AN EXPERIMENTAL STUDY OF GAP AND THICKNESS INFLUENCE ON THE VIBRATION RESPONSE AND DAMPING OF FLEXIBLE FLUID-COUPLED COAXIAL CYLINDERS

by

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I. Introduction

As alternate energy resources play a larger role in the vitality of the economy of the USA, stringent requirements on the safety of nuclear power systems will demand that the varied characteristics of such systems be clearly understood.

Although there are many safety requirements that nuclear systems must meet before they can be certified, it is clear that, in many cases, analytic capabilities, to verify that the systems do in fact satisfy these requirements, are now in the state of the art phase and must be improved so that a higher level of confidence in the analytical results can be assured. The nuclear power industry now possesses sophisticated analytical capabilities in the form of finite element programs which can determine stress and deformation states due to various mechanical and thermal situations. In addition to static loadings, vibration and seismic loadings may result in structural components being subjected to critical stresses which will render the components ineffective if not adequately designed. Presently, dynamic and vibration calculations are made on a routine basis for systems which are assumed to be in air. However, many components reside in or are adjacent to fluid environments.

Analytical capabilities for these fluid-solid situations are in the process of being developed. However, since the overall phenomenon of fluid-solid interaction is not thoroughly understood, it was the objective of this study to develop and carry out an experimental program which would provide additional insight into the mechanics of fluid-solid interaction. Since one of the primary components in a nuclear power system is the reactor vessel and the thermal liner, the experimental study described in this work was limited to the response of a set of coaxial cylinders with water in the annulus. The effects of cylinder thickness and the fluid filled annulus gap size on the resonant frequencies and mode shapes of the cylinders are presented in this paper; also included is an evaluation of damping for selected gaps and cylinder thicknesses. Details of the experimental setup and procedures are also outlined.

II. Review of Literature

The influence of a fluid environment on the dynamic characteristics of structures has been of interest for hundreds of years. In 1779 Pierre Louis Gabriel Du Buat (1) investigated the effects on the frequency response of pendulum bobs vibrating in water. Summaries of nineteenth century investigations can be found in the works of Lord Rayleigh (2) and Lamb (3). More recent experiments by Stelson (4), Keane (5), and Jones (6) were used to explore the virtual mass concept for beams in fluids. Analytical work on the interaction of a shell in a fluid medium as surveyed by Geers (7). Fritz (8), in an experimental investigation of a cylinder bounded externally by a fluid gap and a rigid shell, showed the use of the virtual mass approach for hydroelasticity and bouyancy effect, although damping and compressibility were neglected.

More recent experimental work which is of particular significance to this work is the coupled breathing vibrations of two long, thin coaxial cylinders in fluid by Levin and Milan (9), vibration of a flexible cantilever cylinder surrounded by a water gap and a rigid cylinder by Au-Yang (10), the vibration of a thin cantilever cylinder surrounded by water and a rigid cylinder with varying gap sizes by Mulchay, et al (11), and the vibration study of a flexible set of cylinders by Au-Yang (12). In the Levin-Milan test, only two sets of flexible cylinders were evaluated with a large gap ((radius/gap)$\sim$5) and an inner cylinder with a variable thickness. The Roman test consisted of a single cylinder test with about the same radius to gap ratio. The Mulchay tests included the variation of the gap size but did not include coupling of flexible cylinders.

Much analytical work has been performed since the 1960's by such authors as S. S. Chang (13), Fritz (14), Krajcinovic (15), Sharp-Wenzel (16), Au-Yang(17), Levy (18), Kalinowski (19), Everstine (20), Schroeder (21), Chung (22), and Bowers-Horvay (23),(24). These analyses have varied from classical formulations to numerical descretizations. The use of the finite element method is presently the most popular technique due to the generality of the finite element method. Some advantages and disadvantages of finite element eigenvalue solid-fluid methods with respect to flexible cylinders is discussed by Brown (25).

III. Experimental Set-Up

Test Cylinders - In order to assess the influence of the cylinder thickness, annulus gap size, and boundary conditions on the frequency response, a total of fourteen cylinders, twelve acrylic and two steel, of various diameters, thicknesses, and rigidities was tested in various combinations. Table 1 gives the nominal sizes of the test cylinders. Figure 1 shows several of the acrylic cylinders and the inner steel cylinder. The material properties of the acrylic and steel cylinders are given in Table 1. The 12-inch diameter acrylic cylinders were obtained as cast cylinders while the remaining cylinders were fabricated from acrylic sheets. A study of the thickness variation and radius variation on the acrylic cylinders indicated that the variations were 0.4% to 5% and 0.08% to 3.4% respectively. The cast cylinders consistently showed less variation than the fabricated acrylic cylinders.

Figure 1 shows that the inner acrylic cylinders were manufactured with acrylic caps while the outer acrylic cylinders were fitted with flanges. The acrylic caps and flanges were 0.25 inches thick, with 6 bolt holes in the caps and 12 bolt holes in the flanges. Figure 2 shows a typical longitudinal section for an inner and outer cylinder pair as attached to the test frame. The flanges and caps were designed to provide nearly rigid lateral restraints on the cylinder ends while allowing relatively high rotational freedoms so that simply supported cylinder conditions are approximated. Metal disks were glued circumferentially at 10^0 increments around the circumference at three levels, namely 1/4, 1/2, and 3/4 of the height of each acrylic cylinder and at 1.5" increments along longitudinal lines at four locations, θ = 0, 90, 180, and 270 degrees. These disks were used to provide monitoring points for eddy current probes for determining the mode shapes of vibration.

Test Set-Up - Figure 3 shows the test frame which consisted of two rigid steel stands. The stand on the right was used to mount the cylinders while the stand on the left housed the 10-lb shaker. Both stands were rigidly bolted on an isolated vibration test table, which minimized the influence of external excitations. Extensive tests were conducted to determine the fundamental resonance of the test frame. These tests showed that the frame's response was very high and outside the range of interest for the experiments on the acrylic cylinders. Points of inner and outer cylinders were bolted to the frame at each end (except for certain tests for which the inner cylinder was left free at the top) using steel bolting rings and discs to circumferentially distribute the restraints about the flanges, as indicated in Figure 2. The "O" rings under the flanges sealed the fluid in the cylinder gap.

Instrumentation - Figure 4 gives an overall view of the instrumentation while Figure 5 is a schematic of the experimental procedure. As shown in Figure 5, the outer cylinder was "point" excited by a 10-lb shaker which was controlled by a B&K vibration exciter. Constant force excitation was maintained by a feedback signal from a load cell mounted in series with the exciting rod which instantaneously measured the excitation force. The frequency of oscillation of the cylinder was monitored by a B&K mini-accelerometer mounted on the surface of the cylinder and displayed on an electronic digital counter.

To obtain the mode shapes, the displacement responses of a cylinder were picked up by mapping eddy probes. Outer cylinders were mapped on the outer surfaces and inner cylinders were mapped on the inner surfaces. Time history responses of both inner and outer cylinders were simultaneously displayed on a dual trace scope for monitoring and phase relationship determination. The r.m.s. values of responses were instantaneously recorded by a B&K level chart recorder during frequency "sweeping" operation to determine the natural frequencies. The signals were also stored in analog form by an Ampex F. M. Recorder for future analysis.

IV. Experimental Procedures

Table 2 gives the series of tests performed in this study. As previously mentioned, these tests were designed to determine the influences of cylinder thickness and annulus gap size on the resonant frequencies and mode shapes of the coupled coaxial cylinder systems. The effect of damping is also assessed.

