Dialectical Abstract Argumentation: A Characterization of the Marking Criterion

Nicolás D. Rotstein and **Martín O. Moguillansky** and **Guillermo R. Simari**
Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET)
Artificial Intelligence Research and Development Laboratory (LIDIA)
Universidad Nacional del Sur (UNS), Bahía Blanca, Argentina
e-mail: {ndr,mom,grs}@cs.uns.edu.ar

Abstract

This article falls within the field of abstract argumentation frameworks. In particular, we focus on the study of frameworks using a proof procedure based on dialectical trees. These trees rely on a marking procedure to determine the warrant status of their root argument. Thus, our objective is to formulate rationality postulates to characterize the marking criterion over dialectical trees. The behavior of the marking procedure is closely tied to the alteration of trees, which is the keystone of any model of change based on dialectical argumentation. Hence, the results achieved in this work will benefit research on dynamics in argumentation.

1 Introduction and Motivation

The notion of warrant in abstract argumentation can be analyzed from two standpoints: on behalf of a semantics applied to the graph of arguments [Baroni and Giacomin, 2007; Martínez *et al.*, 2007], or from the construction of a dialectical tree of arguments [Chesñevar and Simari, 2007]. In this article we take the latter approach, focusing on the warrant of the argument in the root. In this way, a marking criterion is defined to label arguments in the tree in order to set their status: usually it is either "defeated" or "undefeated", but other alternatives could be considered. This marking determines whether the root argument can be warranted, after weighing all the related contradictory knowledge. Our aim is to study an abstract characterization of the marking criterion over argumentation frameworks using dialectical trees to perform reasoning.

Dialectical trees are composed of argumentation lines, *i.e.,* sequences of conflicting arguments where each one attacks its predecessor in the sequence. In particular, we will distinguish those lines that determine the defeat of the root argument, which we call *attacking lines*. The study on the marking of argumentation lines gives place to three families of *postulates*, regarding: the marking of argumentation lines (family M); the marking of attacking lines (A); and relating marking with the warrant status of the root (W). The objective of such a characterization is to aid the definition of models of change over argumentation systems, such as Argument Theory Change [Rotstein *et al.*, 2008; Moguillansky *et al.*, 2008]. Argumentation models of change handle the dynamics of argumentative knowledge through the variation of the set of arguments or the attacks among them, to be able to control and direct change.

After giving the abstract characterization, a specific marking criterion is presented, against which the rationality postulates are checked. The importance of studying the marking of argumentation lines lies in the definition of argumentative models of change. For instance, in Argument Theory Change, dynamics over the argumentation theory is handled through the *alteration* of some argumentation lines. Therefore, any mechanism of alteration would benefit from the formalization on the lines marking given here. Alterations could be carried out in a variety of ways: a simple kind of alteration of a line λ is the removal of an argument from λ, whereas a more complex choice is to add a defeater to some argument in λ. In this article, we will assume the former kind of alteration, as the latter introduces unnecessary difficulty.

2 Abstract Argumentation Framework

Intuitively, an *argument* may be interpreted as *a consistent, indivisible piece of knowledge giving support to a claim from a set of premises*. However, in this article we abstract away the inner structure of arguments, without making any formal reference to claims nor premises for arguments. The analysis will be based on the widely accepted Dung's framework.

Definition 1 (Argumentation Framework) *An* ***argumentation framework*** *(AF) is a tuple $\langle \mathbb{A}, \hookrightarrow \rangle$, where $\mathbb{A}$ is a finite set of arguments, and $\hookrightarrow \subseteq (\mathbb{A} \times \mathbb{A})$, the attack relation over $\mathbb{A}$.*

We will write $\mathcal{A} \hookrightarrow \mathcal{B}$ to denote that an argument $\mathcal{A}$ attacks or defeats an argument $\mathcal{B}$. As said before, our analysis will be focused on the warrant status of a single argument. The following abstract framework for dialectical argumentation is inspired by [Chesñevar and Simari, 2007].

Definition 2 (Argumentation Line) *Given an AF $\phi = \langle \mathbb{A}, \hookrightarrow \rangle$, and $\mathcal{B}_1, \ldots, \mathcal{B}_n \in \mathbb{A}$, an* ***argumentation line*** *λ in ϕ is any (non-empty) finite sequence of arguments $[\mathcal{B}_1, \ldots, \mathcal{B}_n]$ such that $\mathcal{B}_i \hookrightarrow \mathcal{B}_{i-1}$, for $1 < i \leq n$. We will say that λ is* ***rooted in*** *$\mathcal{B}_1$, and that $\mathcal{B}_n$ is the* ***leaf*** *of λ. The domain of all argumentation lines in ϕ is denoted as $\mathfrak{Lines}_\phi$.*

The set $\mathfrak{Lines}_\phi$ defines a domain onto which different constraints can be defined. As such constraints are related to sequences which resemble an argumentation dialogue between two parties, we call them *dialectical constraints* and will be useful to determine if an argumentation line is *acceptable*.

Definition 3 (Dialectical Constraint) *Let ϕ be an AF. A* ***dialectical constraint*** $\mathbf{C}$ *in the context of ϕ is any function* $\mathbf{C} : \mathfrak{Lines}_\phi \rightarrow \{true, false\}$.

In what follows, we will assume a dialectical constraint that avoids the construction of circular argumentation lines. Hence, when building trees with *exhaustive lines* (*i.e.,* those to which no more arguments can be added), the non-circularity constraint will keep them finite. The framework enriched with dialectical constraints will be referred to as an *argumentation theory*. These constraints lead to the notion of *acceptable argumentation line*.

