HEURISTIC ALGORITHMS FOR AUTOMATED SPACE PLANNING

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Summary

This paper reviews the major characteristics of existing programs for automatically solving space planning problems. It outlines general features of this class of problem and introduces a set of heuristic decision rules called Constraint Structured Planning, which in preliminary tests have efficiently solved a variety of problems. Developments allowing even greater efficiencies are also outlined.

Introduction

I wish to focus on computer programs allowing automatic resolution of problems which humans solve using orthographic drawing. Typical problems include the layout of floor plans, the arrangement of equipment in rooms, site planning, and other forms of two-dimensional design tasks. In such problems, distance, adjacency, and other functions of arrangement are a principal concern. To distinguish spatial arrangement tasks from other types of design, we call them space planning problems.

Several programs are currently running or are under development which allow formulation and automatic resolution of space planning problems (1, 5, 6, 11, 12). Within them, a variety of computer data structures have been developed which can represent space and objects, and operations have been implemented for manipulating and testing the resulting arrangements. While it is straightforward to use such systems in an interactive design mode, the benefit and challenge of such programs comes from their potential for automated space planning, for automatically combining manipulation and test operations in such a way that acceptable arrangements are quickly generated. To date the complex and ill-formed structure of most space planning tasks has allowed the development of algorithms which do only poorly when compared with an experienced human draftsman.

The relationship between the techniques implemented in the different programs for automatic space planning has not been at all clear. Each must be considered an ad hoc attempt to build from scratch a theory and program with capabilities originally only the province of humans.

This paper examines the structure of the space planning task and proposes improved methods for dealing with it. Several general issues inherent to the task are identified and the methods used for dealing with them in the existing programs are reviewed. The space planning task is organized into its component decision rules. One set of such rules is described which we call Constraint Structured Planning. Aspects of Constraint Structured Planning have been implemented in the General Space Planner (GSP) program in operation at Carnegie-Mellon University (1, 2). Initial trials with it indicate that Constraint Structured Planning has significant potential for efficiently solving a wide variety of space planning problems. The logic behind the aspects of GSP not yet implemented suggest that a program incorporating all its features will be more efficient than those now in operation.

Problem Formulation

In previously published studies of how humans solve space planning problems (3,9) it has been recognized that designers do not treat their problems within the traditional framework of optimization. While on the one hand their concerns are usually complexly related and involve multiple objectives, the small amount of information available about a problem does not permit meaningful development of linear or non-linear parameter weighting schemes.

In a general way, their problems are normally self-defined in terms of constraints. The total set of constraints defines a feasible solution domain. The task is to find a feasible solution within this domain. Optimization of a sort does take place during iteration of the search sequence. When the initial set of constraints results in a trivial search, either new constraints are added or old constraints are replaced with new more restrictive ones. Conversely, when a current problem seems intractable, the constraints are often relaxed. Optimization takes the form of iteratively modifying the problem definition until an appropriate balance between tractability and quality of results are achieved. Problem solving effort thus plays an important role determining the final problem formulation.

Implicit recognition of the above procedure of working has led the designers of all existing computer assisted space planning programs to rely on the following general formulation. Some of the current programs also facilitate iterative definition and resolution of

a problem.

GIVEN:

a	a Space;
$\{b_1, b_2, \ldots b_m\}$	a set of elements to arrange in that space;
[1] $\{c_1, c_2, \ldots c_n\}$	a set of desired spatial relations required for any acceptable arrangement of elements;
$\{d_1, d_2, \ldots d_p\}$	a set of operators for manipulating the location of elements within the Space;
e^o	some initial design condition (which simply may be a)

FIND:

a set of transformations s such that e^s is generated where

$$e^s \leftrightarrow \{c_1, c_2, \ldots c_n\}$$

where $\leftrightarrow$ represents a matching operation.

We call this formulation the single level space planning task. It can depict many typical drafting problems. One example is shown in Figure One. It describes a typical mechanical room layout problem. The elements to be included in the space and the set of relations to be satisfied are shown at the bottom of the figure.

The Representation of Objects and Relations

The representation of spaces and elements that have been implemented in the various space planning programs all define shape. A major concern has been the accuracy with which shapes are defined. Several are limited to rectangles.* Some implementations include predicates assigned to domains within the shape to define the type of space being represented. This information is useful for automatically evaluating possible locations and eliminating those with overlap conflicts. Usually, only overlaps of certain kinds of spaces are allowed. If domains of space and their type are included in the shape representation, then the shape can be made up of more than one type of space. This allows the area adjacent to an element and required for its use to be included in its initial shape definition, e.g., the ■pace that must remain free behind a desk so that the drawers can be pulled out. Other implementations leave the prohibition against certain overlaps to be defined by a relation. Several of the space planning implementations also include points that may be defined within a shape (e.g. the location of an electrical outlet) and reference can be made to particular sides of objects.