Resonant Frequencies - The approximate resonant frequencies of each cylinder system, i.e., an inner and outer cylinder combination, were first determined using the standard "sweeping" technique. The outer cylinder was excited from 5 Hz to 1K Hz at a slow sweeping rate ($\approx$ 40 deg/min.) The peaks of the r.m.s. spectral plot of the response from a stationary eddy probe opposite the excitation point (anti-node) inside the inner cylinder provided the approximate locations of the resonant frequencies of a cylinder system. The "exact" resonant frequencies were then pinpointed by vernier scanning in the vicinity of the "peaks". Three to four of the lowest resonant frequencies were determined for each cylinder system. A typical spectra plot is shown in Figure 6 for one of the preliminary tests which was conducted in air.

Mode Shapes - After the resonant frequencies were determined, the phase relationship between the inner and outer cylinder responses at each particular frequency was determined by simultaneously displaying both signals on a dual trace scope. The mode shapes were then determined by mapping with an eddy probe pointed at the metal disks which were distributed both circumferentially and axially.

During initial testing, both the inner and outer cylinder mode shapes were mapped independently and were found to be identical in shape without exception. In the latter part of the testing, only the inner cylinder mode shapes were determined. Figures 7 and 8 show the mode shapes of the inner cylinder 1 and 2 inside of outer cylinder 10 for the six lowest resonant frequencies.

Equivalent Viscous Damping Ratio (ε) - The half-power technique as given by Thompson (23) was used to determine the viscous damping ratio for a given natural mode. The damping ratio as developed for the half-power point technique is:

$$\varepsilon \simeq \frac{f_{b_1} - f_{b_2}}{2f_n}$$

where

f_n = natural frequency with a response x_n

f_{b_1} = lower side band frequency with a response,

$X_{b_1} = 0.707\ x_n$

f_{b_2} = higher sideband frequency with a response,

$X_{b_2} = 0.707\ x_n$

The response x_n was first measured by an eddy probe at the antinode for a given natural frequency f_n. Then the sideband frequencies f_{b1} and f_{b2} were obtained by "Vernier" scanning of both the left and right sides of the "peak" where the responses were 0.707 X_n as read through a digital meter. Damping ratios for the seven cylinder combinations of test series 7, 8, and 9 of Table 2 are given in Figures 33-35. These will be discussed in the next section.

V. Results and Observations

Figures 9-37 represent data from the 54 cylinder combinations which correspond to test series 1 to 12 of Table 2. For each test approximately 8 resonant frequencies and the corresponding mode shapes were determined beginning with the lowest frequency. The cylinder combinations were developed in the following manner. First, the resonant frequencies for each cylinder were determined (test series 1-3). Then the nine inner cylinders were combined with each of the three outer cylinders yielding 27 tests. By allowing the inner cylinders to be simply supported during one phase (test series 4-6) and free in the second phase (test series 7-9), a total of 75 tests were performed. In test series 4-9, the annular gap between the outer and inner cylinder was filled with water. In all tests the outer cylinders were supported at both ends.

Figures 35-37 represent the results of test series 10-12 in which the damping characteristics of 7 cylinder combinations were studied. Test series 10 consisted of the rigid outer cylinder with flexible cylinders of 0.125 inch-thickness of different diameters. Test series 11 involved the rigid inner cylinder and the 0.125 inch thick outer cylinder while test series 12 used the 0.125 inch thick outer diameter and the 0.125 inch thick inner cylinders with different diameters. In this series of tests both the inner and outer cylinders were supported at both ends.

During test series 4-9, it was observed that two distinct responses from the cylinder combinations occurred. It was found that the cylinder combinations vibrated either in-phase or out-of-phase. During in-phase motion, the inner and outer cylinders both experienced the same sign on their radial component of motion at the same circumferential location. During out-of-phase motion, the inner and outer cylinders experienced different signs on their radial component of motion at the same circumferential location.

In the following the variation of the resonant frequencies will be analyzed with respect to the various parameters previously given.

Variation of Frequency with Annular Gap - Figures 11-28 show the variation in the resonant frequency with gap for various model responses. Figures 11-19 cover the out-of-phase data and Figures 20-28 cover the in-phase data. Each figure represents the variation of frequency with gap for a constant inner and outer shell thickness. These results show that as the gap size decreases the resonant frequency of the cylinder system decreases for the out-of-phase modes, while for in-phase modes the resonant frequency increases as the gap size decreases. It is also seen that the resonant frequency is a nonlinear function of the gap size. Interestingly, a close correspondence of results were found for similar inner-outer and outer-inner cylinder thickness, e.g. the 0.25 inch outer - 0.187 inch inner cylinders have similar resonant frequencies as the 0.187 inch outer - 0.25 inch inner combination as shown in Figures 12 and 14. Similar results also exist for other cylinder combinations as seen in Figures 13, 17 and Figures 16, 18 for the out-of-phase modes. For in-phase modes, the cylinder combinations in Figures 21, 23 and Figures 22, 26 and Figures 25, 27 show the same effects as those observed for the out-of-phase modes.

Variation of Frequency with Inner Cylinder Thickness - Figures 29-34 show the variation in the resonant frequencies with the thickness of the inner cylinder for constant gap size. Figures 29, 31 are the out-of-phase modes for gaps of 0.9375 inch, 0.64375 inch, and 0.125 inch respectively. Figures 32, 34 show the results for the in-phase modes for the same gap sizes given above.

These results indicate that for both in-phase and out-of-phase modes the resonant frequency of the cylinder combinations tend to decrease with decreasing thickness. These results also show that the in-phase modes have significantly higher resonant frequencies associated with them.

Another observation which can be made involves the frequency change as a function of the inner cylinder thickness. It is noted for the thinness outer cylinder O_3, thickness of 0.125 inch, that the variation in the resonant frequency does not depend on the inner cylinder thickness.

Variation of Damping Ratios with the Annular Gap - Figures 32 and 36 show the variation of the damping ratio with gap for the out-of-phase and in-phase modes, respectively, of two flexible concentric cylinders with a 0.125 inch thicknesses and a varying gap. In Figure 35 the upper curve is labeled with ε_2 and ε_3 since the resonant frequencies were the same for this particular thickness. These results indicate that for the out-of-phase modes the damping ratios significantly increase from about 5% to about 15% as the gap decreases, while for the in-phase modes the damping ratios generally remain within the 4 - 5% range as the gap size becomes smaller.

Figure 37 shows the variation of damping ratios with gap for a rigid outer cylinder concentric with a flexible inner cylinder. Again it is observed that the damping ratio increases drastically from about 4% to 32% as the gap between the two cylinders decreases.

VI. Conclusions and Recommendations

The results of this study have provided significant data and understanding on the mechanics of solid-fluid interactions of two concentric flexible cylinders with water in the annular gap. The most significant findings of this study can be briefly stated as follows:

1) The frequency varies nonlinearly with gap size. For out-of-phase modes, the resonant frequency increases as the gap size decreases and for in-phase modes the resonant frequency approaches a limiting value as the gap size decreases.
2) The resonant frequency decreases as the thickness of the inner cylinder decreases for both the in-phase and out-of-phase modes.
3) There is a significant increase in the damping ratios of the cylinder system as the gap decreases for the out-of-phase modes but only a moderate increase for the in-phase modes.
4) There is significantly greater damaging damping associated with flexible-to-rigid cylinder pairs compared to flexible-to-flexible cylinder pairs.

