Defeasible Inclusions in Low-Complexity DLs: Preliminary Notes

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Abstract

We analyze the complexity of reasoning with circumscribed low-complexity DLs such as DL-lite and the $\mathcal{EL}$ family, under suitable restrictions on the use of abnormality predicates. We prove that in circumscribed DL-lite$_R$ complexity drops from NExp$^{\mathrm{NP}}$ to the second level of the polynomial hierarchy. In $\mathcal{EL}$, reasoning remains ExpTime-hard, in general. However, by restricting the possible occurrences of existential restrictions, we obtain membership in Σ_2^p and Π_2^p for an extension of $\mathcal{EL}$.

1 Introduction

The ample literature on nonmonotonic extensions of description logics (DLs) witnesses a long-standing interest for this topic (for some early approaches see [Brewka, 1987; Straccia, 1993; Baader and Hollunder, 1995]). Recently, fresh motivations came from the construction of ontologies for biomedical domains (cf. [Rector, 2004; Stevens *et al.*, 2007]) and from the use of description logics as policy languages [Uszok *et al.*, 2004; Kagal *et al.*, 2003; Tonti *et al.*, 2003] where nonmonotonic reasoning is needed to properly encode default policies and authorization inheritance (cf. [Bonatti and Samarati, 2003]). Several recent works [Donini *et al.*, 1998; 1997; 2002; Bonatti *et al.*, 2006; Giordano *et al.*, 2008] improved our understanding of the complexity of nonmonotonic description logics based on default logic, autoepistemic logic, and circumscription. Unfortunately, nonmonotonic DLs are typically very complex. For example, reasoning with circumscribed $\mathcal{ALC}$ knowledge bases is NExp$^{\mathrm{NP}}$-hard [Bonatti *et al.*, 2006], and a tableaux calculus for reasoning with autoepistemic knowledge bases is in 3-ExpTime [Donini *et al.*, 2002]. Besides such complexity results, it turns out that some theoretical properties that are very important for the implementation of reasoning in "classical" DLs—such as the tree model property for example— do not carry over to nonmonotonic DLs.

Independently from the works on nonmonotonic DLs, low-complexity (monotonic) DLs of practical interest have been recently studied. Here we will focus on DL-lite$_R$ [Calvanese *et al.*, 2005] and the $\mathcal{EL}$ family [Baader, 2003; Baader *et al.*, 2005], whose inferences are in PTIME. The former is motivated by efficient query processing over large bodies of semantic web knowledge. The latter is interesting because it is spontaneously adopted in major biomedical ontologies. It is interesting to investigate whether the syntactic restrictions obeyed by such logics decrease the complexity of reasoning also in a nonmonotonic context.

In this paper, we identify less complex circumscribed DLs by (i) using the constructs supported by DL-lite$_R$ and by the $\mathcal{EL}$ family, and (ii) restricting the use of abnormality predicates by hiding them into "defeasible" inclusion axioms, similar to those adopted by [Straccia, 1993]. The latter restriction is also expected to make the formalism easier to use. Under such restrictions, we prove that (i) satisfiability checking for circumscribed knowledge bases (KB) is equivalent to classical KB satisfiability, and hence in P (sometimes even trivial) for the logics we consider here: DL-lite$_R$, $\mathcal{EL}$, and $\mathcal{EL}^\perp$; (ii) concept satisfiability, instance checking, and subsumption over circumscribed DL-lite$_R$ and *left local* $\mathcal{EL}^\perp$ KBs remain within the second level of the polynomial hierarchy; (iii) the same reasoning tasks for circumscribed $\mathcal{EL}^\perp$ KBs, unfortunately, remain ExpTime-hard.

Further related approaches are [Cadoli *et al.*, 1990; Straccia, 1993]. In [Cadoli *et al.*, 1990], a fragment of $\mathcal{ALE}$ under minimal entailment (an instance of circumscription where all predicates are minimized with the same priority) is proved to belong to Π_2^p. Our approach adopts different DLs and more general forms of circumscription, supporting priorities as well as fixed and variable predicates. In [Straccia, 1993] the underlying nonmonotonic logic is a prioritized version of default logic. The paper contains NP-hardness results for extremely simplified DLs.

The rest of the paper is organized as follows: In Section 2, we recall the basics of DLs. Section 3 introduces the specialized circumscription framework we adopt here. After some auxiliary results (Section 4), sections 5 and 6 illustrate the results on DL-lite$_R$ and the $\mathcal{EL}$ family, respectively. Section 7 concludes the paper with a summary of the results and some directions for future work.

2 Preliminaries

In DLs, *concepts* are inductively defined with a set of *constructors*, starting with a set $\mathsf{N_C}$ of *concept names*, a set $\mathsf{N_R}$ of *role names*, and (possibly) a set $\mathsf{N_I}$ of *individual names* (all countably infinite). We use the term *predicates* to refer to ele-

Name	Syntax	Semantics
inverse role	R^-	$(R^-)^\mathcal{I} = \{(d,e) \mid (e,d) \in R^\mathcal{I}\}$
nominal	$\{a\}$	$\{a^\mathcal{I}\}$
negation	$\neg C$	$\Delta^\mathcal{I} \setminus C^\mathcal{I}$
conjunction	$C \sqcap D$	$C^\mathcal{I} \cap D^\mathcal{I}$
existential restriction	$\exists R.C$	$\{d \in \Delta^\mathcal{I} \mid \exists (d,e) \in R^\mathcal{I} : e \in C^\mathcal{I}\}$
top	$\top$	$\top^\mathcal{I} = \Delta^\mathcal{I}$
bottom	$\bot$	$\bot^\mathcal{I} = \emptyset$

Figure 1: Syntax and semantics of some DL constructs

ments of $\mathsf{N_C} \cup \mathsf{N_R}$. Hereafter, letters A and B will range over $\mathsf{N_C}$, P will range over $\mathsf{N_R}$, and a, b, c will range over $\mathsf{N_I}$. The concepts of the DLs dealt with in this paper are formed using the constructors shown in Figure 1. There, the inverse role constructor is the only role constructor, whereas the remaining constructors are concept constructors. Letters C, D will range over concepts and letters R, S over (possibly inverse) roles.