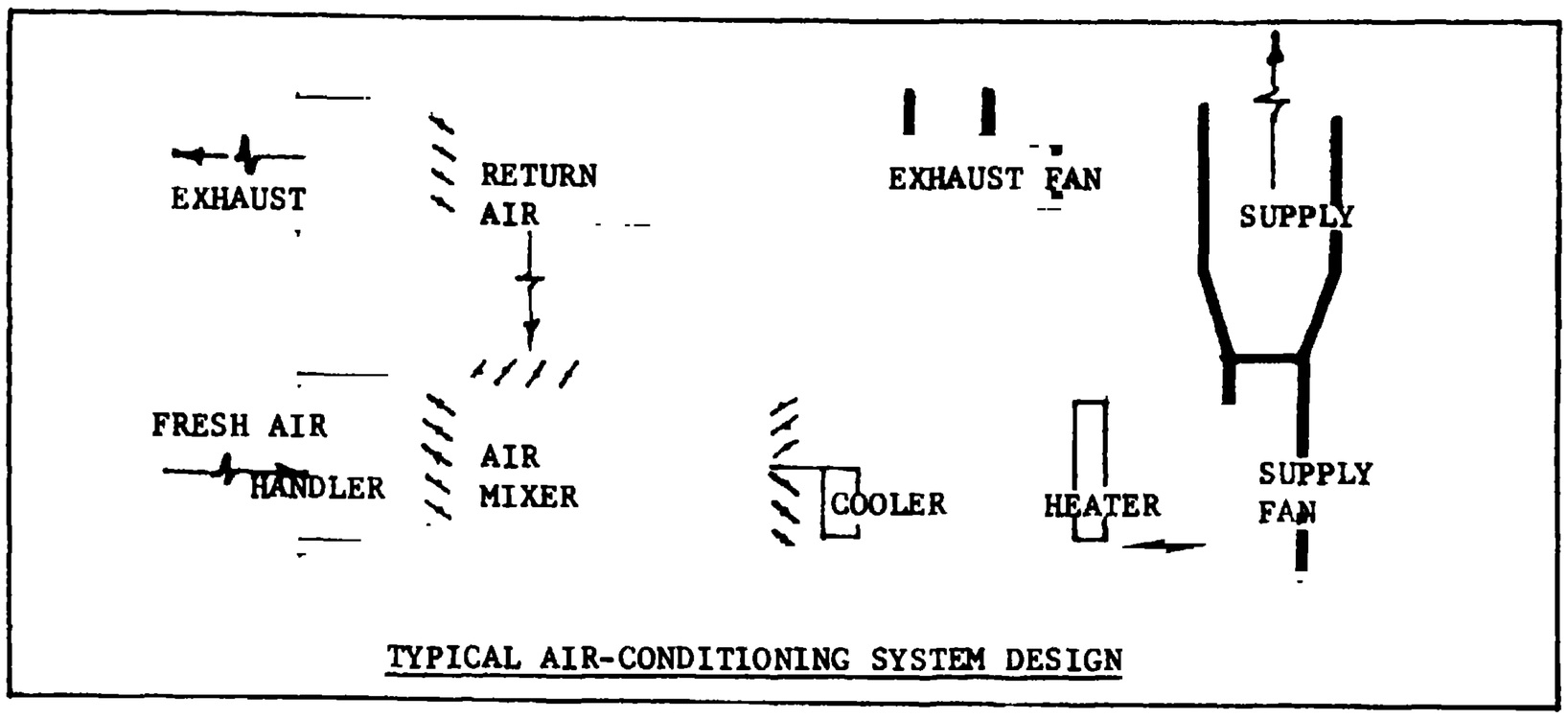
For any particular problem, one can identify a set of relations that should be satisfied if an arrangement is to be accepted. For automatic treatment, the task becomes one of defining a general set of tests with appropriate predicates which are able to depict the wide variety of specific relations required in any particular class of space planning problems. (Throughout the rest of this paper, we will call such tests space Relations - or S-Relations.) In the systems implemented to date, S-Relations useful for room arrangement (5), equipment arrangement within a room (1, 12), and building arrangement on a site (6,11), have been considered. The set of S-Relations that have been implemented in the GSP space planning program are presented in Table One. The task of defining a set of S-Relations that are able to depict the whole set of space planning tasks, or even a single class of such tasks, remains as a problem for further research.

For ease of description, we say that a S-Relation which has as one of its predicates an element i "belongs" to element i. Most S-Relations implemented in programs thus far act to disqualify certain arrangements of <u>pairs</u> of elements. Thus most S-Relations belong to two elements. (A few belong to a single element, e.g., requiring that the element face southward. The IMAGE system by Johnson et al (6) incorporates two S-Relations which belong to three elements.) The reason space planning is inherently difficult is the large (potentially infinite) number of location and orientation combinations that are available for any single element. The difficulty involved in evaluating locations for a single element is severely compounded by the interdependencies among elements imposed by the S-Relations. The exhaustive enumeration of all combinations of locations for each element is impractical for all but the most trivial arrangement problems.