From the results of this study, it is clear that the gap size is an extremely important parameter in either increasing or decreasing the frequency response and/or the damping ratios of the system. The selection of cylinder thickness is important in tuning the coaxial cylinders. The results of this study indicate that, in designing reactor systems composed of coaxial cylinders which reside in a fluid environment, it is imperative that analytical formulations incorporate the additional features which account for the effects of the fluid. It is felt that the work presented here provides benchmark problems which can be used in the verification of computer codes. Further studies on the seismic response of fluid-coupled coaxial cylinders and the effects of fluids with varying viscosities on the resonant characteristics of coupled systems will be reported in later papers.

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Table 1 TEST CYLINDERS

Cylinder	Material	OD(in)	ID(in)	t(in)	R/t	L(in)	Type*	Code**
1	acrylic	12.0		0.250	24	24	E	I_{11}
2	acrylic	12.0		0.187	32.1	24	E	I_{12}
3	acrylic	12.0		0.125	48	24	E	I_{13}
4	acrylic	13.0		0.250	26	24	R	I_{21}
5	acrylic	13.0		0.187	34.8	24	R	I_{22}
6	acrylic	13.0		0.125	52	24	R	I_{23}
7	acrylic	13.625		0.250	27.3	24	R	I_{31}
8	acrylic	13.625		0.187	36.4	24	R	I_{32}
9	acrylic	13.625		0.125	54.5	24	R	I_{33}
10	acrylic		13.875	0.250	28.8	24	R	O_1
11	acrylic		13.875	0.187	38.1	24	R	O_2
12	acrylic		13.875	0.125	56.5	24	R	O_3
13	steel	13.625		0.25		24	R	I_R
14	steel		13.875	0.25		24	R	O_R

* E = extruded (catalogue item)

R = rolled (special order)

** I = inner cylinder

O = outer cylinder

Material Properties @ 70°F

A) Acrylic Cylinders

E (Young's Modulus) = 450,000 psi

ν (Poisson's ratio) = 0.375

ρ (density) = 0.04297 #in³

B) Steel Cylinders

E = 30 x 10^6psi

ν = 0.3

ρ = 0.2841 #/in³

C) Water

K (bulk modulus) = 319,000 psi

ρ = 0.03605 #in³

Table 2 TESTING PROGRAM

Test Series	Description	Purpose	Boundary Conditions**	Tests
1	I_{ij} in air	Frequencies*	SS	9
2	I_{ij} in air	Frequencies	C	9
3	O_i in air	Frequencies	SS	3
4	O_1 with I_{ij} (water in gap)	Frequencies	O_1(SS):I_{ij}(SS)	9
5	O_2 with I_{ij} (water in gap)	Frequencies	O_2(SS):I_{ij}(SS)	9
6	O_3 with I_{ij} (water in gap)	Frequencies	O_3(SS):I_{ij}(SS)	9
7	O_1 with I_{ij} (water in gap)	Frequencies	O_1(SS):I_{ij}(C)	9
8	O_2 with I_{ij} (water in gap)	Frequencies	O_2(SS):I_{ij}(C)	9
9	O_3 with I_{ij} (water in gap)	Frequencies	O_3(SS):I_{ij}(C)	9
10	O_R with I_{i3} (water in gap)	Damping Ratios	O_R(SS):I_{i3}(SS)	3
11	O_3 with I_R (water in gap)	Damping Ratios	O_3(SS):I_R(SS)	1
12	O_3 with I_{i3} (water in gap)	Damping Ratios	O_3(SS):I_{i3}(SS)	3

* Frequencies denote not only the magnitude of the frequencies but also the associated mode shapes.

