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A b s t r a c t

Nonperturbative large scale fluctuations of gluonic field which are present in the exact vacuum of QCD result in modification of interaction between quarks in quarkonium. Such effects are considered for heavy quarks in cases when the size of the quarkonium system is much smaller than that of the dominating gluonic fluctuations. Green's function of relative motion in heavy quarkonium is calculated for energy below thresh/old with the account of leading nonperturbative effects. It is shown that once nonperturbative gluonic degrees of freedom are taken into consideration they can never be absorbed into a sort of interaction potential between quarks.

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1. Introduction

Since the discovery of instanton solutions in QCD [1] (see also [2] and references therein) and of gauge ambiguities [3] it has been realized that the spectrum and structure of eigenstates in QCD should be drastically different from the customary perturbation theory pattern. In particular these peculiarities of the theory should result in reconstruction of the vacuum state. At the level of present understanding the exact vacuum of QCD should contain large scale fluctuations not describable in usual perturbation theory. In ref. [2] were discussed such vacuum fluctuations caused by instanton solutions of moderate scale size so that in a sense weak coupling theory remained justified. However a complete investigation of the problem at all distances requires overcoming the strong coupling barier and is inaccessible at present.

On the other hand it is widely believed (see e.g. [2] and [4]) that nonperturbative fluctuations of gluonic field result in an essential modification of interaction between a heavy quark and an antiquark. Therefore such fluctuations might play an important role in dynamics of a heavy quark-antiquark bound system called quarkonium. In fact the same is true for light quarks (u, d, s) as well. But there is an important difference between heavy and light quarks. The former can be considered as external objects placed into gluonic vacuum, while the fields of the latter also undergo nonperturbative vacuum fluctuations themselves [2], and that is why for light quarks the situation is far more complicated.

In this paper it will be shown that for heavy enough quarks the effect of nonperturbative fluctuations of gluonic field on certain properties of heavy quarkonium can be calculated to some extent without a detailed knowledge of the structure of the vacuum state and of exited ones in QCD. In the approach to be developed below nonperturbative effects in dynamics of heavy quarkonium are expressed in terms of vacuum mean values of local operators constructed from gluonic field operators. In particular the leading effects are proportional to the vacuum matrix element of the form

$$\langle 0 | F^a_{\mu\nu}(0) F^a_{\mu\nu}(0) | 0 \rangle \tag{1}$$

where $F^a_{\mu\nu}(x)$ is the gluonic field tensor.

In fact the procedure outlined below is very close to Wilson operator expansion [5] but provides a somewhat more general view on the properties of quarkonium. In particular below in Sec.3 the nonrelativistic Green's function of relative motion in quarkonium will be calculated up to terms proportional to the value (1). The Green's function calculated in Sec.3 generates expressions for a set of two-point vacuum amplitudes generated by currents of heavy quarks including corrections due to the matrix element (1).

Effects of the vacuum mean value (1) in phenomenology of charmonium and of light mesons were first considered in Refs. [6] and [7] in terms of Wilson operator expansion. From comparison of sum rules for charm production in the e^+e^- annihilation with the experimental data the numerical estimate was found [6,7]

$$\langle 0 | F^a_{\mu\nu} F^a_{\mu\nu} | 0 \rangle \simeq 0.12 \ GeV^4 \tag{2}$$

with a possible (20–40)% uncertainty*.

Alternatively one can try to estimate the numerical value of the vacuum matrix element (1) by considering instanton contribution. Such a calculation [8] invcives integration over scale size of instantons ρ and the corresponding integral is badly divergent at large ρ. In the other words the value (1) recieves dominant contribution from instantons of large scale size which in turn fail to be good approximations to actual field fluctuations [2]. Thus one is again led into the strong coupling region and the problem of theoretical calculation of the numerical value of (1) remains unsolved.

To avoid possible confusions two points should be noted. First is that the (ultraviolet divergent) perturbation theory part of the vacuum average (1) is implied as usually to be subtracted by definition so that a nonzero value of the matrix element (1) is a purely nonperturbative effect. In the other words one can understand the operator product in eq. (1) as N-product $:FF:$ Second point is that the operator $F^a_{\mu\nu}(x)\,F^a_{\mu\nu}(x)$ has zero anomalous dimension [7] so that one need not specify the normalization point for the gluonic operators. (In this sense the value (1) is a universal constant).

The major starting point of this paper is the simple

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*) Throughout this paper a somewhat non-standard but convenient normalisation for $F^a_{\mu\nu}$ is used. It is fixed by the convention that the Yang-Mills Lagrangian is written in the form

$$\mathcal{L}_{YM} = -\frac{1}{g^2} F^a_{\mu\nu} F^a_{\mu\nu}$$

with $g^2 \equiv 4\pi\alpha_s$. $F^a_{\mu\nu}$ is related to the conventio-
(see p.4)

fact that for large mass m of a quark one can consider a quark-antiquark pair localized at a relative distance r such that on one hand $r \gg m^{-1}$, so that the momenta of quarks are nonrelativistic, $p \sim r^{-1} \ll m$, while on the other hand r is small in the scale of strong interactions, so that methods of short distance QCD are applicable. Surely, such a situation requires large mass of a quark; even charmed quarks are not enough heavy in this sense, while for quarks composing Υ-family ($m \simeq 4.5$Gev) this situation seems to be conceivable. The relevance of the approach developed below improves with growing mass of quark and it can be helpful for description of still heavier quarkonium systems which are widely believed to be discovered in future.