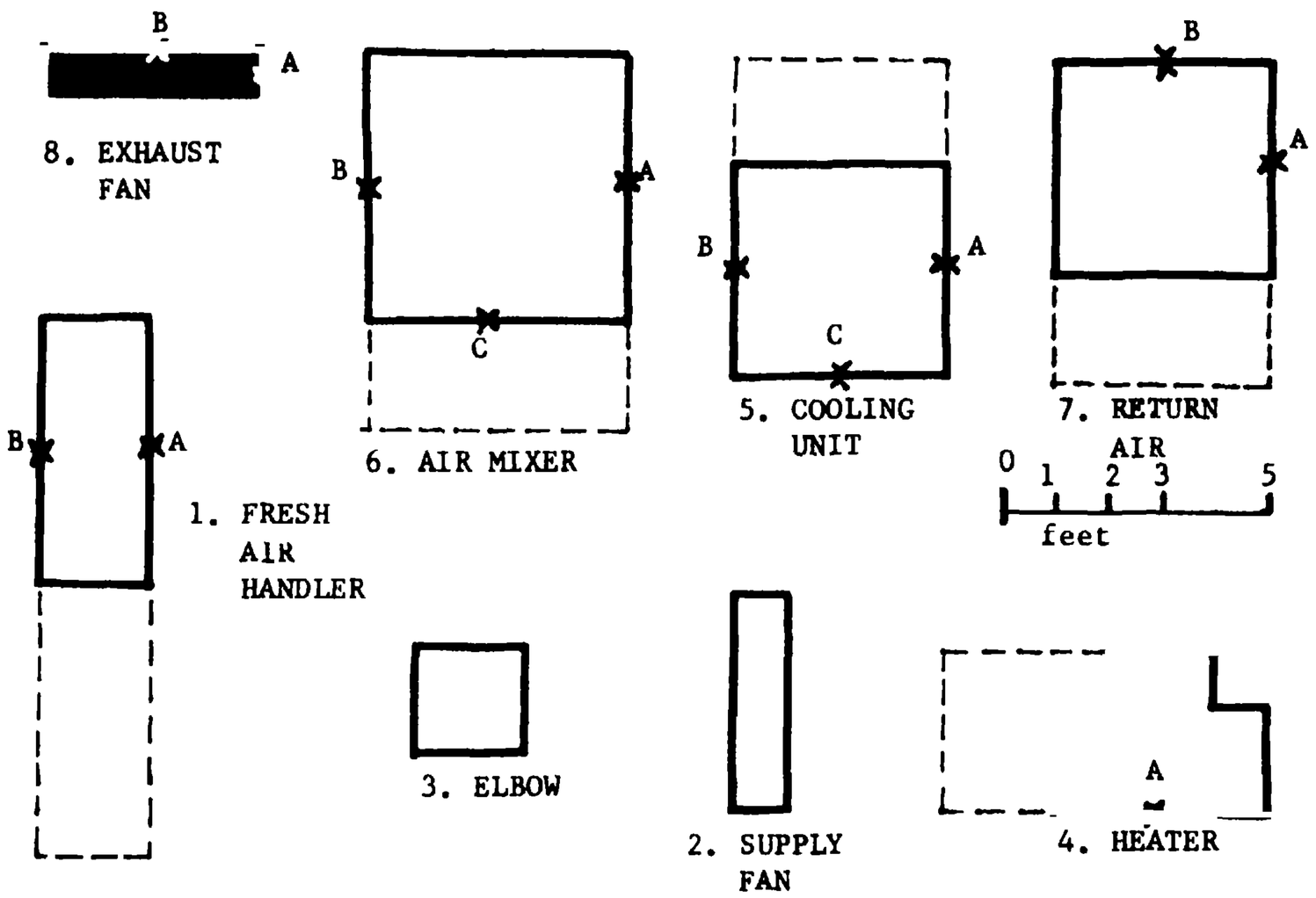
Location Operations

In any space planning program, the types of S-Relations that can be resolved and the characteristics of the location operators available are intimately related. Let us consider for a moment the kinds of locations that should be generated if different S-Relations are to be satisfied. Almost all S-Relations that have been incorporated into space planning programs accept either a line of locations (generated along the perimeter of an element, for example) or an area, possibly disjoint, in which any location

* Of the representations developed to date, Pfeffercorn's is the most accurate. His is likely to be the model for future sophisticated space planning systems (12).



TYPICAL AIR-CONDITIONING SYSTEM DESIGN



REQUIRED RELATIONS

1. Element 2 oriented toward supply.
2. Element 2 <5 from supply
3. Element 3 adjacent to side 3 of element 2.
4. Element 3 adjacent to side 2 of element 4.
5. Point A on element 4 <1 from point B on element 5.
6. Point A on element 5 <1 from point B on element 6.
7. Point A on element 6 <1 from from point B on element 1.
8. Point A on element 1 <3 from outside air.
9. Side 4 on element 7 adjacent to side 1 on element 6.
10. Point B on element 7 <1 from point B on element 8.
11. Point B on element 8 <3 from return air.
12. Side 1 of element 8 oriented toward return air.
13. Visibility from door to point C on element 6.

FIGURE ONE- An example of a space planning problem from the area of building engineering.

is equally acceptable. For example, the area and line allowed by a SIGHT and an ADJACENT relation are shown in Figure Two. The actual area projected by a S-Relation is only defined if one of the elements it belongs to is located. In the following discussion, we limit consideration to those S-Relations for which an area can be defined. (This important limitation is considered in the final section of this paper.)

S-Relations restricting the locations for an element i will define a set of lines and areas, denoted as $S_{i,\delta}$. It is easily proved that the area intersection of any set, S., can be found by examining only the points of intersection between pairs of its areas or lines.* Of course, if two S-Relations belonging to S_δ define lines which are not parallel, then only the point defined by their intersections need be considered. It is important to note that it is never necessary to consider location points that are not at the intersection of two boundaries of areas in S_δ.

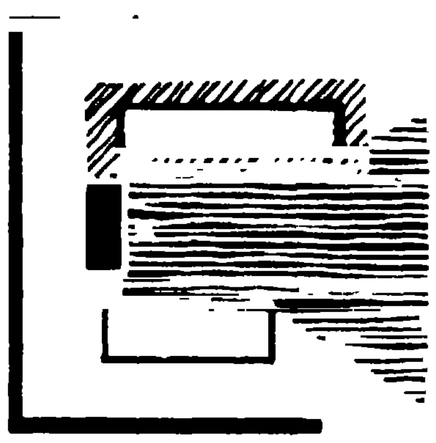


FIGURE TWO - The areas projected by a sightline and an adjacent S-Relation.

In theory, one should be able to project those lines that define the boundaries of the areas in S_δ and to search their point intersections for one satisfying all relations. No space planning program has implemented such an approach to locating elements. Two different approaches have thus far been relied upon.

One approach responds to the interdependency of S-Relations and locations by binding them together, one-to-one, much like the recognition and implementation aspects of an interpreter. The IMAGE system, by Johnson et al at M.I.T. (6), starts with an initial (arbitrary) arrangement and upon finding a S-Relation not satisfied, evokes an operator which satisfies it by finding a new location for an element. This approach simplifies the control structure of a space planning system and guarantees that each S-Relation has at least one effective means for responding to it. Its disadvantage is that the operators used are not able to systematically explore the area defined by any S-Relation.g Thus neither the intersection of the area in S. nor any other systematic search of locations can be undertaken.** Grason's program similarly binds S-Relations to location operators, but incorporates operators which generate all locations satisfying a S-Relation (5). Each time an operator is evoked, it produces another location within the area defined by its S-Relation. To facilitate search, each S-Relation operator pair is contextually independent. This independence is achieved by restricting the S-Relations that can be represented.

Eastman (1) and Pfeffercorn (12), on the other hand, keep the binding between S-Relations and location operators loose. In their programs, a limited number of location operators are provided which serve to satisfy whole sets of S-Relations. In Pfeffercorn's DPS program all the possible locations for an element are generated in one pass and stored on a list for evaluation. A single location operator generates the list and thus it is relied on for satisfying all S-Relations. Eastman's, on the other hand, incorporates operators which sequentially produce locations, one at a time, like Crason's. A set of operators is provided, each producing a different sequence of locations for an element. Thus matching is possible between a set of S-Relations belonging to an element and the most appropriate location operator for locating it. In both programs, no one-to-one relation exists and thus each location produced must be evaluated.

