,$

$$taxlabsh(i) = CTLAB1(i) \cdot V1SSCEE(i) \cdot revsscee(i) + \quad \text{(A88)}$$

$$+ CTLAB1(i) \cdot V1SSCER(i) \cdot revsscer(i) + CTLAB1(i) \cdot V1PIT(i) \cdot revpit(i) -$$

$$- CTLAB1(i) \cdot V1NGWAGE(i) \cdot ngwageleg(i) - CTLAB1(i) \cdot V1NGWAGE(i) \cdot x1lab(i) +$$

$$+ CTLAB2(i) \cdot x1tot(i) + CTLAB3(i) \cdot taxlab\_exog(i) + CTLAB3(i) \cdot taxlabshADD(i)$$

$$ngwageleg(i) = ngwage(i) + taxlabsh(i) \quad \text{(A89)}$$

$$\left( \sum_{i \in IND} V1NGWAGELEG(i) \right) ngwageleg = \quad \text{(A90)}$$

$$\sum_{i \in IND} (V1NGWAGELEG(i) \cdot (ngwage(i) + taxlabsh(i)))$$

where

$$CTLAB1(i) = \frac{-ALPHALAB * \left(TAXLABSH(i) - TAXLABSH(i)^2\right)}{V1NGWAGELEG(i)}$$
$$CTLAB2(i) = -BETA * \left(TAXLABSH(i) - TAXLABSH(i)^2\right)$$
$$CTLAB3(i) = -GAMMA * \left(TAXLABSH(i) - TAXLABSH(i)^2\right)$$
$$V1NGWAGELEG(i) = TAXLABSH(i) \cdot V1LAB(i) - V1SSCER(i)$$
$$ALPHALAB = 0.3125$$
$$BETA = -2.75$$
$$GAMMA = -6.25$$

**Share of users paying the VAT and excise tax**

$$\forall c \in COM,$$
$$taxprodsh(c) = CTPROD1(c) \cdot tvat(c) + CTPROD2(c) \cdot texc(c) + CTPROD3(c) \cdot rgdp + \\ + CTPROD4(c) \cdot taxprod\_exog(c) \tag{A91}$$

where

$$CTPROD1(c) = \frac{-ALPHAPROD * \left(TAXPRODSH(c) - TAXPRODSH(c)^2\right)}{TEXC\_S(c) + TVAT(c)}$$
$$CTPROD2(c) = TEXC\_S(c) \cdot CTPROD1(c)$$
$$CTPROD3(c) = -BETA * \left(TAXPRODSH(c) - TAXPRODSH(c)^2\right)$$
$$CTPROD4(c) = -GAMMA * \left(TAXPRODSH(c) - TAXPRODSH(c)^2\right)$$
$$TEXC\_S(c) = \frac{\sum_{s \in SCR} (TEXC(c,s) \cdot V0BAS(c,s)) - TEXC(c,"dom") \cdot V8BAS(c,"dom")}{\sum_{s \in SCR} V0BAS(c,s) - V8BAS(c,"dom")}$$
$$ALPHAPROD = 0.3125$$

### A.2.13 Pension system

**Remaining life expectancy**

$$\forall a \in AGE \backslash \{100\},$$
$$lifeexp(a) = (1 - DRATE(a)) \cdot lifeexp(a+1) - \\ - (1 + LIFEEXP(a)) \cdot \sum_{g \in GDR} (POPSH(a,g) \cdot drate(a,g) + CEXP(a,g) \cdot pop(a,g)) \tag{A92}$$

$$a \in \{100\},$$
$$lifeexp(a) = - \sum_{g \in GDR} (POPSH(a,g) \cdot drate(a,g) + CEXP(a,g) \cdot pop(a,g)) \tag{A93}$$

where

$$POPSH(a,g) = \frac{POP(a,g)}{\sum\limits_{g \in GDR} POP(a,g)}$$

$$CEXP(a,g) = \left( DRATE(a,g) - \sum_{g \in GDR} (DRATE(a,g) \cdot POPSH(a,g)) \right) \cdot POPSH(a,g)$$

**First pillar (PAYG)**

$$\begin{aligned}
&\forall g \in GDR, \forall a \in AGE \backslash \{100\}, \\
&kpayg(a,g) = CPENS1(a,g) \cdot (1 - DRATE(a,g)) \cdot kpayg_{-1}(a-1,g) + \qquad \text{(A94)} \\
&+ KPAYG_{-1}(a-1,g) \cdot CPENS1(a,g) \cdot (1 - DRATE(a,g)) \cdot pensretpayg + \\
&+ KPAYG_{-1}(a-1,g) \cdot CPENS1(a,g) \cdot drate(a,g) + \\
&+ KPAYG_{-1}(a-1,g) \cdot PENSRETPAYG \cdot (1 - DRATE(a,g)) \cdot pensage(a,g) + \\
&+ CPENS2(a,g) \cdot tpayg(a) + CPENS2(a,g) \cdot TPAYG(a) \cdot (x1lab(a,g) + ngwageleg) \\
&LIFEEXP(a) \cdot POP(a,g) \cdot penspayg(a,g) = PENSAGE(a,g) \cdot kpayg_{-1}(a-1,g) + \qquad \text{(A95)} \\
&+ KPAYG_{-1}(a-1,g) \cdot pensage(a,g) + CPENS3(a,g) \cdot penspayg_{-1}(a-1,g) + \\
&+ CPENS3(a,g) \cdot 0.25 \cdot PENSPAYG_{-1}(a-1,g) \cdot pensind + \\
&+ CPENS3(a,g) \cdot 0.75 \cdot PENSPAYG_{-1}(a-1,g) \cdot pensind_{-1} + \\
&+ KPAYG_{-1}(a-1,g) \cdot PENSAGE(a,g) \cdot pop(a,g) + \\
&+ \frac{PENSAGE(a,g) \cdot KPAYG_{-1}(a-1,g)}{LIFEEXP(a)} \cdot lifeexp(a)
\end{aligned}$$

where

$$CPENS1(a,g) = (1 - PENSAGE(a,g)) \cdot PENSRETPAYG$$

$$CPENS2(a,g) = \frac{ARATE(a,g) \cdot POP(a,g)}{ATOT} \cdot \sum_{i \in IND} V1NGWAGELEG(i)$$

$$CPENS3(a,g) = LIFEEXP(a) \cdot POP(a,g) \cdot (0.25 \cdot PENSIND + 0.75 \cdot PENSIND_{-1})$$