The expansion parameters in the approach considered are typically the "old" $\alpha_s(k \sim 1/r)$ and the "new" one (r/R) (some power) where R is an intrinsic scale size of nonperturbative vacuum fluctuations. The former parameter describes usual perturbation theory effects while the latter one refers to nonperturbative corrections we are mainly interested in. There is in fact no formal definition of R. In terms of refs. [2,8] it is of order of critical size of instantons ρ_c. (According to estimates of ref. [8] $\rho_c^{-1} \simeq 0.17$Gev.). In terms of the approach of this paper the scale of the expansion parameter is given by the ratio of vacuum matrix elements of different dimension,

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nally normalized field tensor $G^a_{\mu\nu}$ as follows $F = \frac{1}{2} g G$.

say

$$\mathcal{R}^{-2} \sim \left| \frac{\langle 0|DFDF|0\rangle}{\langle 0|FF|0\rangle} \right|$$

with $\mathcal{D}$ being the covariant derivative.

For a colourless quark-antiquark pair separated ba a distance $r \ll \mathcal{R}$ the interaction with fluctuations of scale size $\sim \mathcal{R}$ can be expanded into multipole series with dipole term being the leading one. Furthermore, for nonrelativistic quarks the interaction can also be expanded in powers of $v/c \sim (mr)^{-1}$, so that only the electric-like dipole interaction survives in the leading approximation. Therefore, if a quantum mechanical object is considered in which heavy quarks are controlled to be nonrelativistic and separated by a distance short in comparison with $\mathcal{R}$, a sort of perturbation theory in powers of $(r/\mathcal{R})$ and $(mr)^{-1}$ emerges.

In Sec. 2 we shall consider contribution of nonperturbative vacuum fluctuations to characteristics of lowest levels of a superheavy quarkonium. By superheavy is meant a quarkonium for which asymptotic freedom is valid for lowest bound states which are dominantly determined by the Coulomb-like gluonic exchang . Obviously, this requires the following condition to be satisfied:

$$\alpha_s (k \simeq m\alpha_s) \ll 1. \qquad (3)$$

This inequality sets in for quark mass equal to few tens of Gev. The wave function of n-th level is proportional to exp. $(-k_0 r/n)$ with $k_0 = \frac{2}{3} m \alpha_s (k_0)$ being the Bohr momentum. Hence, for $(k_0/n)^{-1}$ small in scale of $\mathcal{R}$ our approach becomes applicable since a contribution of distances $r \gg (k_0/n)^{-1}$ is exponentially small. Surely, this situation does not seem to be of a paramount physical interest, since first there is enough time to wait for a discovery of quarkonium with tens of Gev mass and second, the effects we calculate are only small corrections to essentially Coulomb-like characteristics of quarkonium levels comparable with corrections of Breit-Fermi type and even with purely radiative corrections having relative magnitude of order α_s^3. Nevertheless, it seems reasonable to start with a discussion of this somewhat idealized example, since in this case the basic assumptions and the method are illustrated most transparently.

In Sec. 3 we consider the Green's function of relative motion of quarks in quarkonium at nonphysical energies far enough below threshold. In this object the quarks are controlled at short distances by the condition that the energy is negative (i.e. below the threshold) and large: $\mathcal{E} = -k^2/m$,

$$k \gg R^{-1} \quad \text{and} \quad \alpha_s(k) \ll 1. \tag{4}$$

The latter condition asures validity of usual perturbation theory.

The physical interest of considering Green's function is first, that through spectral representation it generates sum rules for spectral densities of various operators constructed from the fields of heavy quarks in hand and second, that knowing the Green's function even for unphysical energies one can judge upon quark-antiquark interaction, say in terms of potential picture.

The consideration presented below brings also an interesting byproduct. Namely our approach inevitably requires an existence of states composed from quark-antiquark pair in a colour octet state and an explicitly coloured gluonic exitation, which will be discussed in Sec.4. Such states should be located in narrow energy band around the quark-antiquark threshold. However with our method we can make no quantitative conclusions about such states since again this problem requires overcoming the strong coupling barier. The Green's function calculated in Sec. 3 can be applied already for discussion of properties of the Υ-family, say, if one takes $k \simeq 1.5 - 2$Gev. In this case the sum rules generated by the Green's function coincide with the nonrelativistic limit of the sum rules derived in refs.[6,7,9,10] for the case of charmonium. Thus the approach developed here can also be considered as a simple nonrelativistic method of derivation of the sum rules.

2. Nonperturbative Effects in Superheavy Quarkonium

In this section we consider a superheavy quark-antiquark pair so that the condirion (3) is satisfied and in the lowest bound states the quark and the antiquark are located at distances

$$r \sim \left[m \alpha_s \left(k \approx m\alpha_s\right)\right]^{-1} \qquad (5)$$

which are small as compared to the scale of strong interaction.

At short distances a nonrelativistic quark and an antiquark interact with each other by the Coulomb-like potential

$$V(r) = -\frac{4}{3}\,\frac{\alpha_s(1/r)}{r}P_c + \frac{2}{3}\,\frac{\alpha_s(1/r)}{r}P_8\,, \qquad (6)$$

where P_c and P_8 are the projection operators for colour singlet and colour octet states respectively. The lowest levels of the superheavy quarkonium are determined by the colour singlet attractive part of the potential (6).

We are interested here in corrections due to nonperturbative gluonic fluctuations in the exact vacuum of QCD. To account them for one must consider the total Hilbert space including states of heavy quarks as well as of gluons. The Hamiltonian acting in this space can be classified into three terms:

$$\mathcal{H} = \mathcal{H}_Q + \mathcal{H}_g + \mathcal{H}_{int} \tag{7}$$

where $\mathcal{H}_Q$ is the Hamiltonian acting on dynamical variables of quarks only, including the gluonic exchange interaction between them. In the leading nonrelativistic approximation it can be written in the form

$$\mathcal{H}_Q = -\frac{\vec{\nabla}^2}{m} + V(r) \tag{8}$$

where V(r) is given by eq.(6) and the $\vec{\nabla}$ operator is acting on the relative coordinate $\vec{r} = \vec{r}_Q - \vec{r}_{\bar{Q}}$ (We exclude the overall motion of quarkonium by placing the coordinate system origin into the centre of mass of the system). $\mathcal{H}_g$ is the Hamiltonian for gluonic degrees of freedom, and finally, $\mathcal{H}_{int}$ is the Hamiltonian describing interaction of quarkonium with gluonic degrees of freedom.

