

Relational Random Forests Based on Random Relational Rules

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Abstract

Random Forests have been shown to perform very well in propositional learning. FORF is an upgrade of Random Forests for relational data. In this paper we investigate shortcomings of FORF and propose an alternative algorithm, R⁴F, for generating Random Forests over relational data. R⁴F employs randomly generated relational rules as fully self-contained Boolean tests inside each node in a tree and thus can be viewed as an instance of dynamic propositionalization. The implementation of R⁴F allows for the simultaneous or parallel growth of all the branches of all the trees in the ensemble in an efficient shared, but still single-threaded way. Experiments favorably compare R⁴F to both FORF and the combination of static propositionalization together with standard Random Forests. Various strategies for tree initialization and splitting of nodes, as well as resulting ensemble size, diversity, and computational complexity of R⁴F are also investigated.

1 Introduction

In propositional learning Random Forests [Breiman, 2001] are one of the best performing off-the-shelf methods, competitive with both support vector machines and boosted decision trees. In relational learning ensemble methods in general and Random Forests specifically have not been investigated widely. Both [Quinlan, 2000] and [Assche *et al.*, 2006] specifically report issues with excessive runtimes, and comparatively small increases over non-ensemble approaches. In this paper we try to address these issues by using a form of dynamic propositionalization in the spirit of [Landwehr *et al.*, 2006] using random relational rules [Anderson and Pfahringer, 2007] as self-contained boolean tests inside decision tree nodes. We introduce a new algorithm that grows forests in a parallel fashion which minimizes costly coverage computation at the potential expense of diversity. Several variants of the plain algorithm can achieve satisfactory diversity without compromising the parallel generation process.

The next section describes the new algorithm and discusses its complexity. Experimental results are reported and dis-

cussed in Section 3. Finally, Section 4 summarises and points out directions for future research.

2 Random Forests using Random Relational Rules

Propositional Random Forests [Breiman, 2001] are a combination of Bagging with randomized decision tree induction: at each bagging iteration one randomized decision tree is generated for the respective bootstrap sample of the training data. Tree induction can be randomized in various different ways; in Random Forests it is done in the following way: instead of choosing the best test to split data at internal tree nodes, first a subset of all possible attributes is drawn at random, and then the best possible test out of that subset is selected. The size of this subset is a parameter specified by the user, but generally a default of $\sqrt{\text{num_attributes}}$ has been found to work well.

In [Assche *et al.*, 2006], Random Forests were upgraded to relational problem domains by replacing the propositional decision tree learner with a randomized version of a relational decision tree learner (TILDE). This approach exhibits two problems:

- Interpretation of relational trees is not straightforward: every node in a tree is either a test or a predicate, which might introducing new variables. The scope of these variables is limited to the successful (or “yes”) branches of the tree. Each path from the root to a leaf in the tree can then be interpreted as a logical rule, but care must be taken with negated (or “no”) branches, which must be represented by complex negations to ensure proper variable treatment.
- The second and more practical issue is a consequence of the specific form of randomization chosen in FORF. In propositional Random Forest generation the total number of attributes available is a constant known upfront and it is therefore straightforward to specify the size for and compute random subsets thereof. When growing a relational decision tree, the number of possible tests or predicates at a given node is variable and a function of the current path from the root to the respective node. Also, in general this number tends to be both large and to grow quickly with the depth of the tree. In [Assche *et al.*, 2006], a sampling approach was taken to estimate the number of possible tests and predicates applicable

at a node, and then a user-specified percentage of these literals was actually evaluated to determine the single “best” literal out of this subset. This approach is problematic for various reasons. First of all it is computationally expensive, as both the estimation and the subsequent coverage computation for the subset are expensive and have to be repeated for every single node in every tree in the ensemble. In their empirical investigation they have found that they need high percentages (25% and more) to achieve good performance, contrary to propositional Random Forests, which do well with much smaller percentages. Not surprisingly, FORF is therefore much slower than its propositional counterpart, and also does not seem to yield such impressive accuracy improvements as Random Forests do. The latter might be a consequence of the large number of tests and predicates being explored; there is some anecdotal evidence, but unfortunately no good publication, that suggests that even Random Forests struggle to cope with very large numbers of attributes, as tree construction runs out of examples exponentially fast.