The semantics of the above concepts is defined in terms of *interpretations* $\mathcal{I} = (\Delta^\mathcal{I}, \cdot^\mathcal{I})$. The *domain* $\Delta^\mathcal{I}$ is a non-empty set of individuals and the *interpretation function* $\cdot^\mathcal{I}$ maps each concept name $A \in \mathsf{N_C}$ to a set $A^\mathcal{I} \subseteq \Delta^\mathcal{I}$, each role name $r \in \mathsf{N_R}$ to a binary relation $r^\mathcal{I}$ on $\Delta^\mathcal{I}$, and each individual name $a \in \mathsf{N_I}$ to an individual $a^\mathcal{I} \in \Delta^\mathcal{I}$. The extension of $\cdot^\mathcal{I}$ to inverse roles and arbitrary concepts is inductively defined as shown in the third column of Figure 1. An interpretation $\mathcal{I}$ is called a *model* of a concept C if $C^\mathcal{I} \neq \emptyset$. If $\mathcal{I}$ is a model of C, we also say that C is *satisfied* by $\mathcal{I}$.

A *(strong) knowledge base* is a finite set of (i) *concept inclusions (CIs)* $C \sqsubseteq D$ where C and D are concepts, (ii) *concept assertions* $A(a)$ and *role assertions* $P(a,b)$, where a, b are individual names, $P \in \mathsf{N_R}$, and $A \in \mathsf{N_C}$, (iii) *role inclusions (RIs)* $R \sqsubseteq R'$. An interpretation $\mathcal{I}$ *satisfies* (i) a CI $C \sqsubseteq D$ if $C^\mathcal{I} \subseteq D^\mathcal{I}$, (ii) an assertion $C(a)$ if $a^\mathcal{I} \in C^\mathcal{I}$, (iii) an assertion $R(a,b)$ if $(a^\mathcal{I}, b^\mathcal{I}) \in r^\mathcal{I}$, and (iv) a RI $R \sqsubseteq R'$ iff $R^\mathcal{I} \subseteq R'^\mathcal{I}$. Then, $\mathcal{I}$ is a *model* of a strong knowledge base $\mathcal{S}$ iff $\mathcal{I}$ satisfies all the elements of $\mathcal{S}$.

We write $C \sqsubseteq_\mathcal{S} D$ iff for all models $\mathcal{I}$ of $\mathcal{S}$, $\mathcal{I}$ satisfies $C \sqsubseteq D$.

The logic *DL-lite$_R$* [Calvanese *et al.*, 2005] restricts concept inclusions to expressions $C_L \sqsubseteq C_R$, where

$$C_L ::= A \mid \exists R \qquad R ::= P \mid P^- \qquad C_R ::= C_L \mid \neg C_L$$

(as usual, $\exists R$ abbreviates $\exists R.\top$).

The logic $\mathcal{EL}$ [Baader, 2003; Baader *et al.*, 2005] restricts knowledge bases to assertions and concept inclusions built from the following constructs:

$$C ::= A \mid \top \mid C_1 \sqcap C_2 \mid \exists P.C$$

(note that inverse roles are not supported). The extension of $\mathcal{EL}$ with $\bot$, role hierarchies, and nominals (respectively) are denoted by $\mathcal{EL}^\bot$, $\mathcal{ELH}$, and $\mathcal{ELO}$. Combinations are allowed: for example $\mathcal{ELHO}$ denotes the extension of $\mathcal{EL}$ supporting role hierarchies and nominals. Finally, $\mathcal{EL}^{\neg A}$ denotes the extension where negation can be applied to concept names.

3 Defeasible knowledge

A *defeasible inclusion* (DI) is an expression $A \sqsubseteq_n C$ whose intended meaning is: *A's elements are normally in C*.

A *defeasible knowledge base* (DKB) in a logic $\mathcal{DL}$ is a pair $(\mathcal{S}, \mathcal{D})$ where $\mathcal{S}$ is a strong $\mathcal{DL}$ knowledge base, and $\mathcal{D}$ is a set of DIs $A \sqsubseteq_n C$ such that C is a $\mathcal{DL}$ concepts.

Example 3.1 The sentences: *"in humans, the heart is usually located on the left-hand side of the body; in humans with situs inversus, the heart is located on the right-hand side of the body"* [Rector, 2004; Stevens *et al.*, 2007] can be formulated with the following $\mathcal{EL}^\bot$ inclusions

$$\mathtt{Human} \sqsubseteq_n \exists\mathtt{has\_heart}.\exists\mathtt{has\_position}.\mathtt{Left}\,;$$
$$\mathtt{Situs\_Inversus} \sqsubseteq \exists\mathtt{has\_heart}.\exists\mathtt{has\_position}.\mathtt{Right}\,;$$
$$\exists\mathtt{has\_heart}.\exists\mathtt{has\_position}.\mathtt{Left} \sqcap \exists\mathtt{has\_heart}.\exists\mathtt{has\_position}.\mathtt{Right} \sqsubseteq \bot\,.$$ ∎

Intuitively, a model of $(\mathcal{S}, \mathcal{D})$ is a model of $\mathcal{S}$ that maximizes the set of individuals satisfying the defeasible inclusions in $\mathcal{D}$, resolving conflicts by means of specificity whenever possible.

In order to formalize this idea, we first have to specify how DIs are prioritized. We determine specificity based on classically valid inclusions. For all DIs $\delta_1 = (A_1 \sqsubseteq_n C_1)$ and $\delta_2 = (A_2 \sqsubseteq_n C_2)$, we write

$$\delta_1 \prec_\mathcal{S} \delta_2 \text{ iff } A_1 \sqsubseteq_\mathcal{S} A_2 \text{ and } A_2 \not\sqsubseteq_\mathcal{S} A_1\,.$$

For the sake of readability, the subscript $\mathcal{S}$ will be omitted when clear from context.

Second, we have to specify how to deal with the predicates occurring in the knowledge base: is their extension allowed to vary in order to satisfy defeasible inclusions? A discussion of the effects of letting predicates vary vs. fixing their extension can be found in [Bonatti *et al.*, 2006]; they conclude that the appropriate choice is application dependent. Here we let roles vary to avoid undecidability problems (cf. [Bonatti *et al.*, 2006]). The set of concept names $\mathsf{N_C}$, on the contrary, can be arbitrarily partitioned into two sets F and V containing fixed and varying predicates, respectively; we denote this semantics with Circ_F.