The most unknown term here is $\mathcal{H}_g$, and the only property of it assumed for a while is that it enjoys full rights of a Hamiltonian operator in the absence of heavy quarks.

In case $\mathcal{H}_{int}$ term is neglected, the total Hilbert space of the problem splits into a direct product of two independent subspaces, one for quarkonium states and the other for gluonic states. Thus, in this approximation the states of the whole system are factorized in the form

$$|N\rangle = |n_Q\rangle |n_g\rangle \tag{9}$$

with $|n_Q\rangle$ being the eigenstates of the Hamiltonian (8) and $|n_g\rangle$ are the same for the $\mathcal{H}_g$. In fact as it has been already mentioned in the Introduction little is known about the spectrum and properties of $|n_g\rangle$ and there is enough room for assumptions. We make here two assumptions about the spectrum of gluonic states. The first one is that it has the lowest energy state $|0\rangle$ called vacuum and that this state is invariant under Lorentz transformations as well as under global colour rotations. The Hamiltonian $\mathcal{H}_g$ is taken to be normalized in such a way that

$$\mathcal{H}_g |0\rangle = 0. \tag{10}$$

The second assumption will be described later.

Thus, without the $\mathcal{H}_{int}$ term the total spectrum of the problem contains the pure quarkonium states $|n_Q\rangle |0\rangle$ and states with explicit gluonic exitations $|n_g\rangle$. Now we are going to switch on the interaction $\mathcal{H}_{int}$ and by considering it as a perturbation calculate the energy shifts of quarkonium states, i.e. of $|n_Q\rangle |0\rangle$. To do this we need an explicit form of $\mathcal{H}_{int}$. The leading nonrelativistic term is given by interaction with the time-like component of the gluonic 4-potential A^a_μ. As it has been discribed in the Introduction we also expand the interaction in multipoles. The first two terms of this expansion are:

the "charge"

$$\mathcal{H}_0 = 2 t^a A_0^a(0) \tag{11}$$

and the dipole

$$\mathcal{H}_1 = - \xi^a \vec{r} \cdot \vec{E}^a(0) \tag{12}$$

where $t^a = t_1^a + t_2^a$ (a = 1, 2, ... 8) are the colour SU(3) generators for the quark-antiquark system (t_1^a and t_2^a refer to the quark and the antiquark respectively; for a quark $t_1^a = \lambda^a/2$), ξ^a are operators orthogonal to t^a: $\xi^a = t_1^a - t_2^a$, and finally,

$$E_i^a = - F_{0i}^a \tag{13}$$

(Note, that the coupling constant $\frac{1}{2} g$ is included into normalization of gluonic field).

We will consider the perturbation expansion in $\mathcal{H}_{int}$ up to the second order, starting from colourless states $|n_Q\rangle$. Therefore the "charge" term (11) will be not operative, since

$$t^a |\text{colourless state}\rangle = 0,$$

and only the dipole (12) survives.

For the same reasons in the approximation considered we will have no problems with gauge invariance since (12) contains only gauge invariant operator $\vec{E}^a$. Therefore we need not specify the gauge used.

Now we have everything to write down the energy shift due to $\mathcal{H}_{int}$ of a quarkonium level, which in the zeroth order has the form $|N^0\rangle = |n_Q\rangle|0\rangle$ with $|n_Q\rangle$ being colourless. To the first order we have

$$\delta\mathcal{E}_n = \langle N^\circ | \mathcal{H}_1 | N^\circ \rangle = -\langle n_Q | \xi^a \tau_i | n_Q \rangle \underbrace{\langle 0 | E_i^a | 0 \rangle}_{=0} =$$

There are many reasons for this expression to be equal to zero. One is that $\langle 0 | E_i^a | 0 \rangle = 0$ because of the vacuum state invariance under space and colour rotations.

The second-order expression is

$$\delta\mathcal{E}_n = \sum_{|M\rangle} \frac{|\langle N^\circ | \mathcal{H}_1 | M \rangle|^2}{\mathcal{E}_{N^\circ} - \mathcal{E}_M} = \tag{14}$$

$$= \sum_{|m_Q\rangle |m_g\rangle} \frac{\langle n_Q | \xi^a \tau_i | m_Q \rangle \langle m_Q | \tau_k \xi^b | n_Q \rangle \langle 0 | E_i^a | m_g \rangle \langle m_g | E_k^b | 0 \rangle}{\mathcal{E}_{n_Q} - \mathcal{E}_{m_Q} - \mathcal{E}_{m_g}}$$

where $\mathcal{E}_{n_Q}$, $\mathcal{E}_{m_Q}$ are energy eigenv-lues of $\mathcal{H}_Q$ and $\mathcal{E}_{m_g}$ is the same for $\mathcal{H}_g$. Note, that the operators ξ^a link colour singlets to colour octets only. Therefore the states $|m_Q\rangle$ refer to the continuum spectrum of the Hamiltonian (8), (6) in the colour octet sector, thus, in (14) $\mathcal{E}_{m_Q} > 0$. Therefore, if $|n_Q\rangle$ is taken to be one of the lowest bound colourless states, the energy difference $\mathcal{E}_{n_Q} - \mathcal{E}_{m_Q}$ is negative and large for large enough mass of the quark.