In these programs, search of the boundary of the areas defined by different S-Relations is approximated by location operators that respond to particular arrangement characteristics, e.g., the projected edges of elements and the perimeters of elements. Thus they are usually able to generate some of the locations satisfying a set of S-Relations. (The location operations are described in more detail later.) The limitation imposed by the approximations used by these programs has not been determined.

Algorithms for Space Planning

Two alternative but fundamental approaches have been utilized thus far in constructing algorithms for solving space planning problems. The first takes *AB* its initial step the generation and storage of all the possible locations for each individual element. The second step is then to eliminate all conflicting locations resulting from overlaps or S-Relations until only

*Consider the area that is the intersection of S_δ. Each of its edges will becdefined by one boundary of one area in F. . Its corners will then be defined by the intersection of two of its edges.

**IMAGE relies on "averaging" the locations proposed by each S-Relation using a least-squares-means-fit procedure. The new location is likely to be different from any of those that are generated by the S-Relations (6).

S-RELATIONS:

(1) ADJACENT (A, B, SA, SB) - This S-Relation specifies that element B must have all of its side SB adjacent to side SA of element A. (A may also be a space.) Parameters SA and SB are optional; if one or both are not specified, any side of the corresponding element may be adjacent;

(2) SIGHT (A, B, PTA, PTB) - This S-Relation specifies that Point PTA on element A be visible from Point PTB on element B; no solids may lie in the space dirently between them. Again, the points are optional and if not defined, all of the element facing the other element must be visible;

(3) DISTANCE (A, B, F, PTA, PTB) - This S-Relation defines that Point PTA on element A be less than F units away from Point PTB on element B. PTA and PTB are both optional parameters. If either or both are omitted, distance is measured from the center of the element;

(4) ACCESS (A, B, F) - This S-Relation requires that a pathway exist between element A and element B. The pathway must be at least F units wide along its whole length;

(5) ORIENT (A, B, SA, SB) - This relation defines required orientations between elements B and A. Side SB of element B must face element A and side SA of element A must face element B. Either SA or SB (but not both) are optional parameters.

LOCATION OPERATORS:

SCAN (A, B) - Sequentially generates the complete set of locations for element B, identified by the projection of edges in the space A. (See Figure Three.) It sequentially considers placement of the element in each; all four orientations of the element are considered. SCAN is the most general location operator available in GSP;

PERIMETER (A, B, SB) - This operator sequentially generates the significant locations for element B along the boundary of a specified and located element in the space, denoted by A. The side SB of element B is placed along the common border. One or more calls of this operator can satisfy the ADJACENT S-Relation.

TABLE ONE

Elements of General Space Planner. The above S-Relations and Operators are those implemented in the most current version of GSP.

one or more feasible sets remain. The difficulties with this Approach are (a) the large amount of memory consumed in storing all location combinations; (b) the large amounts of computer time required to search the combinations; and (c) the strong assumptions that must be made about which locations from among all those possible should be stored. Pfeffercorn's program relies on this method to locate single elements. The one effort which totally relied on this approach utilized a grid for defining shapes and potential locations (9).*

The second approach, and the one we will examine in detail, views the space planning task as one of sequentially generating locations for each element, and adding elements or changing their locations, one at a time, until a single acceptable arrangement is produced. In order that the perturbation of elements to locations not be random, consider them as initially perturbed and evaluated in lexiographic order. The task then is to alter the initial lexiographic order, using available information, so as to make the search sequence as efficient as possible.

The decisions defining the perturbation sequence and thus the efficiency of space planning in this approach are:

1. the sequence of locations considered for a particular element when it is added to the arrangement;

2. the sequence in which the elements are added;

3. the sequence in which S-Relations are applied to test an arrangement or location and when in the perturbation sequence they are applied;

4. when a location operator cannot find a location, or whan a S-Relation is tested and fails, the rule or rules which define what object/location combination is to be perturbed to allow further progress on the problem.

In terms of the formulation represented in eq. [1], this approach relies on operations of the form

$$[2] \qquad (e^{t}, d^{t+1}, b^{t+1}) \rightarrow e^{t+1}$$

and can be depicted as a search tree.

Of the existing programs for space planning, only two attempt to structure these four decision rules in an efficient manner.** In the following, we present bases on which efficient procedures may be developed for responding to the four decision conditions. In most cases, the concepts described have been incorporated in the GSP program now in operation at Carnegie-Mellon University. Thus the following can be considered a description of the automated design procedures in GSP. Where the decision rules described have not been implemented, we shall explicitly say so. Also included in the discussion is Pfeffercorn's method for treating these four issues. To augment the presentation, the S-Relations and location operators incorporated in GSP are described in Table One. Those interested in a fuller description of CSP should refer to (1, 2).

The Sequence of Locations

In the previous discussion of location operations initial consideration was given to the selection of operators and appropriate locations. In the following, we examine more fully rules for sequencing locations.

The basis for selecting a location operator and/or location for an element is the interaction between two or more of the S-Relations belonging to the element. If the kinds of S-Relations that are used to specify arrangement objectives are kept very simple, many generalizations concerning combinations of S-Relations are possible. For example, suppose adjacency relations require only that any two sides of the elements it belongs to must be in contact. If we also assume that distance relations are always measured from the nearest perimeter of the two objects, then a distance relation of zero is equivalent to an adjacency relation between the same two elements. In cases where the distance relation is not zero, the locations available to the adjacency relation would be a subset of those defined by the distance relation and could be ignored.

If S-Relations are defined in such a way as to represent the full complexity of meaningful problems, then the interaction among them cannot be so simply specified. For example, in the S-Relations that have been incorporated in GSP, the DISTANCE S-Relations allows distance between specified points; the ADJACENT S-Relation allows specification of the side to be adjacent. If points and sides are used as references, the

* Recent contributions in the area of constraint elimination using known alternatives suggest that this approach may still have merit. See (10).