Definition 4 (Argumentation Theory) *An* ***argumentation theory*** *(AT) T is a pair $(\phi, \mathbf{DC})$, where ϕ is an AF and $\mathbf{DC}$ is a finite set of dialectical constraints.*

Definition 5 (Acceptable Argumentation Line) *Given an AT T, an argumentation line λ is* ***acceptable*** *wrt. T iff λ satisfies every constraint in $\mathbf{DC}$. The domain of all acceptable argumentation lines in T is denoted as $\mathfrak{Lines}_\mathsf{T}$.*

Given a theory T, when referring to an argument $\mathcal{A}$ belonging to a line $\lambda \in \mathfrak{Lines}_\mathsf{T}$, we will overload the membership symbol and write "$\mathcal{A} \in \lambda$", and will assume every line is acceptable unless stated otherwise. Since argumentation lines are an exchange of opposing arguments, we could think of it as *two parties engaged in a dispute*, which we call *pro* and *con*.

Definition 6 (Set of *Con* (*Pro*) Arguments) *Given an AT T and an argumentation line $\lambda \in \mathfrak{Lines}_\mathsf{T}$, the* ***set of* con** *(resp.,* **pro***) arguments λ^- (resp., λ^+) of λ is the set containing all the arguments placed on* ***even*** *(resp.,* ***odd****) positions in λ.*

Acceptable argumentation lines rooted in a common argument will be identifiable through bundle sets. This notion will allow the formalization of dialectical trees, which in turn will be the source of analysis for any adopted marking criterion.

Definition 7 (Upper Segment) *Given an AT T and an acceptable argumentation line $\lambda \in \mathfrak{Lines}_\mathsf{T}$ such that $\lambda = [\mathcal{B}_1, \ldots, \mathcal{B}_n]$, the* ***upper segment*** *of λ wrt. $\mathcal{B}_i$ $(1 < i \leq n)$, is defined as $\lambda^\uparrow(\mathcal{B}_i) = [\mathcal{B}_1, \ldots, \mathcal{B}_{i-1}]$. The upper segment of λ wrt. $\mathcal{B}_1$ is undefined.*

Definition 8 (Bundle Set) *Given an AT T, a set $\mathcal{S}_\mathsf{T}(\mathcal{A}) = \{\lambda_1, \ldots, \lambda_n\}$ of all the acceptable argumentation lines from $\mathfrak{Lines}_\mathsf{T}$ rooted in a given argument $\mathcal{A}$ is called a* ***bundle set*** *for $\mathcal{A}$ from T iff for each $\lambda_i \in \mathcal{S}_\mathsf{T}(\mathcal{A})$ there is no $\lambda_j \in \mathcal{S}_\mathsf{T}(\mathcal{A})$ such that $\lambda_i^\uparrow(\mathcal{B}) = \lambda_j$, for some $\mathcal{B} \in \lambda_i$ $(1 \leq i, j \leq n)$.*

Note that a bundle set $\mathcal{S}_\mathsf{T}(\mathcal{A}) \subseteq \mathfrak{Lines}_\mathsf{T}$ is the maximal subset of argumentation lines rooted in $\mathcal{A}$ wrt. set inclusion. Therefore, from $\mathcal{S}_\mathsf{T}(\mathcal{A})$ we can build a *dialectical tree*.

Definition 9 (Dialectical Tree) *Given a AT $\mathsf{T} = (\langle \mathbb{A}, \hookrightarrow \rangle, \mathbf{DC})$, a* ***dialectical tree*** *$\mathcal{T}_\mathsf{T}(\mathcal{A})$ rooted in $\mathcal{A}$ is determined by a bundle set $\mathcal{S}_\mathsf{T}(\mathcal{A})$. For an* ***inner node*** *$\mathcal{B} \in \lambda \in \mathcal{S}_\mathsf{T}(\mathcal{A})$, a* ***child*** *of $\mathcal{B}$ is every argument $\mathcal{D} \in \lambda' \in \mathcal{S}_\mathsf{T}(\mathcal{A})$ such that $\mathcal{D} \hookrightarrow \mathcal{B}$ and $\lambda'^\uparrow(\mathcal{D}) = \lambda^\uparrow[\mathcal{B}]$. The* ***leaves*** *in $\mathcal{T}_\mathsf{T}(\mathcal{A})$ are the leaves of each argumentation line in $\mathcal{S}_\mathsf{T}(\mathcal{A})$. The domain of all dialectical trees in T will be noted as $\mathfrak{Tree}_\mathsf{T}$.*

We will overload the membership symbol again to write "$\lambda \in \mathcal{T}_\mathsf{T}(\mathcal{A})$" when the line λ belongs to the bundle set associated to the tree $\mathcal{T}_\mathsf{T}(\mathcal{A})$. Dialectical trees allow to determine whether the root node is warranted. A *marking function* would define an acceptance criterion applied to each argument in the tree. This could be done by obtaining the mark of an inner node of the tree from its children (*i.e.,* its defeaters). Once each argument has been marked, a *warranting function* will determine the status of the tree from the mark of the root argument. These concepts will remain abstract until the definition of a concrete criterion in Section 4.

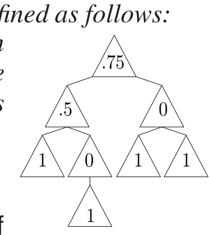
A domain including totally ordered marking values will be assumed, denoted by $\mathfrak{Mark}$. The lower and upper endpoints of this domain will be interpreted as "defeated" (noted D) and "undefeated" (noted U), respectively. Within these two values, an arbitrary range of marking values can be defined. For instance when a marking procedure is based on a degree of defeat, $\mathfrak{Mark}$ could be defined as the interval of reals between 0 and 1, interpreting 0 as D, and 1 as U. Another alternative might be a simple U/D marking, and a variation could be to consider a marking domain $[D, B, U]$, where B stands for "blocked", representing an argument involved in a mutual attack with another. Finally, marking strategies similar to the well-known *minimax* algorithm are also valid.

