Study on Source Inversion Technology for Nuclear Accidents Based on Gaussian Puff Model and EnKF

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ABSTRACT

For nuclear power plant (NPP) accident, the assessment of the radiation consequences plays an important role in the emergency response system. However, the source characteristics which greatly influence thhe accuracy of the assessment result is poorly known or even unknown at the early phase of accident, wich can cause poorly understanding of the situation and delay the response activities. In this paper, source inversion technology in analyzing nuclear accidents based on Gaussian puff model and ensemble Kalman filter (EnKF) is proposed. The method is validated with simulated measurements and the results show that it can give reasonable estimations of the change in release rate and height simultaneously, though the first guess of release rate is 10^2 larger than the true value. The investigation of the influence of sharp change in source term shows that the method is robust to capture the sharp change, but there is a delay of response when the release height increases simultaneously.

Keywords

Nuclear accident, source term estimation, Gaussian puff model, ensemble Kalman filter.

INTRODUCTION

The consequence assessment of an accidental atmospheric release of radioactive materials plays an important role in the emergency response system. Atmospheric dispersion models used in the consequence assessment of an accidental release of radioactive materials are highly dependent on the information of source term. However, the source term is usually poorly known or even unknown at the early phase of emergency. For instance, in the accident of Fukushima Daiichi power plant, the damages caused by the earthquake and the tsunami rendered the backup power system inoperative, so there was no information from the reactor monitoring system. In this respect, source inversion technologies should be developed to estimate the source term using the environmental monitoring data (Rojas-Palma, Madsen, Gering, Puch, Turcanu *et al.*, 2003; Drews, Lauritzen, Madsen and Smith, 2004; Zheng, Leung, Lee and Lam, 2007). This kind of reconstruction of the source term depending on the balance between the information obtained by the environmental measurements and those obtained by the dispersion model predictions is also called as data assimilation (DA), which has been widely used in the field of numerical weather prediction (NWP) over the past two decades (Kalnay, 2003). Recently, the DA method has been applied in the nuclear emergency response system. Rojas-Palma reported some preliminary results from Kalman filter sequential data assimilation method in the real time on-line decision support system (RODOS) for off-site nuclear emergency management in Europe (Rojas-Palma *et al.*, 2003; Rojas-Palma, Aage, Astrup, Bargholz, Drews *et al.*, 2004). Drews et al. (Drews *et al.*, 2004) used Gaussian plume model and Kalman filter method to conduct on-line estimation of the source term for short-range atmospheric dispersion of radioactive materials. Zheng (Zheng *et al.*, 2007) has developed Monte Carlo dispersion model and ensemble Kalman filter (EnKF) (Evensen, 1994) based DA method to conduct simultaneous state and parameter estimation. In those studies, the fluctuations of the estimated parameters are moderate, but the parameters may change greatly due to the explosion or other events during real accidents. In this study, nuclear accident source term estimation method based on Gaussian puff model and ensemble Kalman filter is proposed to estimate the source term and other main parameters using the environmental monitoring data. The method is validated with simulated measurements. Another important purpose of this study is to investigate the impact of sharp change in the estimated parameters on the performance of the inverse modeling method.

DISPERSION MODEL

Atmospheric dispersion modeling refers to the mathematical description of contaminant transport in the atmosphere. Gaussian plume and Gaussian puff model are two widely used standard approaches for studying the transport of airborne contaminants due to turbulent diffusion and advection by the wind (Stockie, 2011). In this study, Gaussian puff model (Krysta, Bocquet, Sportisse and Isnard, 2006) is used, because the plume model only applies to the steady state conditions with little air pollutant emissions and meteorological changes, which cannot be satisfied in our study. In the puff model, a puff will be released to the atmosphere at each time step, and it carries the radioactive materials that have leaked between two successive time steps. As a result, the concentration C at location (x, y, z) at time t comes from all the puffs that have been released, so the model can consider both the emissions and meteorological changes. Gaussian puff model can be described as:

$$C(x,y,z,t)=\sum_{i=1}^{N_{\text{puff}}}\frac{Q_i\Delta t}{(2\pi)^{3/2}\sigma^i_{x,t}\sigma^i_{y,t}\sigma^i_{z,t}}\times\exp\left(-\frac{(x-x^i_t)^2}{2(\sigma^i_{x,t})^2}\right)\exp\left(-\frac{(y-y^i_t)^2}{2(\sigma^i_{y,t})^2}\right)$$
$$\times\left[\exp\left(-\frac{(z-z^i_t)^2}{2(\sigma^i_{z,t})^2}\right)+\exp\left(-\frac{\left(z-\left(2\cdot z^i_g-z^i_t\right)\right)^2}{2(\sigma^i_{z,t})^2}\right)+\exp\left(-\frac{\left(z-\left(2\cdot\left(h_{ABL}+z^i_g\right)-z^i_t\right)\right)^2}{2(\sigma^i_{z,t})^2}\right)\right] \quad (1)$$

Where Q_i is the radioactive content that the ith puff carries, and (x^i_t, y^i_t, z^i_t) is the center of the ith puff at time t. $t = N_{\text{puff}}\Delta t$, where N_{puff} is the number of the released puffs, and Δt is the time step between two successive puffs. The puff is driven by the wind, so the center of the puff evolves according to: $x^i_t= x^i_{t-1}+u^i_{t-1}\Delta t$, $y^i_t= y^i_{t-1}+v^i_{t-1}\Delta t$, $z^i_t= z^i_{t-1}+w^i_{t-1}\Delta t$, where ($u^i_{t-1}$, v^i_{t-1}, w^i_{t-1}) is the wind velocity at the center of the ith puff at time t-1. The Doury model (Krysta *et al.*, 2006) is used to describe the growth of the standard deviations in the horizontal direction:

$$\sigma^i_{x,t}=\sigma^i_{x,t-1}+k_hA_h(\sigma^i_{x,t-1})^{(k_h-1)/k_h}\cdot\Delta t;\qquad \sigma^i_{y,t}=\sigma^i_{x,t} \quad (2)$$

Where k_h and A_h are the coefficients of the model. In the vertical direction, the standard deviation evolves according to:

$$\sigma^i_{z,t}=\sigma^i_{x,t-1}+(1-\text{stab})k_z^{\text{ND}}A_z^{\text{ND}}(\sigma^i_{x,t-1})^{(k_z^{\text{ND}}-1)/k_z^{\text{ND}}}\Delta t$$
$$+\text{stab}\cdot k_z^{\text{WD}}A_z^{\text{WD}}(\sigma^i_{x,t-1})^{(k_z^{\text{WD}}-1)/k_z^{\text{WD}}}\Delta t \quad (3)$$

Where ND and WD mean the normal and weak diffusion respectively. h_{ABL} is the boundary layer height, and z_g is the ground level. The modifications of radioactive decay and deposition are also added to the model.

ENSEMBLE KALMAN FILTER

Generally speaking, the state and observation of a dynamic system could be expressed as

$$\text{x}(t_k)=M(\text{x}(t_{k-1}))+\eta \quad (4)$$

$$\text{y}_o(t_k)=H(\text{x}(t_k))+\varepsilon \quad (5)$$

Where, x is the state vector of the dynamic system, M is the forecast model, η is the prediction error of the model, y_o is the observation vector, H is observation model, and ε is the measurement error. The standard Kalman filter (KF) operates recursively on streams of noisy input data to produce a statistically optimal estimate of the underlying system state (Grewal and Andrews, 1993), but it is only applicable to the linear models and observations. Ensemble Kalman filter (Evensen, 1994, 2003) is a sequential data assimilation technique for use with nonlinear models of the system dynamics and nonlinear observations. The EnKF uses an ensemble of state estimates instead of a single state estimate to provide a representation of nonlinearity. The state matrix is

$$\text{X}=\frac{1}{\sqrt{N-1}}\begin{pmatrix}\text{x}_1 & \text{x}_2 & \dots & \text{x}_N\end{pmatrix} \quad (6)$$