** The breakdown of the remaining programs are as follows: Graaon's (5) relies on an exhaustive search; Negroponte's URBAN5 (11) operates in the interactive design mode only (?); and Johnson's IMAGE (6) relies on a least-squares-means-fit procedure for combining S-Relations.

relation between the two S-Relations cannot be easily specified. After careful analysis of the interaction of the S-Relations in GSP, the author has concluded that only four warrant special attention in its space planning algorithms.*

It is not possible in GSP to generate the boundaries of areas defined by S-Relations. GSP relies instead on selecting a location operator that is likely to quickly generate some of the locations in the intersection. It includes an operator that generates locations responsive to the one line producing S-Relation, ADJACENT. It generates locations along the perimeter of a specified or element. Points on the perimeter are selected that are defined by projection of the edges in the current arrangement. See Figure Three. If no ADJACENT S-Relation belongs to the element to be located, a general operator called SCAN which tries all corners defined by projections of edges within the space. (See Figure Three.) Pfeffercorn's program relies on one operator similar to SCAN for making all locations.

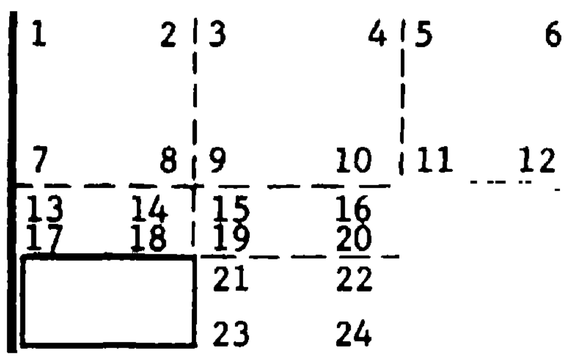


FIGURE THREE - In the twenty-four locations shown, each element would be considered in its four orientations. PERIMETER (in relation to the space) would generate 1-2-3-4-5-6-12-11-16-20-22-24-23-17-13-7 in that order.

* The design of S-Relations which can depict the complexities of the real world and which interact in well specified ways is an important research problem. The interactions specially considered in GSP are that ORIENT defines a subcondition of ADJACENT and SIGHT. Only location-orientation combinations which agree with ORIENT need be considered in solving SIGHT. Two ADJACENT S-Relations interact to define one of three conditions: (1) a single or possibly two locations; (2) a null location set; (3) if the distance between the two elements are the exact width of the third, a line of locations results. Last, two ORIENT S-Relations belonging to the same element must agree, else they define a null intersection.

The Sequence of Elements

In general, an efficient space planner is one which can find a feasible arrangement as quickly as possible. An equivalent objective, is to show in as few trials as possible that no arrangement is possible.** In the following discussion we use this second objective as a substitute of the first.

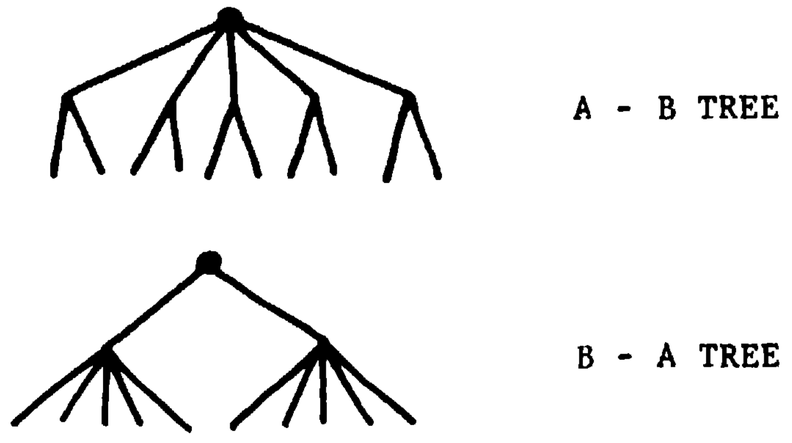


FIGURE FOUR - Effect of order on the number of operations required to search a tree.

The sequence in which elements are added to an arrangement can significantly effect search efficiency. For instance, consider an arrangement involving two elements, A and B. We ignore consideration of all issues but the number of locations for each element; element A can be located in five locations, B in two. Two search trees are possible. Both are shown in Figure Four. In order to show that no arrangement is possible, the number of location operations required to search the A-B tree is 15, while the B-A tree requires 12. In general, the number of operations required to enumerate the alternating location possibilities of a set of elements, with a fixed number of location

** The relationship between these two objectives may not be obvious. Consider each sequentially generated arrangement of elements as identified by Its index n, where $1 < n < N$. In order to prove that no arrangement satisfies all S-Relations, all N arrangements must be generated and evaluated. On the other hand, if M arrangements exist that satisfy the S-Relations, their locations within the ordering of n can be considered random and uniformly distributed (assuming an exhaustive enumeration algorithm). Thus the average number of locations that must be evaluated is equal to

$$P(x_n) = \prod_{j=1}^{n} \frac{M^{x_j}\,(N-M-j+1)^{1-x_j}}{(N-j+1)} \geq 1/2$$

where M = the number of feasible solutions that exist, $M \leq N$. $p(x_n)$ corresponds to the accumulated probability that by the nth draw, one of the M arrangements will be generated. See (14), Chapter 8.

possibilities is equal to W where

$$[3] \qquad W = \sum_{j=1}^{m} \left(\prod_{j=1}^{i} p_{[j]} \right)$$

where $p_{[1]}$, $P_{[2]'}...p_{[m]}$ is a Particular ordering of the location possibilities associated with each of the m elements. It is easily proved that W is minimized when elements are added to an arrangement in ascending order of their p. Thus if we can determine the number of potential locations for each element, we can derive the sequence for searching their combinatorial locations most efficiently.*

In reality, no fixed number of locations can be specified for an element; the number changes according to the arrangement of the other elements. We may approximate the ascending order of p by dynamically evaluating the relative number of locations available after each change in the existing arrangement. After each change, we add to the arrangement next that element with the relatively fewest number of locations available to it.

The locations available to an element are those within the intersection of areas and lines defined by each S-Relation belonging to that element. These areas can be defined for any S-Relation which has one of its element predicates already located; for those without one the resulting acceptable area cannot be defined.

