Preference Learning with Extreme Examples*

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Abstract

In this paper, we consider a general problem of semi-supervised preference learning, in which we assume that we have the information of the extreme cases and some ordered constraints, our goal is to learn the unknown preferences of the other places. Taking the potential housing place selection problem as an example, we have many candidate places together with their associated information (*e.g.*, position, environment), and we know some extreme examples (*i.e.* several places are perfect for building a house, and several places are the worst that cannot build a house there), and we know some partially ordered constraints (*i.e.* for two places, which place is better), then how can we judge the preference of one potential place whose preference is unknown beforehand? We propose a Bayesian framework based on Gaussian process to tackle this problem, from which we not only solve for the unknown preferences, but also the hyperparameters contained in our model.

1 Introduction

The problem of finding out the preferences of an individual exists in many real world applications. For example, a real estate developer evaluating the potential housing places, a customer judges the value of a book, a user assess his/her interests to a movie. Clearly, evaluating the preferences of all the individuals is quite time consuming and almost impossible. Therefore the development of automatic preference prediction (or *preference learning*) algorithms is an important and valuable research topic.

In recent years, many preference learning algorithms have been emerged in artificial intelligence [Doyle, 2004], machine learning [Bahamonde *et al.*, 2004][Chu and Ghahramani, 2005][Chu and Keerthi, 2007], data mining [Agarwal *et al.*, 2006][Yu, 2005] and information retrieval [Herbrich *et al.*, 1998][Nuray and Can, 2003][Xi *et al.*, 2004][Zheng *et al.*, 2007] fields. Most of these algorithms takes preference learning as a supervised learning problem, *i.e.*, we are given a set of instances $\{\mathbf{x}_i\}_{i=1}^n$ which are associated with a partial or complete order relation. Our goal is to learn a "ranking function" from whose data such that it can predict the ranks of new testing instances. However, those type of methods usually suffer from two main problems:

1. Generally training a model in a supervised way needs a large amount of "labeled" data (*i.e.* the data with known orders or preferences). However, in most of the real world cases, we may only known partial order information contained in the data set.
2. There are usually some free parameters contained in the preference prediction model. How to tune those parameters automatically is a headache for most of the algorithms. Generally these parameters are set empirically.

Based on the above considerations, in this paper, we investigate a novel problem called *semi-supervised preference learning* (*SSPL*). In SSPL, we use both labeled (or ordered) data and unlabeled (or unordered) data to train a preference prediction model, that is because in most of the cases, unlabeled data is far easier to obtain (*e.g.*, by crawling the web). These unlabeled data are invaluable resources and how to use them to aid the classification (or regression) task has widely been researched in semi-supervised learning field [Chapelle *et al.*, 2006], but the similar problem in preference learning has rarely been touched. In SSPL, we consider the following two types of supervised information

- Partially ordered information. This type of information the same as in traditional preference learning algorithms. We assume that we know some ordered information on a small portion of data items.
- Extreme preferences. We also suppose that we know some preference information on the extreme cases (*i.e.*, the cases are extremely preferred or disliked by the user, that is, the cases with the highest and lowest preference scores). This type of information is also easy to obtain in many applications. To demonstrate the effectiveness of this type of information, we give an illustrative example in Fig.1.

By incorporating those supervised information, we propose a Bayesian probabilistic kernel approach for preference learning using Gaussian process in this paper. We impose a Gaussian process prior distribution on the latent preference

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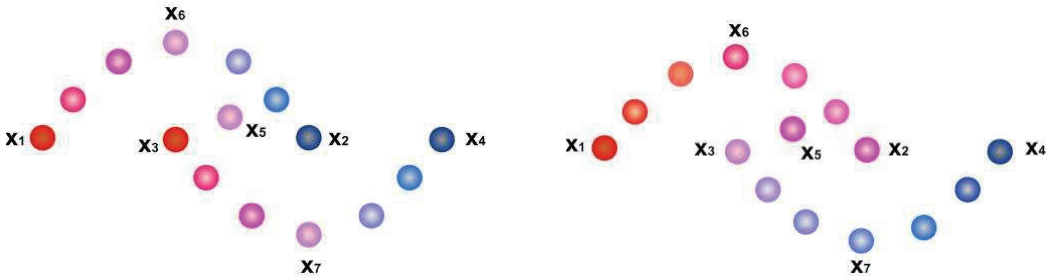