Where the subscript i is used to denote the individual state vector of an ensemble and N denotes the size of an ensemble. In this study, the state vector includes radioactive content and the central positions of the puffs, so there are four parameters for each puff. The model error covariance matrix is an important quantity for the calculation of Kalman gain which will be used to update the state in the analysis step. However, the real model error can never be obtained. In EnKF, the ensemble covariance matrix P_e, which is calculated from the state ensemble, is used as a reasonable substitution of model error covariance matrix.

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$$P_e = X'X'^{T};\ X' = \frac{1}{\sqrt{N-1}}\left(x_1-\bar{x} \quad x_2-\bar{x} \quad \dots \quad x_N-\bar{x}\right);\ \bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i \tag{7}$$

There are two main steps in EnKF, the forecast step and analysis step. The forecast step evolves each ensemble member forward according to the model of the dynamic system:

$$x_i^f(t_k) = M(x_i^a(t_{k-1})) \tag{8}$$

Where, the superscript *f* denotes the forecast result, and superscript *a* means the analysis result, which is statistically optimal estimate of the underlying system state. *M* represents the model of the system, which is Gaussian puff dispersion model. The forecast observations can be predicted based on the observation model.

$$y_i^f = H(x_i^f);\ Y^f = \frac{1}{\sqrt{N-1}}\left(y_1^f \quad y_2^f \quad \dots \quad y_N^f\right) \tag{9}$$

$$Y'^f = \frac{1}{\sqrt{N-1}}\left(y_1^f-\bar{y} \quad y_2^f-\bar{y} \quad \dots \quad y_N^f-\bar{y}\right); \tag{10}$$

When the observation data is available, the analysis step can be conducted. An ensemble of observations Y should be produced by introducing the pseudo-random observation perturbations Y to the observation y_o.

$$Y = \left(y_o+\varepsilon_1 \quad y_o+\varepsilon_2 \quad \dots \quad y_o+\varepsilon_N\right);\ Y' = \left(\varepsilon_1 \quad \varepsilon_2 \quad \dots \quad \varepsilon_N\right) \tag{11}$$

For the linear observation operator, the ensemble Kalman gain of the analysis step is

$$K_e = P_e^f H^T (H P_e^f H^T + R_e)^{-1};\ R_e = Y'Y'^{T} \tag{12}$$

Where, R_e is the observation ensemble covariance matrix, and H is the observation matrix. Each member of the ensemble can be updated using the gain, and it can be calculated as

$$x_i^a = x_i^f + K_e(y_i - Hx_i^f) \tag{13}$$

For the nonlinear observation operators, the state vector can be extended as Evensen (Evensen, 2003)

$$\hat{x} = \begin{pmatrix} x \\ y \end{pmatrix};\quad y = \hat{H}\hat{x} \tag{14}$$

Now, in the augmented state space $\hat{x}$, the nonlinear problem is reduced to the linear one. Similar as (12), the gain and updated augmented state vector can also be expressed as

$$\hat{K}_e = \hat{X}'^f (\hat{H}\hat{X}'^f)^T ((\hat{H}\hat{X}'^f)(\hat{H}\hat{X}'^f)^T + Y'Y'^{T})^{-1} = \hat{X}'^f (Y'^f)^T (Y'^f (Y'^f)^T + Y'Y'^{T})^{-1} \tag{15}$$

$$\hat{x}_i^a = \hat{x}_i^f + \hat{K}_e(y_i - \hat{H}\hat{x}_i^f) = \hat{x}_i^f + \hat{K}_e(y_i - y_i^f) \tag{16}$$

Taking the first *n* rows of these equations we obtain the gain and updated augmented state vector:

$$K_e = X'^f (Y'^f)^T (Y'^f (Y'^f)^T + Y'Y'^{T})^{-1} \tag{17}$$

$$x_i^a = x_i^f + K_e(y_i - y_i^f) \tag{18}$$

When the observation model is nonlinear, the analysis result in (18) is only an approximation to the exact and optimal estimate of the unknown state, because the valid states of system only occupy a submanifold of augmented state space instead of the whole space. The process of data assimilation is shown in Figure 1.