Definition 10 (Marking Function) *Given an AT $\mathsf{T} = (\langle \mathbb{A}, \hookrightarrow \rangle, \mathbf{DC})$, a* ***marking function*** $\mathsf{Marking} : \mathbb{A} \times \mathfrak{Lines}_\mathsf{T} \times \mathfrak{Tree}_\mathsf{T} \rightarrow \mathfrak{Mark}$ *assigns a marking value to each argument in a line $\lambda \in \mathcal{T}$, from a tree $\mathcal{T} \in \mathfrak{Tree}_\mathsf{T}$.*

The function $\mathsf{Marking}$ has three parameters because the only way to individualize an argument is through the line and tree it belongs to. Recall that the marking function assigns a mark to each argument in a tree. Once this is performed, the function $\mathsf{Warranting} : \mathfrak{Tree}_\mathsf{T} \rightarrow \{true, false\}$ evaluates the root's mark and decides whether the root argument should be considered as warranted. In this section both the marking criterion and the warranting function will remain unspecified. An example of them is provided below.

Definition 11 (Warrant) *Given an AT T and a dialectical tree $\mathcal{T}_\mathsf{T}(\mathcal{A})$, argument $\mathcal{A}$ is* ***warranted*** *from T iff $\mathsf{Warranting}(\mathcal{T}_\mathsf{T}(\mathcal{A})) = true$ and $\mathcal{T}_\mathsf{T}(\mathcal{A})$ is a* ***warranting tree***.

Example 1 *Let $\mathfrak{Mark}$ be the interval of real numbers $[0, 1]$. Given an AT T, a marking criterion is defined as follows:* (1) $\mathsf{Marking}(\mathcal{B}_n, \lambda, \mathcal{T}_\mathsf{T}(\mathcal{A})) = 1$, *with $\mathcal{B}_n$ the leaf of λ;* (2) *for an inner node $\mathcal{B} \in \lambda \in \mathcal{T}_\mathsf{T}(\mathcal{A})$ and its set of defeaters $\mathsf{Def} = \{\mathcal{D}_1, \ldots, \mathcal{D}_m\}$:*
$\mathsf{Marking}(\mathcal{B}, \lambda, \mathcal{T}_\mathsf{T}(\mathcal{A})) = 1 - \overline{\mathsf{Def}}$,
$\overline{\mathsf{Def}} = \frac{1}{m} \sum_{i=1}^{m} \mathsf{Marking}(\mathcal{D}_i, \lambda_i, \mathcal{T}_\mathsf{T}(\mathcal{A}))$.
Finally, $\mathsf{Warranting}(\mathcal{T}_\mathsf{T}(\mathcal{A})) = true$ iff $\mathsf{Marking}(\mathcal{A}, \lambda, \mathcal{T}_\mathsf{T}(\mathcal{A})) > \mathfrak{m}$, *for a warranting threshold* $\mathfrak{m}$. *Assuming a theory that yields the tree on the right with* $\mathfrak{m} = .5$, *the root is warranted, whereas if* $\mathfrak{m} = .8$, *it is not.*



3 Characterizing the Marking Criterion

In order to be able to analyze argumentation lines wrt. the marking of each of their arguments, we need to consider their *marking sequence*.

Definition 12 (Marking Sequence) *Given an AT* T*, and an argumentation line* $\lambda = [\mathcal{B}_1, \ldots, \mathcal{B}_n]$ *from the dialectical tree* $\mathcal{T}_{\mathsf{T}}(\mathcal{B}_1)$*, a* ***marking sequence function*** $\mathsf{MarkSeq} : \mathfrak{Lines}_{\mathsf{T}} \times \mathfrak{Tree}_{\mathsf{T}} \rightarrow \mathfrak{Mark}^n$*, where* $n = |\lambda|$*, determines a sequence:*

$$\mathsf{MarkSeq}(\lambda, \mathcal{T}_{\mathsf{T}}(\mathcal{B}_1)) = [\mathsf{Marking}(\mathcal{B}_1, \lambda, \mathcal{T}_{\mathsf{T}}(\mathcal{B}_1)), \ldots, \mathsf{Marking}(\mathcal{B}_n, \lambda, \mathcal{T}_{\mathsf{T}}(\mathcal{B}_1))].$$

Although the definition for a marking sequence allows for an arbitrary sequence of elements from $\mathfrak{Mark}$, these sequences should be restricted by the marking criterion. The following paragraph describes the intuition behind a family of postulates regarding the marking of argumentation lines.

Every leaf in a dialectical tree should be marked as an undefeated argument, since they certainly do not have defeaters. Furthermore, when an argument $\mathcal{A}$ *is defeated by an argument marked as undefeated,* $\mathcal{A}$ *should not be considered also as undefeated, since it would be compromising the meaning of the notion of attack. Even when considering the presence of more defeaters,* $\mathcal{A}$ *must not be undefeated. If we consider a domain* $\mathfrak{Mark}$ *as in Example 1, the presence of several defeaters marked as defeated (*i.e.*, mark 0) along with a single undefeated defeater could make the marking of* $\mathcal{A}$ *to approach to 1, but it must never reach the status of undefeated, no matter what the marking criterion is. Nonetheless, for some marking criteria it is possible for a defeated argument* $\mathcal{B}$ *to have a defeated defeater, since the marking of* $\mathcal{B}$ *could be determined by another non-defeated defeater, as will be analyzed in Section 4. Finally, it is natural to think that an argument should be defeated (undefeated) whenever all of its defeaters are undefeated (defeated).*

From the stated above, four rationality postulates are presented to characterize the marking criterion defined for an argumentation line $\lambda = [\mathcal{B}_1, \ldots, \mathcal{B}_n]$:

$(M1)$ $\mathsf{Marking}(\mathcal{B}_n, \lambda, \mathcal{T}_{\mathsf{T}}(\mathcal{B}_1)) = U$.

