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EARLY TIME FIREBALL GROWTH FROM A NUCLEAR EXPLOSION

MASTER

Gerald L. Nutt

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EARLY TIME FIREBALL GROWTH FROM A NUCLEAR EXPLOSION

Abstract

The early motion of an x-ray fireball from a nuclear detonation in the atmosphere is studied using the equations of radiation transport coupled to the equations of hydrodynamics. The solutions are compared with observations of fireball motion in a free homogeneous atmosphere and near the ground surface. Comparison permits us to determine the amount of energy coupled to the ground by means of x-ray deposition from a surface shot.

1. Introduction

One of the earliest observable effects after the detonation of a nuclear explosive is the rapid growth of a radiation front through the surrounding atmosphere. For the first 20 to 30 μs this growth involves negligible hydrodynamic motion. The character of the radiation front is that of a supercritical or isothermal shock wave. The fireball continues to expand until it reaches the critical temperature, which for air is about 25 eV.

During this early period one can think of the fireball as an isothermal sphere of x rays with a temperature at its equilibrium value and cooling as the sphere expands. Since the cool air surrounding the fireball is opaque to the x rays, the edge of the fireball advances at a speed determined by its temperature by means of a diffusion process.

The main perturbation on this model for times less than 30 μs from detonation is the transfer of energy from the sphere of radiation to the air engulfed by the fireball.

It is reasonable to expect that at sufficiently early times the temperature of the air is not in equilibrium with the radiation and this effect will be more noticeable at the radiation front than at the center of the fireball. Thus we would expect to see a gradient in the air temperature increasing from the edge of the fireball to the center.

In the case of a device detonated on the surface, the above picture can be modified in the first cut by giving the fireball an energy density at a given radius of twice that of a free air burst at the same radius. This assumes that the surface acts as a perfect reflector of the radiation. This effectively doubles the radiant yield of the device as far as the motion of the fireball is concerned. In the next approximation, we add to the energy loss mechanism in the free air burst the radiation absorbed by the ground. Thus, by comparing the early motion of a fireball in a uniform atmosphere with the motion of a fireball centered on the Earth's surface, we should be able to detect any significant absorption of x-ray energy by the surface.

All this suggests a model of an important type of energy coupling to the surface from a nuclear explosive. Namely, energy is transferred through an area defined by the intersection of the fireball with the Earth's surface. This ground surface area is heated by the radiation from the fireball, which is at a uniform but time-dependent temperature.

There are other mechanisms for transferring energy into the Earth's surface including neutron capture, device debris impact and gamma-ray deposition.

Figure 1 shows the available data on early time fireball growth. Fireball radius as a function of time out to about 20 μs is given for four shots. Two of the shots, MOHAWK and HOOD, can be considered free air shots for our purposes. MOHAWK was placed on a tower and HOOD in a balloon. ZUNI was a surface shot over land and APACHE was placed on a barge. The data are from streak camera records taken by Woodward of LLL.

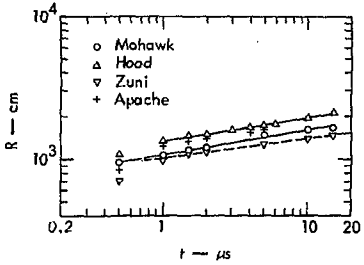

Fig. 1. Observations of fireball growth for various surface shots and free air shots.

The radius is scaled in such a way as to make the energy density in each fireball the same assuming the ground acts as a perfect reflector. The air bursts are scaled to 100 kt while the surface bursts are scaled to 50 kt.

The important features of these data are that for both air bursts the fireball radius grows as $t^{0.185}$. The surface shots grow with a much weaker time dependence. Over soil $t^{0.122}$ seems to apply and if anything the shot over water has an even weaker rate of expansion.

We would like to suggest that the different laws of expansion of fireballs in a homogeneous atmosphere and along the Earth's surface can be accounted for by the deposition of x-ray energy into the ground or water. It is hoped that a simple theoretical description of early time fireball growth will permit a quantitative evaluation of the rate of energy transfer to the ground.