We can compute the approximate area for those S-Relations which define them. We can also combine these areas by assuming that the points in each area are distributed independently of the points in other areas. (Independence is assumed for all but the exceptions listed in the first footnote on the previous page.) We call the total area available to an element i, S_i^*. For an unlocated element i there is a seto of S-Relations c., that belong to it. c0 denotes the S-Relations from among the c which have their other predicate*element already located.c The set of areas S identified by c will define an intersection which is equal to

$$[4] \qquad S_i^* \prod_{\delta \epsilon n} \frac{S_i^{\delta}}{S_i^*}$$

Because the size of an element also restricts the number of locations available to it, we divide the area available to locate an element by its area, or

$$[5] \qquad \frac{S_i^*}{A_i} \prod_{\delta \epsilon n} \frac{S_i^{\delta}}{S_i^*}$$

which defines the relative number of disjoint locations available to element i, given the current arrangement.

For each unlocated element, GSP computes the above function and selects the element with the lowest value to locate in the arrangement next.

Pfeffercorn's program includes elements in decreasing order of their size (12). Thus, it approximates the lefthand part of eq. [5].

Sequencing of S-Relations Applied to a Location

Slagle has shown that the most efficient sequence for applying a set of conjunctive boolean tests is to apply them in ascending order of their cost divided by their probability of failing (13). Slagle's results indicate that in testing any location proposed for an element we should base our decision on θ_j, where

$$[6] \qquad \theta_j = \frac{\text{the cost in computing time for the S-Relation}}{\text{available area} - \text{area allowed by the S-Relation}}$$

The S-Relations would then be applied in increasing order of their 0 . Pfeffercorn also locates elements in order of their severity, (though he does not state his function for determining severity).

In applying the S-Relations to locations, it is useful to distinguish between two types, those that are not affected by the arrangement of elements which are not its predicates, and those that are. An adjacency relation between two elements, no matter how qualified by arguments, will not be affected by the placement of other elements. On the other hand a sightline relation, defining that an open line of vision must exist between two points or areas, may be affected by any change in the total arrangement of elements. We call the first type of S-Relation local, and the others global. A local S-Relation can be evaluated immediately after placement of the elements which are its predicates. If it is satisfied, no later evaluations are required. Global relations can be affected by any perturbation of the arrangement and thus must be tested after any alteration of the arrangement. In GSP, ADJACENT, ORIENTATION, and DISTANCE are local S-Relations.

* In the above, we assume that the cost of generating locations for all elements is equal. See (13).

Backup Procedures.

When a location operator fails to generate an acceptable location for an element, it is due to one of two causes. Either the operator failed to generate a location within an existing intersection of the S-Relation areas, or the intersection of the relevant S-Relations was null. In both GSP and Pfeffercorn's DPS program, the first source of failure is assumed to be negated with careful selection of the location operator. (DPS has only one operator; thus no mistake is possible). Thus any failure is assumed to be caused by a null intersection.

The likelihood that a S-Relation will cause failure of a location operator is inversely proportional to the area it defines; the S-Relation with the smallest area is thus most critical. One way to resolve failure, then, is to alter the area defined by the most critical S-Relation. If that S-Relation happens to be of the local type, then the only way to change its area is to alter the location of its previously located element. If the S-Relation is global, then moving any one of several elements may alter the S-Relation's acceptable area. GSP, in both cases, selects to relocate first the most restrictive element related to the failed one by a local S-Relation. If none is found, then the global S-Relation projecting the smallest acceptable area is considered and its other element selected for re-location. This backup procedure can be applied recursively.

Pfeffercorn's DPS program backtracks by removing both elements of the S-Relation assumed to be most restrictive. It ingeniously composes an arrangement of the two elements that satisfies their S-Relation and attempts to relocate a composite pair back into the arrangement (12).

Implementation of Constraint Structured Planning in GSP

GSP incorporates no means for directly computing the area projected by a S-Relation. Thus it cannot take advantage of the location generation method outlined in the description of location operations. When considering which element to add next to the arrangements GSP also can only approximate the areas projected by the different S-Relations.

For each S-Relation, GSP has a heuristic function which estimates its allowed area. These estimates make assumptions about shapes (that all are rectangular) and the distribution of elements. They are static, in that they rely on the particular problem definition and not on any particular arrangement. The current form of these estimating functions are presented in Table Two. We expect to improve these methods for estimating areas as our experience with GSP grows.

We have found that incorporation of the previously described decision rules is facilitated if the major information they utilize is represented in the form of a directed graph, hence the name Constraint Structured Planning. Each element or space defined as part of the problem is represented as a node in the graph. Each S-Relation is represented as one or two directed edges. If a S-Relation belongs to two elements, it defines two edges, one in each direction between them. If a S-Relation is unary or between an element and the space, then only one edge is defined. Each edge has a value corresponding to the area allowed for its successor element when its predecessor element is already located. Each node then takes a 0 - 1 value denoting whether it has been located. For example, the problem shown in Figure One can be represented by the graph shown In Figure Five.

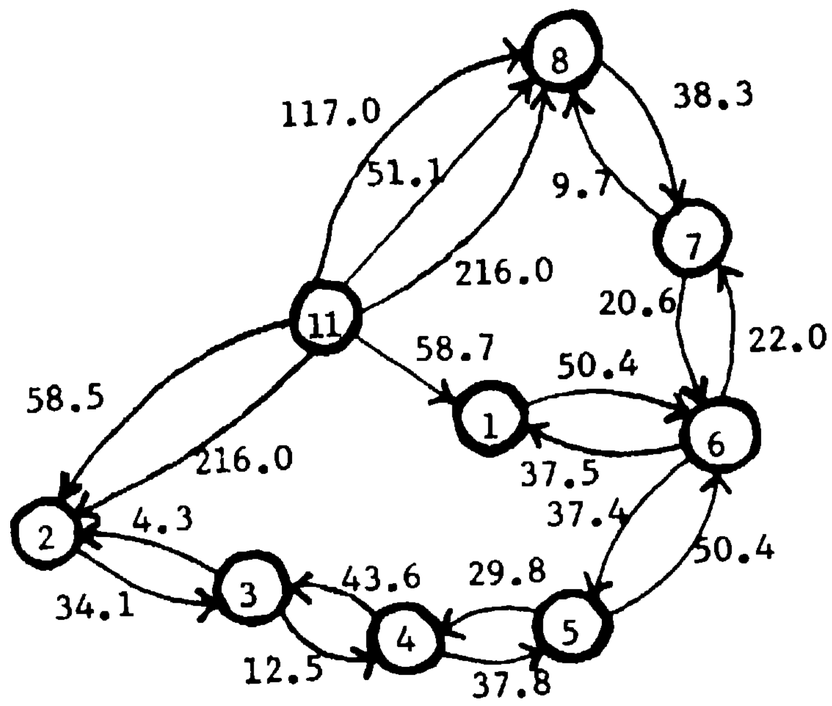


FIGURE FIVE - Constraint Graph for the problem described in Figure One.