(a) True preferences (b) Predicted preferences

Figure 1: A toy example that illustrates the effectiveness of extreme examples, where the data points are denoted by filled circles, and its color suggests its preference. The closer the color to red (blue), the more (less) the corresponding point is preferred. (a) shows the true data preferences which is distributed as two half-moons with a noise point $\mathbf{x}_5$ in the middle, and $\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{x}_3$, $\mathbf{x}_4$ being extreme examples. The figure shows that the preference of $\mathbf{x}_6$ and $\mathbf{x}_7$ are the same. However, if only given the pairwise constraints pref($\mathbf{x}_1$) $>$ pref($\mathbf{x}_6$) $>$ pref($\mathbf{x}_5$) $>$ pref($\mathbf{x}_4$), we can get a preference distribution in (b), which shows that pref($\mathbf{x}_6$) $>$ pref($\mathbf{x}_7$), which is not correct.

prediction function, and employ an appropriate likelihood function which is composed of two parts: one is a Gaussian function which measures the prediction loss on extreme preferences; the other is a generalized probit function measuring the consistency of the predicted preferences and the known ordered information. Moreover, the *expectation propagation* (*EP*) technique [Minka, 2001] is adopted to automatically tune the hyperparameters contained in the model. Finally we apply our method to a practical housing potential application problem which demonstrates the effectiveness of our method.

It is worthwhile to highlight several aspects of the proposed approach here:

1. Our model is a semi-supervised model, which can make use of more information compared to traditional supervised model. Moreover, the supervised model can be regarded as a special case of our semi-supervised model with all the data items having their relevant supervised information.

2. Unlike in traditional supervised models where we need to set the model parameters empirically, the model parameters can be self-adapted in our approach.

3. Unlike some traditional semi-supervised approach (*e.g.*, [Zhou *et al.*, 2004]) which can only predict the preference of those "unlabeled" data, our method can be easily extended to predict the preferences of new testing data items.

The rest of this paper is organized as follows. In section 2 we will introduce our Bayesian inference framework in detail, the experimental results on benchmark data sets will be introduced in section 3. In section 4 we will introduce the background of a real world housing potential application problem, and show the detailed procedure of how to apply our model to solve the problem, and demonstrate the results, followed by the conclusions in section 5.

2 Semi-supervised Preference Learning Under a Bayesian Framework

Consider a set $\mathcal{X}$ of n distinct data items $\mathbf{x}_i \in \mathbb{R}^d$, in which $\mathcal{X}_\mathcal{L} = \{\mathbf{x}_i\}_{i=1}^l$ are the extreme cases and we know their associated preferences in prior. Besides, we also have a set of observed pairwise preference relations on data items $\mathcal{E} = \{(\mathbf{x}_u, \mathbf{x}_v)\}$, such that if $(\mathbf{x}_u, \mathbf{x}_v) \in \mathcal{E}$, then $f(\mathbf{x}_u) \leq f(\mathbf{x}_v)$, which means that data item $\mathbf{x}_u$ is less preferred than $\mathbf{x}_v$, and f is the latent preference prediction function. For example, in the housing potential place selection problem, $f(\mathbf{x}_u) \leq f(\mathbf{x}_v)$ means it is more appropriate to build a house on place $\mathbf{x}_v$ than place $\mathbf{x}_u$.