To combat these issues, R^4F takes a very different approach to relational Random Forest construction inspired by propositionalization. Each node in the tree uses one fully self-contained logical rule as a boolean test. If an example is covered by the rule, the test succeeds, and the example is passed down the “yes” branch; otherwise the test fails, and the example is passed down the “no” branch. This immediately alleviates the interpretation problem mentioned above and also simplifies and speeds up the randomized tree construction: every node simply needs to choose the best of a number of random relational rules, which from the tree construction point of view is simply choosing one boolean attribute from a random set of boolean attributes. Thus R^4F can be viewed as a Random Forest, where split attribute selection has been replaced with selecting one out of a small set of boolean features which are generated by some form of oracle on the fly as needed. Every one of these random features or rules is only evaluated for coverage once on the full dataset, and can then be used by any split node in any of all the trees of the ensemble, thus cutting down drastically on coverage computation compared to FORF. Such coverage computation is the single most expensive step in most relational learning systems.

The following subsections will explain the construction of random relational rules, detail the construction of the random forests based on these random rules, and discuss its computational complexity.

2.1 Rule Construction

The random relational rules employed here are definite clauses, which comprise both predicates containing variables, as well as tests on and comparisons between these variables and so-called theory constants. Neither functors nor recursion are permitted. For example, the Mutagenesis dataset [Srinivasan *et al.*, 1994] comprises the following three predicates:

```
molecule(MolID, Class)
atom(MolID, AtmID, El, Quanta, Charge)
```

```
bond(MolID, AtmID1, AtmID2, BondType)
```

A molecule is described by two parameters: a unique identifier and a class label (active or inactive). An atom is described by five parameters: the identifier of the molecule it belongs to, a unique identifier, its element type, its quanta type, and its electrical charge. A bond is described by four parameters: the identifier of the molecule that it belongs to, the unique identifiers for the two atoms it is linking, as well as its own bond type. An example of a rule generated on that dataset is:

```
active(MolID):-
  atom(MolID,_,_,_,Charge),
  Charge >= 0.078,
  bond(MolID,_,AtomID1,BondType1),
  bond(MolID,_,AtomID2,BondType2),
  BondType1 != BondType2,
  AtomID1 = AtomID2.
```

This rule describes all compounds that contain an atom with a charge above 0.078 and two bonds of different types that both include a particular atom. Underscores are used here for clarity, to denote variables not used in this rule.

Such random rules are generated in the following way: every rule has a user-specified length; at each stage a predicate or test is chosen uniformly at random with the following restrictions: for a predicate exactly one variable (or parameter) must already appear in the rule; all other variables are new. This ensures that clauses are linked. Tests on the other hand may not add any new variables. Tests include the usual equal and not-equal comparisons to other variables or theory constants, as well as range comparisons for numeric arguments. All prefixes of a rule can also be used as (shorter) random rules. Coverage computation can produce coverage information for all prefixes of the full rule at no additional cost.

2.2 Forest Construction

We apply Random Relational Rules to random forests by using randomly generated rules as the splitting condition or boolean test in internal tree nodes. As the rule generation process is independent of the current tree state it is straightforward to parallelize tree and indeed forest generation. The pseudocode for the simplified Random Relational Forest algorithm (Random Relational Rules - Random Forests, or R^4F) is given in Algorithm 1.