The Constraint Graph depicting any particular problem is treated not as an alternative representation of the problem, but as a planning representation that allows analysis of important characteristics. Thus we use the Constraint Graph in a manner similar to Gelernter's planning model for geometry theorem proving (4).

The matrix form of the Constraint Graph is utilized and consists of an edge by node matrix, denoted I((n + 3, m + 1). The extra rows and columns are used as follows: R(n + 1 , i) = the estimated area of element i; R(n + 2, i) denotes whether the element has been located and R (j, m + 1) denotes the type of S-Relation involved. See Figure Six.

On selecting a new element to locate, GSP

In general:

A_i = area of element i

A_s = area of the space

$A_i^* = A_x + A_i - \sum_{j=1}^{m} A_j$ = area available for locating element A

w_{sa} = length of side SA of element A

ADJACENT:

$$A = 2A_b \left(\frac{x_s + y_s \frac{A_s}{w_{sa}}}{d} \right) \frac{A_b^*}{A_s}$$ where $d = w_{sb}$ if $w_{sb} > 0$ else $d = \frac{x_b + y_b}{8}$;

DISTANCE:

$$A = \left(A_b + \frac{4F + g}{2g + 1} \right) \frac{4A_b^*}{A_s}$$ where $g = A_a$ if $a \neq 11$ else $g = 0$;

SIGHT:

$$A = A_s + A_b + \left(\frac{c - 4}{2} \right) \sum_{j=1}^{m} A_j$$ where $c = 2$ iff PTA $\neq 0$ and PTB $\neq 0$; $c = 1$ iff PTA $\neq 0$ or PTB $\neq 0$; else $c = 0$;

ACCESS:

$$A = 4A_b^* - F (x_s + y_s) ;$$

ORIENT:

$$A = A_b^* .$$

TABLE TWO

Current functions for estimating the area allowed by an S-Relation.
The area is multiplied by the number of orientations allowed.

	1	2	3	4	5	6	7	8	11	m+1
1		216.0							0.1	5
2		51.1							0.1	3
3			4.3	34.1						1
4				43.6	12.5					1
5					29.8	37.8				3
6						37.5	50.4			3
7	37.5						50.4			3
8	58.7								0.1	3
9							20.6	22.0		1
10								38.3	9.7	3
11									51.1 0.1	3
12									216.0 0.1	5
13									117.0	2
m+1	20	4	4	15	30	30	20	4	216	
m+2									2	
m+3	2.9 12.8								6.2	

FIGURE SIX - Matrix form of the Constraint Graph for the problem in Figure One.

computes for each element $1 \leq i \leq m$ the intersection of known areas belonging to it, defined in terms of the estimated disjoint locations allowed, e.g.

$$[7] \qquad \frac{R(n+1, S)}{R(n+1, i)} \prod_{\delta \epsilon n} \frac{R(\delta, i)}{R(n+1, S)}$$

where S = the reference number of the space and $\delta \epsilon n$ where $R(n+2, k) > 0$ and $R(\delta, k) > 0$

Eq. [7] corresponds to eq. [5].

After an element has been selected and a location proposed, we wish to sequentially evaluate it according to Slagle's criterion, e.g. eq. [6], Currently, we assume all tests to have the same cost. Thus we make

c_{min} = largest area of a S-Relation (belonging to element i) tested during this iteration.

and select j which

$$\min R(j, i) \quad 1 \leq j \leq n$$

$$[8] \quad \text{subject to} \quad R(j, i) \geq c_{min}$$

In practice, we apply this algorithm to the constraint graph twice, first subject to the additional constraint that R(j, m4+1) designates a local S-Relation, then again without it.

If an element i cannot be located acceptably, the backup procedure is implemented by relocating that element i*, selected from among those where

$$R(j, i^*) > 0 \quad 1 \leq j \leq n, \text{ which}$$

$$[9] \quad \min R(j, i).$$

As above, two iterations may be required, first for local then for global S-Relations.

An Example

Constraint Structured Planning seems to have

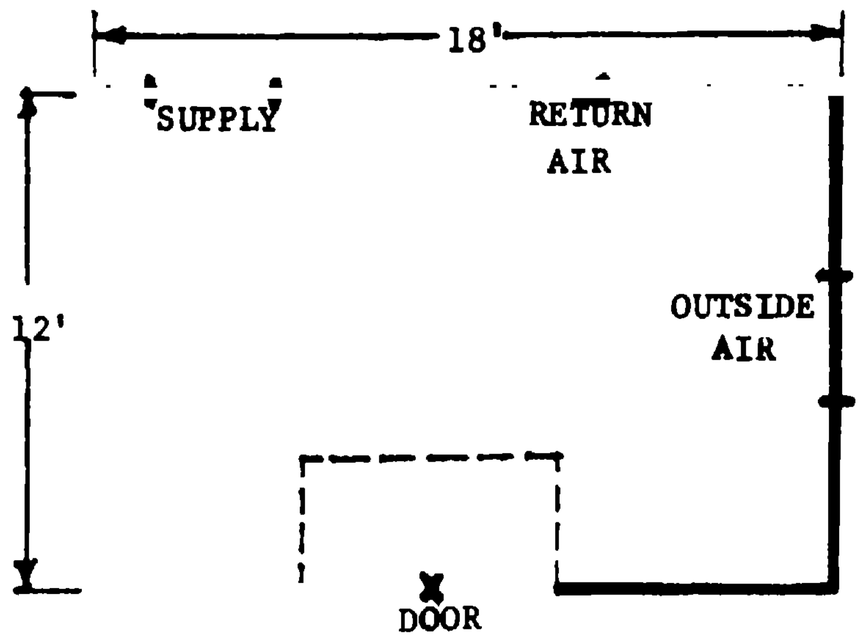


FIGURE SEVEN (a)