2. Self-Similar Solutions of Hydrodynamic and Radiative Transfer Equations

SIMILARITY CONDITIONS IN REDUCED FORM

We shall proceed in a manner suggested by Elliott in treating a spherically expanding Marshak-type radiation front.[1]

1. L. A. Elliott, Proc. Roy. Soc. London 258 A, 827 (1960).

For spherical symmetry, the equations we would like to solve are

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \frac{\rho}{r^2}\frac{\partial}{\partial r}(r^2 u) = 0 \qquad \text{(continuity)}, \quad (1)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0$$

(conservation of momentum), (2)

$$\frac{\partial E}{\partial t} + u\frac{\partial E}{\partial r} + p\frac{t}{\partial t}\left(\frac{1}{p}\right) + pu\frac{\partial}{\partial r}\left(\frac{1}{p}\right) + \frac{1}{pr^2}\frac{\partial}{\partial r}(Fr^2)$$

$$= W \quad \text{(energy balance)}, \qquad (3)$$

$$F = -\frac{c\lambda}{3}\frac{\partial}{\partial r}(aT^4)$$

(radiative transfer), (4)

and

$$E = E_M + E_R; \quad p = p_M + p_R. \qquad (5)$$

Here W represents an arbitrary energy sink and F is the radiation flux. If T is the local equilibrium radiation temperature, then

$$\rho E_R = aT^4 = 3p_R. \qquad (6)$$

We define a specific heat ratio for the system as

$$\rho E_M = \frac{p_M}{\gamma - 1}. \qquad (7)$$

Equations (1) through (4) can be reduced to four ordinary differential equations in the dimensionless independent variable x,

$$x = \frac{r}{R},$$

by the following substitutions:

$$u(r,t) = \dot{R}\overline{u}(x),$$

$$\rho(r,t) = \rho_0\overline{\rho}(x)$$

$$p(r,t) = \rho_0\dot{R}^2\overline{p}(x),$$

$$E(r,t) = \dot{R}^2\overline{E}(x) \qquad (8)$$

$$F(r,t) = \rho_0\dot{R}^3\overline{F}(x),$$

$$W = \dot{R}^3R^{-1}\overline{W}(x).$$

Here R(t) is some characteristic length that depends only upon time. R is required to satisfy the following differential equation

$$\dot{R}^2 = A^2R^{-\alpha} \qquad (9)$$

and ρ_0 refers to the ambient density outside the shock.

The *substitution of Eqs.* (8) and (9) *into* Eqs. (1) through (4) yield the following reduced equations:

$$(\overline{u}-x)\frac{\overline{\rho}'}{\rho} + \left(\frac{2\overline{u}}{x} + \overline{u}'\right) = 0, \qquad (1')$$

$$\overline{u}'(\overline{u}-x) - \frac{\alpha}{2}u = \frac{\overline{p}'}{\rho}, \qquad (2')$$

$$-\alpha\overline{E} + (\overline{u}-x)\overline{E}' + \frac{\overline{p}'}{\overline{\rho}}\left(\frac{2\overline{u}}{x} - \overline{u}'\right) + \frac{1}{\rho}\frac{1}{x^2}\frac{d}{dx}(x^2\overline{F}) = \overline{W}, \qquad (3')$$

and

$$\overline{p}_{R'} = -\Lambda^{-1}\overline{F}. \qquad (4')$$

The *primes* refer to derivative with respect to x. Equation (4') is for constant Λ provided we restrict λ, the radiation mean free path, as follows:

$$c\lambda = \Lambda\dot{R}R = \Lambda AR^{1-\frac{\alpha}{2}}. \qquad (10)$$

In our problem this restriction means that λ is constant over the whole sphere and dependent upon the size or the temperature of the sphere.

From Eqs. (6), (8) and (9), we find

$$R = \left(\frac{3\rho_0}{a}A^2\overline{p}_R\right)^{1/\alpha} T^{-4/\alpha},$$

which together with Eq. (10) gives

$$c\lambda = \Lambda A\left(\frac{3\rho_0}{a}A^2\overline{p}_R\right)^{(1/\alpha)(1-\alpha/2)} \times T^{(-4/\alpha)(1-\alpha/2)}. \qquad (11)$$

Equations (1) through (7) give us eight equations in the nine independent variables $\overline{\rho}$, $\overline{u}$, $\overline{F}$, $\overline{E}$, $\overline{F}_M$, $\overline{E}_R$, $\overline{p}$, $\overline{p}_M$, and $\overline{p}_R$. We have not included in our catalogue of equations an equation of state in terms of temperature.

If we are to assume that the local material temperature T_M and the radiation temperature were equal, an equation of state of the type

$$p_M = p_M\,(\rho, T_M),\ \ T_M = T\ ,$$

would give us an additional relationship between p_M and p_R. Unfortunately, at very early stages in the growth of the fireball we may find that the radiation and matter are far from being in equilibrium.