Let $\mathcal{O}$ be the observations including $\mathcal{X}_\mathcal{L}$ and $\mathcal{E}$, then the posterior of the latent preference function vector $\mathbf{f} = [f(\mathbf{x}_1), f(\mathbf{x}_2), \cdots, f(\mathbf{x}_n)]^\top$ given the observation $\mathcal{O}$ is

$$P(\mathbf{f}|\mathcal{O}, \mathcal{X}) \propto P(\mathbf{f}|\mathcal{X})P(\mathcal{O}|\mathbf{f}) = P(\mathbf{f}|\mathcal{X})P(\mathcal{X}_\mathcal{L}|\mathbf{f})P(\mathcal{E}|\mathbf{f}) \quad (1)$$

The term, $p(\mathbf{f})$ is the prior. It enforces a smoothness constraint and depends upon the underlying data manifold. Similar to the spirit of graph regularization [Zhu, 2005][Zhou *et al.*, 2004], we use similarity graphs and their transformed Laplacian to induce priors on the preferences $\mathbf{f}$. The second and third term, $p(\mathcal{X}_\mathcal{L}|\mathbf{f})$ and $p(\mathcal{E}|\mathbf{f})$ are the likelihoods that incorporate the prior information provided by the extreme examples and the data in $\mathcal{E}$.

2.1 Gaussian Process Prior

The latent function values $f(\mathbf{x}_i)$ are assumed to be a realization of random variables in a zero-mean Gaussian Process, then this Gaussian process can be fully specified by the covariance matrix. To define a proper covariance matrix, we recall a basic principle in graph based methods [Zhu, 2005]: the predicted data labels should be sufficiently smooth with respect to the data graph. The smoothness of f with respect to the data graph could be measured by

$$\mathcal{S}(f) = \sum_{ij} K_{ij} \left(\frac{f(\mathbf{x}_i)}{\sqrt{d_{ii}}} - \frac{f(\mathbf{x}_j)}{\sqrt{d_{jj}}} \right)^2 = \mathbf{f}^\top \mathbf{L} \mathbf{f} \quad (2)$$

where K_{ij} is the value of the (i,j)-th element in the data kernel matrix $\mathbf{K} \in \mathbb{R}^{n\times n}$, $\mathbf{L} = \mathbf{I} - \mathbf{D}^{-1/2}\mathbf{K}\mathbf{D}^{-1/2} \in \mathbb{R}^{n\times n}$ is the normalized graph Laplacian [Zhou *et al.*, 2004]. $\mathbf{D}$ is the diagonal degree matrix with the i-th element on its diagonal line $d_{ii} = \sum_j K_{ij}$. Then we can construct a data dependent prior distribution of $\mathbf{f}$ as

$$P(\mathbf{f}|\mathcal{X}) = \frac{1}{Z_{\mathbf{f}}} \exp\left(-\frac{1}{2}\mathbf{f}^\top \widetilde{\mathbf{L}}\mathbf{f}\right) \tag{3}$$

where $\widetilde{\mathbf{L}} = \mathbf{L} + \zeta\mathbf{I}$ is the diagonal-jittered Laplacian matrix which is positive definite with some constant ζ [Kapoor *et al.*, 2005], $Z_{\mathbf{f}}$ is normalizing constant which makes $P(\mathbf{f})$ a probability distribution. Such a prior in fact encodes some geometrical information of data distribution $P(\boldsymbol{x})$ [Belkin *et al.*, 2006], *e.g.* if two data items $\mathbf{x}_i$ and $\mathbf{x}_j$ are close in the intrinsic geometry of $P(\boldsymbol{x})$, then the conditional distributions of $P(f_i|\mathbf{x}_i)$ and $P(f_j|\mathbf{x}_j)$ should be similar, where f_i denotes the preference of $\mathbf{x}_i$.

2.2 Likelihood

The likelihood of the observations $\mathcal{O} = \{\mathcal{X}_\mathcal{L}, \mathcal{E}\}$ given the latent function f should be composed of two parts, one part measures the loss between the predictions and the actual preferences (denoted by $\{y_i\}$) of $\mathcal{X}_\mathcal{L}$, the other measures the loss between the predictions and $\mathcal{E}$. The loss between $f(\mathbf{x}_i)$ and y_i can be simply computed by a square function, therefore the likelihood of $\{y_i\}$ given $\mathbf{f}$ can be evaluated as

$$P(\{y_i\}|\mathbf{f}) = \prod_{i=1}^{l} P(y_i|f_i) = \exp\left(-\frac{1}{2}\sum_{i=1}^{l}(y_i - f_i)^2\right) \tag{4}$$