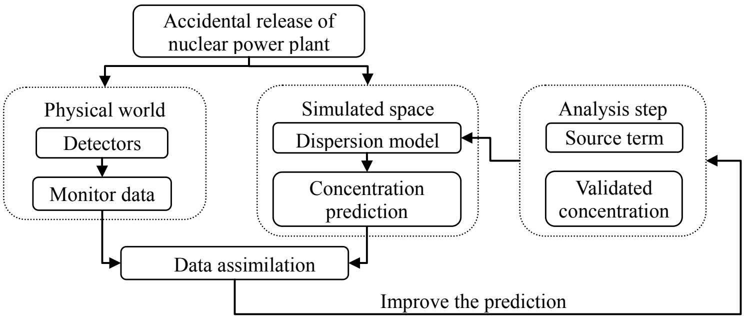

Figure 1. Process of the EnKF Data Assimilation Method

RESULTS AND DISCUSSION

The Gaussian puff model and the data assimilation method are realized by Matlab. The source term estimation method is validated with simulated measurements. The scenario is shown in Figure 2(A), where the radioactive material is released from a NPP. The ground level concentrations are measured by 10 detectors which locate approximately 1km downwind from the NPP. The contour lines show the distribution of the radioactive material concentration 25 minutes after the start of the accident. Gaussian white noise is added to the measurements to simulate the measurement error. Two cases with different source terms are simulated. In the case 1, the release rate and the release height are respectively shown in Figure 3(A) and Figure 3(B). Two different types of changes in source rate are assumed to investigate the performance of the estimation method: a discontinuous step increase of release rate which happens at 100 minutes due to explosion and a gradual continuous decrease of release rate which last from 250 to 500 minutes due to the countermeasures. The release height in case 1 remains constant during the accident. In case 2, the change of release rate is same as that in case 1, but the release height also changes as shown in Figure 4(B). In both cases, the time step of the dispersion model is 30s. Every 10minutes, the concentrations are measured by the detectors, and the measurement lasts for 750 minutes.

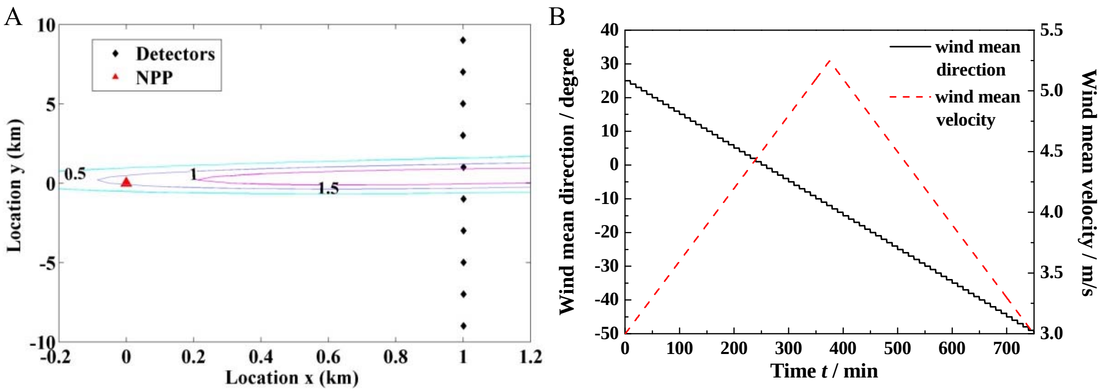

Figure 2. (A) The Simulated Scenario of Nuclear Power Plant Accident. (B) Changes of the Wind Parameters