We feel it is best, therefore, to make the assumption that E_M and E_R are connected by

$$E_M(r, t) = \phi(x)\, E_R(r, t)\ . \tag{12}$$

The practical advantage to this formulation of the problem is that the radiation and matter need not be in equilibrium. Equation (12) does not provide the necessary additional condition since it introduces the new variable $\phi(x)$.

Three necessary boundary conditions that must be satisfied for the above differential equations are provided by the Hugoniot analogues at the position $r = R$:

$$\rho_1(\dot{R} - u_1) = \rho_0 \dot{R}\ , \tag{13}$$

$$p_1 - p_0 = \rho_0 \dot{R} u_1\ , \tag{14}$$

and

$$E_1 + \frac{p_1}{\rho_1} + 1/2(\dot{R} - u_1)^2$$
$$= E_0 + \frac{p_0}{\rho_0} + 1/2\ R^2\ . \tag{15}$$

The subscripts 0 and 1 refer to ambient and shock conditions, respectively. We have assumed stationary ambient conditions in the fluid with no radiation flux. These equations can be expressed in reduced form.

ENERGY BUDGET

The total amount of energy inside the fireball at any time is the sum of the kinetic and internal energies including the radiation energy.

$$E_T = 4\pi \int_0^R (E + 1/2\ u^2)\rho r^2 dr \tag{16}$$

$$= 4\pi\rho_0 R^3 \dot{R}^2 B\ , \tag{17}$$

where

$$B = \int_0^1 (\overline{E} + 1/2\overline{u}^2)\overline{\rho} x^2 dx = \text{const.} \tag{18}$$

It is clear from Eqs. (9) and (17) that if E_T is a constant, that is, if the total energy is conserved inside our disturbance at radius $R(t)$, then

$$\alpha = 3\ . \tag{19}$$

Solving Eq. (9),

$$R = \left[A\left(\frac{\alpha}{2} + 1\right)(t - t_0) + R_0^{\frac{\alpha}{2}+1}\right]^{\frac{2}{\alpha+2}}\ . \tag{20}$$

Thus, R behaves like $t^{0.4}$ in a self-similar disturbance in which the total energy is conserved. Conversely, if the energy in the disturbance is not constant, R cannot behave like $t^{0.4}$. An example of a self-similar disturbance in which the total energy is constant is the Taylor solution for a blast wave.

If the disturbance is associated with a source or sink of energy, that is, if W is not identically equal to zero, then we can write

$$\frac{dE_T}{dt} = 4\pi\int_0^R W\rho r^2 dr \tag{21}$$

$$= 4\pi\rho_0 \dot{R}^3 R^2 C , \tag{22}$$

where

$$C = \int_0^1 \overline{W}\bar{\rho}x^2 dx = \text{const.} \tag{23}$$

By dividing Eq. (22) by Eq. (17) and integrating, we get

$$\dot{R}^2 = A^2 R^{C/B-3}$$

with the result

$$\alpha = 3 - C/B . \tag{24}$$

Thus, we show that the motion of the front of the disturbance is closely connected with the rate at which energy is added to or depleted from the wave.

This leads us to some pretty definite conclusions about what we are observing in the streak camera records:

- The time dependence of fronts in the interval $1\ \mu s < t < 20\ \mu s$ are of the form of Eq. (20). This suggests that the motion is self-similar to a good approximation.
- We are not observing a disturbance in which the energy is conserved. The time dependences indicate that C is negative and

$$C/B = \begin{cases} -5.81;\ \alpha = 8.81 \text{ for air shots (MOHAWK, HOOD)} \\ -11.4;\ \alpha = 14.4 \text{ for surface shots (ZUNI)} \end{cases} \tag{25}$$

- Because of the very low mean free path of the x-rays in cold air we can assume that energy is *not* escaping through the visible front. Hence, the energy sink for the free air shot must be caused by the absorption of radiant energy by the engulfed air. The fact that C/B is smaller for free air shots than for surface shots indicates the presence of an additional energy sink, namely, the ground surface.

We must still examine the question of whether or not the restriction of the temperature dependence of λ by Eq. (11) is a reasonable one. Substituting for α,

$$\lambda \propto \begin{cases} T^{1.55} \text{ for air shots} \\ T^{1.72} \text{ for surface shots .} \end{cases} \tag{26}$$

Comparing this temperature dependence with that of the Rosseland mean free path in air at standard density as shown in Fig. 2, we find acceptably close agreement in the temperature range $0.1\ \text{keV} < T < 1\ \text{keV}$ for both the free air and surface shots. This adds strength to the assumption that the observed wave is nearly self-similar.