Following [Chu and Ghahramani, 2005], the ideal noise-free case for the likelihood of $(u,v) \in \mathcal{E}$ is

$$P((u_k,v_k)|(f_{u_k},f_{v_k})) = \begin{cases} 1, & if\ f_{u_k} \geqslant f_{v_k} \\ 0, & otherwise \end{cases} \tag{5}$$

In real world case, the preferences are usually contaminated by some noise. If we assume such noise are a zero mean Gaussians, then

$$\begin{aligned}
&P((u_k,v_k)|(f_{u_k},f_{v_k})) \\
&= \iint P(\delta_{u_k})P(\delta_{v_k})P((u_k,v_k)|(f_{u_k}+\delta_{u_k}, f_{v_k}+\delta_{v_k}))d\delta_{u_k}d\delta_{v_k} \\
&= \iint \mathcal{N}(\delta_{u_k};0,\rho^2)\mathcal{N}(\delta_{v_k};0,\rho^2)P((u_k,v_k)|(f_{u_k}+\delta_{u_k}, f_{v_k}+\delta_{v_k}))d\delta_{u_k}d\delta_{v_k} \\
&= \Phi(z_k)
\end{aligned}$$

where $z_k = \frac{f_{v_k} - f_{u_k}}{\sqrt{2}\rho}$, and $\Phi(z) = \int_{-\infty}^{z}\mathcal{N}(\delta;0,1)d\delta$. Note that we use k to denote the entry index in $\mathcal{E}$. Then the total likelihood of $\mathcal{E}$ given $\mathbf{f}$ becomes

$$P(\mathcal{E}|\mathbf{f}) = \prod_k \Phi(z_k) \tag{6}$$

Therefore the total likelihood of the observations $\mathcal{O}$ given $\mathbf{f}$ becomes

$$\begin{aligned}
P(\mathcal{O}|\mathbf{f}) &= P(\{y_i\}|\mathbf{f})P(\mathcal{E}|\mathbf{f}) \\
&= \exp\left(-\frac{1}{2}\sum_{i=1}^{l}(y_i - f_i)^2\right)\prod_k \Phi(z_k)
\end{aligned} \tag{7}$$

2.3 Approximate Inference

In this paper, we use *Expectation Propagation* (*EP*) to obtain a Gaussian approximation of the posterior $P(\mathbf{f}|\mathcal{O})$. Although, the prior derived in section 2.1 is a Gaussian distribution, the exact posterior is not a Gaussian due to the form of the likelihood. We use EP to approximate the posterior as a Gaussian. EP has been previously used [3] to train a Bayes Point Machine, where EP starts with a Gaussian prior over the classifiers and produces a Gaussian posterior. Our task is very similar and we use the same algorithm. In our case, EP starts with the prior defined in Eq.(3) and incorporates likelihood to approximate the posterior $P(\mathbf{f}|\mathcal{O},\mathcal{X}) \sim \mathcal{N}(\bar{\mathbf{f}}, \boldsymbol{\Sigma}_{\mathbf{f}})$, where $\mathcal{N}$ denotes a normal distribution.

2.4 Hyperparameter Learning

We apply an EM-EP style algorithm [Kim and Ghahramani, 2006] to estimate the hyperparameters in our algorithm, which is also referred to as *evidence maximization* [Kapoor *et al.*, 2005]. Denote the parameters of the kernel as Θ_K, then the parameters contained in our algorithm are $\Theta = \{\Theta_K, \zeta, \rho\}$. Under the EM-EP framework, in the E-step of the EM algorithm, we use EP to approximate the posterior $q(\mathbf{f})$; in the M-step, we maximize the following lower bound according to the Jensen's inequality

$$\begin{aligned}
\mathcal{F} &= \int_{\mathbf{f}} q(\mathbf{f}) \log \frac{P(\mathbf{f}|\mathcal{X},\Theta)P(\mathcal{O}|\mathbf{f})}{q(\mathbf{f})} \\
&= -\int_{\mathbf{f}} q(\mathbf{f})\log q(\mathbf{f}) + \int_{\mathbf{f}} q(\mathbf{f}) \log \mathcal{N}(\mathbf{f};\mathbf{0},\widetilde{\mathbf{L}}^{-1}) \\
&\quad + \frac{1}{2}\sum_{i=1}^{l}\int_{f_i} q(f_i)(f_i - y_i)^2 \\
&\quad + \sum_k \iint_{f_{u_k},f_{v_k}} q(f_{u_k},f_{v_k})\log\Phi(z_k)
\end{aligned} \tag{8}$$

The EM procedure alternates between the E-step and the M-step until convergence.