**Second pillar (FF)**

$$\forall g \in GDR, \forall a \in AGE \backslash \{100\},$$

$$kff(a,g) = CPENS4(a,g) \cdot (1 - DRATE(a,g)) \cdot kff_{-1}(a-1,g) + \tag{A96}$$
$$+ KFF_{-1}(a-1,g) \cdot CPENS4(a,g) \cdot (1 - DRATE(a,g)) \cdot pensretff +$$
$$+ KFF_{-1}(a-1,g) \cdot CPENS4(a,g) \cdot drate(a,g) +$$
$$+ KFF_{-1}(a-1,g) \cdot PENSRETFF \cdot (1 - DRATE(a,g)) \cdot pensage(a,g) +$$
$$+ CPENS2(a,g) \cdot tff(a) + CPENS2(a,g) \cdot TFF(a) \cdot (x1lab(a,g) + ngwageleg)$$

$$LIFEEXP(a) \cdot POP(a,g) \cdot pensff(a,g) = PENSAGE(a,g) \cdot kff_{-1}(a-1,g) + \tag{A97}$$
$$+ KFF_{-1}(a-1,g) \cdot pensage(a,g) + CPENS3(a,g) \cdot pensff_{-1}(a-1,g) +$$
$$+ CPENS3(a,g) \cdot 0.25 \cdot PENSFF_{-1}(a-1,g) \cdot pensind +$$
$$+ CPENS3(a,g) \cdot 0.75 \cdot PENSFF_{-1}(a-1,g) \cdot pensind_{-1} +$$
$$+ KFF_{-1}(a-1,g) \cdot PENSAGE(a,g) \cdot pop(a,g) +$$
$$+ \frac{PENSAGE(a,g) \cdot KFF_{-1}(a-1,g)}{LIFEEXP(a)} \cdot lifeexp(a)$$

where

$$CPENS4(a,g) = (1 - PENSAGE(a,g)) \cdot PENSRETFF$$

**Return on pension capital and pension indexation**

$$pensretpayg = \frac{7}{12} \cdot (revpens - revpens_{-1}) + \frac{5}{12} \cdot (revpens_{-1} - revpens_{-2}) \tag{A98}$$

$$pensretff = \frac{1}{IRATE} \cdot irate + p2tot \tag{A99}$$

$$pensind = CPENSIND1 \cdot \left( \frac{7}{12} \cdot p2tot + \frac{5}{12} \cdot p2tot_{-1} \right) - \tag{A100}$$
$$- CPENSIND2 \cdot \left( \frac{7}{12} \cdot p2tot_{-1} + \frac{5}{12} \cdot p2tot_{-2} \right)$$

where

$$CPENSIND1 = \Phi \left( \frac{PENSINDALL - 1}{0.000001^{0.5}} \right) \cdot e^{0.000001}$$
$$CPENSIND2 = \Phi \left( \frac{PENSINDALL_{-1} - 1}{0.000001^{0.5}} \right) \cdot e^{0.000001}$$

**Aggregate contributions and pensions**

$$\sum_{a\in AGE}\sum_{g\in GDR}(EMP(a,g)\cdot(TPAYG(a)+TFF(a)))\cdot revpens= \tag{A101}$$
$$=\sum_{a\in AGE}\sum_{g\in GDR}(EMP(a,g)\cdot(TPAYG(a)+TFF(a)))\cdot ngwageleg+$$
$$+\sum_{a\in AGE}\sum_{g\in GDR}(EMP(a,g)\cdot(TPAYG(a)+TFF(a))\cdot x1lab(a,g))+$$
$$+\sum_{a\in AGE}\sum_{g\in GDR}(EMP(a,g)\cdot(tpayg(a)+tff(a)))$$

$$\left(\sum_{a\in AGE}\sum_{g\in GDR}POP(a,g)\cdot PENSAGE(a,g)\cdot PENSPAYG(a,g)\right)\cdot penspaygtot= \tag{A102}$$
$$=\sum_{a\in AGE}\sum_{g\in GDR}(POP(a,g)\cdot PENSAGE(a,g)\cdot penspayg(a,g))+$$
$$+\sum_{a\in AGE}\sum_{g\in GDR}(POP(a,g)\cdot PENSAGE(a,g)\cdot PENSPAYG(a,g)\cdot pop(a,g))+$$
$$+\left(\sum_{a\in AGE}\sum_{g\in GDR}POP(a,g)\cdot PENSAGE(a,g)\cdot PENSPAYG(a,g)\right)\cdot penstotADD$$

$$\left(\sum_{a\in AGE}\sum_{g\in GDR}POP(a,g)\cdot PENSAGE(a,g)\cdot PENSFF(a,g)\right)\cdot pensfftot= \tag{A103}$$
$$=\sum_{a\in AGE}\sum_{g\in GDR}(POP(a,g)\cdot PENSAGE(a,g)\cdot pensff(a,g))+$$
$$+\sum_{a\in AGE}\sum_{g\in GDR}(POP(a,g)\cdot PENSAGE(a,g)\cdot PENSFF(a,g)\cdot pop(a,g))+$$
$$+\left(\sum_{a\in AGE}\sum_{g\in GDR}POP(a,g)\cdot PENSAGE(a,g)\cdot PENSFF(a,g)\right)\cdot penstotADD$$