MOTION OF THE RADIATION WAVE

Let us restrict our considerations to the case where there is no hydrodynamic motion. The state of affairs during the first 20 to 30 μs of fireball growth is shown in machine calculations with the MEG code. Assuming $\bar{u} \equiv 0$ and $\bar{\rho} = 1$, Eq. (3') becomes

$$\frac{1}{x^2}\frac{d}{dx}(\Lambda x^2 \bar{p}_{R'}) - 3x\bar{p}_{R'} - 3\alpha\bar{p}_{R'}$$

$$= \overline{W} + 3x(\phi p_{R'} + \phi'\bar{p}_R) + 3\alpha\phi\bar{p}_R \tag{27}$$

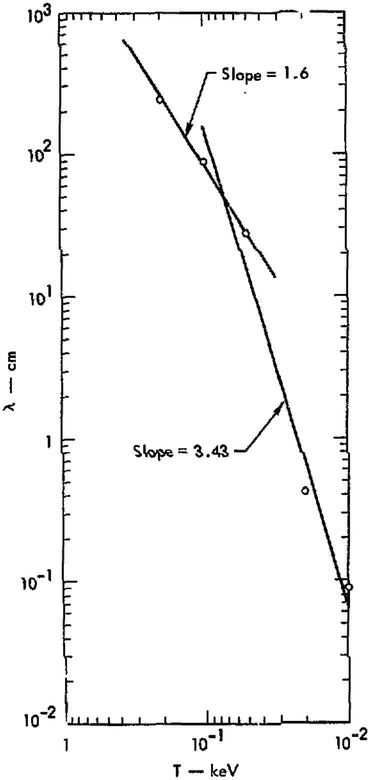

Fig. 2. Temperature dependence of Rosseland mean free path in air at standard density.

using Eqs. (6), (4'), and (12). All the terms involving the internal energy of the medium have been placed on the right hand side of the equation together with the sink-source term $\overline{W}$. Let

$$\bar{p}_{R''} + \left(\frac{2}{x} + \frac{3}{\Lambda} x\right) \bar{p}_{R'} + \frac{3\alpha}{\Lambda} \bar{p}_R = \frac{\overline{W}}{\Lambda} \qquad (28)$$

In the absence of a specific functional dependence of $\phi(x)$, we have simply put all terms on the right side of Eq. (28) in some generalized sink $\overline{W}$. This equation has a regular singular point at $x = 0$, so that for reasonable behavior in $\overline{W}(x)$ we can find a solution.

We must require of the solution that the radiation flux at the origin vanish so that

$$\bar{p}_{R'}(x) = -\Lambda^{-1}\overline{F}(x) \to 0, \text{ as } x \to 0 . \qquad (29)$$

Returning to Eq. (3') and the condition $\bar{\rho} = 1$, we find

$$x^2\overline{F} = \int_0^x (\overline{W} x^2 + \alpha\overline{E} x^2 + \overline{E}' x^3)\, dx .$$

We can reasonably assume that $\overline{W}(x)$, $\overline{E}(x)$ and $\overline{E}'(x)$ are regular at the origin. Hence we can write

$$\frac{\overline{F}(x)}{x} \underset{x\to 0}{\to} \frac{1}{3}(\overline{W}(0) + \alpha\overline{E}(0)) - \frac{1}{x^3}\int_0^x \overline{E}' x^3 dx .$$

The last term vanishes and from Eq. (29)

$$\bar{p}_{R'}(x)/x \underset{x\to 0}{\to} -\frac{\overline{W}(0)}{3\Lambda} - \frac{\alpha}{3\Lambda}\overline{E}(0) . \qquad (30)$$

Returning to Eq. (2') we get, with the restrictions of no fluid motion,

$$\bar{p}' = 0 , \qquad (31)$$

which has the solution

$$\bar{p} = \left[1 + 3(\gamma - 1)\phi(x)\right]\bar{p}_R(x) = p_0 = \text{const.} \qquad (32)$$