- **E-Step**. Given the current parameters Θ^i, approximate the posterior $q(\mathbf{f}) \sim \mathcal{N}(\bar{\mathbf{f}}, \Sigma_{\mathbf{f}})$ by EP.
- **M-Step**. Update

$$\Theta^{i+1} = arg\max_{\Theta} \int_{\mathbf{f}} q(\mathbf{f}) \log \frac{P(\mathbf{f}|\mathcal{X},\Theta)P(\mathcal{O}|\mathbf{f})}{q(\mathbf{f})} \tag{9}$$

In the M-step the maximization with respect to the Θ cannot be computed in a closed form, but can be solved using gradient descent. The gradients of the lower bound with respect to the parameters are as follows:

$$\frac{\partial\mathcal{F}}{\partial\Theta_K} = \frac{1}{2}\left(tr\left(\widetilde{\mathbf{L}}^{-1}\frac{\partial\mathbf{L}}{\partial\Theta_K}\right) - \bar{\mathbf{f}}^\top\frac{\partial\mathbf{L}}{\partial\Theta_K}\bar{\mathbf{f}} - tr\left(\frac{\partial\mathbf{L}}{\partial\Theta_K}\right)\right) \tag{10}$$

$$\frac{\partial\mathcal{F}}{\partial\zeta} = \frac{1}{2}\left(tr\left(\widetilde{\mathbf{L}}^{-1}\right) - \bar{\mathbf{f}}^\top\bar{\mathbf{f}} - tr\left(\Sigma_{\mathbf{f}}\right)\right) \tag{11}$$

$$\begin{aligned}
\frac{\partial\mathcal{F}}{\partial\rho} = -\int_{\mathbf{f}_k} &\frac{\mathbf{a}^\top\mathbf{f}_k \exp\left(-\frac{\boldsymbol{\mu}_k^\top\mathbf{a}\mathbf{a}^\top}{4\rho^2}(2\mathbf{f}_k - \boldsymbol{\mu}_k)\right)}{2\rho^2\sqrt{\pi}\left|\mathbf{I} + \Sigma_k\frac{\mathbf{a}\mathbf{a}^\top}{2\rho^2}\right|^{1/2}\Phi(z_k)} \\
&\cdot\mathcal{N}\left(\mathbf{f}_k;\boldsymbol{\mu}_k,\left(\Sigma_k + \frac{\mathbf{a}\mathbf{a}^\top}{2\rho^2}\right)^{-1}\right)d\mathbf{f}_k
\end{aligned} \tag{12}$$

where in the last equation, $\mathbf{f}_k = [f_{v_k}, f_{u_k}]^\top$, $\mathbf{a} = [-1, 1]^\top$, $\boldsymbol{\mu}_k = \mathbf{B}_k\bar{\mathbf{f}}$, and $\Sigma_k = \mathbf{B}_k\Sigma_{\mathbf{f}}\mathbf{B}_k^\top$ and $\mathbf{B}_k = [\mathbf{e}_{u_k}^\top, \mathbf{e}_{v_k}^\top]^\top \in \mathbb{R}^{2\times n}$ and $\mathbf{e}_{u_k}$, $\mathbf{e}_{v_k} \in \mathbb{R}^{1\times n}$ are indicator vectors for u_k and v_k with all their elements being 0 except for the u_k-th (or v_k-th) element being 1. This complicated integration can be approximated by Gaussian quadrature or Romberg integration at some appropriate accuracy.