where

$$EMP(a,g)=\frac{ARATE(a,g)\cdot POP(a,g)}{ATOT}\cdot\sum_{i\in IND}EMP(i)$$

### A.2.14 Fiscal block

**Value added tax revenues**

$$GOVREVVAT \cdot govrevvat = \sum_{c \in COM} \sum_{s \in SRC} (CVAT1(c,s) \cdot pprod(c,s)) + \tag{A104}$$
$$+ \sum_{c \in COM} \sum_{s \in SRC} (CVAT2(c,s) \cdot x2(c,s)) + \sum_{c \in COM} \sum_{s \in SRC} (CVAT3(c,s) \cdot x3(c,s)) +$$
$$+ \sum_{c \in COM} \sum_{s \in SRC} (CVAT4(c,s) \cdot x6(c,s)) + \sum_{c \in COM} (CVAT5(c) \cdot taxprodsh(c)) +$$
$$+ \sum_{c \in COM} CVAT6(c) \cdot tvat(c)$$

where

$$GOVREVVAT = \sum_{c \in COM} \sum_{s \in SRC} (V2PROD(c,s) + V3PROD(c,s) + V6PROD(c,s)) \cdot$$
$$\cdot TVAT(c) \cdot TAXPRODSH(c)$$
$$CVAT1(c,s) = (V2PROD(c,s) + V3PROD(c,s) + V6PROD(c,s)) \cdot TVAT(c) \cdot TAXPRODSH(c)$$
$$CVAT2(c,s) = V2PROD(c,s) \cdot TVAT(c) \cdot TAXPRODSH(c)$$
$$CVAT3(c,s) = V3PROD(c,s) \cdot TVAT(c) \cdot TAXPRODSH(c)$$
$$CVAT4(c,s) = V6PROD(c,s) \cdot TVAT(c) \cdot TAXPRODSH(c)$$
$$CVAT5(c) = \sum_{s \in SRC} (V2PROD(c,s) + V3PROD(c,s) + V6PROD(c,s)) \cdot TVAT(c)$$
$$CVAT6(c) = \sum_{s \in SRC} (V2PROD(c,s) + V3PROD(c,s) + V6PROD(c,s)) \cdot TAXPRODSH(c)$$
$$V2PROD(c,s) = V2BAS(c,s) \cdot (1 + TPROD(c) + TEXC(c,s) \cdot TAXPRODSH(c))$$
$$V3PROD(c,s) = V3BAS(c,s) \cdot (1 + TPROD(c) + TEXC(c,s) \cdot TAXPRODSH(c))$$
$$V6PROD(c,s) = V6BAS(c,s) \cdot (1 + TPROD(c) + TEXC(c,s) \cdot TAXPRODSH(c))$$

**Excise revenues**

$$GOVREVEXC \cdot govrevexc = \sum_{c \in COM} \sum_{s \in SRC} \sum_{i \in IND} (CEXC1(c,s,i) \cdot x1(c,s,i)) + \tag{A105}$$
$$+ \sum_{c \in COM} \sum_{s \in SRC} (CEXC2(c,s) \cdot x2(c,s)) + \sum_{c \in COM} \sum_{s \in SRC} (CEXC3(c,s) \cdot x3(c,s)) +$$
$$+ \sum_{c \in COM} \sum_{s \in SRC} (CEXC4(c,s) \cdot x4(c,s)) + \sum_{c \in COM} \sum_{s \in SRC} (CEXC5(c,s) \cdot x5(c,s)) +$$
$$+ \sum_{c \in COM} \sum_{s \in SRC} (CEXC6(c,s) \cdot x6(c,s)) + \sum_{c \in COM} \sum_{s \in SRC} (CEXC7(c,s) \cdot x7(c,s)) +$$
$$+ \sum_{c \in COM} (CEXC8(c) \cdot texc(c)) + \sum_{c \in COM} (CEXC9(c) \cdot taxprodsh(c))$$

where

$$GOVREVEXC = \sum_{c \in COM} \sum_{s \in SRC} (V0BAS(c,s) - V8BAS(c,s)) \cdot TEXC(c,s) \cdot SHADOW(c)$$
$$CEXC1(c,s) = V1BAS(c,s,i) \cdot TEXC(c,s) \cdot TAXPRODSH(c)$$
$$CEXC2(c,s) = V2BAS(c,s) \cdot TEXC(c,s) \cdot SHADOW(c)$$
$$CEXC3(c,s) = V3BAS(c,s) \cdot TEXC(c,s) \cdot SHADOW(c)$$
$$CEXC4(c,s) = V4BAS(c,s) \cdot TEXC(c,s) \cdot SHADOW(c)$$
$$CEXC5(c,s) = V5BAS(c,s) \cdot TEXC(c,s) \cdot SHADOW(c)$$
$$CEXC6(c,s) = V6BAS(c,s) \cdot TEXC(c,s) \cdot SHADOW(c)$$
$$CEXC7(c,s) = V7BAS(c,s) \cdot TEXC(c,s) \cdot SHADOW(c)$$
$$CEXC8(c) = \sum_{s \in SRC} (V0BAS(c,s) - V8BAS(c,s)) \cdot TEXC(c,s) \cdot SHADOW(c)$$
$$CEXC9(c) = \sum_{s \in SRC} (V0BAS(c,s) - V8BAS(c,s)) \cdot TEXC(c,s)$$