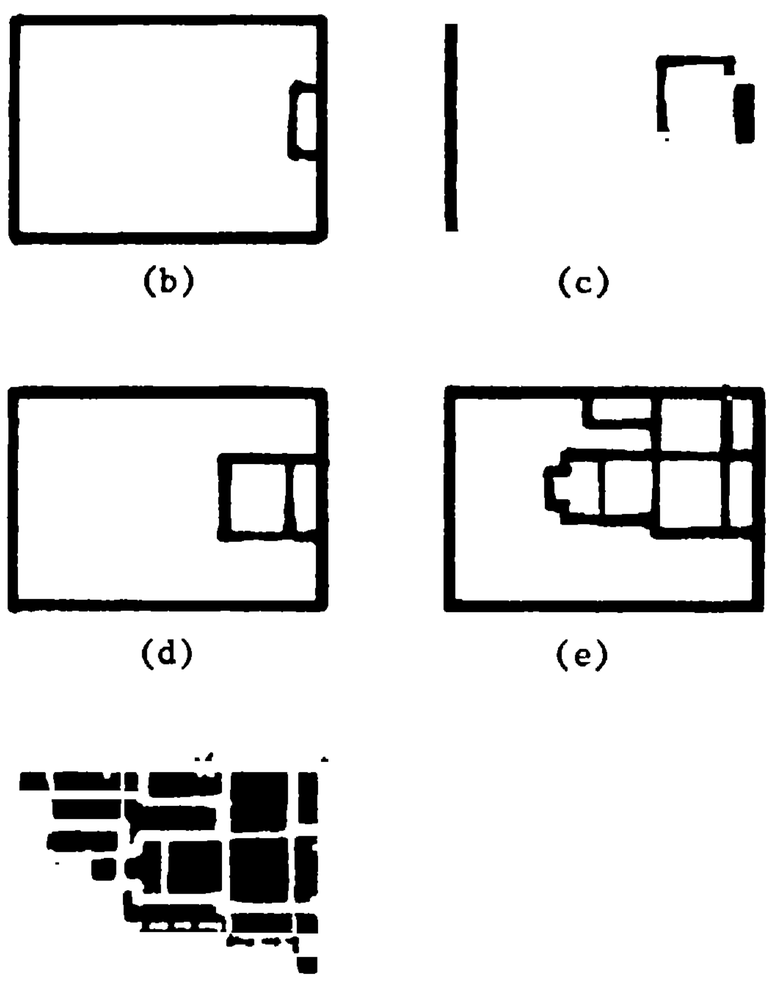


FIGURE SEVEN - Example problem and the sequence of alternative states generated by GSP to solve it.

powerful capabilities for dealing with a general class of space planning problems. Formal evaluation and empirical testing of its capabilities are currently underway. In order for the reader to better understand the characteristics of its operation, we offer a detailed description of its operation on one trial problem. We use the problem presented in Figure One and whose Constraint Graph is shown in Figures Five and Six. We assign to GSP the problem of arranging the machinery in Figure One to the space shown in Figure Seven (a), subject to the S-Relations defined in Figure One.

Row n+3 in Figure Six shows the computed intersection areas for the S-Relations upon initiation of the program. Element One is selected. Locations for it are generated by SCAN, which rotates it and locates it as shown in Figure Seven (6). The intersections of allowed areas (eq. [7]) are recomputed for each element. Element Six is selected next, which is located with SCAN as shown in Figure Seven (c). Of the six remaining elements, Seven has the smallest allowed area. Because it must be adjacent to element Six, PERIMETER attempts to locate it but fails to do so for lack of room. The backup algorithm, eq. [9], is initiated and it identifies element Six as the best candidate for relocation. It is moved to the location shown in Figure Seven (d), the only other acceptable one. Element Seven is then located, followed sequentially by Eight, Five and Four. The result is shown in Figure Seven (e). The last two elements are added, first Three then Eight, after Eight is rotated to satisfy the ORIENT S-Relation. The accepted arrangement is shown in Figure Seven (f), as printed on our plotter. In preliminary tests, specification, solution, and all plotting for this task took about three and a half minutes of CPU time on Carnegie-Mellon University's IBM 360/67 computer, using TSS FORTRAN.

Its general performance indicates that Constraint Structured Planning works most efficiently when the task is highly structured. Search is significantly speeded up if many ADJACENT S-Relations are involved. If no S-Relations project areas for unlocated elements, they are chosen according to their approximate size, biggest first. GSP is known to fail on some problems, due to poor handling of global S-Relations. Debugging and re-coding are expected to significantly lower current running times.

Conclusion

While Constraint Structured Planning, as implemented in GSP, begins to effectively deal with several of the issues inherent in space planning, many remain for more detailed exploration. Within the problem domain considered here, issues still remain in each of the four decision processes. In the element selection process, only S-Relations which have one of their predicate elements located can currently be brought into the analysis, yet the other S-Relations act to delimit allowed areas also. No means has yet allowed us to estimate the restrictions imposed by all S-Relations.

GSP was implemented before the author appreciated or understood how S-Relations and location operators interact. Thus no means is built into it for projecting areas defined by S-Relations, nor for looking at the boundary intersections between such areas. We anticipate that a system allowing these capabilities would greatly increase the efficiency of search, as well as to allow more varied shapes of elements. Pfeffercorn's method of representation would easily handle these requirements.

Effective backup procedures have been developed for treating local S-Relations, but there

has been little progress in characterizing or treating global S-Relations; thev seem to be the cause of most search difficulties.

Future efforts at expanding GSP will include the introduction of variable shape elements and the appropriate operators and tests to deal with them. We also propose to expand GSP so as to handle hierarchical problem domains, e.g. "arrange the equipment in the rooms, which are arranged and shaped within a building, which is arranged and shaped on a site" (1). Such formulations clearly open up new problem domains and exciting challenges for future research.

Applications for space planning programs should be obvious. As greater capabilities develop for processing spatial arrangement tasks, better analyzed, better structured, and even more beautiful physical environments should result.

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