The set F, the DIs $\mathcal{D}$, and their ordering $\prec$ induce a strict partial order over interpretations, defined below. As we move down the ordering we find interpretations that are more and more normal w.r.t. $\mathcal{D}$. For all $\delta = (A \sqsubseteq_n C)$ and all interpretations $\mathcal{I}$ let the set of individuals *satisfying* δ be:

$$\mathsf{sat}_\mathcal{I}(\delta) = \{x \in \Delta^\mathcal{I} \mid x \notin A^\mathcal{I} \text{ or } x \in C^\mathcal{I}\}\,.$$

Definition 3.2 For all interpretations $\mathcal{I}$ and $\mathcal{J}$, and all $F \subseteq \mathsf{N_C}$, let $\mathcal{I} <_{\mathcal{D},F} \mathcal{J}$ iff:

1. $\Delta^\mathcal{I} = \Delta^\mathcal{J}$;
2. $a^\mathcal{I} = a^\mathcal{J}$, for all $a \in \mathsf{N_I}$;
3. $A^\mathcal{I} = A^\mathcal{J}$, for all $A \in F$;
4. for all $\delta \in \mathcal{D}$, if $\mathsf{sat}_\mathcal{I}(\delta) \not\supseteq \mathsf{sat}_\mathcal{J}(\delta)$ then there exists $\delta' \in \mathcal{D}$ such that $\delta' \prec \delta$ and $\mathsf{sat}_\mathcal{I}(\delta') \supset \mathsf{sat}_\mathcal{J}(\delta')$;
5. there exists a $\delta \in \mathcal{D}$ such that $\mathsf{sat}_\mathcal{I}(\delta) \supset \mathsf{sat}_\mathcal{J}(\delta)$.

The subscript $\mathcal{D}$ will be omitted when clear from context.

Definition 3.3 [Model] Let $\mathcal{KB} = (\mathcal{S}, \mathcal{D})$ and $F \subseteq \mathsf{N_C}$. An interpretation $\mathcal{I}$ is a *model* of $\mathsf{Circ}_F(\mathcal{KB})$ iff $\mathcal{I}$ is a (classical) model of $\mathcal{S}$ and for all models $\mathcal{J}$ of $\mathcal{S}$, $\mathcal{J} \not<_F \mathcal{I}$.

Remark 3.4 This semantics is a special case of the circumscribed DLs of [Bonatti *et al.*, 2006]. The correspondence can be seen by (i) introducing for each DI $A \sqsubseteq_n C$ a fresh atomic concept Ab, playing the role of an abnormality predicate; (ii) replacing $A \sqsubseteq_n C$ with $A \sqcap \neg Ab \sqsubseteq C$; (iii) minimizing the predicates Ab introduced above according to the priorities over defeasible inclusions.

In order to enhance readability, we will use the following notation for the special cases in which all concept names are varying and the case in which they are all fixed: $<_{\mathsf{var}}$ and $\mathsf{Circ}_{\mathsf{var}}$ stand for $<_\emptyset$ and $\mathsf{Circ}_\emptyset$, respectively; $<_{\mathsf{fix}}$ and $\mathsf{Circ}_{\mathsf{fix}}$ stand respectively for $<_{\mathsf{N_C}}$ and $\mathsf{Circ}_{\mathsf{N_C}}$.

In this paper, we consider the following standard reasoning tasks over defeasible DLs:

Knowledge base consistency Given a DKB $\mathcal{KB}$, decide whether $\mathsf{Circ}_F(\mathcal{KB})$ has a model.

Concept consistency Given a concept C and a DKB $\mathcal{KB}$, check whether C is *satisfiable w.r.t.* $\mathcal{KB}$, that is, there exists a model $\mathcal{I}$ of $\mathsf{Circ}_F(\mathcal{KB})$ such that $C^\mathcal{I} \neq \emptyset$.

Subsumption Given two concepts C, D and a DKB $\mathcal{KB}$, check whether $\mathsf{Circ}_F(\mathcal{KB}) \models C \sqsubseteq D$, that is, for all models $\mathcal{I}$ of $\mathsf{Circ}_F(\mathcal{KB})$, $C^\mathcal{I} \subseteq D^\mathcal{I}$.

Instance checking Given $a \in \mathsf{N_I}$, a concept C, and a DKB $\mathcal{KB}$, check whether $\mathsf{Circ}_F(\mathcal{KB}) \models C(a)$, that is, for all models $\mathcal{I}$ of $\mathsf{Circ}_F(\mathcal{KB})$, $a^\mathcal{I} \in C^\mathcal{I}$.

We conclude this section with an example taken from [Bonatti *et al.*, 2006].

Example 3.5 The following inclusions model a policy with authorization inheritance and multiple overridings:

$\texttt{User} \sqsubseteq_n \neg\exists\texttt{hasAccessTo.ConfidentialFile}$
$\texttt{Staff} \sqsubseteq \texttt{User}$
$\texttt{Staff} \sqsubseteq_n \exists\texttt{hasAccessTo.ConfidentialFile}$
$\texttt{BlacklistedStaff} \sqsubseteq \texttt{Staff} \sqcap \neg\exists\texttt{hasAccessTo.ConfidentialFile}$.

Let $\mathcal{S}$ contain the second and fourth inclusions plus the assertion Staff(John), and let $\mathcal{D}$ consist of the first and third inclusions. Let $\mathcal{KB} = (\mathcal{S}, \mathcal{D})$. Due to the second inclusion, the DI for Staff has greater priority than the DI for User. In all models of $\mathsf{Circ}_{\mathsf{var}}(\mathcal{KB})$, John belongs to ∃hasAccessTo.ConfidentialFile and not to BlacklistedStaff. On the contrary, there exist models of $\mathsf{Circ}_{\mathsf{fix}}(\mathcal{KB})$ where John does not belong to ∃hasAccessTo.ConfidentialFile because John belongs to BlacklistedStaff and $\mathsf{Circ}_{\mathsf{fix}}$ does not allow to change the extension of BlacklistedStaff to satisfy the DI for Staff. ∎

4 Auxiliary results

The logics we deal with enjoy the finite model property.

Lemma 4.1 *Let $\mathcal{KB} = (\mathcal{S}, \mathcal{D})$ be a DKB in DL-lite$_R$ or $\mathcal{ELHO}^{\bot,\neg}$. For all $F \subseteq \mathsf{N_C}$, $\mathsf{Circ}_F(\mathcal{KB})$ has a model only if $\mathsf{Circ}_F(\mathcal{KB})$ has a finite model whose size is exponential in the size of $\mathcal{KB}$.*

Proof. A simple adaptation of a result for $\mathcal{ALCIO}$ [Bonatti *et al.*, 2006], taking role hierarchies into account. ∎

As a consequence, these logics preserve classical consistency (because all $<_{\mathcal{D},F}$-descending chains of models originating from a finite model must be finite):

Theorem 4.2 *Let $\mathcal{KB} = (\mathcal{S}, \mathcal{D})$ be a DKB in in DL-lite$_R$ or $\mathcal{ELHO}^{\bot,\neg}$. For all $F \subseteq \mathsf{N_C}$, $\mathcal{S}$ is (classically) consistent iff $\mathsf{Circ}_F(\mathcal{KB})$ has a model.*

Under very mild assumptions, Circ_F and $\mathsf{Circ}_{\mathsf{fix}}$ (which is a special case of the former) are equally expressive.