** SS = Simple support

C = Clamped

Figure 1 Acrylic and Inner Steel Cylinder

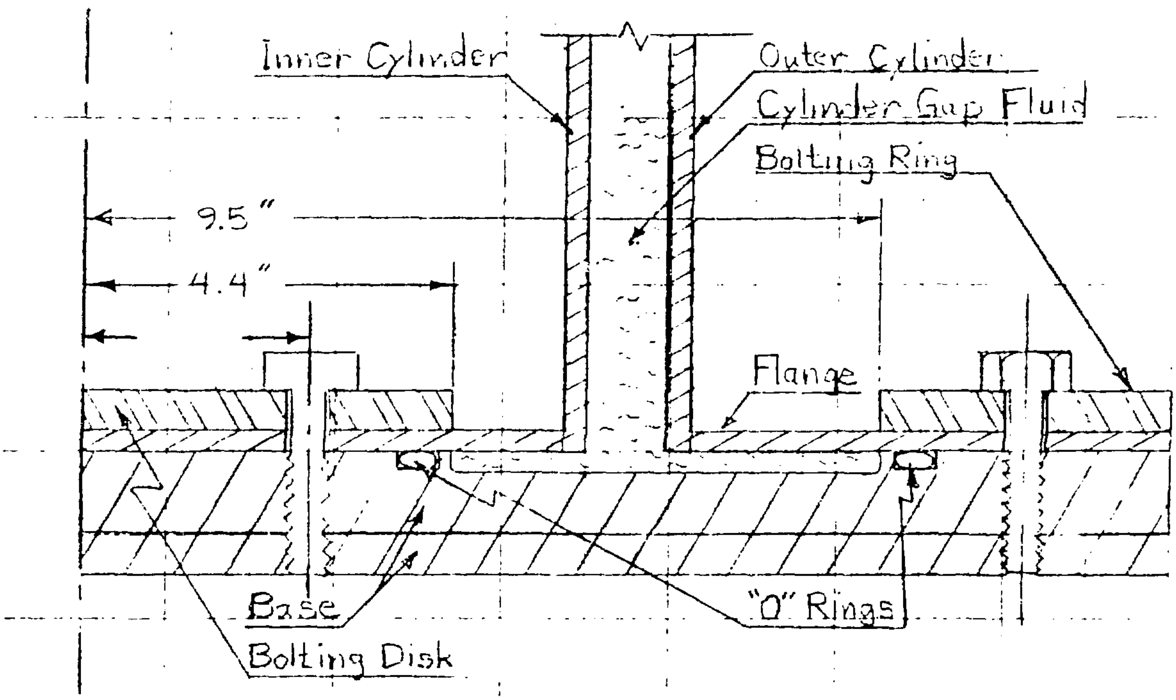


Figure 2 Cylinder Support Configuration

Figure 3 Inner Cylinder Mounted In Test Frame

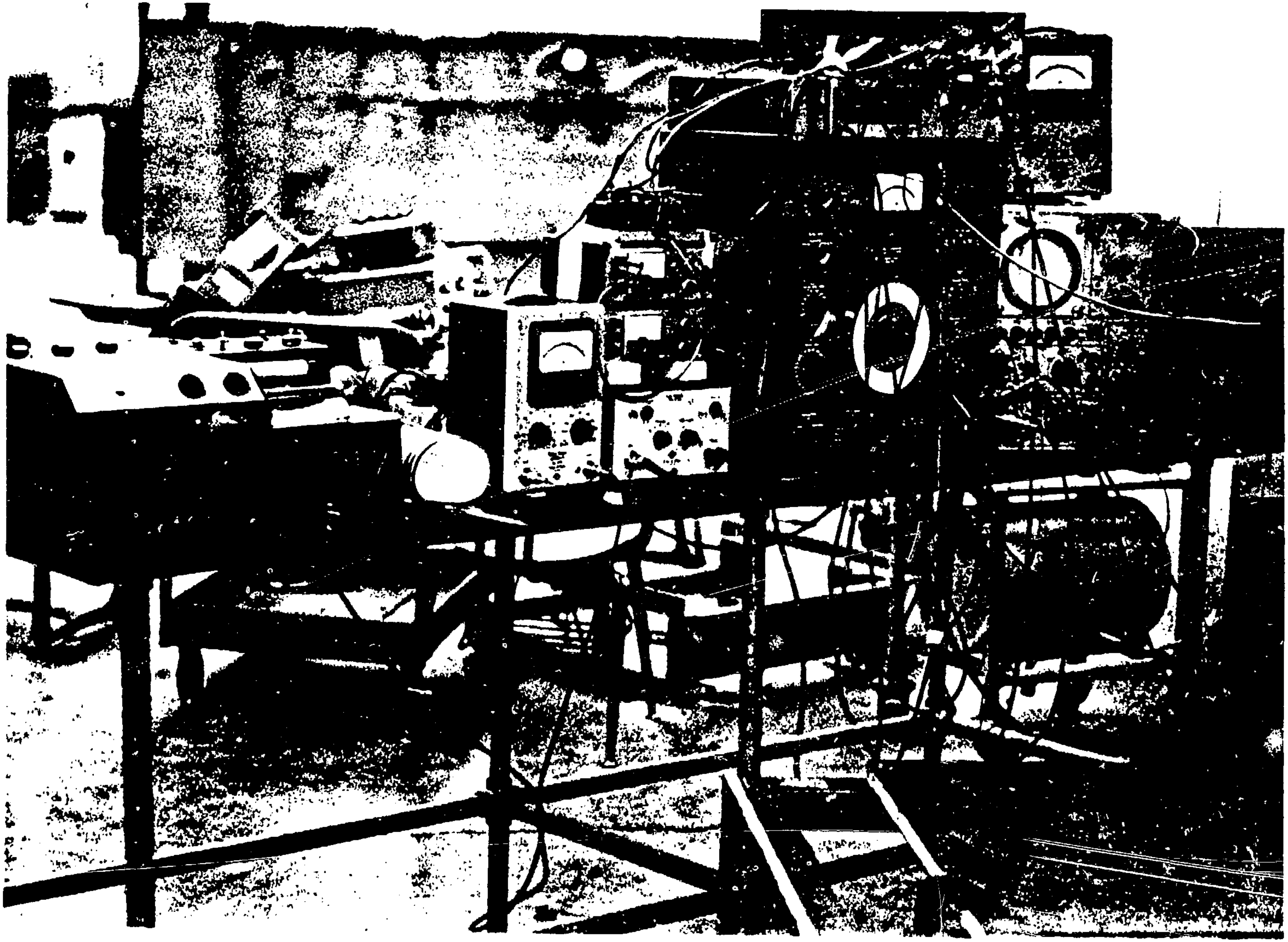
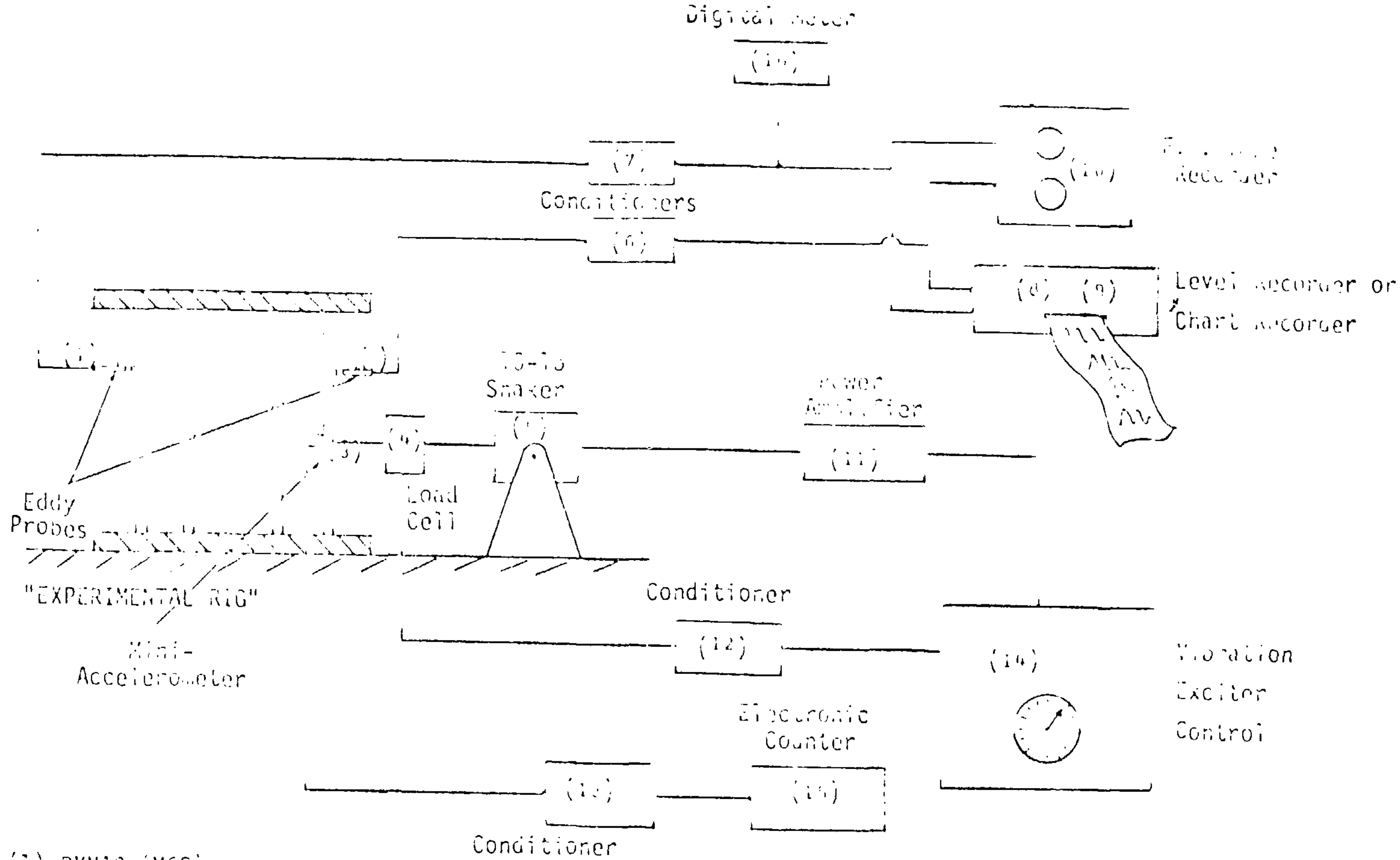


Figure 4 Electronic Recording Equipment for Resonant Frequency Study



(1) DYMAC (M60)
(2) DYMAC (M60)
(3) B & K (8307)
(4) B & K (8200)
(5) B & K (4809)
(6) DYMAC (M600)
(7) DYMAC (M600)
(8) B & K (2606)
(9) GOULD-BRUSH (440)
(10) AMPEX (FR 1300-A)
(11) B & K (2706)
(12) B & K (2622)
(13) B & K (2622)
(14) B & K (1019)
(15) HP (5512-A)
(16) KEITHLEY ([illegible])