**Personal income tax revenues**

$\forall i \in IND,$

$$revpit(i) = ngwage(i) + x1lab(i) + revpitADD \tag{A106}$$

$$\left(\sum_{i \in IND} V1PIT(i)\right) \cdot govrevpit = \sum_{i \in IND} (V1PIT(i) \cdot revpit(i)) \tag{A107}$$

**Social security contribution revenues**

$\forall i \in IND,$

$$revsscee(i) = ngwage(i) + x1lab(i) + revssceeADD \tag{A108}$$
$$revsscer(i) = ngwage(i) + x1lab(i) + revsscerADD \tag{A109}$$

$$\left(\sum_{i \in IND} V1SSCEE(i)\right) \cdot govrevsscee = \sum_{i \in IND} (V1SSCEE(i) \cdot revsscee(i)) \tag{A110}$$
$$\left(\sum_{i \in IND} V1SSCER(i)\right) \cdot govrevsscer = \sum_{i \in IND} (V1SSCER(i) \cdot revsscer(i)) \tag{A111}$$
$$\left(\sum_{i \in IND} (V1SSCEE(i) + V1SSCER(i))\right) \cdot govrevssc =$$
$$= \left(\sum_{i \in IND} V1SSCEE(i)\right) \cdot govrevsscee + \left(\sum_{i \in IND} V1SSCER(i)\right) \cdot govrevsscer \tag{A112}$$

**Other revenues**

$$govrevoth = ngdp \tag{A113}$$

**Total government revenues**

$$GOVREV \cdot govrev = GOVREVVAT \cdot govrevvat + GOVREVEXC \cdot govrevexc + \tag{A114}$$
$$+ \left( \sum_{i \in IND} V1PIT(i) \right) \cdot govrevpit + \left( \sum_{i \in IND} (V1SSCEE(i) + V1SSCER(i)) \right) \cdot govrevssc +$$
$$+ GOVREVOTH \cdot govrevoth$$

where

$$GOVREV = GOVREVVAT + GOVREVEXC + \left( \sum_{i \in IND} V1PIT(i) \right) +$$
$$+ \left( \sum_{i \in IND} (V1SSCEE(i) + V1SSCER(i)) \right) + GOVREVOTH$$

**Social expenditures**

$$govexpsocpens = penspaygtot \tag{A115}$$
$$govexpsocinv = poptot + \frac{1}{6} \cdot ngwageleg + \frac{1}{3} \cdot ngwageleg_{-1} + \frac{1}{3} \cdot ngwageleg_{-2} + \tag{A116}$$
$$+ \frac{1}{6} \cdot ngwageleg_{-3} + govexpsocinvADD$$
$$govexpsocpar = xtotlab + \sum_{g \in GDR} \left( \frac{POP0(g)}{\sum_{g \in GDR} POP0(g)} \cdot pop0(g) \right) + \tag{A117}$$
$$\frac{1}{3} \cdot ngwageleg + \frac{2}{3} \cdot ngwageleg_{-1} + govexpsocparADD$$
$$govexpsocill = xtotlab + \frac{1}{3} \cdot ngwageleg + \frac{2}{3} \cdot ngwageleg_{-1} + govexpsocillADD \tag{A118}$$
$$govexpsocunempl = utot + \frac{1}{3} \cdot ngwageleg + \frac{2}{3} \cdot ngwageleg_{-1} + govexpsocunemplADD \tag{A119}$$
$$govexpsocotheur = govexpsocothADD \tag{A120}$$
$$GOVEXPSOCOTH \cdot govexpsocoth = GOVEXPSOCOTHEUR \cdot govexpsocotheur \tag{A121}$$
$$GOVEXPSOC \cdot govexpsoc = GOVEXPSOCPENS \cdot govexpsocpens + \tag{A122}$$
$$+ GOVEXPSOCINV \cdot govexpsocinv + GOVEXPSOCPAR \cdot govexpsocpar +$$
$$+ GOVEXPSOCINV \cdot govexpsocill + GOVEXPSOCUNEMPL \cdot govexpsocunempl +$$
$$+ GOVEXPSOCOTH \cdot govexpsocoth$$

**Government expenditures**

$\forall c \in COM$,

$$govexpcon1(c) = x3\_s(c) + ppur3\_s(c) \tag{A123}$$
$$govexpcon2(c) = x4\_s(c) + pprod4\_s(c) \tag{A124}$$
$$govexpinv(c) = x7\_s(c) + pprod7\_s(c) \tag{A125}$$

$$govirate = \frac{1}{IRATE} \cdot irate + p2tot \quad \text{(A126)}$$

$$govexpint = govirate + \frac{1}{GOVDEBT} \cdot govdebt \quad \text{(A127)}$$

$$\left(\sum_{c\in COM}\sum_{s\in SRC}(V3PUR(c,s) + V4PROD(c,s))\right) \cdot govexpcon = \quad \text{(A128)}$$

$$= \sum_{c\in COM}\left(\sum_{s\in SRC} V3PUR(c,s) \cdot (x3\_s(c) + ppur3\_s(c))\right) +$$

$$+ \sum_{c\in COM}\left(\sum_{s\in SRC} V4PROD(c,s) \cdot (x4\_s(c) + pprod4\_s(c))\right)$$

$$\left(\sum_{c\in COM}\sum_{s\in SRC} V7PROD(c,s)\right) \cdot govexpinv = \quad \text{(A129)}$$

$$= \sum_{c\in COM}\left(\sum_{s\in SRC} V3PUR(c,s) \cdot (x7\_s(c) + pprod7\_s(c))\right)$$

$$GOVEXP \cdot govexp = \quad \text{(A130)}$$

$$\left(\sum_{c\in COM}\sum_{s\in SRC}(V3PUR(c,s) + V4PROD(c,s))\right) \cdot govexpcon +$$

$$\left(\sum_{c\in COM}\sum_{s\in SRC} V7PROD(c,s)\right) \cdot govexpinv + GOVEXPSOC \cdot govexpsoc +$$

$$+ GOVEXPINT \cdot govexpint + GOVEXPOTH \cdot govexpoth$$

where

$$GOVEXP = \left(\sum_{c\in COM}\sum_{s\in SRC}(V3PUR(c,s) + V4PROD(c,s))\right) +$$

$$+ \left(\sum_{c\in COM}\sum_{s\in SRC} V7PROD(c,s)\right) + GOVEXPSOC + GOVEXPINT + GOVEXPOTH$$