Theorem 4.3 *If $\mathcal{DL}$ is a description logic supporting unqualified existential restrictions ($\exists R$), then concept consistency, subsumption, and instance checking in $\mathsf{Circ}_F(\mathcal{DL})$ can be reduced in polynomial time to concept consistency, subsumption, and instance checking (respectively) in $\mathsf{Circ}_{\mathsf{fix}}(\mathcal{DL})$.*

The idea behind the proof is simple: Let $\mathcal{KB}$ be any given DKB. Introduce a new role name R_A for each (variable) concept name $A \notin F$. Then replace each occurrence of any $A \notin F$ with $\exists R_A$. The details of the proof are omitted here for space limitations.

5 Complexity of circumscribed DL-lite$_R$

In this section we focus on DL-lite$_R$ DKBs $(\mathcal{S}, \mathcal{D})$ that consist in a DL-lite$_R$ KB $\mathcal{S}$ and a set $\mathcal{D}$ of inclusions $A \sqsubseteq_n C$ such that $A \sqsubseteq C$ is a (classical) DL-lite$_R$ CI. Our complexity results for DL-lite$_R$ rely on the possibility of extracting a small (polynomial-size) model from any model of a circumscribed DKB. We start with $\mathsf{Circ}_{\mathsf{var}}$:

Lemma 5.1 *Let $\mathcal{KB}$ be a DL-lite$_R$ knowledge base. For all models $\mathcal{I}$ of $\mathsf{Circ}_{\mathsf{var}}(\mathcal{KB})$ and all $x \in \Delta^\mathcal{I}$ there exists a model $\mathcal{J}$ of $\mathsf{Circ}_{\mathsf{var}}(\mathcal{KB})$ such that (i) $\Delta^\mathcal{J} \subseteq \Delta^\mathcal{I}$, (ii) $x \in \Delta^\mathcal{J}$, (iii) for all DL-lite$_R$ concepts C, $x \in C^\mathcal{I}$ iff $x \in C^\mathcal{J}$, and (iv) $|\Delta^\mathcal{J}|$ is polynomial in the size of $\mathcal{KB}$.*

Proof. Assume that $\mathcal{KB} = (\mathcal{S}, \mathcal{D})$, $\mathcal{I}$ is a model of $\mathsf{Circ}_{\mathsf{var}}(\mathcal{KB})$, and $x \in \Delta^\mathcal{I}$. Let $\mathsf{cl}(\mathcal{KB})$ be the set of all concepts and individual names occurring in $\mathcal{KB}$. Choose a minimal set $\Delta \subseteq \Delta^\mathcal{I}$ containing: (i) x, (ii) all $a^\mathcal{I}$ such that $a \in \mathsf{N_I} \cap \mathsf{cl}(\mathcal{KB})$, (iii) for each concept $\exists R$ in $\mathsf{cl}(\mathcal{KB})$ satisfied in $\mathcal{I}$, a node y_R such that for some $z \in \exists R^\mathcal{I}$, $(z, y_R) \in R^\mathcal{I}$.

Now define $\mathcal{J}$ as follows: (i) $\Delta^\mathcal{J} = \Delta$, (ii) $a^\mathcal{J} = a^\mathcal{I}$ (for $a \in \mathsf{N_I} \cap \mathsf{cl}(\mathcal{KB})$), (iii) $A^\mathcal{J} = A^\mathcal{I} \cap \Delta$ ($A \in \mathsf{N_C} \cap \mathsf{cl}(\mathcal{KB})$), and (iv) $P^\mathcal{J} = \{(z, y_P) \mid z \in \Delta \text{ and } z \in \exists P^\mathcal{I}\} \cup \{(y_P, z) \mid z \in \Delta \text{ and } z \in \exists P^{-\mathcal{I}}\}$ ($P \in \mathsf{N_R}$).

Note that by construction, for all $z \in \Delta^\mathcal{J}$ and for all $C \in \mathsf{cl}(\mathcal{KB})$, $z \in C^\mathcal{J}$ iff $z \in C^\mathcal{I}$; consequently, $\mathcal{J}$ is a classical model of $\mathcal{S}$. Moreover, the cardinality of $\Delta^\mathcal{J}$ is linear in the size of $\mathcal{KB}$ (by construction). So we are only left to show that $\mathcal{J}$ is a $<_{\mathcal{D},\mathsf{var}}$-minimal model of $\mathcal{KB}$.

Suppose not, and consider any $\mathcal{J}' <_{\mathcal{D},\mathsf{var}} \mathcal{J}$. Define $\mathcal{I}'$ as follows: (i) $\Delta^{\mathcal{I}'} = \Delta^\mathcal{I}$, (ii) $a^{\mathcal{I}'} = a^\mathcal{I}$, (iii) $A^{\mathcal{I}'} = A^{\mathcal{J}'}$, (iv) $P^{\mathcal{I}'} = P^{\mathcal{J}'}$. Note that the elements in $\Delta^\mathcal{I} \setminus \Delta^{\mathcal{J}'}$ satisfy no left-hand side of any DL-lite$_R$ inclusion (be it classical or defeasible), therefore all inclusions are vacuously satisfied. Moreover, the restriction of $\mathcal{I}'$ to $\Delta^{\mathcal{J}'}$ is $<_{\mathcal{D},\mathsf{var}}$-smaller than the corresponding restriction of $\mathcal{I}$ in the interpretation ordering. It follows that $\mathcal{I}' <_{\mathcal{D},\mathsf{var}} \mathcal{I}$, and hence $\mathcal{I}$ cannot be a model of $\mathsf{Circ}_{\mathsf{var}}(\mathcal{KB})$ (a contradiction). ∎

The above proof can be refined and adapted to $\mathsf{Circ}_{\mathsf{fix}}$.