Figure 3 shows the estimated source release rate for case 1. The first guess of the release rate and height are 10^8Bq/s and 110m respectively, which are far away from the true values. Noises are added to the wind direction and velocity as in (Zheng *et al.*, 2007), because there is model error or observation error in the prediction or measurement of atmospheric parameters in real situation. The disturbed parameter value is calculated as $max(0, p\times(1+e))$, where p is the "true" parameter value, e is the noise. Firstly, 25% white Gaussian noise with standard deviation of $0.25p$ is added to the wind direction and velocity respectively. It takes about 50 minutes to retrieve the true source term from the first guess, and the estimation method can well capture the sharp increase in the release rate, the results are better for the continuous decrease phase. Then, 5% overestimated noise, which is uniformly distributed between (0, 0.05), is added, making the velocity and angle of the wind larger than the true ones. The estimation can also well capture the sharp increase, but the results are larger, because the colored noised strong wind can blow away the radioactive material more quickly and more contents should be emitted from the source to make the predicted concentration close to the measurements.

A B

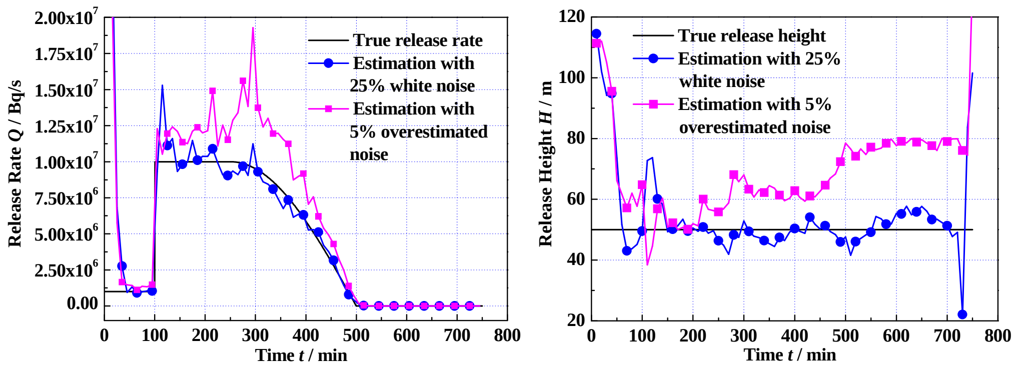

Figure 3. (A) Estimation of Release Rate for Case 1. (B) Estimation of Release Height for Case 1

In case 2, both the release rate and height have changed during the accident. The estimations are shown in Figure 4. The estimated release rate cannot capture the sharp increase as well as that in case 1 due to the simultaneous increase of release height. Since the puffs are emitted higher, it takes more time for the radioactive material to reach the ground, and less information is transmitted to the detectors before the puffs are blown away beyond the detectors. As a result, it takes more time for the estimation method to respond to the sharp increase. We suppose that under this circumstance, using the measurement further away from the source and enlarging the assimilation time step may improve the estimation, so dynamic arrangement of the monitoring network is required. Similar to the results in case 1, the estimated release rate is also larger when 5% overestimated noise is added to the wind parameters. Despite the decease of accuracy in release rate estimation, it is still encouraging to find that the method can give reasonable estimations of the changes in release rate and height simultaneously. The results show the potential ability of the source term estimation method for nuclear accident.

