The Physics, Mathematics and Common Sense of Cosmic Dark Energy and Spacetime Extra Dimensions

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Abstract – **Contrary to conventional views, this Letter to the Editor argues that the dark energy sector of the cosmos agrees not only with the theory and observations but also with ordinary common sense in the deep meaning of the words as well as with the idea of extra spacetime dimensions.**

The present work is a concerted effort to present the subject of dark cosmic energy [1-9] and its intimate connection to spacetime extra dimensions in as simple a manner as possible but no more [9-12]. Our plan is a synergistic exposition where physics, mathematics as well as empirical evidence based on measurements and observations build a monolithic unit with the chief aim of bringing to the fore the somewhat overshadowed common sense side of the inevitability of a cosmic dark energy sector in any universe admitting extra dimensions [3,10,11]. In particular we will show why we favour in that respect a combined Einstein-Kaluza fractal spacetime [15,16] which as we will see, possesses all the attributes which we need to explain the naturalness of having three types of energy, namely the 4.5% ordinary directly measurable cosmic energy [8,9,12] and the two components of the 95.5% dark energy which we cannot measure directly and which splits again into two weakly coupled types of cosmic energy, namely the attractive dark matter energy of about 22% and the pure dark energy constituting about 73.5% of the total overall energy density of the universe [1-9]. The last type of energy is at a minimum renowned to be the most likely candidate for the driving force behind the famous and relatively recently discovered accelerated cosmic expansion [10-34].

Our first starting point is the almost self evident fact that if there are extra dimensions for spacetime, then they must be, as surmised by T. Kaluza long ago, hidden dimensions [12,15,16,17]. In turn if there are hidden dimensions then there must be some volume of spacetime to which we have no easy access and consequently some energy will appear to be missing [17-21]. This point squares perfectly with another even more obvious fact that is truly physical, mathematical, number theoretical [23,24] as well as being a common sense conclusion. The said point is related to the most famous and beautifully simple equation in physics, namely E = mc² [18,22]. This equation is historically linked to the creator of the special and general theory of relativity, A. Einstein but it goes as far back as the work of at least Poincare and Lorenz and possibly earlier eminent scientists [18,22]. As well known at the time when this equation was developed (or discovered), the only messenger particle known to physics was the photon of Newton and Einstein [3,12,22]. The word messenger particle was of course not in use then since the standard model of high energy elementary particles was not developed yet. In other words from the 12 degrees of freedom of the standard model only the photon was known and familiar as the constituents of light and the other 11 photon-like particles were totally unknown [6,12,22]. This is an important point that we need to stop at and dwell upon in some detail. The 12 degrees of freedom of the standard model corresponds to the dimensions of the Lie symmetry groups dim U(1) for the photon of electromagnetism [25], dim SU(2) = 3 for the electroweak w^+, w^-, Z_o and dim SU(3) = 8 for the strong force messenger particles, i.e. the eight gluons [3],[6-12]. It is thus not difficult to reason that if Einstein had written a Lagrangian which he did not and found at the end as the corresponding eigenvalue the same result he found, namely E = mc², then this E must be the largest possible. That means ignoring the $12 - 1 = 11$ unknown "generalized coordinates" lead to a gross over estimation of the eigenvalue E [12,22]. It is also reasonable to conjecture that the factor of reduction, which should be applied to E, must be a scaling proportional to $(1/2)(1/11) = \lambda$. This was explained elsewhere in more detail [12] and it is sufficient here to make a superficial comparison between Newton's kinetic energy $E(N) = (1/2)mv^2$ with Einstein's maximal energy density E = mc² and realize that except for replacing the velocity v by the maximal velocity of light c, the difference between the two equations is a scaling factor of $(1/2)$ [12,22]. It is then not unreasonable and is in fact an intuitively viable conjecture to assume that the same situation holds for the scaling effect of the missing $12 - 1 = 11$ generalized coordinates of E leading to an educated guess that it should be $E = (1/2)(1/11)mc^2 = mc^2/22$. This shrewd guess work turned out to be deadly right [12,22] and some highly sophisticated mathematics showed beyond doubt that this value corresponds to the correct ordinary energy density of the cosmos and is given exactly by [7-9]