In the neighborhood of the radiation front ($x = 1$), Eq. (2') is not strictly correct since terms involving the momentum transfer of the radiation flux have not been included in Eq. (2). This approximation should be extremely good near the center of the fireball. Using Eqs. (12) and (32) in Eq. (30) we find

$$\frac{\bar{p}_R'}{x}\underset{x\to 0}{\to} \frac{1}{3\Lambda}\left\{\overline{W}(0) + \frac{3\gamma-4}{\gamma-1}\,\alpha\bar{p}_R(0) + \frac{\alpha}{\gamma-1}p_0\right\}. \tag{33}$$

Now let us introduce an assumption regarding the sink term W:

$$W = \dot{R}^3 R^{-1}\delta\bar{p}_R(x), \tag{34}$$

where δ is a constant determining the strength of the sink. $\bar{p}_R$ is proportional to T^4. Thus we can allow the sink to vary as the local *radiation energy* density. Physically this is reasonable.

Letting $E = E_R$ in Eq. (18) and using (34) in (23), we get

$$\frac{C}{B} = \frac{\int_0^1 \delta\bar{p}_R x^2 dx}{\int_0^1 3\bar{p}_R x^2 dx} = \frac{\delta}{3}. \tag{35}$$

Thus, we determine the strength of the absorption of radiation in the fireball from the motion of the front.

The constant p_0 can be divided out of the above equations by introducing the variable

$$p_D(x) = \frac{\bar{p}_R(x)}{p_0}. \tag{36}$$

Equation (28) becomes

$$p_D'' + \left(\frac{2}{x} + \frac{3}{\Lambda}x\right)p_D' + \frac{9}{\Lambda}p_D = 0. \tag{37}$$

The interesting thing about Eq. (37) is that *it is independent of* α

Solutions of Eq. (37) are shown in Fig. 3 for representative values of Λ. It is clearly shown that for longer mean free paths the fireball is *more nearly isothermal*. The values of Λ can be determined from the data in Fig. 1 through the use of Eqs. (10) and (20):

$$\Lambda = c\lambda\left(\frac{\alpha}{2} + 1\right)\frac{t}{R^2}. \tag{38}$$

Typical values of Λ range from 50 to 150 assuming $\lambda \approx 250$ *cm*. Clearly such values of Λ give a fireball that is very nearly isothermal.

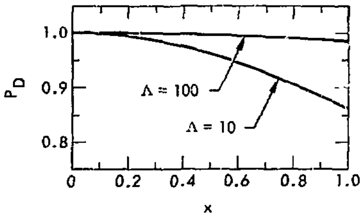

Fig. 3. Reduced radiation pressure as a function of x for different radiation mean free paths.

3. Energy Deposit in the Ground

We can now compare the energy in a hemispherical fireball from a surface shot with the energy in one half the spherical fireball of a free air burst and thereby calculate the energy deposited in the ground.

Modifying Eq. (22) for a hemisphere, the total *energy in a radiation* wave is determined by the differential equation

$$dE_T = 2\pi\rho_0 C\ A^3\ R^{2-\frac{3}{2}\alpha}\ \dot{R}^{-1}\ dR$$

$$= 2\pi\rho_0 C\ A^2\ R^{2-\alpha}\ dR \qquad (39)$$

Integrating from R_0 where $E_T = Y$,

$$E_T = Y + 2\pi\rho_0 BA^2\left(R^{C/B} - R_0^{\ C/B}\right) \qquad (40)$$

where we have used Eq. (24). The difference between E_T as calculated for the air burst and equivalent surface shot should be the energy transferred into the surface:

$$E_G = 2\pi\rho_0 B\left\{A^2\left(R^{C/B} - R_0^{C/B}\right) - A'^2\left(R'^{C'/B} - R_0'^{C'/B}\right)\right\}, \qquad (41)$$

where the primes refer to quantities derived from surface shots. Since $\overline{p}_R$ is independent of α, the value of B, which depends only upon $\overline{p}_R$, should be the same for both cases. As a result B appears as a common factor in Eq. (41).

Using the data on ZUNI and MOHAWK in Eqs. (41) and (20), we find that E_G has the time dependence shown in Fig. 4. The peak value of the energy deposited in the ground is reached at t about 1 μs. Until this time the fireball has been depositing energy in the ground. After 1 μs the ground begins to reradiate into the fireball. The ratio of the peak energy in the ground to the energy at 10 μs is 1.81.

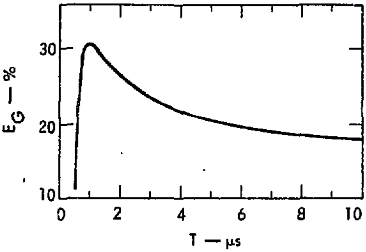

Fig. 4. Fraction of radiant yield deposited in ground as a function of time.