**Government budget balance**

$$govbb = GOVREV \cdot govrev - GOVEXP \cdot govexp \quad \text{(A131)}$$

$$govbbcum = govbbcum_{-1} + govbb \quad \text{(A132)}$$

$$NGDP \cdot govbbgdp = govbb - (GOVREV - GOVEXP) \cdot ngdp \quad \text{(A133)}$$

$$govdebt = -govbbgdp \quad \text{(A134)}$$

where

$$NGDP = NCON + NGOV + NINV + NEXP - NIMP$$

$$NGOV = \sum_{c \in COM} \sum_{s \in SRC} (V3PUR(c,s) + V4PROD(c,s))$$

$$NINV = \sum_{c \in COM} \sum_{s \in SRC} (V5PROD(c,s) + V7PROD(c,s)) +$$

$$+ \sum_{c \in COM} V6PROD(c, "dom")$$

$$NEXP = \sum_{c \in COM} \sum_{s \in SRC} V8BAS(c,s) + \sum_{c \in COM} V9PUR(c, "dom")$$

$$NIMP = \sum_{c \in COM} \sum_{i \in IND} V1BAS(c, "imp", i) + \sum_{c \in COM} V2BAS(c, "imp") +$$

$$+ \sum_{c \in COM} V3BAS(c, "imp") + \sum_{c \in COM} V4BAS(c, "imp") +$$

$$+ \sum_{c \in COM} V5BAS(c, "imp") + \sum_{c \in COM} V7BAS(c, "imp") +$$

$$+ \sum_{c \in COM} V8BAS(c, "imp") + \sum_{c \in COM} V9BAS(c, "imp")$$

$$V9PUR(c, "dom") = V9BAS(c, "dom") \cdot (1 + TPROD(c) + TEXC(c, "dom") \cdot TAXPRODSH(c)) \cdot$$

$$\cdot (1 + TVAT(c) \cdot TAXPRODSH(c))$$

### A.2.15 Accounting identities

**Private consumption**

$$NCON \cdot rcon = \sum_{c \in COM} \sum_{s \in SRC} (V2PUR(c,s) \cdot x2(c,s)) + \quad \text{(A135)}$$

$$+ \sum_{c \in COM} (V9BAS(c, "imp") \cdot x9(c, "imp"))$$

$$NCON \cdot p2tot = \sum_{c \in COM} \sum_{s \in SRC} (V2PUR(c,s) \cdot ppur(c,s)) + \quad \text{(A136)}$$

$$+ \sum_{c \in COM} (V9BAS(c, "imp") \cdot pbas(c, "imp"))$$

**Government consumption**

$$NGOV \cdot ngov = \sum_{c \in COM} \sum_{s \in SRC} (V3PUR(c,s) \cdot (x3(c,s) + ppur(c,s))) + \quad \text{(A137)}$$

$$+ \sum_{c \in COM} \sum_{s \in SRC} (V4PROD(c,s) \cdot (x4(c,s) + pprod(c,s)))$$

$$NGOV \cdot rgov = \sum_{c \in COM} \sum_{s \in SRC} (V3PUR(c,s) \cdot x3(c,s)) + \quad \text{(A138)}$$

$$+ \sum_{c \in COM} \sum_{s \in SRC} (V4PROD(c,s) \cdot x4(c,s))$$

**Investments**

$$NINV \cdot ninv = \sum_{c \in COM} \sum_{s \in SRC} (V5PROD(c,s) \cdot (x5(c,s) + pprod(c,s))) + \tag{A139}$$

$$+ \sum_{c \in COM} (V6PROD(c, "dom") \cdot (x6(c, "dom") + pprod(c, "dom"))) +$$

$$+ \sum_{c \in COM} \sum_{s \in SRC} (V7PROD(c,s) \cdot (x7(c,s) + pprod(c,s)))$$

$$NINV \cdot ninv = \sum_{c \in COM} \sum_{s \in SRC} (V5PROD(c,s) \cdot x5(c,s)) + \tag{A140}$$

$$+ \sum_{c \in COM} (V6PROD(c, "dom") \cdot x6(c, "dom")) + \sum_{c \in COM} \sum_{s \in SRC} (V7PROD(c,s) \cdot x7(c,s))$$

**Exports**

$$NEXP \cdot nexp = \sum_{c \in COM} \sum_{s \in SRC} (V8BAS(c,s) \cdot (x8(c,s) + pbas(c,s))) + \tag{A141}$$

$$+ \sum_{c \in COM} (V9PUR(c, "dom") \cdot (x9(c, "dom") + ppur(c, "dom")))$$

$$NEXP \cdot nexp = \sum_{c \in COM} \sum_{s \in SRC} (V8BAS(c,s) \cdot x8(c,s)) + \tag{A142}$$

$$+ \sum_{c \in COM} (V9PUR(c, "dom") \cdot x9(c, "dom"))$$

**Imports**

$$NIMP \cdot nimp = \sum_{c \in COM} \sum_{i \in IND} (V1BAS(c, "imp", i) \cdot (x1(c, "imp", i) + pbas(c, "imp"))) + \tag{A143}$$