Figure 5. Instrumentation for Natural Frequency and Mode Shape Determination

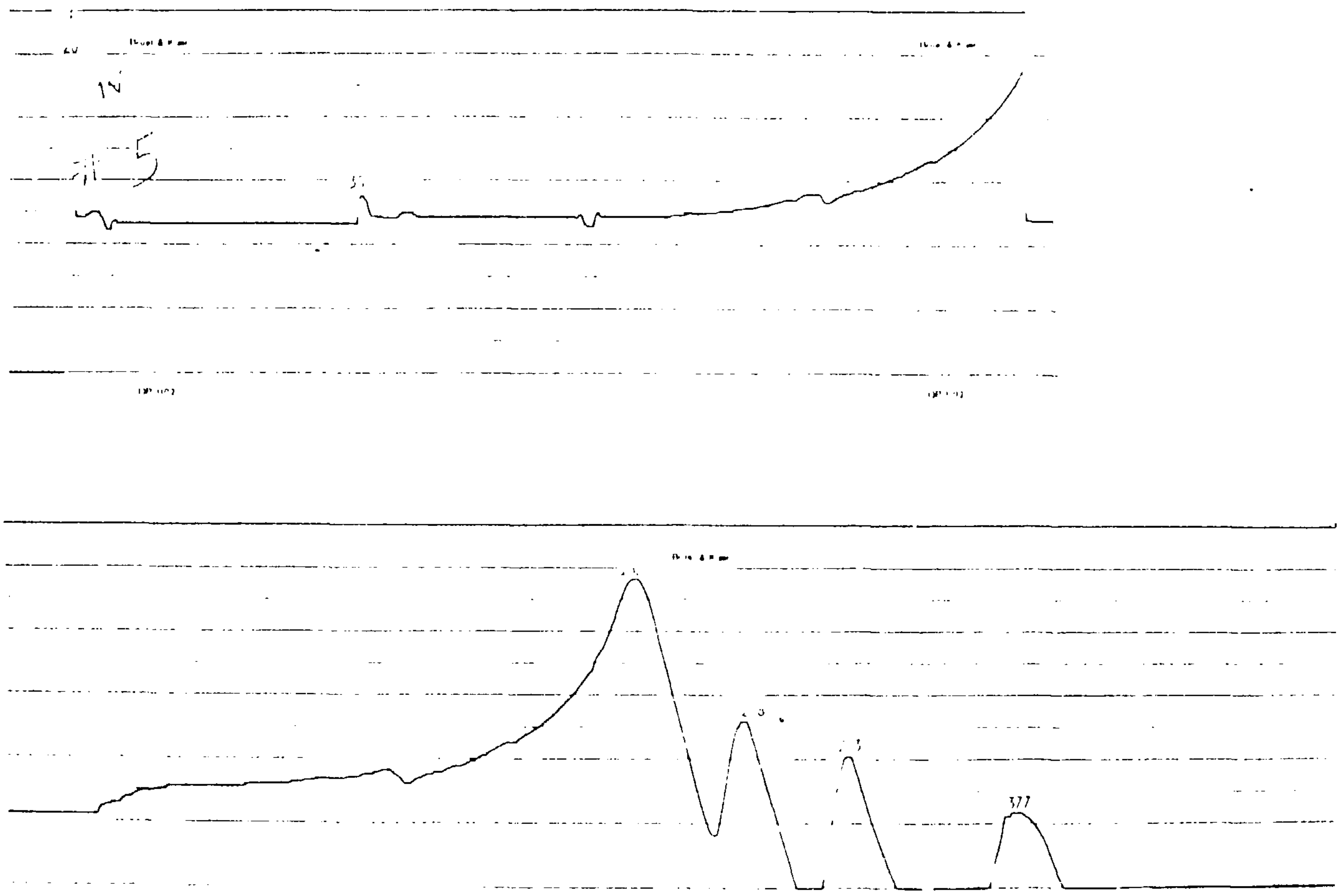


Figure 6. Strip Chart Recording for Simple Supported Inner Cylinder 5 in Air

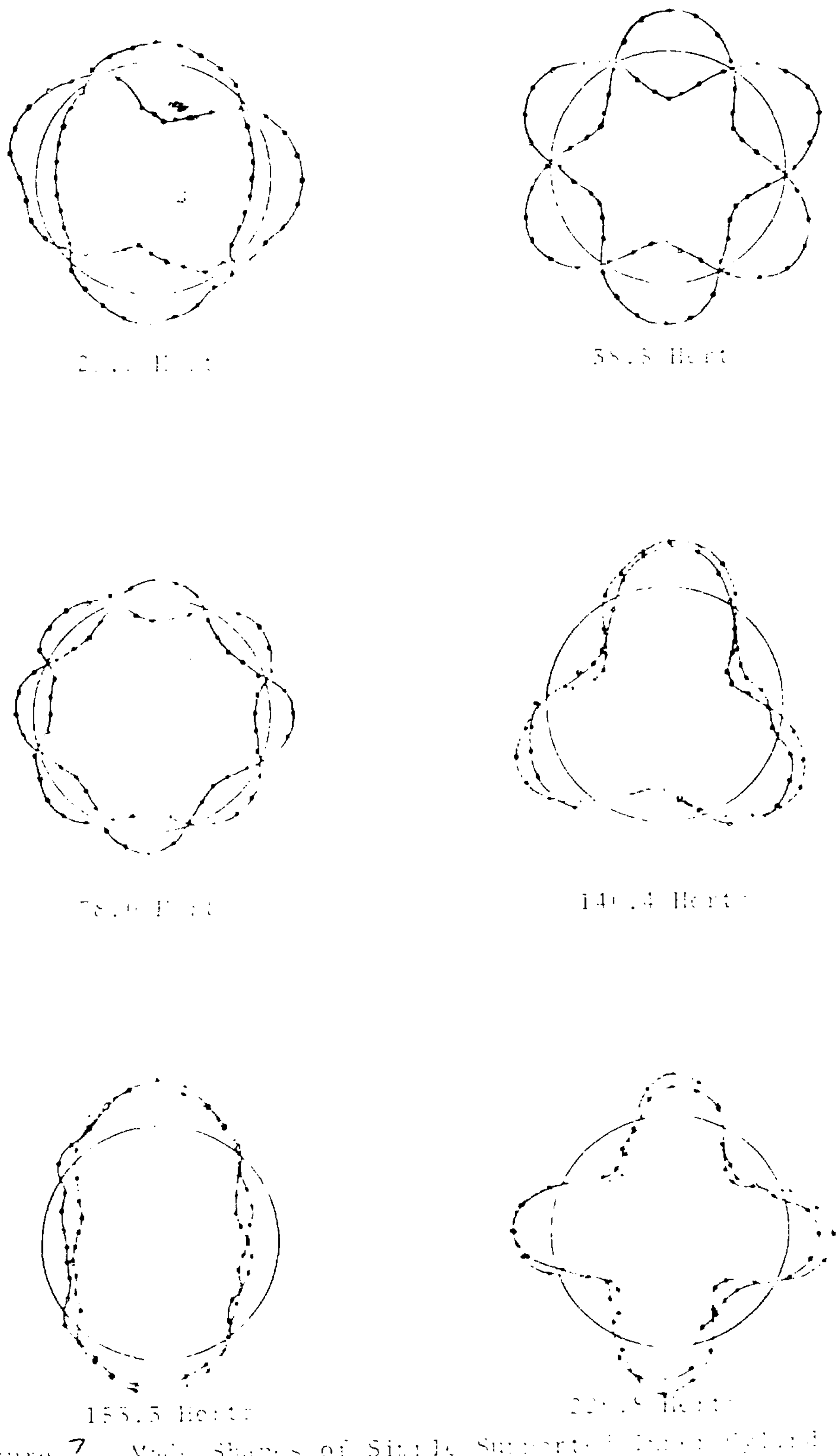


Figure 7 Mode Shapes of Single Supported [illegible]
with Outer Cylinder 10

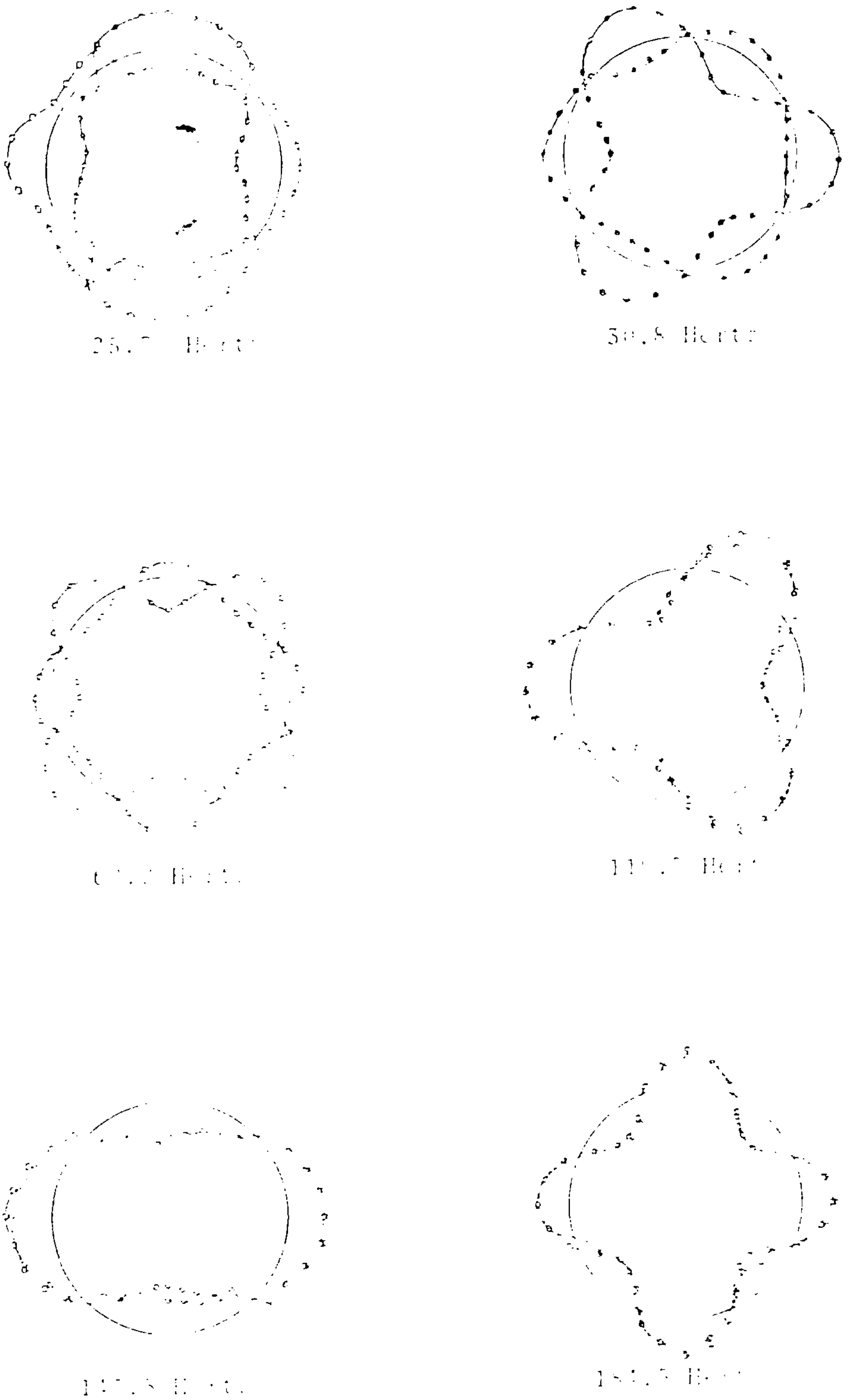


Figure 8. [illegible] Simply Supported [illegible] with Outer Cylinder [illegible]

- RESONANT FREQUENCIES FOR SIMPLE SUPPORTED INNER CYLINDERS IN AIR

Cylinder	Frequency (Hertz)			
1	198 (3)	365 (4)	550 (5)	
2	160 (3)	194 (2)	265 (4)	373 (5)
3	132 (3)	183 (4)	399 (4)	
4	200 (3)	335 (4)	416 (5)	
5	168 (3)	208 (2)	263 (4)	377 (5)