Usually cover computation is the most time-consuming operation a relational learner needs to perform. This costly operation is executed exactly once for each random rule on the full dataset, and then every node on the waiting list can efficiently check whether the current rule actually properly splits its subset of the full data. This way all nodes of all trees of the ensemble can be grown in parallel. Clearly this operation would lead to identical trees, if all trees would be started simultaneously on the full dataset. To introduce the diversity necessary for good ensemble performance the algorithm staggers the start of individual trees, therefore different tree nodes see different subsets of random rules before making a split decision. Additional diversity is ensured by the use of bagging, i.e. each root node is initialized with a randomly drawn bootstrap sample instead of the full training set. Other

Algorithm 1 Pseudocode for the R^4F algorithm

```
Initialize the forest
while #(open leaves) > 0 do
  Generate a Rule and compute total cover
  if #(root nodes) < Maximum #(trees) then
    Initialize the next root node and add to open leaves
  end if
  for all Open Leaves do
    for all possible prefixes do
      Calculate information gain for the prefix
    end for
    if prefix with best infogain splits the leaf then
      if said prefix is better than current best then
        update current best prefix
      end if
    end if
    increment Rule Count
  end if
  if Rule Count == Maximum Rule Count then
    if bestPrefix == NULL then
      Close the leaf
    else
      Split the leaf using bestPrefix
      add children to open leaves
    end if
  end if
end for
end while
```

options for inducing more diversity into the ensemble were also tried, and will be discussed and evaluated in the next section. Once nodes are initialized, and are not class-pure, they are put onto a list and will wait for rules that will split their data into two non-empty sets. After a user-defined maximum number of rules have been seen (MRC , or maximum rule count), a node will either be split on the best rule seen, or will be turned into a leaf predicting an appropriate class distribution, if no rule was found to split this node. Root nodes are an exception as they ignore the MRC setting and split on the first non-trivial rule seen. Together with bagging this ensures sufficient diversity of the trees, even though they are grown in parallel from the same stream of boolean features.

To clarify the forest building procedure, Figures 1 to 3 show three exemplary stages of forest construction, with the trees designated by letters, and internal nodes marked with an identifier corresponding to the rule that split them. Nodes are described by the tree designation followed by a numeric identifier. The training data consists of 20 instances, 10 each of two classes. The class distribution at each node is given to the node's left. For simplicity we assume that MRC is set to one, i.e. splitting is done immediately, if possible at all. We also assume that full rules (not prefixes) lead to the highest information-gain in all cases.

Figure 1 shows the state of the forest after the first rule, Rule R_1 , has been added. Initially, the root node of Tree A, A_1 , was the only node on the Open Leaf List. Now, A_1 has been split, and two leaves A_2 and A_3 created. A_2 , A_3 and the root node of Tree B, B_1 have been added to the Open Leaf list and A_1 has been removed from the list.

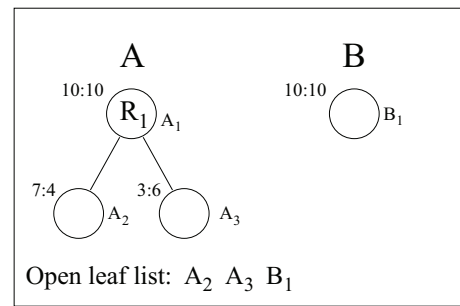


Figure 1: Example of R^4F Forest Construction, Stage 1

Figure 2 shows the state of the forest after another rule, R_2 has been processed. Both A_2 and A_3 have been split, B_1 has been split, and most of the new leaves thus created (A_5 through A_7 and B_2 through B_3), along with the root node of Tree C, C_1 , have been added to the Open Leaf list. A_4 now contains instances of only one class (denoted by the double circle) and so was not added to the Open Leaf list, and will not be split any further.

Figure 3 shows the state of the forest after the third rule R_3 has been processed. Nodes A_5 and A_6 have been split, adding A_8 through A_{11} to the Open Leaf list. Node A_7 has not been split by R_3 , and therefore had its Fail count incremented. We set the maximum fail count to 1 (to keep this example concise), and thus A_7 (marked by the crossed circle) will now be removed from the Open Leaf list and will not be split any further. A_7 will predict a class distribution of (1/3, 2/3) for test examples. Nodes B_2 and B_3 have been split, producing B_4 through B_7 , of which B_4 , B_6 and B_7 will be added to the Open Leaf list, while B_5 will not, as it is class-pure. C_1 has been split, producing C_2 and C_3 , and the root of tree D, D_1 , has also been added to the Open Leaf list.