Lemma 5.2 *Let $\mathcal{KB}$ be a DL-lite$_R$ knowledge base. For all models $\mathcal{I}$ of $\mathsf{Circ}_{\mathsf{fix}}(\mathcal{KB})$ and all $x \in \Delta^{\mathcal{I}}$ there exists a model $\mathcal{J}$ of $\mathsf{Circ}_{\mathsf{fix}}(\mathcal{KB})$ such that (i) $\Delta^{\mathcal{J}} \subseteq \Delta^{\mathcal{I}}$, (ii) $x \in \Delta^{\mathcal{J}}$, (iii) for all DL-lite$_R$ concepts C, $x \in C^{\mathcal{I}}$ iff $x \in C^{\mathcal{J}}$ (iv) $|\Delta^{\mathcal{J}}|$ is polynomial in the size of $\mathcal{KB}$.*

Proof. We will employ a refined definition of Δ. It should be a $\subseteq$-minimal set containing: (i) x, (ii) all $a^{\mathcal{I}}$ such that $a \in \mathsf{N_I} \cap \mathsf{cl}(\mathcal{KB})$, (iii) for each concept $\exists R$ in $\mathsf{cl}(\mathcal{KB})$ satisfied in $\mathcal{I}$, a node y_R such that $y_R \in (\exists R^-)^{\mathcal{I}}$, and finally (iv) for all inclusions $C \sqsubseteq \exists R$ or $C \sqsubseteq_n \exists R$ in $\mathcal{KB}$ such that $(C \sqcap \exists R)^{\mathcal{I}} \neq \emptyset$, a node $z \in (C \sqcap \exists R)^{\mathcal{I}}$.

Define $\mathcal{J}$ as in the previous lemma, using the above Δ. Recall that for all $z \in \Delta^{\mathcal{J}}$ and for all $C \in \mathsf{cl}(\mathcal{KB})$, $z \in C^{\mathcal{J}}$ iff $z \in C^{\mathcal{I}}$; consequently, $\mathcal{J}$ is a classical model of $\mathcal{S}$. Moreover, the cardinality of $\Delta^{\mathcal{J}}$ is linear in the size of $\mathcal{KB}$ (by construction). So we are only left to show that $\mathcal{J}$ is a $<_{\mathcal{D},\mathsf{fix}}$-minimal model of $\mathcal{KB}$.

Suppose not, and consider any $\mathcal{J}' <_{\mathcal{D},\mathsf{fix}} \mathcal{J}$. Define $\mathcal{I}'$ as follows: (a) $\Delta^{\mathcal{I}'} = \Delta^{\mathcal{I}}$, (b) $a^{\mathcal{I}'} = a^{\mathcal{I}}$, (c) $A^{\mathcal{I}'} = A^{\mathcal{I}}$, (d) each $R^{\mathcal{I}'}$ is a minimal set such that (d1) $R^{\mathcal{I}'} \supseteq R^{\mathcal{J}'}$, (d2) for all $z \in \Delta^{\mathcal{I}} \setminus \Delta^{\mathcal{J}}$, and for all inclusions $C \sqsubseteq \exists R$ or $C \sqsubseteq_n \exists R$ in $\mathcal{KB}$ such that $z \in (C \sqcap \exists R)^{\mathcal{I}}$, if $R^{\mathcal{J}'}$ contains a pair (v, w), then $(z, w) \in \mathcal{R}^{\mathcal{I}'}$; finally, (d3) each $P^{\mathcal{I}'}$ is closed under the role inclusion axioms of $\mathcal{KB}$. Note that, by construction,

(*) for all $z \in \Delta^{\mathcal{I}'} \setminus \Delta^{\mathcal{J}'}$, $z \in \exists R^{\mathcal{I}'}$ only if $z \in \exists R^{\mathcal{I}}$;

(**) for all $z \in \Delta^{\mathcal{I}'} \setminus \Delta^{\mathcal{J}'}$, $z \in \exists R^{\mathcal{I}'}$ only if there exists $v \in \Delta^{\mathcal{J}'}$ such that $v \in \exists R^{\mathcal{J}'}$.

Now we prove that $\mathcal{I}'$ is a model of the CIs of $\mathcal{KB}$. By construction, the edges (z, w) introduced in (d2) do not change the set of existential restrictions satisfied by the members of $\Delta^{\mathcal{J}'}$; as a consequence—and since $\mathcal{J}'$ is a model of $\mathcal{KB}$—the members of $\Delta^{\mathcal{J}'}$ satisfy all the CIs of $\mathcal{KB}$.

Now consider an arbitrary element $z \in \Delta^{\mathcal{I}'} \setminus \Delta^{\mathcal{J}'}$ and any CI γ of $\mathcal{KB}$. If γ is $\exists$-free, then $\mathcal{I}$ and $\mathcal{I}'$ give the same interpretation to γ by definition, therefore z satisfies γ. If γ is $\exists R \sqsubseteq A$, $\exists R \sqsubseteq \neg A$, $\exists R \sqsubseteq \neg\exists S$, or $A \sqsubseteq \neg\exists R$ (and considering that $\mathcal{I}$ satisfies γ) z fails to satisfy γ only if for some $R' \in \{R, S\}$, $z \notin (\exists R')^{\mathcal{I}}$ and $z \in (\exists R')^{\mathcal{I}'}$; this is impossible by (*). Next, suppose γ is $\exists R \sqsubseteq \exists S$. If $z \in (\exists R)^{\mathcal{I}'}$, then by (**) there exists a $v \in \Delta^{\mathcal{J}'}$ satisfying $(\exists R)^{\mathcal{J}'}$ and hence $(\exists S)^{\mathcal{J}'}$ (as $\mathcal{J}'$ is a model of $\mathcal{KB}$), therefore $z \in (\exists S)^{\mathcal{I}'}$ (by d2). We are only left to consider $\gamma = A \sqsubseteq \exists R$: If $z \in A^{\mathcal{I}'} = A^{\mathcal{I}}$, then there exists $w_A \in A^{\mathcal{J}'}$ (by construction of Δ). Then $z \in (\exists R)^{\mathcal{I}'}$ (by d2). Therefore, in all possible cases, z satisfies γ.

This proves that $\mathcal{I}'$ satisfies all the CIs of $\mathcal{KB}$. It is not hard to verify that $\mathcal{I}'$ satisfies also all role inclusions of $\mathcal{KB}$. Therefore, in order to derive a contradiction, we are left to prove that $\mathcal{I}' <_{\mathcal{D},\mathsf{fix}} \mathcal{I}$ (which implies that $\mathcal{I}$ is not a model of $\mathsf{Circ}_{\mathsf{fix}}(\mathcal{KB})$).

Claim: For all $\delta \in \mathcal{D}$, if $\mathsf{sat}_{\mathcal{J}}(\delta) \subseteq \mathsf{sat}_{\mathcal{J}'}(\delta)$, then $\mathsf{sat}_{\mathcal{I}}(\delta) \subseteq \mathsf{sat}_{\mathcal{I}'}(\delta)$.

Suppose $\mathsf{sat}_{\mathcal{J}}(\delta) \subseteq \mathsf{sat}_{\mathcal{J}'}(\delta)$. It suffices to prove that for all $z \in \Delta^{\mathcal{I}'} \setminus \Delta^{\mathcal{J}'}$, if $z \in \mathsf{sat}_{\mathcal{I}}(\delta)$ then $z \in \mathsf{sat}_{\mathcal{I}'}(\delta)$.