6	129 (3)	174 (4)	210 (5)	
7	196 (3)	312 (4)	420 (5)	486 (5)
8	162 (3)	212 (2)	246 (4)	358 (5)
9	128 (3)	159 (4)	207 (5)	243 (5)

FIG. 9

- RESONANT FREQUENCIES FOR PENDULUM MODE OF INNER CYLINDERS IN AIR

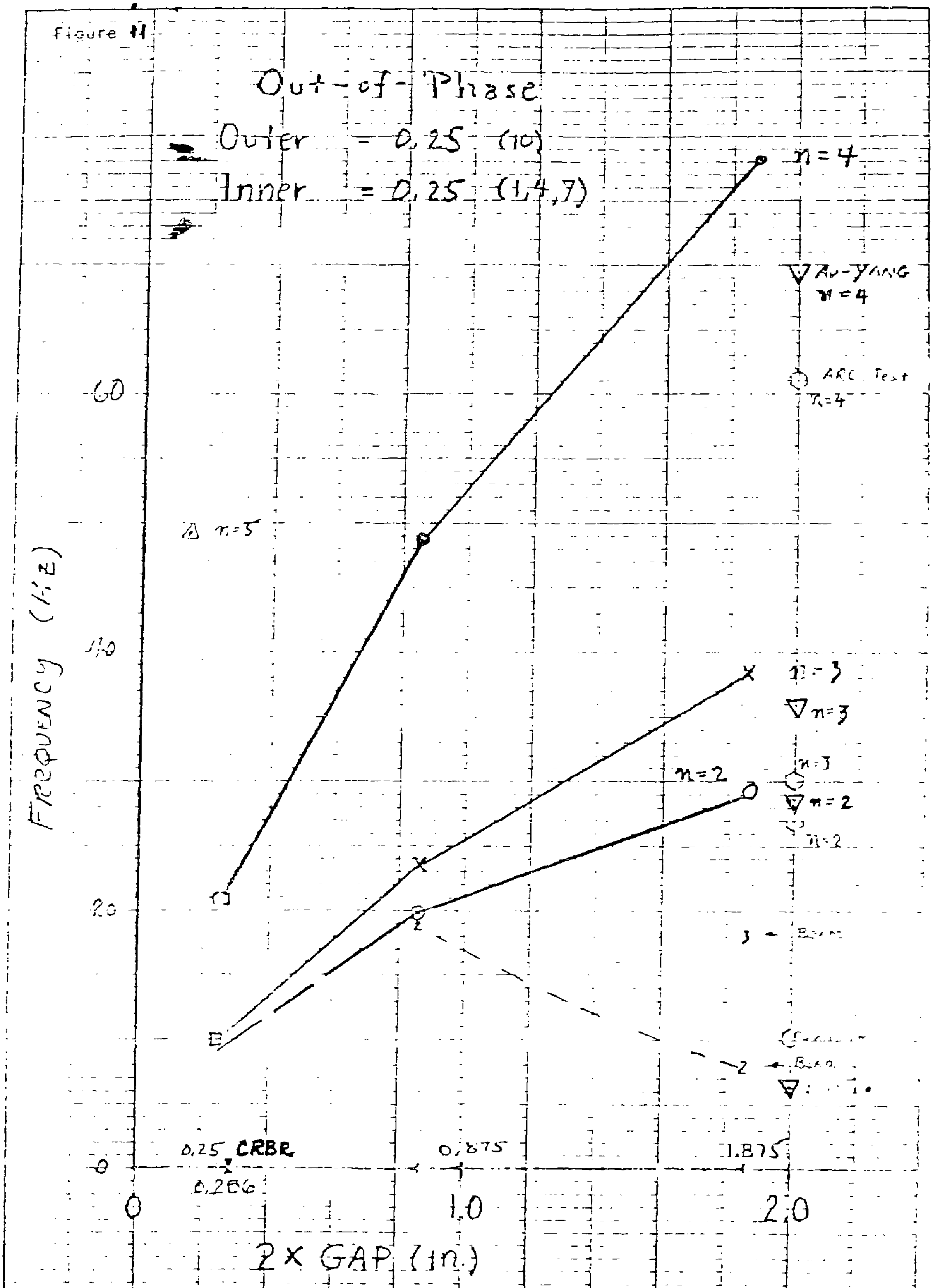
Cylinder	Frequency (Hertz)			
1	36.0 (1, 1)	115 (1, 1)	210 (1, 1)	228 (3,1)
2	38.2 (1, 1)	79.4 (1, 1)	174 (3, 1)	198 (2, 1)
3	40.9 (1, 1)	77 (3, 1)	148 (5, 1)	198 (5, 1)
4	27.8 (1, 1)	215 (3, 1)	344 (4, 1)	423 (3, 2)
5	27.2 (1, 1)	182 (3, 1)	218 (2, 1)	278 (4, 1)
6	29.5 (1, 1)	92 (3, 1)	146 (3, 1)	186 (4, 1)
7	21.3 (1, 1)	205 (3, 1)	227 (2, 1)	325 (4, 1)
8	21.6 (1, 1)	112 (2, 1)	173 (3, 1)	222 (2, 1)
9	21.3 (1, 1)	109 (2, 1)	119 (6, 1)	142 (3, 1)