2.5 Induction for Out-of-Sample Data

We denote the optimal parameter setting inferred from the EM-EP procedure to be Θ^*. For a new coming case $\mathbf{x}$, its latent preference value $f_{\mathbf{x}}$ together with the latent preference vector $\mathbf{f} \in \mathbb{R}^{n\times 1}$ of the training samples follows a joint multivariate Gaussian prior, i.e.,

$$\begin{bmatrix} \mathbf{f} \\ f_{\mathbf{x}} \end{bmatrix} \sim \mathcal{N}\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \Sigma & \mathbf{k} \\ \mathbf{k}^\top & k(\mathbf{x},\mathbf{x}) \end{pmatrix}\right] \quad (13)$$

where $\mathbf{k} = [k(\mathbf{x},\mathbf{x}_1), k(\mathbf{x},\mathbf{x}_2), \cdots, k(\mathbf{x},\mathbf{x}_n)]^\top$ and $k(\cdot,\cdot)$ is some pre-defined kernel function, where $\Sigma = \widetilde{\mathbf{L}}^{-1}$. The conditional distribution of $f_{\mathbf{x}}$ given $\mathbf{f}$ is also a Gaussian, denoted as $P(f_{\mathbf{x}}|\mathbf{f}, \Theta^*)$, with mean $\mathbf{f}^\top\Sigma^{-1}\mathbf{k}$ and variance $k(\mathbf{x},\mathbf{x}) - \mathbf{k}^\top\Sigma^{-1}\mathbf{k}$. The predictive distribution of $P(f_{\mathbf{x}}|\mathcal{O}, \mathcal{X}, \Theta^*)$ can be computed as an integral over the $\mathbf{f}$-space, i.e.,

$$P(f_{\mathbf{x}}|\mathcal{O}, \mathcal{X}, \Theta^*) = \int P(f_{\mathbf{x}}|\mathbf{f}, \Theta^*)P(\mathbf{f}|\mathcal{O}, \mathcal{X}, \Theta^*)d\mathbf{f} \quad (14)$$

where $P(\mathbf{f}|\mathcal{O}, \mathcal{X}, \Theta^*)$ is a Gaussian posterior distribution of f approximated by the EM-EP procedure, then the predictive distribution (14) can be simplified as a Gaussian $\mathcal{N}(f_{\mathbf{x}}; \mu_{\mathbf{x}}, \sigma_{\mathbf{x}}^2)$ with

$$\mu_{\mathbf{x}} = \mathbf{k}^\top\Sigma_{\mathbf{f}}^{-1}\bar{\mathbf{f}} \quad (15)$$

$$\sigma_{\mathbf{x}}^2 = k(\mathbf{x},\mathbf{x}) - \mathbf{k}^\top\Sigma_{\mathbf{f}}^{-1}\mathbf{k} \quad (16)$$

3 Experiments on Benchmark Data Sets

In this section, we will present the results of applying our algorithm to several benchmark data sets.

3.1 Data Sets

We test the performance of our algorithm on five benchmark data sets downloaded from `http://www.liacc.up.pt/~ltorgo/Regression/DataSets.html`, whose target values were discretized into ordinal quantities using equal-length binning. These bins divide the range of target values into a given number of intervals that are of same length. The resulting rank values are ordered, representing these intervals of the original metric quantities. Table 1 summarizes the basic characteristics of those data sets, where "Size" denotes the number of instances in the data set; "Dimension" indicates the dimensionality of the data points; "# Training" is the number of instances used for training; "# Testing" is the number of instances used for testing.

3.2 Methods for Comparison

Beside our method, we also implemented some other competitive methods for comparison including

Table 1: Description of the data sets.