Now we are in position to make the second assumption about the spectrum of gluonic states $|m_g\rangle$. Namely we assume that in the situation considered the r.h.s. of eq.(14) can be expanded in powers of $\mathcal{E}_{m_g} / (\mathcal{E}_{n_Q} - \mathcal{E}_{m_Q})$

Then the first term of this expansion has the form

$$\delta\mathcal{E}_n = \sum_{|m_g\rangle} \langle 0 | E_i^a | m_g \rangle \langle m_g | E_k^b | 0 \rangle \; \times$$

$$\times \sum_{|m_Q\rangle} \frac{\langle n_Q | \xi^a \tau_i | m_Q \rangle \langle m_Q | \xi^b \tau_k | n_Q \rangle}{\mathcal{E}_{n_Q} - \mathcal{E}_{m_Q}} = \tag{15}$$

$$= \langle 0 | E_i^a E_k^b | 0 \rangle \sum_{|m_Q\rangle} \frac{\langle n_Q | \xi^a \tau_i | m_Q \rangle \langle m_Q | \xi^b \tau_k | n_Q \rangle}{\mathcal{E}_{n_Q} - \mathcal{E}_{m_Q}} .$$

Thus one sees that with the latter assumption the sum over the unknown spectrum of gluonic states $|m_g\rangle$ is reduced to the vacuum mean value $\langle 0|E_i^a E_k^b|0\rangle$. One can employ colour and Lorentz invariance of the vacuum state to relate the latter to the manifestly Lorentz invariant value (1). Indeed, one has

$$\langle 0|E_i^a E_k^b|0\rangle = \frac{1}{3}\delta_{ik}\,\frac{1}{8}\,\delta^{ab}\langle 0|\vec{E}^c\vec{E}^c|0\rangle.$$

Now it is convenient to introduce the 4-vector q_μ which is the total 4-momentum of the quark-antiquark system considered. In the rest frame to which all the previous relations refer q has only a time-like component q_0, so that from eq. (13) one has

$$\vec{E}^a\vec{E}^a = -(q_\alpha F_{\mu\alpha})(q_\beta F_{\mu\beta})/q^2 .$$

Due to the Lorentz invariance of the vacuum

$$\langle 0|F_{\mu\alpha}^a F_{\mu\beta}^a|0\rangle = \frac{1}{4}g_{\alpha\beta}\,\langle 0|F_{\mu\nu}^a F_{\mu\nu}^a|0\rangle ,$$

and thus we have

$$\langle 0|E_i^a E_k^b|0\rangle = \frac{1}{24}\,\delta_{ik}\,\delta^{ab}\langle 0|\vec{E}^c\vec{E}^c|0\rangle = \qquad (16)$$
$$= -\frac{1}{96}\,\delta_{ik}\,\delta^{ab}\langle 0|F_{\mu\nu}^c F_{\mu\nu}^c|0\rangle .$$

Note the negative sign of the last expression. It shows that with positive value of the quantity (1) (see eq. (2)) the fluctuations of the gluonic field correspond to imaginary "electric" component, or in the other words that it is more adequate to treat the fluctuations in the Euclidean space, which fits very well with the instanton philosophy.

The assumption of the validity of the expansion in powers of $\varepsilon_{m_g}/(\varepsilon_{n_a}-\varepsilon_{m_a})$ refers in fact to the structure of gluonic fluctuations. Indeed, the assumption implies that the sum over the intermediate gluonic states in eq. (15) is saturated by states $|m_g\rangle$ with energies low as compared to $|\varepsilon_{n_a}|$. In this sense the gluonic system is assumed to be "soft" in camparison with the quark-antiquark system. This conjecture can be analized somehow by considering subsequent terms of the expension. The first of them is proportional to

$$\langle 0 | E_i^a \mathcal{H}_g E_i^a | 0 \rangle \tag{17}$$

which is zero because of kinematical reasons. Indeed, using eq. (10) one has

$$\langle 0 | E_i^a \mathcal{H}_g E_i^a | 0 \rangle = \langle 0 | E_i^a [\mathcal{H}_g , E_i^a] | 0 \rangle =$$

$$= i \langle 0 | E_i^a (\frac{\partial}{\partial t} E_i^a) | 0 \rangle = -i \frac{q_\alpha q_\gamma q_\beta}{(q^2)^{3/2}} \langle 0 | F_{\mu\alpha}^a \partial_\gamma F_{\mu\beta}^a | 0 \rangle$$

and the vacuum mean value in the last expression in this chain is vanishing since it should have an odd number of Lorentz indices. The next term is proportional to

$$\langle 0 | E_i^a \mathcal{H}_g \mathcal{H}_g E_i^a | 0 \rangle = \langle 0 | (\partial_0 E_i^a)(\partial_0 E_i^a) | 0 \rangle$$

which is reduced to the vacuum mean value of the $d=6$ operator $\langle 0 | DF^a DF^a | 0 \rangle$. Thus our assumption is valid in case

$$| \langle 0 | DF^a DF^a | 0 \rangle / \langle 0 | F^a F^a | 0 \rangle | \ll \varepsilon_{n_a}^2 .$$

In the vein of ref.[2] the ratio on the l.h.s. is of order ρ_c^{-2}, where ρ_c is the cutoff parameter for instanton scale size.

Thus the perturbation theory in $\mathcal{H}_{int}$ and the expansion in powers of $\mathcal{E}_{m_g}$ (which is in a sense a perturbation expansion in $\mathcal{H}_g$) both result in a series in vacuum mean values of operators of increasing dimension. This expansion seems to make sense as long as the vacuum matrix elements are determined by large distances, or in the other words as long as the gluonic system can be considered as soft.