$$+ \sum_{c \in COM} (V2BAS(c, "imp") \cdot (x2(c, "imp") + pbas(c, "imp"))) +$$

$$+ \sum_{c \in COM} (V3BAS(c, "imp") \cdot (x3(c, "imp") + pbas(c, "imp"))) +$$

$$+ \sum_{c \in COM} (V4BAS(c, "imp") \cdot (x4(c, "imp") + pbas(c, "imp"))) +$$

$$+ \sum_{c \in COM} (V5BAS(c, "imp") \cdot (x5(c, "imp") + pbas(c, "imp"))) +$$

$$+ \sum_{c \in COM} (V7BAS(c, "imp") \cdot (x7(c, "imp") + pbas(c, "imp"))) +$$

$$+ \sum_{c \in COM} (V8BAS(c, "imp") \cdot (x8(c, "imp") + pbas(c, "imp"))) +$$

$$+ \sum_{c \in COM} (V9BAS(c, "imp") \cdot (x9(c, "imp") + pbas(c, "imp")))$$

$$NIMP \cdot rimp = \sum_{c \in COM} \sum_{i \in IND} (V1BAS(c, "imp", i) \cdot x1(c, "imp", i) +) + \tag{A144}$$

$$+ \sum_{c \in COM} (V2BAS(c, "imp") \cdot x2(c, "imp")) +$$

$$+ \sum_{c \in COM} (V3BAS(c, "imp") \cdot x3(c, "imp")) +$$

$$+ \sum_{c \in COM} (V4BAS(c, "imp") \cdot x4(c, "imp")) +$$

$$+ \sum_{c \in COM} (V5BAS(c, "imp") \cdot x5(c, "imp")) +$$

$$+ \sum_{c \in COM} (V7BAS(c, "imp") \cdot x7(c, "imp")) +$$

$$+ \sum_{c \in COM} (V8BAS(c, "imp") \cdot x8(c, "imp")) +$$

$$+ \sum_{c \in COM} (V9BAS(c, "imp") \cdot x9(c, "imp"))$$

**Gross domestic product**

$$NGDP \cdot ngdp = NCON \cdot ncon + NGOV \cdot ngov + NINV \cdot ninv +$$

$$+ NEXP \cdot nexp - NIMP \cdot nimp \tag{A145}$$

$$NGDP \cdot rgdp = NCON \cdot rcon + NGOV \cdot rgov + NINV \cdot rinv +$$

$$+ NEXP \cdot rexp - NIMP \cdot rimp \tag{A146}$$

$$pgdp = ngdp - rgdp \tag{A147}$$

## A.3 Calibrated parameters

Table A3: Calibrated parameters by commodity

| CPA 2008 commodity | $SIGMA1$ | $SIGMA1SUP$ | $SIGMA2$ | $SIGMA3$ | $SIGMA5$ | $SIGMA7$ | $SIGMA8$ | $EPSILON$ |
|---|---|---|---|---|---|---|---|---|
| 01: Products of agriculture, hunting and related services | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 | 2.00 | 0.40 |
| 02: Products of forestry, logging and related services | 1.10 | 1.10 | 1.10 | 1.10 | 1.10 | 1.10 | 1.10 | 0.55 |
| 03: Fish and other fishing products; aquaculture products; support services to fishing | 1.10 | 1.10 | 1.10 | 1.10 | 1.10 | 1.10 | 1.10 | 0.63 |
| B: Mining and quarrying | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 2.00 | 0.58 |
| 10–12: Food products; beverages; tobacco products | 1.10 | 1.10 | 1.10 | 1.10 | 1.10 | 1.10 | 1.10 | 0.75 |
| 13–15: Textiles; wearing apparel; leather and related products | 0.80 | 1.60 | 1.60 | 1.60 | 1.60 | 1.60 | 1.60 | 0.43 |
| 16: Wood and of products of wood and cork, except furniture; articles of straw and plaiting materials | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 | 1.50 | 0.78 |
| 17: Paper and paper products | 1.10 | 1.10 | 1.10 | 1.10 | 1.10 | 1.10 | 1.10 | 0.80 |
| 18: Printing and recording services | 1.10 | 1.10 | 1.10 | 1.10 | 1.10 | 1.10 | 1.10 | 0.65 |
| 19: Coke and refined petroleum products | 1.10 | 1.10 | 1.10 | 1.10 | 1.10 | 1.10 | 1.10 | 0.30 |
| 20: Chemicals and chemical products | 2.00 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 1.20 |
| 21: Basic pharmaceutical products and pharmaceutical preparations | 1.10 | 1.10 | 1.10 | 1.10 | 1.10 | 1.10 | 1.10 | 1.03 |
| 22: Rubber and plastics products | 1.80 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 1.28 |
| 23: Other non-metallic mineral products | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 2.00 | 0.58 |
| 24: Basic metals | 0.50 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.80 | 1.08 |
| 25: Fabricated metal products, except machinery and equipment | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 1.00 |
| 26: Computer, electronic and optical products | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 1.00 |
| 27: Electrical equipment | 1.60 | 1.60 | 1.60 | 1.60 | 1.60 | 1.60 | 1.60 | 1.08 |
| 28: Machinery and equipment n.e.c. | 1.60 | 1.60 | 1.60 | 1.60 | 1.60 | 1.60 | 1.60 | 1.00 |
| 29: Motor vehicles, trailers and semi-trailers | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 1.00 |
| 30: Other transport equipment | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 1.00 |
| 31–32: Furniture; other manufactured goods | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 1.00 |
| 33: Repair and installation services of machinery and equipment | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 1.25 |
| D: Electricity, gas, steam and air conditioning | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.80 |
| 36: Natural water; water treatment and supply services | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 | 1.00 | 1.28 |
| 37–39: Sewerage services, sewage sludge; waste collection, treatment and disposal services, materials recovery services; remediation services and other waste management services | 1.10 | 1.10 | 1.10 | 1.10 | 1.10 | 1.10 | 1.10 | 1.50 |