$(M2)$ If $\mathsf{Marking}(\mathcal{B}_i, \lambda, \mathcal{T}_{\mathsf{T}}(\mathcal{B}_1)) = U$ then $\mathsf{Marking}(\mathcal{B}_{i+1}, \lambda, \mathcal{T}_{\mathsf{T}}(\mathcal{B}_1)) \neq U$, $1 \leq i < n$.

Note that accepting $(M1)$ prohibits a dialectical tree to be composed of a single argument marked as defeated. For every inner node $\mathcal{B}$ in any $\lambda \in \mathcal{T}_{\mathsf{T}}(\mathcal{B}_1)$, given the set $\{\mathcal{D}_1, \ldots, \mathcal{D}_m\}$ of its defeaters, then for every i such that $1 \leq i \leq m$:

$(M3)$ If $\mathsf{Marking}(\mathcal{D}_i, \lambda_i, \mathcal{T}_{\mathsf{T}}(\mathcal{B}_1)) = D$ then $\mathsf{Marking}(\mathcal{B}, \lambda, \mathcal{T}_{\mathsf{T}}(\mathcal{B}_1)) = U$.

$(M4)$ If $\mathsf{Marking}(\mathcal{D}_i, \lambda_i, \mathcal{T}_{\mathsf{T}}(\mathcal{B}_1)) = U$ then $\mathsf{Marking}(\mathcal{B}, \lambda, \mathcal{T}_{\mathsf{T}}(\mathcal{B}_1)) = D$.

The postulates are not devised to envision every possible marking criterion; that is, they are not axioms. Thus, the non-satisfaction of any of them does not undermine the validity of a particular marking. There should be, however, sensible reasons for this to happen. For instance, a particular marking could consider the mark "blocked" and a case might arise where leaves are marked as blocked, violating $(M1)$.

Some argumentation lines can be deemed as being "responsible" for a dialectical tree to be non-warranting. We call them *attacking lines*. In order to recognize attacking lines out of its marking sequence, a concrete *attacking line function* $\alpha : \mathfrak{Mark}^n \rightarrow \{true, false\}$ should be specified from the marking criterion. However, when considering an abstract marking criterion, it is not possible to fully characterize individual attacking lines, since α would be also unspecified.

Example 2 *Considering the marking criterion in Ex. 1, an attacking line function could check if the marks of the* pro *arguments in the sequence are below the warranting threshold:* $\alpha([v_1, \ldots, v_n]) = true$ iff $v_{2k-1} \leq \mathfrak{m}$*, where* $k \in [1, \lceil n/2 \rceil]$*.*

When $\mathfrak{m} = .8$ *the root argument is not warranted, and the only attacking line recognized by* α *is the one with the marking sequence* $[.75, .5, 0, 1]$*, since the two* pro *arguments are marked* $.75$ *and* 0*, that is, below the warranting threshold.*

The individual definition for an attacking line is highly dependent on the marking criterion, therefore instead of giving an abstract definition, we will specify them through the concept of *attacking set*. This set comprehends those lines in a dialectical tree such that, without them, the tree would warrant its root argument. However, due to interdependency among lines, multiple sets could be compliant with this intuition. Therefore, we will rely on a comparison criterion establishing relative relevance among lines (noted "$\succ$") to set a complete order over lines, which will univocally determine the attacking set. This relevance order could be defined from the same comparison criterion used to define attacks.

Definition 13 (Attacking Set) *Given a comparison criterion among lines "*$\succ$*", and a dialectical tree based on a bundle set* $\mathcal{S}_{\mathsf{T}}(\mathcal{A}) \subseteq \mathfrak{Lines}_{\mathsf{T}}$ *from an AT* $\mathsf{T} = (\langle \mathbb{A}, \hookrightarrow \rangle, \mathbf{DC})$*, the* ***attacking set*** $\mathsf{Att}^{\succ}_{\mathsf{T}}(\mathcal{A}) \subseteq \mathcal{S}_{\mathsf{T}}(\mathcal{A})$ *is the minimal subset wrt. set inclusion determined by "*$\succ$*" such that either* $\mathsf{Att}^{\succ}_{\mathsf{T}}(\mathcal{A}) = \mathcal{S}_{\mathsf{T}}(\mathcal{A})$ *or the tree built from* $\mathcal{S}_{\mathsf{T}}(\mathcal{A}) \setminus \mathsf{Att}^{\succ}_{\mathsf{T}}(\mathcal{A})$ *warrants* $\mathcal{A}$*.*

The objective of this definition is to identify attacking lines. Note that the set $\mathcal{S}_{\mathsf{T}}(\mathcal{A}) \setminus \mathsf{Att}^{\succ}_{\mathsf{T}}(\mathcal{A})$ might not conform a bundle set, thus the tree built from it might not be associable to any particular theory. That is, the removal of the attacking lines from the bundle set of a non-warranting tree is not intended to conform a proper change operation. Recall that removing a line implies removing all of its arguments from the theory. Therefore, we have that an argument could belong to several lines, and its removal would affect all of them. Furthermore, if a non-attacking line contains this argument, the line might turn into attacking. In general, unexpected pruning should be considered and treated accordingly. Such side-effects have to be addressed by a change operator, problem that exceeds the scope of this article. This matter is thoroughly discussed in [Rotstein *et al.*, 2008; Moguillansky *et al.*, 2008].