To summarize, R^4F differs from FORF [Assche *et al.*, 2006], in two main ways. First of all FORF does not use full rules in every node, but in contrast paths from the root to each leaf comprise rules. As logical variables can only be shared across positive paths, this complicates both generation as well as interpretation of such trees. Like Breiman's original random forest, FORF randomly restricts the set of possible tests (features) and then picks the best test from that restricted set. R^4F on the other hand uses a fully self-contained randomly generated relational rule as a test. As a consequence, R^4F can easily generate its trees in a staggered parallel fashion, with each new rule being available for all open leaves, while FORF processes both nodes and trees fully sequentially. Even though in theory FORF could also parallelize node and tree generation, it could still not share the expensive cover computation across nodes or trees the way R^4F can.

R^4F can also be seen as an example of dynamic propositionalization [Landwehr *et al.*, 2006], in that the features are generated dynamically on-demand, and do not have to be pre-computed in advance as would be common in static propositionalization [Kramer *et al.*, 2000].

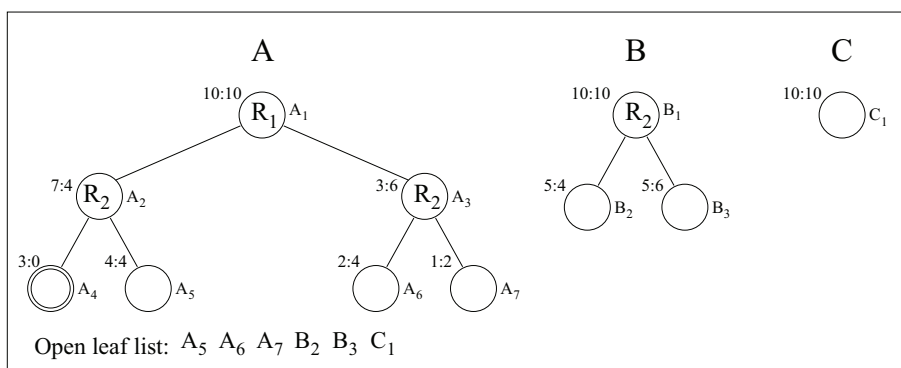


Figure 2: Example of R^4F Forest Construction, Stage 2

2.3 Complexity of Forest Construction

When R^4F generates and evaluates a rule, it then applies the derived test at every open leaf in each active tree. The cost of rule evaluation is the same as for previous uses of Random Relational Rules [Botta *et al.*, 2003; Anderson and Pfahringer, 2007]. As each test is applied to all open leaves, the number of rules required to be evaluated is substantially less than would be required if a new rule were being evaluated for each open leaf, as in standard random forests. The number of rules required for forest construction is heavily influenced by the number of trees in the forest and the Maximum Rule Count. A rough estimate for this value is the sum of the average number of rules required to construct a single tree and the number of trees in the forest, as when the last tree in the forest is completed, the previous trees are also likely to be complete:

$$\text{Rules required for forest generation} \approx (n + s) \quad (1)$$

where n is the number of trees in the forest and s is the average number of rules required to construct a single tree. Empirical evidence confirming Equation 1 can be found in [Anderson, 2009].

Because of the staggered fashion in which the trees are generated, each tree has access to at least one more rule than its immediate successor and so its construction has probably already finished at the time the construction of that successor finishes. Thus, when the final tree is complete, it is likely that all previous trees are complete or nearly so.

The number of rules required to construct a single tree can vary substantially. It is a function of the particular dataset, the Maximum Rule Count and the particular random rules generated. A worst case upper bound is given by:

$$\text{Max \#rules needed per tree} = (t - 1) \times MRC \quad (2)$$

where t is the size of the training set and MRC is the Maximum Rule Count. It is unlikely that this upper bound would be reached under normal circumstances, as it describes the pathological case where each node in the tree is split only after the maximum possible number of rules have been generated, and at every node the split has resulted in one single-instance leaf and one leaf containing all the remaining instances (see Figure 4). In practice the number of rules re-