| F: Constructions and construction works | 1.80 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 0.68 |
| 45: Wholesale and retail trade and repair services of motor vehicles and motorcycles | 1.60 | 1.60 | 1.60 | 1.60 | 1.60 | 1.60 | 1.60 | 1.00 |
| 46: Wholesale trade services, except motor vehicles and motorcycles | 1.50 | 1.60 | 1.60 | 1.60 | 1.60 | 1.60 | 1.60 | 0.65 |
| 47: Retail trade services, except motor vehicles and motorcycles | 1.80 | 1.60 | 1.60 | 1.60 | 1.60 | 1.60 | 1.60 | 0.43 |
| 49: Land transport services and transport services via pipelines | 2.70 | 2.70 | 2.70 | 2.70 | 2.70 | 2.70 | 2.70 | 1.08 |
| 50: Water transport services | 1.70 | 1.60 | 1.60 | 1.60 | 1.60 | 1.60 | 1.60 | 1.13 |
| 51: Air transport services | 1.70 | 1.60 | 1.60 | 1.60 | 1.60 | 1.60 | 1.60 | 1.15 |
| 52: Warehousing and support services for transportation | 1.70 | 1.60 | 1.60 | 1.60 | 1.60 | 1.60 | 1.60 | 1.15 |
| 53: Postal and courier services | 2.30 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 1.23 |
| I: Accommodation and food services | 2.30 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 0.80 |
| 58: Publishing services | 2.30 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 1.00 |
| 59–60: Motion picture, video and television programme production services, sound recording and music publishing; programming and broadcasting services | 1.70 | 1.60 | 1.60 | 1.60 | 1.60 | 1.60 | 1.60 | 1.00 |
| 61: Telecommunications services | 2.70 | 2.70 | 2.70 | 2.70 | 2.70 | 2.70 | 2.70 | 1.40 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 62–63: Computer programming, consultancy and related services; information services | 1.20 | 1.10 | 1.10 | 1.10 | 1.10 | 1.10 | 1.10 | 0.65 |
| 64: Financial services, except insurance and pension funding | 1.20 | 1.10 | 1.10 | 1.10 | 1.10 | 1.10 | 1.10 | 1.75 |
| 65: Insurance, reinsurance and pension funding services, except compulsory social security | 1.20 | 1.10 | 1.10 | 1.10 | 1.10 | 1.10 | 1.10 | 1.15 |
| 66: Services auxiliary to financial services and insurance services | 1.20 | 1.10 | 1.10 | 1.10 | 1.10 | 1.10 | 1.10 | 1.43 |
| 68A: Real estate services (except services of own real estate) | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 0.78 |
| 68B: Rental and operating services of own real estate | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 1.45 |
| 69–70: Legal and accounting services; services of head offices, management consulting services | 1.60 | 1.60 | 1.60 | 1.60 | 1.60 | 1.60 | 1.60 | 1.45 |
| 71: Architectural and engineering services, technical testing and analysis services | 1.60 | 1.60 | 1.60 | 1.60 | 1.60 | 1.60 | 1.60 | 1.45 |
| 72: Scientific research and development services | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 1.23 |
| 73: Advertising and market research services | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 0.85 |
| 74–75: Other professional, scientific and technical services; veterinary services | 2.40 | 2.40 | 2.40 | 2.40 | 2.40 | 2.40 | 2.40 | 1.48 |
| 77: Rental and leasing services | 2.40 | 2.40 | 2.40 | 2.40 | 2.40 | 2.40 | 2.40 | 0.65 |
| 78: Employment services | 2.40 | 2.40 | 2.40 | 2.40 | 2.40 | 2.40 | 2.40 | 0.55 |
| 79: Travel agency, tour operator and other reservation services and related services | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 1.00 |
| 80–82: Security and investigation services; services to buildings and landscape; office administrative, office support and other business support services | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 1.00 |
| O: Public administration and defence services; compulsory social security services | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 1.00 |
| P: Education services | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 1.00 |
| 86: Human health services | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 1.00 |
| 87–88: Residential care services; social work services without accommodation | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 1.00 |
| 90–92: Creative, arts and entertainment services; library, archive, museum and other cultural services; gambling and betting services | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 1.00 |
| 93: Sporting services and amusement and recreation services | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 1.00 |
| 94: Services furnished by membership organisations | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 1.00 |
| 95: Repair services of computers and personal and household goods | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 2.20 | 1.00 |
| 96: Other personal services | 1.60 | 1.60 | 1.60 | 1.60 | 1.60 | 1.60 | 1.60 | 1.00 |