The percentage of the radiant yield in the ground can be calculated if we express Y in the following way:

$$Y = 2\pi\rho_0\ B\ A^2\ R_0^{C/B}. \qquad (42)$$

We can factor out the term $2\pi\rho_0 B$ from Eq. 41 and only calculate $A^2 R_0^{\ C/B}$:

$$A^2 R_0^{C/B} = \begin{cases} 8.97 \times 10^9 & \text{MOHAWK } R_0 \text{ in cm} \\ 10.6 \times 10^9 & \text{ZUNI t in sh.} \end{cases} \qquad (43)$$

Thus, at 19 μs the percentage of radiant yield deposited in the ground is between 18 and 21%.

4. Conclusion

In the preceding section we showed how to calculate the amount of energy deposited in the ground caused by the surface burst of a nuclear explosive by comparing the motion of the fireball with the motion of a fireball for a free air burst. The assumptions behind the formulas just derived are essentially:

- Equations (1) through (4).
- No hydrodynamic motion for times less than 20 μs.
- Solutions of the equations are self-similar.
- W is proportional to the radiant energy density.

With regard to the first assumption, the well-known solution to the Milne problem shows that the radiant flux near a temperature discontinuity is reasonably well described by the diffusion approximation to the radiant transfer equation. The momentum transfer equation has no term representing the radiant flux but it can be shown that these corrections in the order of $(\dot{R}/C)^2$ are at most about one percent.

The second assumption is valid as long as the radiation front is advancing more rapidly than a sound signal in the heated air. Machine calculations confirm that there is virtually no motion of the heated air for up to 30 μs.

Whether or not the self-similar solutions are the correct ones has to be answered in the same way that one answers the question for any other solution; namely, does the solution satisfy all of the boundary conditions.

In addition, we had the restriction on the temperature dependence of λ. We found this latter restriction presented no special problems for air.

We found that Eq. (42), along with the condition that F = 0 at the origin, completely determine the solution of Eq. (37) if we also make use of the data in Fig. 1.

The assumption that the sink of radiant energy, W, is proportional to T^4 is plusible but open to some question. The problem here is that to solve Eq. (27) we need $\phi(x)$. Equation (2') provides a solution for $\phi(x)$ valid in some region around the origin but which breaks down near the edge of the front for reasons discussed above.

This solution, which is Eq. (32), represents a <u>source</u> to the radiation field not a sink. We may ask, under what conditions do the terms in Eq. (27), which contain ϕ, actually represent an energy sink for the radiation?

That is, when does the following condition obtain:

$$3x(\phi\bar{p}_R)' + 3\alpha\phi\bar{p}_R < 0$$

or

$$\frac{d(\phi\bar{p}_R)}{\phi\bar{p}_R} < -\frac{\alpha}{x}\,dx. \qquad (44)$$

A possibility is to require that

$$\frac{d(\phi\bar{p}_R)}{\phi\bar{p}_R} = -\alpha\frac{dx}{x^{1+\epsilon}},$$

with the solution

$$\phi\bar{p}_R = Kx^{-\alpha(1+\epsilon)}. \qquad (45)$$

A sink strength corresponding to Eq. (45) is

$$3K\alpha\epsilon x^{-\alpha(1+\epsilon)} \qquad (46)$$

Thus, if we wish to improve on our assumption of Eq. (34), we could represent the rate of energy absorption in Eq. (27) by using Eq. (32) within some radius x_0, and for $x > x_0$ use Eq. (46). This could be done in a self-similar manner but we would have to fix three new parameters K, ϵ and x_0 in our solution for $\overline{p}_R(x)$. Two necessary conditions would be the continuity of $\phi\overline{p}_R$ and of $\overline{p}_R$.

For the purposes of this paper, such added complication is unnecessary and Eq. (34) is adequate. This analysis does, however, suggest an interesting aspect of the physics in the expanding fireball. In the interior regions the heated air acts as a source and adds to the energy being propagated towards the front. Only in a relatively cold region near the front does the air absorb energy.

Acknowledgment

E. C. Woodward of L-Division, LLL, made the observations and produced the data that appear in this report. It was a difficult and time-consuming task to retrieve and interpret the old film records of these shots. Without his enthusiasm and interest the results just reported would not have been possible.

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