Data Sets	Size	Dimension	# Training	# Testing
Diabetes	2	43	30	13
BreastCancer	32	194	130	64
Pyrimidines	74	27	50	24
Trizazines	186	60	100	86
MachineCPU	209	6	150	59
BostonHouse	506	13	300	206

- **SVM**. This is a support vector method for ranking [Shashua and Levin, 2003]. 5-fold cross validation was used to determine the optimal values of model parameters (including the width of the Gaussian kernel and the regularization parameter), and the test error was obtained using the optimal model parameters for each formulation. The initial search was done on a 7×7 coarse grid linearly spaced by 1.0 in the region $\{(\log_{10} C, \log_{10}\sigma) | -2 \le \log_{10} \le 4, -3 \le \log_{10}\sigma \le 3\}$, followed by a fine search on a 9×9 uniform grid linearly spaced by 0.2 in the $(\log_{10} C, \log_{10}\sigma)$ space.
- **GPPL**. This is the Gaussian process preference learning (GPPL) method in [Chu and Ghahramani, 2005]. The implementation is based on the code `http://www.gatsby.ucl.ac.uk/~chuwei/plgp.htm`, where gradient methods have been employed to maximize the approximated evidence for model adaptation with the initial values of the Gaussian kernel width κ and the noise ρ to be 1 and $1/d$, d is the data dimensionality.
- **GPOR**. This is the Gaussian process ordinal regression method implemented using the EP algorithm as in [Chu and Ghahramani, 2004]. The implementation code is downloaded from `http://www.gatsby.ucl.ac.uk/~chuwei/ordinalregression.html`.
- **SVOR**. This is the support vector ordinal regression method implemented the same as in [Chu and Keerthi, 2007], where we use implicit constraints and the code is downloaded from `http://www.gatsby.ucl.ac.uk/~chuwei/svor.htm`.

For the GPPL method, we generate all the pairwise constraints from the training data, and for our *semi-supervised preference learning* (*SSPL*) method, we further provide the preferences of the extreme cases in the training set. The experimental results, including the mean absolute error and standard deviation, over 20 independent runs are summarized in Table 2. From the table we can clearly see the advantage of our method.

4 Application in Housing Potential Selection

In this section, we present a novel application of our algorithm in computer aided housing potential estimation and location recommendation system. Such an application is important since nowadays, facilities or outlets (*e.g.* bank branches, retail stores, automobile dealers, etc) are crucial for people's daily lives. However, it is usually expensive and time consuming for a company to evaluate the suitability of the facility site locations and optimize the site network to serve more

Table 2: Experimental results on benchmark data sets.

	SVM	GPPL	GPOR	SVOR	SSPL
Diabetes	0.7462±0.1414	0.6763±0.1508	0.6654±0.1373	0.6658±0.1439	0.6318±0.1247
Breast Cancer	1.0031±0.0727	1.0055±0.0868	1.0141±0.0932	1.1243±0.1077	0.8796±0.0754
Pyrimidines	0.4500±0.1136	0.4096±0.1206	0.3917±0.0745	0.6945±0.2032	0.3544±0.1420
Triazines	0.6977±0.0259	0.6783±0.0198	0.6878±0.0295	0.7033±0.0276	0.6248±0.0193
MachineCPU	0.1915±0.0423	0.1793±0.0562	0.1856±0.0424	0.2136±0.1033	0.1536±0.0317
Boston House	0.2672±0.0190	0.2763±0.0314	0.2585±0.0200	0.2887±0.0198	0.1934±0.0156

customers. The most commonly adopted method is to hire or ask for the business consultants to write some evaluation reports on estimating whether there is big value at a location for housing. Generally consultants should investigate several factors around the housing location within a million square meters including (but not limited to)

- The commercial services sites such as shopping centers, banks, supermarkets, carnies, amusement parks, etc.
- The social service sites such as hospitals, hotels, kindergartens, schools, colleges, etc.
- The traffic conveniences such as bus and subway stations, or even the railway and air stations.
- Other facilities such as bars and restaurants.

As an example, we show a map of housing locations along with their impact factors in Fig. 2, where different symbols represent different factors. In the figure there are totally 47 housing potential locations plotted.

After collecting all the relevant information above, the consultants need to integrate them together by assigning different weights to different factors and finally give a overall estimation of the suitability of a place. However, this may not be a good strategy since (1) the number of factors that may affect the suitability of a place could be very large, which makes the determination of their weights a hard and time consuming task; (2) the weights are usually determined manually by the consultants according to their professional experience. Therefore the construction of a mathematical model for evaluating the suitability of each location automatically is a problem worthy of researching due to its practical requirements.