- RESONANT FREQUENCIES FOR SIMPLE SUPPORTED OUTER CYLINDERS IN AIR

Cylinder	Frequency (Hertz)				
10	198 (3, 1)	237 (2, 1)	304 (4, 1)	446 (5, 2)	574 (6, 1)
11	157 (3, 1)	213 (4, 1)	326 (5, 2)	436 (6, 2)	836 (7, 3)
12	128 (3, 1)	167 (4, 1)	215 (5, 1)	327 (6, 3)	399 (7, 3)

FIG. 10

Figure 11



Out-of-Phase
Outer = 0.25 (10)
Inner = 0.25 (1,4,7)
n = 4
Au-Yang n = 4
ARC Test n = 4
n = 5
n = 3
n = 3
n = 3
n = 2
n = 2
n = 2
3 ← Beam
2 ← Beam
FREQUENCY (Hz)
60
40
20
0
0.25 CRBR
0.286
0.875
1.875
0
1.0
2.0
2 × GAP (in.)

46 0780

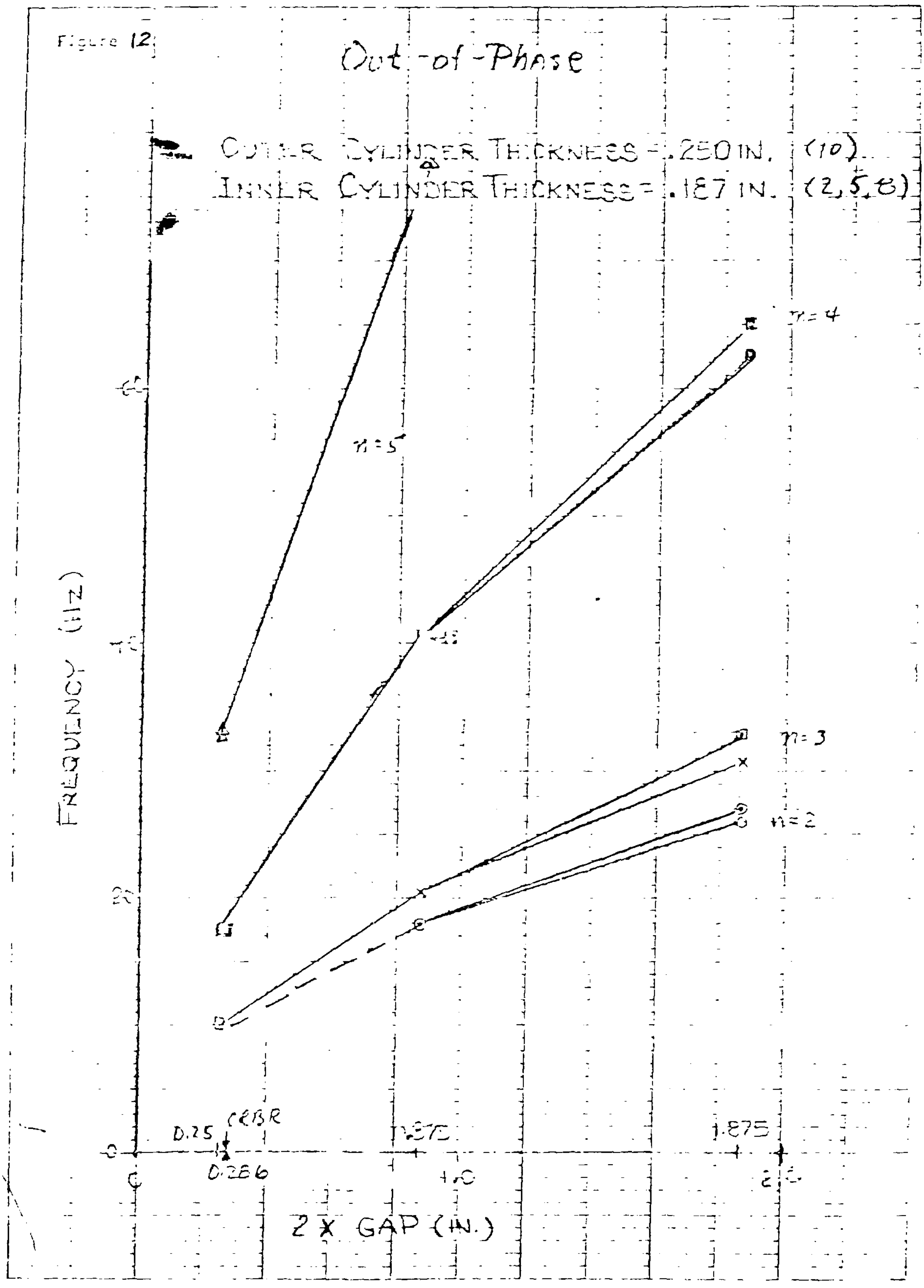
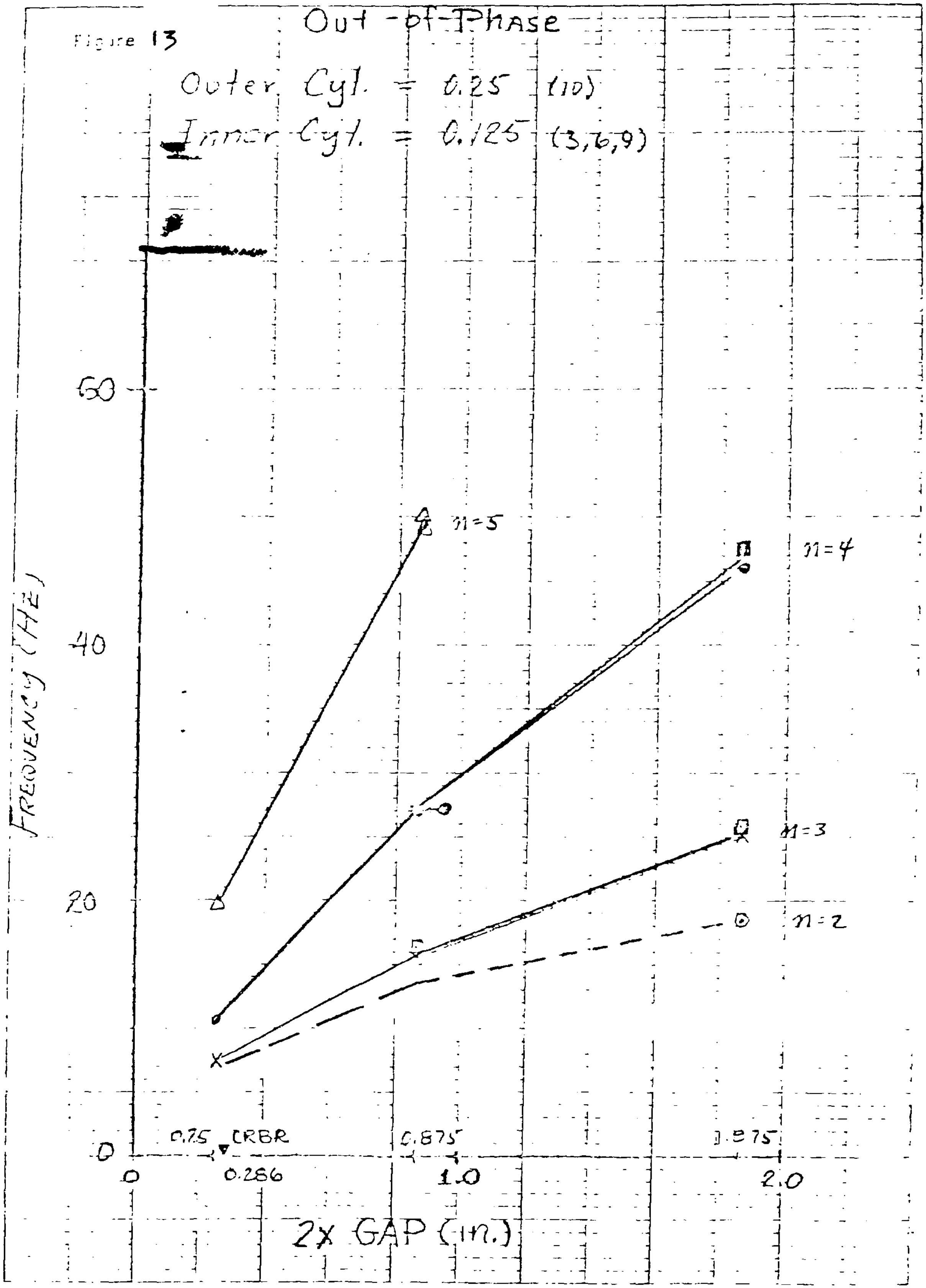