$$\left(\phi^5/2\right) = 1/(22.18033989)$$
$$= 1/(22 + k) \qquad (1)$$
$$\simeq 1/22$$

where ϕ^5 is the famous Hardy's probability of quantum entanglement [26] and $k = \phi^3\left(1 - \phi^3\right)$ is 'tHooft's renormalon topological weight [15,17]. In other words, ordinary energy density is only $1/22 = 4.5\%$ of Einstein's maximal energy and is clearly related to the photon of Maxwell's electromagnetic field [3]. However it is what we consider to be the common sense perception of ordinary energy density of about 4.5% which is more over

in full agreement with cosmic measurements and observations that is of great interest for us here. The reason is what we mentioned right from the beginning, namely that we do see the light ergo we do see the atoms of light, i.e. the photons. However we do not see in any direct or semi direct way the other 11 photon-like messenger particles and therefore it should not come as any great surprise that we do not see the 95.5% energy density associated with them nor can we measure this density in any direct way [28]. Thus the division in two cosmic energy sectors, ordinary and dark is odd but not a mystery. There is in fact another common sense reason why we should have anticipated this result. This is a different type of common sense related to the importance of surveying results from pure mathematics which may pave the way towards a proper understanding of unexpected results in physics. There is indeed a relatively old and well established theorem in probability theory and measure theory due to the outstanding mathematician Aryeh Dvoretzky [27,31,33] which states that for a manifold of sufficiently high dimensionality, meaning five or larger, 96 percent of the volume of the manifold is located at a layer adjacent to the surface while the so called bulk, i.e. the "inside" of the manifold contains only 4% of the volume [5,28,31]. This is the mathematical gift to cosmic dark energy research for obvious reasons which we may summarize in the following manner connected to the sophisticated mathematical derivation we just eluded to earlier on [28]. In short we can reason that ordinary energy density is given by $\phi^5/2$ and therefore dark energy density is [26-33] $1-\left(\phi^5/2\right)=5\phi^2/2\simeq 21/22$.

Now since $\phi^5/2$ is the energy due to the zero set quantum particle and $5\phi^2/2$ is the energy of the empty set quantum wave and noting that the empty set is basically the surface of the zero set [28], it follows then that finding $\phi^5/2\simeq 4.5\%$ and $5\phi^2/2\simeq 95.5\%$ is almost the same conclusion coming out of Dvoretzky's theorem [28-33]. Put differently dark energy may be interpreted as the result of hidden extra dimensions and everything is clearly pointing in this direction of measure concentration due to high dimensionalities.

Recapitulating our discussion so far, we see that ordinary energy is clearly linked to the relatively "visible" photons of our familiar light while the dark sector of cosmic energy is linked to photon-like messenger particles which we cannot see directly in addition to being related to the hidden "dark" extra dimensions of spacetime all apart of being most concentrated at the surface of the universe, i.e. the cosmic hyperbolic horizon [19]. These are actually all intuitive common sense reasons apart of being exact mathematical results confirmed with careful observation and accurate measurement and seen that way they are somewhat surprising, mildly unexpected and relatively unusual results but are really far from being truly mysterious or beyond explanation [1-12].

All that is left for us to do now is to give a simple general quantitative derivation to the preceding argument making it accurate and give a more precise explanation for

the difference between the two components of the dark sector, namely the dark matter energy and the pure dark energy which we know from accurate cosmic measurements to be about 22% and 73.5% respectively as mentioned earlier on [6-9].

The kind of straightforward derivation that we have in mind here is based upon what we called the organizing centre of E-infinity theory [6-9][34]. This central equation basically depends on two other facts which are almost too good to believe, namely the fundamental equation of reconstructing the inverse Sommerfeld electromagnetic fine structure constant $\bar{\alpha}_o=137+k_o$, where $k_o=\phi^5\left(1-\phi^5\right)$ and ϕ^5 is Hardy's probability of quantum entanglement [31-34]

$$\begin{aligned}\bar{\alpha}_o &= \bar{\alpha}_1\left(1/\phi\right)+\bar{\alpha}_2+\bar{\alpha}_3+\bar{\alpha}_4 \\ &= \left(60\right)\left(1/\phi\right)+30+\left(8+1\right)+1 \\ &= 137+k_o\end{aligned} \qquad (2)$$