Certainly, the proposed expansion fails also when the quarkonium system becomes soft itself. This takes place for the near threshold levels when the energy gap $\mathcal{E}_{m_Q}-\mathcal{E}_{n_Q}$ is comparable with the characteristic values of $\mathcal{E}_{m_g}$. We can argue that in this situation a mixing of states of the type $|n_Q\rangle|0\rangle$ with $|m_Q\rangle|m_g\rangle$ containing explicit gluonic exitations becomes essential. In fact it is quite obvious if one considers the first correction to the wave function

$$|N^0\rangle+|\delta N\rangle = |N^0\rangle+\sum_{|M\rangle}\frac{\langle M|\mathcal{H}_1|N^0\rangle}{\mathcal{E}_{N^0}-\mathcal{E}_M}|M\rangle =$$

$$= |n_Q\rangle|0\rangle - \sum_{m_Q,m_g}\frac{\langle m_Q|\xi^a r_i|n_Q\rangle\langle m_g|E_i^a|0\rangle}{\mathcal{E}_{n_Q}-\mathcal{E}_{m_Q}-\mathcal{E}_{m_g}}|m_Q\rangle|m_g\rangle$$

This expression also shows that as far as the dipole interaction (12) is concerned, the mixing occurs with states containing a colour octet quark-antiquark pair and a colour octet gluonic exited state.

There is one serious problem concerning such states. The problem emerges if one invokes the idea of confinement. In this case there are no free states with explicit colour and one should avoid considering them. This however seems impossible at the present level of understanding as long as gluonic operators like $\vec{E}^a$ or $F^a_{\mu\nu}$ are dealt with. These operators require colour octet states since they have no matrix elements between colour singlets. In fact we do not know the language in which QCD should be formulated to include both the idea of gauge colour and the confinement. In the approach considered there is no sign of confinement and therefore we are forced to consider explicitly coloured gluonic states and the same for quarks. Obviously we can and actually do restrict ourselves to consideration of states with no net colour. With the idea of confinement the colourless states made from a coloured quark pair and a coloured gluonic exitation should be bound thus adding a new type of states to quarkonium spectroscopy.

On phenomenological grounds such states were first considered in ref. [11] and there are also calculations considering states with valence gluons in the framework of the MIT bag model [12].

Now we will postpone a further discussion of quarkonium states with gluonic exitations till Sec.4.

Returning to eq. (15) it is worth noting that it can be applied in particular calculations of energy shifts of superheavy quarkonium levels if this problem becomes actual some day. With eq. (16) after noting that $\sum_{|m_Q\rangle} \frac{|m_Q\rangle\langle m_Q|}{\mathcal{E}_{m_Q}-\mathcal{E}_{n_Q}}$ is the Green's function for the repulsive potential $+\frac{2}{3}\alpha_s(t^{-1})/t$ denoted here as $G^{(8)}(\mathcal{E}_{n_Q})$, ($|m_Q\rangle$ are colour octets) and using the relation

$$\langle \text{colourless state} | \xi^a \xi^b | \text{colourless state} \rangle = \frac{4}{3}\delta^{ab},$$

one can rewrite eq. (15) in the form

$$\delta\varepsilon_n = 8\varkappa^4 \int d^3r\, d^3r'\, \psi_n^*(\vec{r})\, r_i\, G^{(8)}(\vec{r},\vec{r}\,',\varepsilon_n)\, r_i'\, \psi_n(\vec{r}\,') \tag{18}$$

where

$$\varkappa^4 = \frac{1}{144} \langle 0 | F_{\mu\nu}^a F_{\mu\nu}^a | 0 \rangle \simeq (170\ \mathrm{Mev})^4 \tag{19}$$

and $G^{(8)}(\vec{r},\vec{r}\,',\varepsilon)$ is the above mentioned Green's function in the coordinate representation obeying the equation:

$$\left(-\frac{\vec{\nabla}^2}{m} + \frac{2}{3}\, \frac{\alpha_s(r^{-1})}{r} - \varepsilon \right) G^{(8)}(\vec{r},\vec{r}\,',\varepsilon) = \delta(\vec{r}-\vec{r}\,')$$

In the same approximation one can also find the corrections to matrix elements of a colour singlet operator $\mathcal{O}$ acting only on quark variables (say, matrix elements for radiative transitions) betw.en lowest states of a superheavy quarkonium. For this one should calculate the corrections to the wave functions up to the second order and then take the matrix element. The final answer is

$$\begin{aligned}
\langle N' | \mathcal{O} | N \rangle = {} & \langle n_Q' | \mathcal{O} | n_Q \rangle \Big[1 + \\
& + 4\varkappa^4 \Big(\langle n_Q | r_i \cdot G^{(8)\prime}(\varepsilon_{n_Q}) \cdot r_i | n_Q \rangle + \\
& + \langle n_Q' | r_i \cdot G^{(8)\prime}(\varepsilon_{n_Q'}) \cdot r_i | n_Q' \rangle \Big) \Big] - \\
& - 8\varkappa^4 \Big[\langle n_Q' | r_i\, G^{(8)}(\varepsilon_{n_Q'}) \cdot \mathcal{O} \cdot G^{(8)}(\varepsilon_{n_Q}) \cdot r_i | n_Q \rangle + \\
& + \langle n_Q' | \mathcal{O} \cdot G^{(0)}(\varepsilon_{n_Q}) \cdot r_i \cdot G^{(8)}(\varepsilon_{n_Q}) \cdot r_i | n_Q \rangle + \\
& + \langle n_Q' | r_i \cdot G^{(8)}(\varepsilon_{n_Q'}) \cdot r_i \cdot G^{(0)}(\varepsilon_{n_Q'}) \cdot \mathcal{O} | n_Q \rangle \Big],
\end{aligned} \tag{20}$$

where $G^{(8)\prime}(\varepsilon)$ is the derivative of $G^{(8)}(\varepsilon)$ over ε : $G^{(8)\prime}(\varepsilon) \equiv \frac{\partial}{\partial\varepsilon} G^{(8)}(\varepsilon)$, and $G^{(0)}(\varepsilon)$ is the Coulomb-like Green's function for the colour singlet state:

$$\left(-\frac{\vec{\nabla}^2}{m} - \frac{4}{3}\,\frac{\alpha_s(r^{-1})}{r} - \varepsilon\right) G^{(0)}(\vec{r},\vec{r}\,';\varepsilon) = \delta(\vec{r}-\vec{r}\,').$$

All the products in eq. (20) inside the Dirac brackets should be understood as operator products.