Unfortunately, it is difficult to construct a good location evaluation model because

1. We usually do not know the exact mechanism of evaluating those locations. As stated before, in most cases we should consider multiple factors simultaneously to evaluate whether a location is good or bad, *e.g.*, in banking, we should consider deposit, loan, financial service revenue, cost, etc. Since different factors have different descriptions and scales, it is difficult to model those factors into a single objective function to be optimized.
2. We usually do not have enough historical rating data of evaluating the housing locations, which makes training a proper model for evaluating new places very hard and unreliable.

Based on the above considerations, the model proposed in this paper could be very suitable for solving the housing location evaluation problem since (1) Usually the information

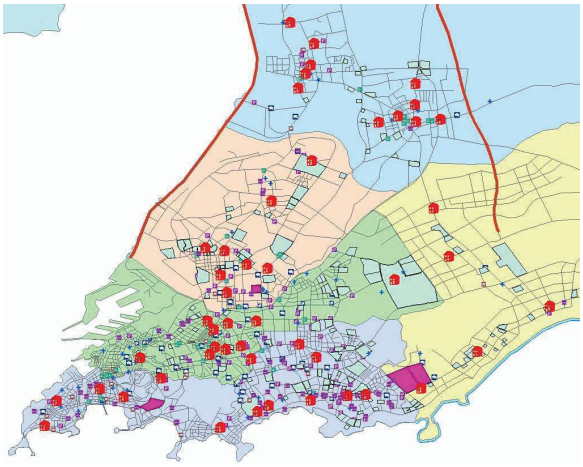

Figure 2: An example of map with housing locations and their impact factors. Factors around the housing location within a million square meters should be considered features for ranking.

of extreme cases are easy to obtain since the extremely good or bad locations are treated as examples to guide the consultants for their evaluations; (2) Our model need not to know the exact preferences (ratings) of each location, but only the ordered relationships between some of the pairwise locations, which is much easier to work out by consulting some experts; (3) Our model is semi-supervised, which means that we need not to collect a large amount of historical rating data, generally a small portion of them is enough; (4) Our model can tune the model parameters adaptively according to the data distributions, so the users need not to worry about how to set the optimal parameters related to different factors.

In our experiments, we just adopted the housing location distribution map shown in Fig.2 as our data set. Therefore there are totally 49 potential housing locations. For each location, we construct a 32 dimensional vector which can summarize the factors that may affect the final evaluation of the suitability of itself[1]. We label four locations as extremely good places for housing and four locations as extremely bad places for housing. For the other 41 locations, we randomly generate 20 pairwise ordered constraints, and this process is repeated 20 times. To demonstrate the superiority of our method, we

[1]The vector is summarized in the following way. We first extract 32 factors (facilities) which may affect the final suitability evaluations of the housing potential locations. For each vector, the value on one dimension is the number of its corresponding facility.

Table 3: Experimental results of different methods on the housing potential location estimation task

	SSDML	GPPL	BSSPL
AUROC	0.7805	0.7934	0.8336
AUROCCH	0.8326	0.8533	0.8848

also conducted two competitive approaches:

- The semi-supervised distance metric learning (SSDML) method [20], which does not make use of the ordered constraints.
- The Gaussian process preference learning (GPPL) method [Chu and Ghahramani, 2005], which cannot take the information on extreme cases into account.

Finally, to compare the performances of those different algorithms, we use the areas under the receiver operating characteristic ROC curve (AUROC) and convex hull of ROC curve (AUROCCH) as our criterions[2]. The final results are summarized in Table 3 (note that all the values in the table are averaged over 20 independent runs), from which we can clearly observe that our method can perform significantly better than the other two methods for this task.

5 Conclusions

In this paper, we propose a novel semi-supervised preference learning method using Gaussian process, where we assume that we are given a set of pairwise ordered constraints together with some extreme examples. We propose an EM-EP algorithm to learn the hyperparameters in our algorithm. Finally the experimental results on both benchmark data sets and real world housing potential place estimation are presented to show the effectiveness of our method.

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[2]When computing these ROC related scores, we actually transform the preference learning problem into a two-class classification problem, i.e., the corresponding place is "good" or "bad" for housing.

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