In all cases but those in which the right-hand side of δ is $\exists R$, the proof is similar to the proof for CIs (it exploits (*) and the fact that all atomic concepts are fixed).

Finally, let δ be $A \sqsubseteq_n \exists R$ and consider an arbitrary $z \in \Delta^{\mathcal{I}'} \setminus \Delta^{\mathcal{J}'}$ such that $z \in \mathsf{sat}_{\mathcal{I}}(\delta)$ and $z \in (A \sqcap \exists R)^{\mathcal{I}}$. By (iv), Δ contains a $v \in (A \sqcap \exists R)^{\mathcal{I}}$, and hence $\Delta^{\mathcal{J}'}$ contains a $v \in (A \sqcap \exists R)^{\mathcal{J}'}$; consequently, by (d2), $z \in (\exists R)^{\mathcal{I}'}$ and hence $z \in \mathsf{sat}_{\mathcal{I}'}(\delta)$. This completes the proof of the claim.

Now, $\mathcal{I}' <_{\mathcal{D},\mathsf{fix}} \mathcal{I}$ follows as a straightforward consequence of the Claim. ∎

Theorem 5.3 *Concept consistency over circumscribed DL-lite$_R$ DKBs is in Σ_2^p. Subsumption and instance checking over circumscribed DL-lite$_R$ DKBs are in Π_2^p.*

Proof. (Sketch) By the above lemmas, it suffices to guess a polynomial model $\mathcal{I}$ of the KB that proves consistency or disproves subsumption/instance checking. Then, with an NP oracle, one can check that $\mathcal{I}$ is minimal w.r.t. $<_{\mathsf{var}}$ or $<_{\mathsf{fix}}$. ∎

6 Circumscribing the $\mathcal{EL}$ family

In $\mathcal{ELHO}$, that cannot express any contradictions, defeasible inclusions cannot be possibly blocked under $\mathsf{Circ}_{\mathsf{var}}$, and circumscription collapses to classical reasoning:

Theorem 6.1 *Let $\mathcal{KB} = (\mathcal{S}, \mathcal{D})$ be an $\mathcal{ELHO}$ DKB. Then $\mathcal{I}$ is a model of $\mathsf{Circ}_{\mathsf{var}}(\mathcal{KB})$ iff $\mathcal{I}$ is a model of $\mathcal{S} \cup \hat{\mathcal{D}}$, where $\hat{\mathcal{D}} = \{A \sqsubseteq C \mid (A \sqsubseteq_n C) \in \mathcal{D}\}$.*

By the results of [Baader *et al.*, 2005], it follows that in $\mathsf{Circ}_{\mathsf{var}}(\mathcal{ELHO})$, concept satisfiability is trivial, subsumption and instance checking are in P.

If we make $\mathcal{EL}$ more interesting by adding $\bot$ as a source of inconsistency, then complexity increases significantly.

Theorem 6.2 *In $\mathsf{Circ}_{\mathsf{var}}(\mathcal{EL}^{\bot})$, concept satisfiability, instance checking, and subsumption are ExpTime-hard. These results still holds if knowledge bases contain no assertion.*[1]

Proof. (Sketch) We first reduce TBox satisfiability in $\mathcal{EL}^{\neg A}$ (which is known to be ExpTime-hard [Baader *et al.*, 2005]) to the complement of subsumption in $\mathsf{Circ}_{\mathsf{var}}(\mathcal{EL}^{\bot})$. Let $\mathcal{T}$ be a TBox (i.e., a set of CIs) in $\mathcal{EL}^{\neg A}$. First introduce for each concept name A occurring in $\mathcal{T}$ a fresh concept name $\bar{A}$ whose intended meaning is $\neg A$. Obtain $\mathcal{T}'$ from $\mathcal{T}$ by replacing each literal $\neg A$ with $\bar{A}$. Let $\mathcal{KB}$ be the DKB obtained by extending $\mathcal{T}'$ with the following inclusions, where U and U_A — for all A occurring in $\mathcal{T}$— are fresh concept names (representing undefined truth values), and R is a fresh role name:

$$
\begin{array}{rcll@{\qquad}rcll}
A \sqcap \bar{A} & \sqsubseteq & \bot & (1) & \top & \sqsubseteq_n & A & (5)\\
A \sqcap U_A & \sqsubseteq & \bot & (2) & \top & \sqsubseteq_n & \bar{A} & (6)\\
\bar{A} \sqcap U_A & \sqsubseteq & \bot & (3) & \top & \sqsubseteq_n & U_A & (7)\\
U_A & \sqsubseteq & U & (4) & \top & \sqsubseteq_n & \exists R.U_A & (8)\\
 & & & & U & \sqsubseteq_n & U_A & (9)
\end{array}
$$

[1] Equivalently, in DL's terminology: *ABoxes are empty*.

It can be verified that $\mathcal{T}$ is satisfiable iff in some model of $\mathsf{Circ}_{\mathsf{var}}(\mathcal{KB})$ all U_A are empty, which holds iff $\mathsf{Circ}_{\mathsf{var}}(\mathcal{KB}) \not\models \top \sqsubseteq \exists R.U$. Consequently, subsumption in $\mathsf{Circ}_{\mathsf{var}}(\mathcal{EL}^{\bot})$ is ExpTime-hard.

Similarly, for any given $a \in \mathsf{N_I}$, $\mathcal{T}$ is satisfiable iff there exists a model $\mathcal{I}$ of $\mathsf{Circ}_{\mathsf{var}}(\mathcal{KB})$ such that $a^{\mathcal{I}} \notin (\exists R.U)^{\mathcal{I}}$. Therefore, instance checking in $\mathsf{Circ}_{\mathsf{var}}(\mathcal{EL}^{\bot})$ is ExpTime-hard as well.