Table A4: Calibrated parameters by industry

| NACE Rev. 2 industry | *SIGMA1PRIM* | *SIGMA1LAB* | *DELTA* |
|---|---|---|---|
| 01: Crop and animal production, hunting and related service activities | 1.55 | 0.50 | 0.1123 |
| 02: Forestry and logging | 1.11 | 0.50 | 0.1250 |
| 03: Fishing and aquaculture | 0.94 | 0.50 | 0.0973 |
| B: Mining and quarrying | 1.11 | 0.50 | 0.0791 |
| 10–12: Manufacture of food products; beverages; tobacco products | 0.50 | 0.50 | 0.1543 |
| 13–15: Manufacture of textiles; wearing apparel; leather and related products | 0.41 | 0.50 | 0.1134 |
| 16: Manufacture of wood and of products of wood and cork, except furniture, manufacture of articles of straw and plaiting materials | 0.41 | 0.50 | 0.1282 |
| 17: Manufacture of paper and paper products | 0.59 | 0.50 | 0.1846 |
| 18: Printing and reproduction of recorded media | 0.41 | 0.50 | 0.1987 |
| 19: Manufacture of coke and refined petroleum products | 0.32 | 0.50 | 0.0906 |
| 20: Manufacture of chemicals and chemical products | 0.32 | 0.50 | 0.1130 |
| 21: Manufacture of basic pharmaceutical products and pharmaceutical preparations | 0.32 | 0.50 | 0.1194 |
| 22: Manufacture of rubber and plastic products | 0.32 | 0.50 | 0.1471 |
| 23: Manufacture of other non-metallic mineral products | 0.24 | 0.50 | 0.1056 |
| 24: Manufacture of basic metals | 0.67 | 0.50 | 0.2568 |
| 25: Manufacture of fabricated metal products, except machinery and equipment | 1.02 | 0.50 | 0.1325 |

| | | | |
|---|---|---|---|
| 26: Manufacture of computer, electronic and optical products | 1.37 | 0.50 | 0.1348 |
| 27: Manufacture of electrical equipment | 1.37 | 0.50 | 0.1638 |
| 28: Manufacture of machinery and equipment n.e.c. | 1.55 | 0.50 | 0.2313 |
| 29: Manufacture of motor vehicles, trailers and semi-trailers | 1.11 | 0.50 | 0.1127 |
| 30: Manufacture of other transport equipment | 0.67 | 0.50 | 0.1091 |
| 31–32: Manufacture of furniture; other manufacturing | 0.67 | 0.50 | 0.1995 |
| 33: Repair and installation of machinery and equipment | 0.67 | 0.50 | 0.1035 |
| D: Electricity, gas, steam and air conditioning supply | 0.50 | 0.50 | 0.1688 |
| 36: Water collection, treatment and supply | 0.50 | 0.50 | 0.1745 |
| 37–39: Sewerage; waste collection, treatment and disposal activities, materials recovery; remediation activities and other waste management services | 1.02 | 0.50 | 0.0363 |
| F: Constructions | 0.85 | 0.50 | 0.3302 |
| 45: Wholesale and retail trade and repair of motor vehicles and motorcycles | 1.02 | 0.50 | 0.1325 |
| 46: Wholesale trade, except motor vehicles and motorcycles | 1.37 | 0.50 | 0.1349 |
| 47: Retail trade, except motor vehicles and motorcycles | 1.37 | 0.50 | 0.1638 |
| 49: Land transport and transport via pipelines | 1.11 | 0.50 | 0.1127 |
| 50: Water transport | 0.67 | 0.50 | 0.1091 |
| 51: Air transport | 0.67 | 0.50 | 0.1995 |
| 52: Warehousing and support activities for transportation | 0.67 | 0.50 | 0.1035 |
| 53: Postal and courier activities | 0.50 | 0.50 | 0.1688 |
| I: Accommodation and food service activities | 1.55 | 0.50 | 0.2312 |
| 58: Publishing activities | 0.50 | 0.50 | 0.1688 |
| 59–60: Motion picture, video and television programme production, sound recording and music publishing activities; programming and broadcasting activities | 0.50 | 0.50 | 0.1688 |
| 61: Telecommunications | 0.50 | 0.50 | 0.1688 |
| 62–63: Computer programming, consultancy and related activities; information service activities | 0.50 | 0.50 | 0.1688 |
| 64: Financial service activities, except insurance and pension funding | 0.50 | 0.50 | 0.1745 |
| 65: Insurance, reinsurance and pension funding, except compulsory social security | 0.50 | 0.50 | 0.1745 |
| 66: Activities auxiliary to financial services and insurance activities | 0.50 | 0.50 | 0.1745 |
| 68A: Real estate activities (except services of own real estate) | 1.02 | 0.50 | 0.0363 |
| 68B: Rental and operating activities of own real estate | 1.02 | 0.50 | 0.0363 |
| 69–70: Legal and accounting activities; activities of head offices; management consultancy activities | 0.85 | 0.50 | 0.3302 |
| 71: Architectural and engineering activities, technical testing and analysis | 0.85 | 0.50 | 0.3302 |
| 72: Scientific research and development | 0.85 | 0.50 | 0.3302 |
| 73: Advertising and market research | 0.85 | 0.50 | 0.3302 |
| 74–75: Other professional, scientific and technical activities; veterinary activities | 0.85 | 0.50 | 0.3302 |
| 77: Rental and leasing activities | 0.85 | 0.50 | 0.3302 |
| 78: Employment activities | 0.85 | 0.50 | 0.3302 |
| 79: Travel agency, tour operator and other reservation service and related activities | 0.85 | 0.50 | 0.3302 |
| 80–82: Security and investigation activities; services to buildings and landscape activities; office administrative, office support and other business support activities | 0.85 | 0.50 | 0.3302 |
| O: Public administration and defence, compulsory social security | 1.20 | 0.50 | 0.0835 |
| P: Education | 1.20 | 0.50 | 0.0940 |
| 86: Human health activities | 1.20 | 0.50 | 0.1704 |
| 87–88: Residential care activities; social work activities without accommodation | 1.20 | 0.50 | 0.1704 |
| 90–92: Creative, arts and entertainment activities; libraries, archives, museums and other cultural activities; gambling and betting activities | 1.11 | 0.50 | 0.1943 |
| 93: Sports activities and amusement and recreation activities | 1.11 | 0.50 | 0.1943 |
| 94: Activities of membership organisations | 1.11 | 0.50 | 0.1943 |
| 95: Repair of computers and personal and household goods | 1.11 | 0.50 | 0.1943 |
| 96: Other personal service activities | 1.11 | 0.50 | 0.1943 |