As said before, we have possibly multiple choices for the attacking set, and a criterion is needed to determine it. Argumentation lines belonging to any of these sets are going to be considered attacking lines. Given the set $\Gamma_{\mathsf{T}}(\mathcal{A})$ of all the possible attacking sets (without considering the comparison criterion), λ is an *attacking line* iff $\lambda \in \bigcup(X), X \in \Gamma_{\mathsf{T}}(\mathcal{A})$. From the following intuition, we propose a new family of postulates describing individual attacking lines.

The marking sequence should reveal whether an argumentation line is an attacking line. Furthermore, every attacking line should end with a con *argument. This implies that an argumentation line ending with a* pro *argument is not an attacking line. However, if the line ends with a* con *argument, it could be the case that it is not an attacking line. Finally, the removal of a* con *argument in an attacking line turns it into non-attacking, and the removal of a* pro *argument in such a* *line yields an upper segment that is an attacking line. That is, the only way to effectively alter an attacking line is by the removal of a* con *argument; removing a* pro *would augment the threat to the root.*

By *effective alteration* we refer to an alteration that turns an attacking line into non-attacking. The following postulates characterize attacking lines within a theory T for an argumentation line $\lambda \in \mathfrak{Lines}_{\mathsf{T}} \in \mathcal{T}_{\mathsf{T}}(\mathcal{A})$, and an attacking line function α:

$(A1)$ λ is attacking line iff $\alpha(\mathsf{MarkSeq}(\lambda, \mathcal{T}_{\mathsf{T}}(\mathcal{A}))) = true$.
$(A2)$ If $\lambda = [\mathcal{B}_1, \ldots, \mathcal{B}_n]$ is an attacking line then $\mathcal{B}_n \in \lambda^-$.
$(A3)$ If λ is an attacking line then
($\lambda^{\uparrow}(\mathcal{B})$ is not attacking line iff $\mathcal{B} \in \lambda^-$).

The concepts of attacking lines, attacking set and warrant are strongly connected and this relation responds to the following intuitions:

It would be ideal to not depend on any argumentation lines criterion to determine the attacking set. To achieve this, there should be only one attacking set, determined just by the marking criterion and containing every attacking line. The main motivation to identify attacking lines is to recognize the parts of the tree that affect the warrant status of the root argument. Thus, if a tree contains at least one attacking line, the root should not be warranted. Regarding the set $\mathsf{Att}^{\succ}_{\mathsf{T}}(\mathcal{A})$*, it is assumed to contain the minimal set of lines such that, without them, the tree would be warranting. Hence, an empty set* $\mathsf{Att}^{\succ}_{\mathsf{T}}(\mathcal{A})$ *should yield a warranted root argument.*

Next we propose a set of postulates to characterize the relation over the concepts of attacking lines, attacking set, and warrant. Given an AT T and a dialectical tree $\mathcal{T}_{\mathsf{T}}(\mathcal{A})$:

$(W1)$ $\lambda \in \mathcal{T}_{\mathsf{T}}(\mathcal{A})$ is an attacking line iff $\lambda \in \mathsf{Att}^{\succ}_{\mathsf{T}}(\mathcal{A})$.
$(W2)$ There exists an attacking line $\lambda \in \mathcal{T}_{\mathsf{T}}(\mathcal{A})$ iff $\mathsf{Warranting}(\mathcal{T}_{\mathsf{T}}(\mathcal{A})) = false$.
$(W3)$ $\mathsf{Att}^{\succ}_{\mathsf{T}}(\mathcal{A}) = \emptyset$ iff $\mathsf{Warranting}(\mathcal{T}_{\mathsf{T}}(\mathcal{A})) = true$.

A dialectical tree with every line ending in a *pro* argument reflects a situation in which every exchange of arguments *pro* and *con* could not be successfully counterargued by a *con* argument (see $(M1)$). Since no *con* argument prevailed, the root argument is not threatened and therefore it is natural to believe it should be warranted. On the contrary, when every leaf in the tree is a *con* argument, the root argument would be impossible to be warranted, and the tree would not satisfy the $\mathsf{Warranting}$ function. This rationale comes from $(M1)$, $(A2)$ and $(W2)$, and the following proposition could be inferred:

Proposition 1 *Given an AT* T*, a dialectical tree* $\mathcal{T}_{\mathsf{T}}(\mathcal{A})$*, if every* $\lambda \in \mathcal{T}_{\mathsf{T}}(\mathcal{A})$ *is such that the leaf of* λ *is in* λ^+ *then* $\mathsf{Warranting}(\mathcal{T}_{\mathsf{T}}(\mathcal{A})) = true$.

Breaking the postulate $(W1)$ would not compromise the objective of this set of postulates, as long as $(W2)$ and $(W3)$ are satisfied. In such a case, it should be clear why either some attacking lines do not belong to the set, or the set contains lines that are not attacking. Satisfying $(W2)$ without satisfying $(W3)$ implies that the set $\mathsf{Att}^{\succ}_{\mathsf{T}}(\mathcal{A})$ would contain lines that are non-attacking and/or some attacking lines were left out of $\mathsf{Att}^{\succ}_{\mathsf{T}}(\mathcal{A})$. On the other hand, satisfying $(W3)$ but not $(W2)$ means that not every attacking line compromises $\mathcal{A}$: the tree is warranting and contains attacking lines.

Proposition 2 *Given a specific marking criterion, if* $(W1)$ *is guaranteed then* $(W2)$ *is equivalent to* $(W3)$.