One can readily verify that the relative magnitude of the corrections discussed is of order $\varkappa^4 m^2/k_0^6$. Therefore, the formulae (15), (18) and (20) can be trusted as long as this parameter is small.

3. Quarkonium Green's Function

In this section we proceed to consideration of a more realistic case of a quarkonium made from quarks of a moderate though large enough mass, say, of the Υ -family or a somewhat heavier. In this case the quarks inside the bound states are no longer located at sufficiently short relative distances and to asure an applicability of our approach we must consider the system at energies where there are no bound states. An adequate object in this situation is the Green's function. Indeed, if one considers the propagation of quarks off a stationary local source producing the quark-antiquark pairs with an energy far below threshold, $\varepsilon = -k^2/m$, then the wave function of the pairs is fading as exp(-kr) and only distances $r \lesssim k^{-1}$ are relevant.

For quarks interacting with gluons we must start with the total Green's function which is an operator in the total Hilbert space involving states of heavy quarks as well as of gluons,

$$\mathcal{G}(\varepsilon) = \sum_N \frac{|N\rangle\langle N|}{\varepsilon_N - \varepsilon} , \tag{21}$$

with the sum running over the complete set of eigenstates $|N\rangle$ of the total Hamiltonian,

$$(\mathcal{H}_Q + \mathcal{H}_g + \mathcal{H}_{int})|N\rangle = \varepsilon_N |N\rangle . \tag{22}$$

The Green's function (21) is in fact a resolvent operator

$$\mathcal{G}(\varepsilon) = \frac{1}{\mathcal{H}_Q + \mathcal{H}_g + \mathcal{H}_{int} - \varepsilon} , \tag{23}$$

and we are going to calculate it by considering the term $\mathcal{H}_g + \mathcal{H}_{int}$ as a perturbation:

$$\mathcal{G}(\varepsilon) = \frac{1}{\mathcal{H}_Q - \varepsilon} - \frac{1}{\mathcal{H}_Q - \varepsilon}(\mathcal{H}_g + \mathcal{H}_{int})\frac{1}{\mathcal{H}_Q - \varepsilon} + \tag{24}$$
$$+ \frac{1}{\mathcal{H}_Q - \varepsilon}(\mathcal{H}_g + \mathcal{H}_{int})\frac{1}{\mathcal{H}_Q - \varepsilon}(\mathcal{H}_g + \mathcal{H}_{int})\frac{1}{\mathcal{H}_Q - \varepsilon} + \\ + \dots$$

The perturbation expansion in $\mathcal{H}_g$ is justified in case the relevant sums over intermediate gluonic states are determined by those with energy much less than $|\varepsilon|$. We suppose as previously that that is the case if one projects $\mathcal{G}(\varepsilon)$ out on the gluonic vacuum state, thus leaving the Green's function to be an operator acting only on quark variables. It is also convenient to project it out on colourless sector of heavy quark-antiquark states and define

$$G(\varepsilon) = P_0 \langle 0|\mathcal{G}(\varepsilon)|0\rangle P_0 = \\ = \sum_N P_0 \frac{\langle 0|N\rangle\langle N|0\rangle}{\varepsilon_N - \varepsilon} P_0 . \tag{25}$$

Note, that $G(\varepsilon)$ is still an operator acting on colour singlet states of quarkonium.

Substituting th expansion (24) into eq.(25) one finds that in terms written out explicitly in eq. (24) $\mathcal{H}_g$ is not operative due to eq. (10) and since it commutes with $\mathcal{H}_Q$. Moreover, the vacuum mean value of $\mathcal{H}_{int}$ also vanishes, and thus the linear in $\mathcal{H}_g + \mathcal{H}_{int}$ term in (24) brings zero contribution to $G(\varepsilon)$. Therefore, with the explicit form of $\mathcal{H}_{int}$ given by eq. (12) one finds

$$G(\varepsilon) = P_0 \frac{1}{\mathcal{H}_Q - \varepsilon} P_0 +$$

$$+ \langle 0 | E_i^a E_k^b | 0 \rangle P_0 \frac{1}{\mathcal{H}_Q - \varepsilon} \xi^a r_i \frac{1}{\mathcal{H}_Q - \varepsilon} \xi^b r_k \frac{1}{\mathcal{H}_Q - \varepsilon} P_0 . \tag{26}$$

Now this expression can be readily adapted for actual calculations if one notes that $(\mathcal{H}_Q - \varepsilon)^{-1}$ is the Green's function for the Coulomb-like problem with the potential (6):

$$\frac{1}{\mathcal{H}_Q - \varepsilon} = G^{(0)}(\varepsilon)\, P_0 + G^{(8)}(\varepsilon)\, P_8 .$$

Thus, using eq. (16) and simple properties of ξ^a operators we find in the coordinate representation

$$G(\vec{x}, \vec{y}, \varepsilon) = G^{(0)}(\vec{x}, \vec{y}, \varepsilon)\, P_0 -$$

$$- 8\varkappa^4 \int d^3 z\, d^3 z'\, G^{(0)}(\vec{x}, \vec{z}, \varepsilon)\, z_i\, G^{(8)}(\vec{z}, \vec{z}', \varepsilon)\, z'_i\, G^{(0)}(\vec{z}', \vec{y}, \varepsilon)\, P_0 \tag{27}$$

(Henceforth the projector P_0 will be omitted, but implied).

A good piece of physical information can be derived from eq. (27). For instance this expression gives in the nonrelativistic limit the correction due to the vacuum mean value (1) to the whole set of amplitudes of the form

$$\Pi(k^2) = i\int dx\, e^{iqx} \langle 0 | T\{ J^{(a)}(x), J^{(a)}(0)\} | 0 \rangle , \qquad (28)$$

$$q^2 \equiv s \equiv 4m^2 - 4k^2 ,$$

with $J^{(a)}(x) = \bar{Q}(x)\, O^{(a)} Q(x)$ being a set of local operators constructed from the field of the heavy quarks $Q(x)$.