Finally, add a fresh concept name B and all the inclusions $B \sqcap \exists R.U_A \sqsubseteq \bot$; call the new DKB $\mathcal{KB}'$. Note that $\mathcal{T}$ is satisfiable iff in some model of $\mathsf{Circ}_{\mathsf{var}}(\mathcal{KB})$ all U_A are empty, which holds iff B is satisfiable w.r.t. $\mathsf{Circ}_{\mathsf{var}}(\mathcal{KB}')$. Consequently, concept satisfiability in $\mathsf{Circ}_{\mathsf{var}}(\mathcal{EL}^{\bot})$ is ExpTime-hard. ∎

Since $\mathsf{Circ}_{\mathsf{var}}$ is a special case of Circ_F, and by Theorem 4.3, the above theorem applies to Circ_F and $\mathsf{Circ}_{\mathsf{fix}}$, too:

Corollary 6.3 *For $X = F, \mathsf{fix}$, concept satisfiability checking, instance checking, and subsumption in $\mathsf{Circ}_X(\mathcal{EL}^{\bot})$ are ExpTime-hard. These results still hold if ABoxes are empty (i.e. assertions are not allowed).*

The above proof can be adapted to $\mathsf{Circ}_F(\mathcal{EL})$. First we have to introduce a new concept name D representing $\top$ and translate each concept C into C^* as follows:

- $C^* = C$ if C is a concept name;
- $C^* = \bar{A}$ if C is $\neg A$ (for all A, $\bar{A}$ is a new concept name);
- $C^* = D \sqcap \exists R.(C_1^* \sqcap D)$ if C is $\exists R.C_1$;
- $C^* = C_1^* \sqcap C_2^*$ if C is $C_1 \sqcap C_2$.

Each $C_1 \sqsubseteq C_2$ in $\mathcal{T}$ is translated into $C_1^* \sqsubseteq C_2^*$. Then we extend the translated TBox with the following inclusions, where Bot (representing $\bot$), all U_A, and Bad are new concept names and R is a new role name:

$A \sqsubseteq D$	(10)	$D \sqsubseteq_n U_A$	(18)
$\bar{A} \sqsubseteq D$	(11)	$D \sqsubseteq D'$	(19)
$U_A \sqsubseteq D$	(12)	$D' \sqsubseteq_n \exists R.U_A$	(20)
$A \sqcap \bar{A} \sqsubseteq Bot$	(13)	$D' \sqsubseteq_n \exists R.Bot$	(21)
$A \sqcap U_A \sqsubseteq Bot$	(14)	$\exists R.U_A \sqsubseteq Bad$	(22)
$\bar{A} \sqcap U_A \sqsubseteq Bot$	(15)	$\exists R.Bot \sqsubseteq Bad$	(23)
$D \sqsubseteq_n A$	(16)		
$D \sqsubseteq_n \bar{A}$	(17)	$D(a)$ (ABox assertion)	(24)

Let $\mathcal{KB}$ be the resulting DKB. Finally set $F = \{D, Bot\}$. Now (24) guarantees that D is nonempty; the translation $(\cdot)^*$, (10) and (11) make sure that by restricting to D any model $\mathcal{I}$ of $\mathsf{Circ}_F(\mathcal{KB})$ where all U_A and Bot are empty one obtains a model of $\mathcal{T}$. Inclusion (19) gives (20) and (21) lowest priority. These two DIs and (22)-(23) include D' into Bad whenever the intended meaning of the atoms $\bar{A}$ is violated. Then $\mathcal{T}$ is satisfiable iff for some model $\mathcal{I}$ of $\mathsf{Circ}_F(\mathcal{KB})$, $Bad^{\mathcal{I}} = \emptyset$. In turn, this happens iff $\mathsf{Circ}_F(\mathcal{KB}) \not\models D \sqsubseteq Bad$, and iff $a^{\mathcal{I}} \notin Bad^{\mathcal{I}}$. Then we have the desired reduction from $\mathcal{EL}^{\neg A}$ TBox satisfiability to the complement of subsumption and instance checking in $\mathsf{Circ}_F(\mathcal{EL})$. As a consequence, and by Theorem 4.3:

Theorem 6.4 *Instance checking and subsumption are ExpTime-hard both in $\mathsf{Circ}_F(\mathcal{EL})$ and in $\mathsf{Circ}_{\mathsf{fix}}(\mathcal{EL})$. The same holds in the restriction of $\mathcal{EL}$ not supporting $\top$.*

Concept consistency is simpler, instead. Call an interpretation $\mathcal{I}$ *maximal* iff for all $A \in \mathsf{N_C}$, $A^{\mathcal{I}} = \Delta^{\mathcal{I}}$, and for all $P \in \mathsf{N_R}$, $P^{\mathcal{I}} = \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$. It is not hard to verify that all $\mathcal{ELHO}$ concepts and all $\mathcal{ELHO}$ inclusions (both classical and defeasible) are satisfied by all $x \in \Delta^{\mathcal{I}}$, therefore maximal models are always models of $\mathsf{Circ}_F(\mathcal{KB})$, for all DKBs $\mathcal{KB}$ and all $F \subseteq \mathsf{N_C}$. As a consequence we have that concept consistency is trivial:

Theorem 6.5 *For all $\mathcal{EL}$ concepts C, DKBs $\mathcal{KB}$, and $F \subseteq \mathsf{N_C}$, C is satisfied by some model of $\mathsf{Circ}_F(\mathcal{KB})$.*

One of the causes of the complexity of instance checking and subsumption for $\mathsf{Circ}_{\mathsf{fix}}(\mathcal{EL}^{\bot})$ is the ability of inferring consequences from qualified existential restrictions $\exists P.B$. By limiting their occurrences, it is possible to reduce significantly the complexity of instance checking and subsumption for $\mathsf{Circ}_{\mathsf{fix}}(\mathcal{EL}^{\bot})$ knowledge bases.

Definition 6.6 An $\mathcal{EL}^{\bot}$ knowledge base is *left local* (LL) if its concepts inclusions are instances of the following schemata:

$$A \sqsubseteq_{[n]} B \quad A \sqsubseteq_{[n]} \exists P.B \quad A_1 \sqcap A_2 \sqsubseteq B$$
$$\exists P \sqsubseteq B \quad \exists P_1 \sqsubseteq \exists P_2.B$$

where A and B can be concept names or $\bot$. A *LL $\mathcal{EL}^{\bot}$ concept* is any concept that can occur in the above inclusions.

Note the similarity with the normal form of $\mathcal{EL}$ inclusions [Baader *et al.*, 2005] that, however, would allow the more general inclusions $\exists P.A \sqsubseteq B$ and $\exists P_1.A \sqsubseteq \exists P_2.B$.