Theorem 1 *Let* $\mathsf{T} = (\langle \mathbb{A}, \hookrightarrow \rangle, \mathbf{DC})$ *be an AT,* $\mathcal{A} \in \mathbb{A}$*, an argument,* $\mathcal{T}_{\mathsf{T}}(\mathcal{A})$*, a marked dialectical tree,* $\mathsf{Att}^{\succ}_{\mathsf{T}}(\mathcal{A})$*, the attacking set, and* $\mathcal{S}_{\mathsf{T}}(\mathcal{A})$*, the bundle set of* $\mathcal{T}_{\mathsf{T}}(\mathcal{A})$*. If the marking criterion verifies* $(W2)$ *and* $(W3)$*, the tree built from* $\mathcal{S}_{\mathsf{T}}(\mathcal{A}) \setminus \mathsf{Att}^{\succ}_{\mathsf{T}}(\mathcal{A})$ *warrants* $\mathcal{A}$*.*

On a theoretical level, we can refer to the W postulates disregarding the families A and M, as we defined the concept of attacking set by abstracting away the marking criterion. This definition was performed considering subsets of the bundle set of the tree at issue. When looking from the standpoint of an implemented argumentation system, this approach seems to be rather impractical. Therefore, it is necessary to specify attacking lines and attacking set relying only in the marking criterion. In such a case, guaranteeing the W postulates would be related to the satisfaction of postulates A and M. Moreover, the W postulates characterize the success of an abstract model of change. In this article we relate the change in argumentation to attacking lines, and therefore, this condition over argumentation lines should also be studied through the A family. Particularly, $(A3)$ reflects the characterization of an appropriate alteration of dialectical trees through attacking lines. This discussion includes part of the ongoing work.

4 Analysis of a Specific Marking Criterion

Defeasible Logic Programming (DELP [García and Simari, 2004]) is a formalism based on an argumentative machinery for reasoning over *defeasible logic programs*. In general, a defeasible logic program is conformed by two different finite sets of rules: the set of *strict rules* and the set of *defeasible rules*. The first represents information that is beyond discussion, and therefore required to be consistent. The set of defeasible rules represents potentially contradictory information that could be used to build *arguments*, which could be *conflicting*. Conflicts among arguments are decided by an *argument comparison criterion*, therefore given two conflictive arguments $\mathcal{A}$ and $\mathcal{B}$, if $\mathcal{A}$ is strictly preferred over $\mathcal{B}$, then $\mathcal{A}$ is said to be a *proper defeater* of $\mathcal{B}$. When there is no preference, each one is a *blocking defeater* for the other.

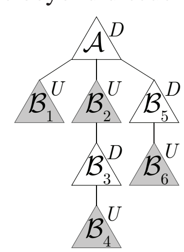


Figure 1: DELP marking example

Sequences of conflicting arguments representing an attack chain are called argumentation lines and are treated as specified in Section 2. Argumentation lines are acceptable when they verify non-circularity (no argument is reintroduced), concordance (the set of *pro* (resp., con) arguments in the same line is consistent), and there is no subsequence of three arguments yielding two consecutive blocking defeats. Let $\mathfrak{Mark}$ be $[D, U]$. The DELP marking criterion is:
(1) all leaves are marked U;
(2) every inner node $\mathcal{B}$ is marked U iff every child of $\mathcal{B}$ is marked D; otherwise, $\mathcal{B}$ is marked D. Finally, $\mathsf{Warranting}(\mathcal{T}_{\mathsf{T}}(\mathcal{A})) = true$ iff $\mathsf{Marking}(\mathcal{A}, \lambda, \mathcal{T}_{\mathsf{T}}(\mathcal{A})) = U$. The dialectical tree in Fig. 1 exemplifies this criterion.

In this section, we reconsider the postulates given in Section 3 from the standpoint of the DELP marking criterion. We will classify argumentation lines according to their marking and will make a special distinction on which kind of DELP lines are attacking lines. The marking of an argumentation line will not be considered individually, but in concordance with the context provided by the tree it belongs to. Thus, the marking sequence assigned to an argumentation line corresponds to the marking given by the dialectical tree. For instance, in the tree from Fig. 1, the line $[\mathcal{A}, \mathcal{B}_5, \mathcal{B}_6]$ does not have the marking sequence $[U, D, U]$ but the marking $[D, D, U]$, since the marking is affected by line $[\mathcal{A}, \mathcal{B}_1]$.

From now on, we will refer to the marking sequence of argumentation lines as strings composed by the letters D and U; then, they will be associable to regular expressions.

Proposition 3 *Considering the* DELP *marking criterion, given an AT* T *and an argumentation line* λ *in a tree* $\mathcal{T}_\mathsf{T}(\mathcal{B}_1)$, *the marking sequence* $\mathsf{MarkSeq}(\lambda, \mathcal{T}_\mathsf{T}(\mathcal{B}_1))$ *corresponds to the regular expression* $U(D^+U)^*|(D^+U)^+$.

Considering the DELP marking criterion it is trivial to prove postulates $(M1)\ldots(M4)$. The marking sequence of a line can be simply a U (never a single D), or begin with a U or a D and then include an arbitrarily long alternation of Ds and Us. At some point it could repeat any number of Ds (never a U). Finally, any marking sequence ends with a U.

Proposition 4 *The* DELP *marking satisfies* $(M1)$ *to* $(M4)$.

When considering a non-warranting tree, as said before, we need to characterize which lines are the ones making the root argument to be defeated. In this sense, we distinguish two types of lines: *attacking* and *D-rep* lines. The latter are lines involving a repetition of the mark D (see Def. 15).

Argumentation lines in a bundle set conform a tree, thus there might be overlapping among them. Two or more argumentation lines are called *adjacent* when they share an upper segment containing two or more arguments.