One can relate the Green's function to e.g. the electromagnetic vacuum polarization function $P(k^2)$ for $k^2 \ll m^2$. In the case $P(k^2)$ is defined as follows

$$(q_\mu q_\nu - q^2 g_{\mu\nu}) P(k^2) = \qquad (28a)$$
$$= i\int dx\, e^{iqx} \langle 0 | T\{ \bar{Q}(x)\gamma_\mu Q(x), \bar{Q}(0)\gamma_\nu Q(0)\} | 0 \rangle ,$$

it is related to $G(\varepsilon)$ by the equation

$$\frac{d}{dk^2} P(k^2) = \frac{3}{2m^2} \lim_{\substack{\vec{x}\to 0 \\ \vec{y}\to 0}} \frac{\partial}{\partial k^2}\, G(\vec{x},\vec{y}, -k^2/m) \qquad (29)$$

where the factor 3 comes from the summation over colour in the electromagnetic current, while the $2m^2$ in the denominator arise because of different normalization of the relativistic and nonrelativistic amplitudes. Note also, that eq. (29) contains derivatives over k^2. On the l.h.s. the differentiation eliminates the unphysical additive constant which depends on the renormalization convention, while on the r.h.s. the kinematical singularity at $\vec{x} = \vec{y}$ also dropps out after differentiation.

The integral in eq. (27) can be readily done in the energy region $|\varepsilon| \gg m\alpha_s^2$, $(k \gg m\alpha_s)$, where the Coulomb-like interaction (6) can be also treated perturbatively, so that neglecting it one can write

$$G^{(0)}(\vec{x},\vec{y},\varepsilon) \simeq G^{(8)}(\vec{x},\vec{y},\varepsilon) \simeq \frac{m}{4\pi}\,\frac{e^{-kr}}{r} \qquad (30)$$

$(k^2 = -m\varepsilon)$, with $r = |\vec{x}-\vec{y}|$. Technically it is more convenient to perform calculations in the momentum representation in which the operator $\vec{r}$ is reduced to a derivative, $\vec{r} = i\,\partial/\partial\vec{p}$, and then transforming back into the coordinate representation we have

$$G(\vec{x},\vec{y},\varepsilon) = \frac{m\,e^{-kr}}{4\pi r}\Big\{1 - \frac{\varkappa^4 m^2 r}{4}\Big[\frac{3}{k^5} + \frac{3r}{k^4} + \frac{2r^2}{k^3} + \frac{r^3}{k^2}\Big]\Big\}. \qquad (31)$$

Now the calculation of the r.h.s. of eq. (29) is trivial and one finds

$$\frac{d}{dk^2}P(k^2) = -\frac{3}{16\pi m k}\left(1 - \frac{15}{4}\,\frac{m^2\varkappa^4}{k^6}\right) \qquad (32)$$

This expression precisely coinsides with the nonrelativistic limit (i.e. at $k^2 << m^2$) of the results obtained in refs. 6, 7 by rather tedious calculations of Feynman diagramms.

One can readily find the analogs of relation (29) for amplitudes of vacuum polarization by other currents $J^{(a)}$ giving to the quark-antiquark pair different angular momenta. Say, for a P-wave current e.g. $\bar{Q}Q$ or $\bar{Q}\gamma_5\overleftrightarrow{\partial}_\mu Q$ one finds that the corresponding amplitude of the type (28) is proportional to the expression

$$\lim_{\substack{\vec{x}\to 0\\ \vec{y}\to 0}} \frac{\partial}{\partial x_i}\,\frac{\partial}{\partial y_k}\left(\frac{\partial}{\partial k^2}\right)^2 G\left(\vec{x},\vec{y},\frac{-k^2}{m}\right)$$

which is immediately calculable with eq. (31).

The physical interest of considering amplitudes of the type (28) is that through spectral representation they provide QCD sum rules for spectral densities of quarkonium states with different quantum numbers [6, 7, 9, 10]. For instance, with the quantity $P(k^2)$ (see eq. (28a)) one explores the vector states of quarkonium which are directly observed in the e^+e^- annihilation. Eq. (32) generates the following sum rules for Q-flavour production in e^+e^--annihilation

$$\int \frac{R_Q(s)\,ds}{(s-4m^2+4k^2)^2} = \frac{9\pi Q_Q^2}{mk}\left(1+\frac{2\pi\alpha_s(k)\,m}{3k}-\frac{15}{4}\frac{m^2\varkappa^4}{k^6}\right);\quad (m^2\varkappa^4)^{1/3}\ll k^2\ll m^2, \tag{33}$$

where $R_Q(s) = \sigma(e^+e^- \longrightarrow Q\text{-flavour})\,3s/4\pi\alpha^2$ and Q_Q is the electric charge of the quarks in hand. In eq. (33) the obvious first Coulomb correction is also included.

4. Connection with the Potential Model Approach and the Spectrum of Quarkonium States

We start this section with a remark that eq. (27) can be taken as the result of first iteration of the equation

$$(\mathcal{H}_Q + U(\varepsilon) - \varepsilon)\cdot G(\varepsilon) = 1 \tag{34}$$

where U is a nonlocal interaction potential

$$U(\varepsilon)\cdot\varphi = 8\varkappa^4\int d^3z' \, (\vec{z}\vec{z}')\,G^{(8)}(\vec{z},\vec{z}',\varepsilon)\,\varphi(\vec{z}') \tag{35}$$

Note also that the expression (18) for the energy shift is also a diagonal matrix element of U. Therefore the potential U might be inferred as a nonperturbative part of interaction potential between quarks in quarkonium. It is interesting that there is no sign of the local linear potential usually employed in the nonrelativistic potential model (see e.g. [13]). This implies that at least at the distances in hand the linear potential is irrelevant.