and the remarkable fact expressed by the following equation

$$\sum_{i=1}^{4}\bar{\alpha}_i=100 \qquad (3)$$

This last equation is deeply connected to the equally remarkable P-Adic expansion of $\bar{\alpha}_o=137$ to basis of two namely

$$\left\|\bar{\alpha}_o\right\|_2=\left\|128+8+1\right\|_2=1 \qquad (4)$$

In other words equation 3 is exactly a hundred fold of equation 4 [25-34]. It is not difficult to see that the fractal version of D = 26 bosonic string theory or alternatively the bosonic sector of the Heterotic superstring theory is given by $D=26+k=26.18066989$ and in view of Einstein's D = 4 we must have $D=26+k-4=222+k$ compactified dimensions then. Consequently we see that the density of these compactified dimensions, which are identified with the density of the dark matter energy [28-34] must be

$$\gamma=\frac{\left(26+k\right)-4}{\sum_1^4\bar{\alpha}_i} \qquad (5)$$
$$= 0.2218033989$$

Therefore the density of the pure dark energy sector is found simply as the difference between the total dark energy density and dark matter energy density given by the following equation [28-33]

$$\begin{aligned}\left(5\phi^2/2\right) &= 0.9549150289-0.2218033989 \\ &= 0.73311163 \\ &\simeq 73.3112\%\end{aligned} \qquad (6)$$

in full agreement with all previous computations as well as conjectured values based on cosmic measurements and observation [6-9]. Taking all the preceding results on board we conclude that while ordinary energy is connected to the photon as a quantum particle, dark energy is connected to the quantum wave in general [28] and may be subdivided to dark matter energy connected to the compactified extra dimensions and finally pure dark energy which is living in the organizing centre of it all,

that is to say living in the normed space with the weight of one hundred dimensions as shown by equations 3 and 4 and exists as a quasi fluid filling the space left between the compactified dimensions [10].

We hope that the preceding intuitive mental pictures can be of help in deepening our understanding of the cosmos in general and the dark cosmic energy in particular. To appreciate the far reaching consequences for the concerted view presented in the present analysis, we just need to say that it is a precursor to a very general type of superstring theory anchored in nonlinear dynamics which we could name general topological superstring theory which incorporates nonlinear dynamics, transfinite set theory and Menger-Urysohn dimensional theory [6-8][34] in the classical superstring theory as stated by E. Witten and C. Fafa some time ago. However this is the subject of another future paper or papers which will relate the work of Witten, Fafa and others [35,36] to the present super quantization of the golden mean number system upon which E-infinity theory is based [6-8][12,37].

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AUTHOR'S PROFILE

Professor **M.S. El Naschie** was born in Cairo, Egypt on 10th October 1943. He received his elementary education in Egypt. He then moved to Germany where he received his college education and then his undergraduate education at the Technical University of Hannover where he earned his (Dipl-Ing) diploma, equivalent to a Master's degree and Chartered Structural Engineer. After that he moved to the UK where he enlisted as a post graduate student in the stability research group of the late Lord Henry Chilver and obtained his Ph.D. degree in structural mechanics under the supervision of Professor J.M.T. Thompson, FRS. After his promotions up to the rank of full professor, he held various positions in the UK, Saudi Arabia and USA and was a visiting professor, senior scholar or adjunct professor in Surrey University, UK, Cornell, USA, Cambridge University, UK and Cairo University, Egypt. In 2012 he ran for the Presidency of Egypt but withdrew at the final stage and returned to academia and his beloved scientific research. He is presently a Distinguished Professor at the Dept. of Physics, Faculty of Science of the University of Alexandria, Egypt.

Professor El Naschie is well known for his research in structural stability in engineering as well as for his work on high energy physics and more recently for his work in cosmology and elucidation of the secrets of dark energy and dark matter as well as for proposing a dark energy Casimir nano reactor. He is the creator of E-infinity theory, which is a physical theory based on random Cantor sets and can be applied to micro, macro and mesoscopic systems.

Professor El Naschie is the single or joint author of about one thousand publications in engineering, physics, mathematics, cosmology and political science. His current h-index is 74 and his i-10 index is 753 according to Google Scholar Citation.