Definition 14 (Adjacent Argumentation Lines) *Let* T *be an AT,* $\lambda_1, \lambda_2 \in \mathfrak{Lines}_\mathsf{T}$, *two argumentation lines, and* $\mathcal{B}_1 \in \lambda_1$ *and* $\mathcal{B}_2 \in \lambda_2$, *two arguments. Lines* λ_1 *and* λ_2 *are said to be* ***adjacent*** *at an argument* $\mathcal{B}$ iff $\lambda_1^{\uparrow}(\mathcal{B}_1) = \lambda_2^{\uparrow}(\mathcal{B}_2) = [\mathcal{A}, \ldots, \mathcal{B}]$, *where* $\mathcal{A} \neq \mathcal{B}$. *Argument* $\mathcal{B}$ *is said to be the* ***adjacency point*** *between* λ_1 *and* λ_2.

Since there cannot be an argumentation line with a marking sequence containing a repetition of Us, then a line starting with a D is either a *D-rep line* (Def. 15) or an attacking line (Def. 16). This classification was introduced and discussed in [Moguillansky *et al.*, 2008]. The former is defined next, as those lines that have repetitions of Ds in their sequence.

Definition 15 (D-rep Line) *Considering the* DELP *marking criterion, given an AT* T *and an argumentation line* λ *in a tree* $\mathcal{T}_\mathsf{T}(\mathcal{B}_1)$, λ *is a* ***D-rep line*** iff $\mathsf{MarkSeq}(\lambda, \mathcal{T}_\mathsf{T}(\mathcal{B}_1))$ *corresponds to the regular expression* $(DU)^*(D^+(DU)^+)^+$.

Next, we reify the notion of individual attacking line according to the DELP marking criterion. Recall that attacking lines in a tree are those such that, without them, the tree would warrant its root argument.

Definition 16 (Attacking Line) *Considering the* DELP *marking, given an AT* T *and a line* λ *in a tree* $\mathcal{T}_\mathsf{T}(\mathcal{B}_1)$, λ *is an* ***attacking line*** iff $\mathsf{MarkSeq}(\lambda, \mathcal{T}_\mathsf{T}(\mathcal{B}_1))$ *matches the regular expression* $(DU)^+$.

Hence, the definition for the attacking line function would check if the marking sequence of the line at issue conforms to the regular expression $(DU)^+$.

Proposition 5 *The* DELP *marking satisfies* $(A1)$.

Having a definition for individual attacking lines allows us to check if postulate $(W2)$ is verified. According to the DELP marking criterion, in order to have no attacking lines, we have two options: either the root argument is marked as U, and trivially there are no attacking lines (they start with a D), or it is marked as D and the tree contains only D-rep lines; however, each D-rep line would have at least one adjacent attacking line (Prop. 8), turning this last option impossible.

Proposition 6 *The* DELP *marking satisfies* $(W2)$.

The following proposition states that, according to the DELP marking criterion, every attacking line ends with a *con* argument, and removing a *pro* argument never turns an attacking line into non-attacking.

Proposition 7 *The* DELP *marking satisfies* $(A2)$ *and* $(A3)$.

Although D-rep lines have the root argument marked D, they are not "responsible" for the mark D of the root argument: this mark is a D due to the attacking set. As stated by Prop. 8, it is useful to recognize D-rep lines, since they reveal the existence of attacking lines adjacent to them. The only option to have a defeater marked D for an argument D is to have a second defeater, that is marked U. Hence, whenever there is repetition of Ds, there exists an adjacent line that either is also D-rep or follows the attacking line pattern. Finally, it is impossible to always have D-reps adjacent to D-reps, since, at some point, there will be no more repetition of Ds, and therefore, at least one attacking line will be involved.

Proposition 8 *Considering the* DELP *marking criterion, given an AT* T *and a D-rep line* $\lambda_d \in \mathcal{T}_\mathsf{T}(\mathcal{B}_1)$, *it holds that* λ_d *has at least one adjacent attacking line* $\lambda_a \in \mathcal{T}_\mathsf{T}(\mathcal{B}_1)$.

Although attacking lines can now be individualized, not every attacking line has to be altered in order to warrant the root argument. As will be clear next, this condition is bound to interdependent attacking lines. This relation corresponds to adjacency among lines. The following proposition describes a particular configuration of adjacent attacking lines that can be "solved" by altering just one of them.

Proposition 9 *Given a dialectical tree* $\mathcal{T}_\mathsf{T}(\mathcal{A})$ *from an AT* $\mathsf{T} = (\langle \mathbb{A}, \hookrightarrow \rangle, \mathbf{DC})$, *two arguments* $\mathcal{A}, \mathcal{B} \in \mathbb{A}$, *and the subset* $\{\lambda_1, \ldots, \lambda_n\} \subseteq \mathsf{Att}_\mathsf{T}^{\succ}(\mathcal{A})$, *where all the* λ_i $(1 \leq i \leq n)$ *are adjacent at* $\mathcal{B}$, *if* $\mathcal{B}$ *is an argument marked as* U *then the effective alteration of some* λ_k $(1 \leq k \leq n)$ *below* $\mathcal{B}$ *turns every* λ_i *into a non-attacking line.*

The requirement for the adjacency point to be marked as U comes from the following analysis: if the adjacency point is marked as U, the next argument $\mathcal{B}$ in each of the (attacking) lines is necessarily marked as D. An alteration on any of these lines (below the adjacency point) would turn $\mathcal{B}$ into a U argument, consequently changing the marking of the adjacency point and breaking the marking of all the involved attacking lines. Finally, none of them remain as attacking. It is important to notice that an alteration above the adjacency point would trivially resolve the situation, since it would be truncating every line involved, leaving a non-attacking portion of the shared upper segment.

Next, we give the DELP-specific version of the concept of *attacking set*. The standpoint from which is given, however, differs from the original version, since now we have characterized the structure of individual attacking lines. Note that, from Prop. 9, we have that it is possible for several attacking lines to be turned into non-attacking by altering just one of them. Such condition would contradict $(W1)$. However desirable, having an attacking set that does not contain every attacking line brings about the possibility of having more than one option to obtain a single minimal attacking set. Consequently, in order to be able to choose the most appropriate line to be altered, we will rely on the argumentation lines comparison criterion introduced in Section 3.