However even with the nonlocal interaction (35) it is impossible to go further suppressing the gluonic degrees of freedom. This is already seen from eq. (20) only a part of which can be derived from the first iteration of the interaction (35). In fact eq. (20) can be obtained in no approach dealing only with quark degrees of freedom. Therefore it seems entirely inadequate to discuss the properties of quarkonium in isolation from gluonic states.

The incompleteness of the description dealing with only quark-antiquark dynamical variables becomes evident if one tries to write down a spectral representation for the Green's function (27) of the form

$$G(\vec{x},\vec{y},\varepsilon) = \sum_n \frac{\psi_n(\vec{x})\,\psi_n^*(\vec{y})}{\varepsilon_n - \varepsilon} \tag{36}$$

with a complete set of orthogonal functions ψ_n. It is rather trivial to show ad absurdum that such a representation is impossible for the function given by (27). Indeed, from (36) and the orthogonality ansatz one finds that a function represented by eq. (36) should obey the necessary condition

$$\frac{\partial}{\partial\varepsilon} G(\vec{x},\vec{y},\varepsilon) - \int G(\vec{x},\vec{z},\varepsilon)\, G(\vec{z},\vec{y},\varepsilon)\, d^3z = 0 \tag{37}$$

On the other hand calculating the same expression for the function (27) or even for that given by eq. (31) one finds that for them the condition (37) is violated by a term proportional to $G^{(0)}(\varepsilon)\cdot\frac{\partial U}{\partial\varepsilon}\cdot G^{(0)}(\varepsilon)$.

Thus, once the relevance of the results (27) and (31) to the approximation considered is trusted, one is led inevitably to the conclusion that the description of quarkonium in terms of quark variables is incomplete and the gluonic degrees of freedom should be explicitly accounted for. Suroly, after inclusion of the latter variables the Green's function $G(\mathcal{E})$ has a perfect spectral representation as given by eq. (25). In the other words, once the nonperturbative fluctuations of the gluonic field are taken into consideration they can never be absorbed into a sort of interaction potential between quarks.

The physical interpretation of the situation described is that in the vein of the confinement idea there should be states made of both heavy quarks and gluons and these states as it has been argueu in Sec. 2 should be located in a narrow energy band near the Q -flavour threshold where the quarkonium system and gluonic spectrum are equally "soft". Note, that without assuming such states to exist it is impossible to reconcile the spectral representation (25) for $G(\varepsilon)$ with the absence of representation (36).

5. C o n c l u s i o n s

In this paper an approa h is developed to consideration of the effects of nonperturbative gluonic fluctuations in the exact vacuum state on the nonrelativistic dynamics of heavy quarkonium. The approach is applicable in case the quark and the antiquark are controlled to be separated by distances small compared to the characteristic scale of vacuum fluctuations, so that the interaction of the quarkonium system with the gluonic field $i.e.\ G$ can is expanded in multipoles. The leading corrections considered are proportional to the vacuum matrix element $\langle 0|F^a_{\mu\nu}(0)F^a_{\mu\nu}(0)|0\rangle$.

For the case of superheavy quarkonium in which the lowest bound states are determined by the short distance Coulomb-like exchange potential the energy shifts due to nonperturbative fluctuations are calculated (see eq. (18)) as well as corrections to matrix elements of a colourless operator acting on quark variables between lowest levels (eq. (20)).

In case of moderately heavy quarks the Green's function of relative motion in quarkonium is found up to terms proportional to $\langle 0|F^a_{\mu\nu}F^a_{\mu\nu}|0\rangle$ at energies below threshold.

An investigation of spectral representation for the Green's function led us to the conclusion that the effects of the nonperturbative fluctuations of the gluonic field can not be expressed in terms of an interaction potential between quarks and that one should consider states which contain both the quark-antiquark pair and a gluonic xitation including those in which each of them is a colour octet.

There is nothing seen to prevent from extending the approach proposed on the terms of order $v^2/c^2 \simeq k^2/m^2$ as well as on contributions of vacuum mean values of operators of higher dimension than F^2. For this one needs to consider:

i) higher multipoles in quarkonium interaction with gluonic field, as well as terms of orders v/c and v^2/c^2 in the interaction,

ii) perform further iterations of $\mathcal{H}_g + \mathcal{H}_{int}$.

It can be believed that a consideration of this sort *will* be helpful in studying properties of the Υ-family and of analogous systems made *from* still heavier quarks.

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References

1. A.A.Belavin et al, Phys.Lett. 59B, 85 (1975).
2. C.Callan, R.Dashen and D.Gross, Preprint COO-2220-115 (1977).
3. V.N.Gribov. Proc. XII LNPI Winter School, p.147. Leningrad (1977).
4. F.Wilczek and A.Zee, Phys.Rev.Lett., 40, 83 (1978).
5. K.Wilson, Phys.Rev. 179, 1499 (1969).
6. A.Vainshtein, V.Zakharov and M.Shifman, Pisma ZhETF 27, 60 (1978).
7. M.Shifman et al, Preprint ITEP - 10 (1978).
8. M.Shifman et al, Preprint ITEP - 15 (1978).
9. V.Novikov et al, Phys.Rev.Lett., 38, 626 (1977), Phys. Lett. 67B, 409 (1977).
10. V.Novikov et al, Preprints ITEP - 79 and 83 (1977).
11. A.Vainshtein and L.Okun. Yad.Fiz. 23, 1347 (1976).
12. R.Jaffe and K.Johnson. Phys.Lett., 60B, 201 (1976). I.Yu.Kobzarev et al, Yad.Fiz. 27, 506 (1978).
13. E.Eichten et al, Cornell preprint CLNS-375 (1978).

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