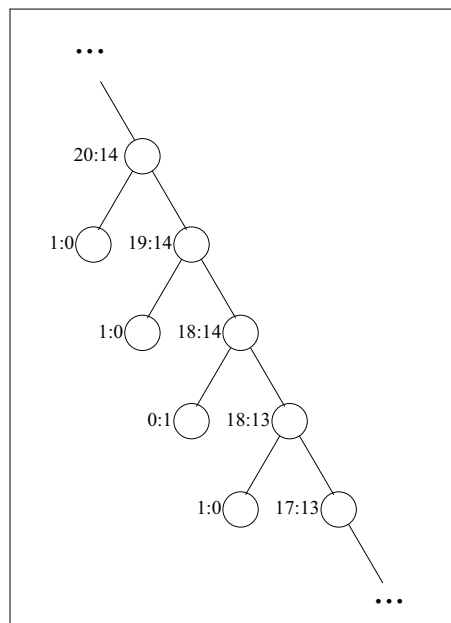


Figure 4: Worst-case tree

quired has always been substantially lower than this worst case upper bound.

3 Empirical Evaluation

R^4F was tested using the following standard ILP datasets: Mutagenesis (with and without regression-unfriendly instances) [Srinivasan *et al.*, 1994], Musk1 [Dietterich *et al.*, 1997], Carcinogenesis [Srinivasan *et al.*, 1997], and Diterpenes [Dzeroski *et al.*, 1997]. Mutagenesis and Carcinogenesis were limited to low-level structural information as represented by atoms and bonds; additional propositional information such as global properties *lumo* or *logP*, or predefined functional groups was deliberately excluded: they are known to improve classification accuracy significantly, thereby potentially masking the relational performance of the investigated algorithms. The current implementation of R^4F is limited to two classes, so the Diterpenes dataset was transformed

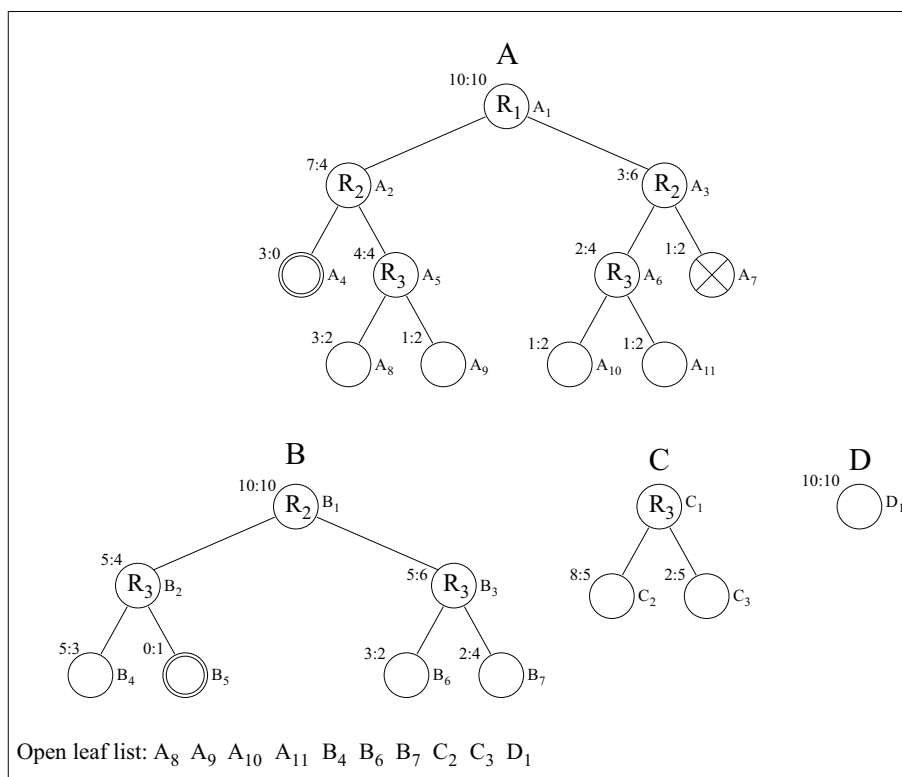


Figure 3: Example of R^4F Forest Construction, Stage 3

Table 1: Accuracy for R^4F using 500 trees, bagged roots, and MRC equal to 50 compared to static propositionalization.