Definition 17 (Attacking Set) *Given a dialectical tree* $\mathcal{T}_{\mathsf{T}}(\mathcal{A})$*, the* ***attacking set*** *is:*

$$\mathsf{Att}^{\succeq}_{\mathsf{T}}(\mathcal{A}) = \{\lambda \mid \lambda \in \mathcal{T}_{\mathsf{T}}(\mathcal{A}) \text{ is an attacking line and every attacking line } \lambda' \in \mathcal{T}_{\mathsf{T}}(\mathcal{A}) \text{ adjacent to } \lambda \text{ at an argument marked as } U \text{ is such that } \lambda' \succ \lambda\}$$

As said before, the attacking set contains those argumentation lines that will be altered by a proper change mechanism. For that purpose, when having to choose from attacking lines adjacent at a U argument, a particular change operation could avoid the erasure of all of them, altering just the *least relevant* one, leaving the rest unaffected, except for their marking. Postulate $(W1)$ proposed in Section 3 stated that it would be desirable for every attacking line to belong to $\mathsf{Att}^{\succeq}_{\mathsf{T}}(\mathcal{A})$. Clearly, this is not verified by the DELP marking criterion.

Proposition 10 *The* DELP *marking does not satisfy* $(W1)$.

However, the reason why this postulate is not verified is justified: the DELP version of the definition for the attacking set attempts to minimize the amount of attacking lines to be altered when looking for the warrant of the root argument. By Prop. 2.1, we have that now postulates $(W2)$ and $(W3)$ are not equivalent. Since the satisfaction of $(W2)$ was already stated in Prop. 6, it remains to be analyzed whether $(W3)$ is satisfied in order to verify Theorem 1. This is quite straightforward: the set $\mathsf{Att}^{\succeq}_{\mathsf{T}}(\mathcal{A})$ is empty only if there are no attacking lines (see Def. 17). In such a case, from Prop. 6, the tree would be warranting.

Proposition 11 *The* DELP *marking satisfies* $(W3)$.

Provided that the DELP marking criterion guarantees both $(W2)$ and $(W3)$ (Prop. 6 and 11), it also satisfies Theorem 1.

5 Discussion and Concluding Remarks

In this paper we proposed three families of postulates to characterize the behavior of any marking criterion. The notion of attacking line was abstractly introduced along with the concept of attacking set, in order to identify the portions of a non-warranting tree that render the root argument defeated. This relation was proven to be useful to associate the marking of a tree wrt. its warrant status. After the abstract characterization, a concrete marking criterion was given. This allowed to check the suitability of the postulates, and also showed that additional problems can arise, corresponding to the particular marking under consideration.

In [Modgil and Caminada, to appear 2009] the authors propose properties to label (what we call 'mark') arguments graphs towards the implementation of proof procedures for argumentation systems. That is, their main focus is put on algorithmic aspects of argumentation. Our paper, however, is devoted to a purely theoretical characterization of the marking criterion through families of postulates at a level of abstraction that would allow for an implementation. A similar approach to [Modgil and Caminada, to appear 2009] was given in [Verheij, 2007], which presented an algorithm for a credulous argumentation semantics. The main difference between these two works is that the former constitutes a more general approach.

This article did not attempt to give an exhaustive characterization for the marking criterion. Hence, future work will continue in this direction, in order to complete this theoretical approach. In particular, we are interested in the formalization of change in the marking of a tree for any kind of alteration (*e.g.,* addition of defeaters). We will also study the connection between the warrant statuses of arguments; for instance, given two arguments $\mathcal{A}$ and $\mathcal{B}$ such that $\mathcal{A} \hookrightarrow \mathcal{B}$, they cannot be simultaneously warranted. Finally, the marking criterion introduced in Ex. 1 will be formalized and checked against the three families of postulates.

References

[Baroni and Giacomin, 2007] P. Baroni and M. Giacomin. On principle-based evaluation of extension-based argumentation semantics. *AIJ*, 171(10-15):675–700, 2007.

[Chesñevar and Simari, 2007] C. Chesñevar and G. Simari. A Lattice-based Approach to Computing Warranted Belief in Skeptical Argumentation Frameworks. In *IJCAI*, pages 280–285, January 2007.

[García and Simari, 2004] A. García and G. Simari. Defeasible Logic Programming: An Argumentative Approach. *TPLP*, 4(1-2):95–138, 2004.

[Martínez *et al.*, 2007] Diego Martínez, Alejandro García, and Guillermo Simari. Modelling well-structured argumentation lines. In *IJCAI*, pages 465–470, 2007.

[Modgil and Caminada, to appear 2009] Sanjay Modgil and Martin Caminada. Proof theories and algorithms for abstract argumentation frameworks. In Iyad Rahwan and Guillermo Simari, editors, *Argumentation in Artificial Intelligence*, chapter 6. Springer-Verlag, to appear (2009).

[Moguillansky *et al.*, 2008] M. Moguillansky, N. Rotstein, M. Falappa, A. García, and G. Simari. Argument Theory Change Applied to Defeasible Logic Programming. In *AAAI*, pages 132–137, 2008.

[Rotstein *et al.*, 2008] N. Rotstein, M. Moguillansky, M. Falappa, A. García, and G. Simari. Argument Theory Change: Revision Upon Warrant. In *COMMA*, pages 336–347, 2008.

[Verheij, 2007] Bart Verheij. A labeling approach to the computation of credulous acceptance in argumentation. In *IJCAI*, pages 623–628, 2007.