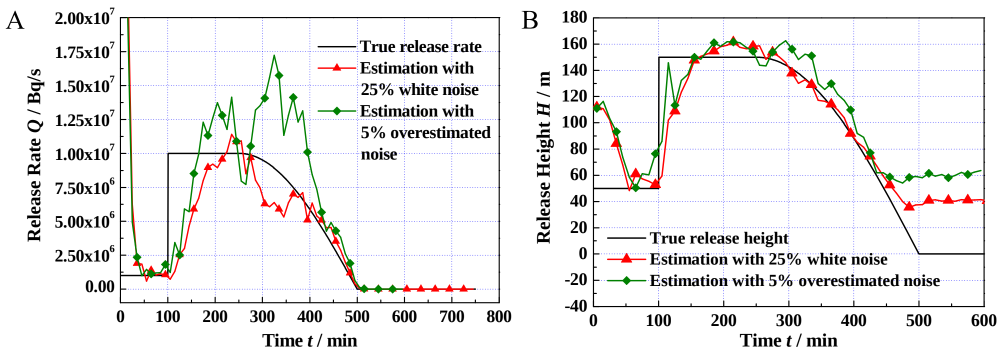

Figure 4. (A) Estimation of Release Rate for Case 2. (B) Estimation of Release Height for Case 2

CONCLUSION

Nuclear accident source term estimation method based on Gaussian puff model and ensemble Kalman filter is proposed. The method is validated with simulated measurements and the results show that the method can give reasonable estimations of the changes in release rate and height simultaneously, though the first guess of release rate is 10^2 larger than the true value. Impact of sharp and continuous changes of estimated parameters on performance of EnKF is investigated, and the results show that the method is robust to capture the sharp change, but there is a delay of response when the release height increases. For further study, a model should be introduced to transform the concentration to gamma dose rate in our next study, because the measurements are usually in the form of gamma dose rate during nuclear accident. The method should also be validated with real measurement data, including the data from real nuclear accidents and atmospheric dispersion experiments.

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REFERENCES

1. Drews, M., Lauritzen, B., Madsen, H. and Smith (2004) Kalman Filtration of Radiation Monitoring Data from Atmospheric Dispersion of Radioactive Materials. *Radiation Protection Dosimetry* 111(3): 257-269.
2. Evensen, G. (1994) Sequential Data Assimilation with a Nonlinear Quasi-Geostrophic Model Using Monte-Carlo Methods to Forecast Error Statistics. *Journal of Geophysical Research-Oceans* 99(C5): 10143-10162.
3. Evensen, G. (2003) The Ensemble Kalman Filter: Theoretical Formulation and Practical Implementation. *Ocean Dynamics* 53(4): 343-367.
4. Grewal, M.S. and Andrews, A.P. (1993) Kalman Filtering Theory and Practise, Prentice Hall.
5. Krysta, M., Bocquet, M., Sportisse, B. and Isnard, O. (2006) Data Assimilation for Short-Range Dispersion of Radionuclides: an Application to Wind Tunnel Data. *Atmospheric Environment* 40(38): 7267-7279.
6. Kalnay, E. (2003) Atmospheric modeling, data assimilation and predictability, Cambridge University Press.
7. Rojas-Palma, C., Aage, H. K., Astrup, P., Bargholz, K., Drews, M., et al. (2004) Experimental Evaluation of Gamma Fluence-rate Predictions from Argon-41 Releases to the Atmosphere over a Nuclear Research Reactor Site. *Radiation Protection Dosimetry* 108(2): 161-168.
8. Rojas-Palma, C., Madsen, H., Gering, F., Puch, R., Turcanu, C., et al. (2003) Data Assimilation in the Decision Support System RODOS. *Radiation Protection Dosimetry* 104(1): 31-40.
9. Stockie, J.M. (2011) The Mathematics of Atmospheric Dispersion Modeling, *SIAM Review.* 53(2): 349-372.
10. Zheng, D. Q., Leung, J. K. C., Lee, B. Y. and Lam, H. Y. (2007) Data Assimilation in the Atmospheric Dispersion Model for Nuclear Accident Assessments. *Atmospheric Environment* 41(11): 2438-2446.