Dataset	R^4F	Static	Significant
Carcinogenesis	61.24	60.91	no
Diterpenes _{52,3}	97.31	96.97	yes
Diterpenes _{52,54}	96.24	94.22	yes
Diterpenes _{54,3}	98.43	97.69	yes
Musk ₁	85.32	89.13	no
Mutagenesis _{All}	79.27	76.66	no
Mutagenesis _{RF}	85.99	84.95	no

into three two-class versions by using all pairwise combinations of the three largest classes called 3, 52 and 54 – Diterpenes_{54,3}, Diterpenes_{52,3} and Diterpenes_{52,54}.

All results were obtained as averages over ten times ten-fold cross-validation. Table 1 compares the accuracy obtained for each dataset, using both R^4F using 500 trees, bagged roots, and MRC equal to 50, and a static propositionalization based on 1000 non-trivial random rules turned into boolean features and processed by an equivalent standard Random Forest.

R^4F has several advantages over a static two-stage method that generates a propositional representation of the data first, and then constructs a Random Forest based on the propositionalized data. The latter approach must generate a suffi-

ciently large number of rules in the first stage without knowing which ones will actually be useful. The propositional representation is potentially very large, but might still not be a good enough approximation of the relational problem. Thus the number of rules to generate will be a critical parameter for the user to set. R^4F on the other hand has a simple stopping condition: completion of the forest, so it never generates more rules than needed. No memory is needed for any intermediate representation, and forest generation is fully parallel. Still, as can be seen in this table, static propositionalization works fairly well for the datasets studied here, and even outperforms R^4F on the Musk dataset, even though that win is not statistically significant due to the small size of that dataset. The only significant differences are the wins for R^4F on each of the three Diterpenes datasets, in all other cases the standard deviations over the ten times ten-fold cross-validation runs are too large.

For comparison with published results for FORF[Assche *et al.*, 2006], we also tested R^4F on Mutagenesis_{All} (Table 2) and the *Financial* dataset (Table 3) using out-of-bag evaluation rather than cross-validation. This was necessary as apparently it is not feasible to evaluate FORF using ten times ten-fold cross-validation. In their paper they used only four datasets, two of which are used here as well, and two were dropped for the following reasons: one is simply too small for any meaningful comparison, as it only comprises 20 examples in total; the other one is a multiple classes problem that the current version of R^4F cannot handle yet. We com-

Table 2: Comparison of R^4F and FORF (out-of-bag evaluation) on *Mutagenesis_{RF}*

Algorithm	Accuracy		
R^4F	79.1 ± 1.1	R^4F is	Significance
FORF	74.7 ± 1.4	better	95%
FORF-SA	78.9 ± 1.8	equal	< 90%
FORF-RA	78.1 ± 1.2	equal	< 90%
FORF-LA	79.0 ± 1.4	equal	< 90%

Table 3: Comparison of R^4F and FORF (out-of-bag evaluation) on *Financial*

Algorithm	Accuracy		
R^4F	87.8 ± 0.7	R^4F is	Significance
FORF	85.7 ± 0.6	better	95%
FORF-SA	99.8 ± 0.2	worse	99%
FORF-RA	99.7 ± 0.4	worse	99%
FORF-LA	99.8 ± 0.4	worse	99%

pare to the highest FORF results listed in their paper, and see that R^4F significantly beats the standard aggregate-less FORF both times. Remember that R^4F currently does not include aggregates, still it is as good as the all FORF-plus-aggregates variants (FORF-LA: lookahead aggregates, FORF-SA: simple aggregates, and FORF-RA: refined aggregates) on one of the two datasets. Clearly, for the second dataset aggregates are essential for near-perfect prediction. Adding aggregates to R^4F will be one major direction for future research.