Lemma 6.7 *Let $\mathcal{KB}$ be an LL $\mathcal{EL}^{\bot}$ knowledge base. For all models $\mathcal{I} \in \mathsf{Circ}_{\mathsf{var}}(\mathcal{KB})$ and $x \in \Delta^{\mathcal{I}}$ there exists a model $\mathcal{J} \in \mathsf{Circ}_{\mathsf{var}}(\mathcal{KB})$ such that (i) $\Delta^{\mathcal{J}} \subseteq \Delta^{\mathcal{I}}$, (ii) $x \in \Delta^{\mathcal{J}}$, (iii) $|\Delta^{\mathcal{J}}|$ is polinomial in the size of $\mathcal{KB}$.*

Proof. The proof is analogous to the proof of Lemma 5.1. Here we start with a slightly different set Δ. Choose a minimal set $\Delta \subseteq \Delta^{\mathcal{I}}$ containing: (i) x, (ii) all $a^{\mathcal{I}}$ such that $a \in \mathsf{N_I} \cap \mathsf{cl}(\mathcal{KB})$, (iii) for each concept $\exists P$ in $\mathsf{cl}(\mathcal{KB})$ satisfied in $\mathcal{I}$, a node y_P such that for some $z \in \exists P^{\mathcal{I}}$, $(z, y_P) \in P^{\mathcal{I}}$ and (iv) for each concept $\exists P.B$ in $\mathsf{cl}(\mathcal{KB})$ satisfied in $\mathcal{I}$, a node $y_{P,B}$ such that for some $z \in \exists P.B^{\mathcal{I}}$, $(z, y_{P,B}) \in P^{\mathcal{I}}$ and $y_{P,B} \in B^{\mathcal{I}}$.

Now define $\mathcal{J}$ as follows: (i) $\Delta^{\mathcal{J}} = \Delta$, (ii) $a^{\mathcal{J}} = a^{\mathcal{I}}$ (for $a \in \mathsf{N_I} \cap \mathsf{cl}(\mathcal{KB})$), (iii) $A^{\mathcal{J}} = A^{\mathcal{I}} \cap \Delta$ ($A \in \mathsf{N_C} \cap \mathsf{cl}(\mathcal{KB})$), and (iv) $P^{\mathcal{J}} = \{(z, y_P) \mid z \in \Delta \text{ and } z \in \exists P^{\mathcal{I}}\} \cup \{(z, y_{P,B}) \mid z \in \Delta \text{ and } z \in \exists P.B^{\mathcal{I}}\}$ ($P \in \mathsf{N_R}$).

The rest of the proof is similar to the proof of Lemma 5.1 and is omitted here for space limitations. We only remark that the restriction to LL KBs is needed to ensure that $\mathcal{J}$ is a classical model of the CIs in $\mathcal{KB}$. ∎

Lemma 6.8 *Let $\mathcal{KB}$ be a LL $\mathcal{EL}^{\bot}$ knowledge base. For all models $\mathcal{I} \in \mathsf{Circ}_{\mathsf{fix}}(\mathcal{KB})$ and $x \in \Delta^{\mathcal{I}}$ there exists a model $\mathcal{J} \in \mathsf{Circ}_{\mathsf{fix}}(\mathcal{KB})$ such that (i) $\Delta^{\mathcal{J}} \subseteq \Delta^{\mathcal{I}}$, (ii) $x \in \Delta^{\mathcal{J}}$, (iii) $|\Delta^{\mathcal{J}}|$ is polinomial in the size of $\mathcal{KB}$.*

		var	F / fix
Concept sat.	$\mathcal{EL}$	trivial up to $\mathcal{ELHO}$ (*Thm 6.5*)	
	$\mathcal{EL}^{\bot}$	$\geq$ ExpTime (*Thm 6.2, Cor 6.3*)	
	DL-lite$_R$, LL $\mathcal{EL}^{\bot}$	$\leq \Sigma_2^p$ (*Thm 5.3, Thm 6.9*)	
Instance checking Subsumption	$\mathcal{EL}$	P (*)	$\geq$ ExpTime (*Thm 6.4*)
	$\mathcal{EL}^{\bot}$	$\geq$ ExpTime (*Thm 6.2, Cor 6.3*)	
	DL-lite$_R$, LL $\mathcal{EL}^{\bot}$	$\leq \Pi_2^p$ (*Thm 5.3, Thm 6.9*)	

(*) Classical up to $\mathcal{ELHO}$ (by *Theorem 6.1*)

Table 1: Summary of complexity results

Proof. Similar to the proof of Lemma 5.2. Use a slightly modified, minimal set $\Delta \subseteq \Delta^{\mathcal{I}}$ containing: (i) x, (ii) all $a^{\mathcal{I}}$ such that $a \in \mathsf{N_I} \cap \mathsf{cl}(\mathcal{KB})$, (iii) for each concept $\exists P$ in $\mathsf{cl}(\mathcal{KB})$ satisfied in $\mathcal{I}$, a node y_P such that for some $z \in \exists P^{\mathcal{I}}$, $(z, y_P) \in P^{\mathcal{I}}$, (iv) for each concept $\exists P.B$ in $\mathsf{cl}(\mathcal{KB})$ satisfied in $\mathcal{I}$, a node $y_{P,B}$ such that for some $z \in \exists P.B^{\mathcal{I}}$, $(z, y_{P,B}) \in P^{\mathcal{I}}$ and $y_{P,B} \in B^{\mathcal{I}}$ and finally (v) for all inclusions $C \sqsubseteq \exists R.B$ or $C \sqsubseteq_n \exists R.B$ in $\mathcal{KB}$ such that $(C \sqcap \exists R.B)^{\mathcal{I}} \neq \emptyset$, a node $z \in (C \sqcap \exists R.B)^{\mathcal{I}}$. The rest of the proof is similar to the proof of Lemma 5.2 and omitted here. ∎

As a consequence of the above lemmata we get:

Theorem 6.9 *Concept consistency over circumscribed LL $\mathcal{EL}^{\bot}$ DKBs is in Σ_2^p. Subsumption and instance checking over circumscribed LL $\mathcal{EL}^{\bot}$ DKBs are in Π_2^p.*

7 Conclusions and further work

The complexity of circumscribed description logics can be significantly reduced by (i) restricting the underlying DL to DL-lite$_R$ and to suitable members of the $\mathcal{EL}$ family, and (ii) restricting nonmonotonic constructs to defeasible inclusions $A \sqsubseteq_n C$. KB satisfiability is equivalent to its classical version (by Theorem 4.2) and hence it is within P (sometimes even trivial) for the logics we investigated. The results for all the other reasoning tasks are summarized in Table 1. Surprisingly, fixed predicates in conjunction with qualified existential restrictions are powerful enough to keep the complexity of instance checking and subsumption ExpTime-hard even for a language like $\mathcal{EL}$, which is not able to express any inconsistency. For DL-lite$_R$ and LL $\mathcal{EL}^{\bot}$, complexity drops to Σ_2^p and Π_2^p, instead.

We are currently sharpening our complexity bounds, and extending them to more expressive logics, looking for alternatives to left-local KBs to confine complexity within the polynomial hierarchy. These theoretical results and the semantic properties emerging from their proofs will be exploited to design suitable calculi and algorithms for reasoning with circumscribed defeasible knowledge bases.

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