In addition to measuring accuracy we have also conducted studies to judge the sensitivity of the algorithm to parameter settings and to alternative split rule selection methods. For lack of space we can only summarize here, claiming that, as expected, larger number of trees consistently produce higher accuracies, but that after a couple of hundred trees accuracy usually levels out on a plateau. The MRC or maximum rule count is more of an optimization parameter, usually with optimal values between 50 and 100, where lower values tend to underfit, and higher values cause overfitting. Measuring tree sizes and tree diversity, fully random trees are usually larger and more diverse, and higher MRC values reduce both the size and diversity of trees. For a lot more detail on these aspects please see [Anderson, 2009].

4 Summary and Future Work

The efficiency and effectiveness of the R^4F algorithm is the result of the application of randomly generated relational rules to the random forests framework. Staggered root initialization allows R^4F to produce diverse trees in parallel, and the experimental results obtained are very competitive with those achieved by another Relational Random Forest algorithm. The main direction for future work will be the inclusion of aggregates into R^4F : FORF benefits significantly from adding aggregates, so R^4F might also be able to take advantage of the additional expressiveness that aggregates provide. As it currently stands, R^4F outperforms the non-aggregate version of FORF and is already equal in performance to the

aggregate-enhanced version of FORF on some datasets. Alternative and more targeted random rule generation methods, which work on explicit seed examples that must be covered by a rule, will also be investigated.

References

- [Anderson and Pfahringer, 2007] Grant Anderson and Bernhard Pfahringer. Clustering relational data based on randomized propositionalization. In *Proceedings International Conference on Inductive Logic Programming (ILP2007)*. Springer, 2007.
- [Anderson, 2009] Grant Anderson. Random relational rules, forthcoming PhD thesis, University of Waikato, 2009.
- [Assche *et al.*, 2006] Anneleen Assche, Celine Vens, Hendrik Blockeel, and Sašo Džeroski. First order random forests: Learning relational classifiers with complex aggregates. *Mach. Learn.*, 64(1-3):149–182, 2006.
- [Botta *et al.*, 2003] Marco Botta, Attilio Giordana, Lorenza Saitta, and Michèle Sebag. Relational learning as search in a critical region. *J. of Machine Learning Research*, 4:431–463, 2003.
- [Breiman, 2001] Leo Breiman. Random forests. *Machine Learning*, 45(1):5–32, 2001.
- [Dietterich *et al.*, 1997] Thomas G. Dietterich, Richard H. Lathrop, and Tomas Lozano-Perez. Solving the multiple instance problem with axis-parallel rectangles. *Artificial Intelligence*, 89(1-2):31–71, 1997.
- [Džeroski *et al.*, 1997] Saso Džeroski, Steffen Schulze-Kremer, Karsten R. Heidtke, Karsten Siems, and Dietrich Wettschereck. Applying ilp to diterpene structure elucidation from 13c nmr spectra. In *ILP '96: Selected Papers from the 6th International Workshop on Inductive Logic Programming*, pages 41–54, London, UK, 1997. Springer.
- [Kramer *et al.*, 2000] Stefan Kramer, Nada Lavrač, and Peter Flach. Propositionalization approaches to relational data mining. *Relational Data Mining*, pages 262–286, 2000.
- [Landwehr *et al.*, 2006] Niels Landwehr, Andrea Passerini, Luc De Raedt, and Paolo Frasconi. kfoil: Learning simple relational kernels. In *AAAI*. AAAI Press, 2006.
- [Quinlan, 2000] Ross Quinlan. Relational learning and boosting. *Relational Data Mining*, pages 292–304, 2000.
- [Srinivasan *et al.*, 1994] Ashwin Srinivasan, Stephen Muggleton, Ross D. King, and Micheal J.E. Sternberg. Mutagenesis: ILP experiments in a non-determinate biological domain. In S. Wrobel, editor, *Proceedings of the 4th International Workshop on Inductive Logic Programming*, volume 237, pages 217–232. Gesellschaft für Mathematik und Datenverarbeitung MBH, 1994.
- [Srinivasan *et al.*, 1997] Ashwin Srinivasan, Ross D. King, Stephen Muggleton, and Michael J.E. Sternberg. Carcinogenesis predictions using ILP. In S. Džeroski and N. Lavrač, editors, *Proceedings of the 7th International Workshop on Inductive Logic Programming*, volume 1297, pages 273–287. Springer, 1997.