Figure 12
Out-of-Phase
OUTER CYLINDER THICKNESS = .250 IN. (10)
INNER CYLINDER THICKNESS = .187 IN. (2,5,8)
n = 5
n = 4
n = 3
n = 2
FREQUENCY (HZ)
60
40
20
0
0.25
CEBR
0.286
.875
1.875
0
1.0
2.0
2 X GAP (IN.)

46 0780

Figure 13



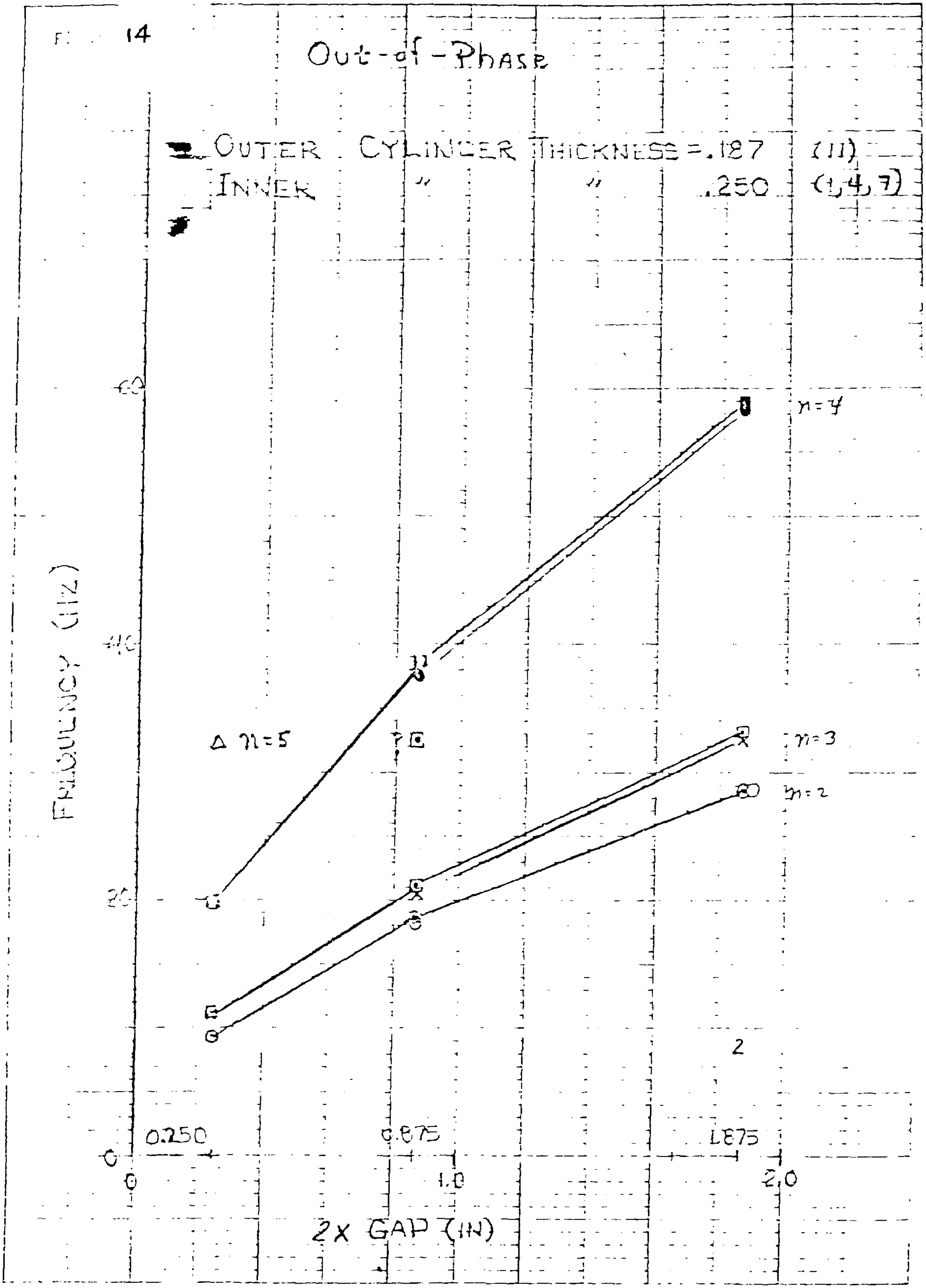
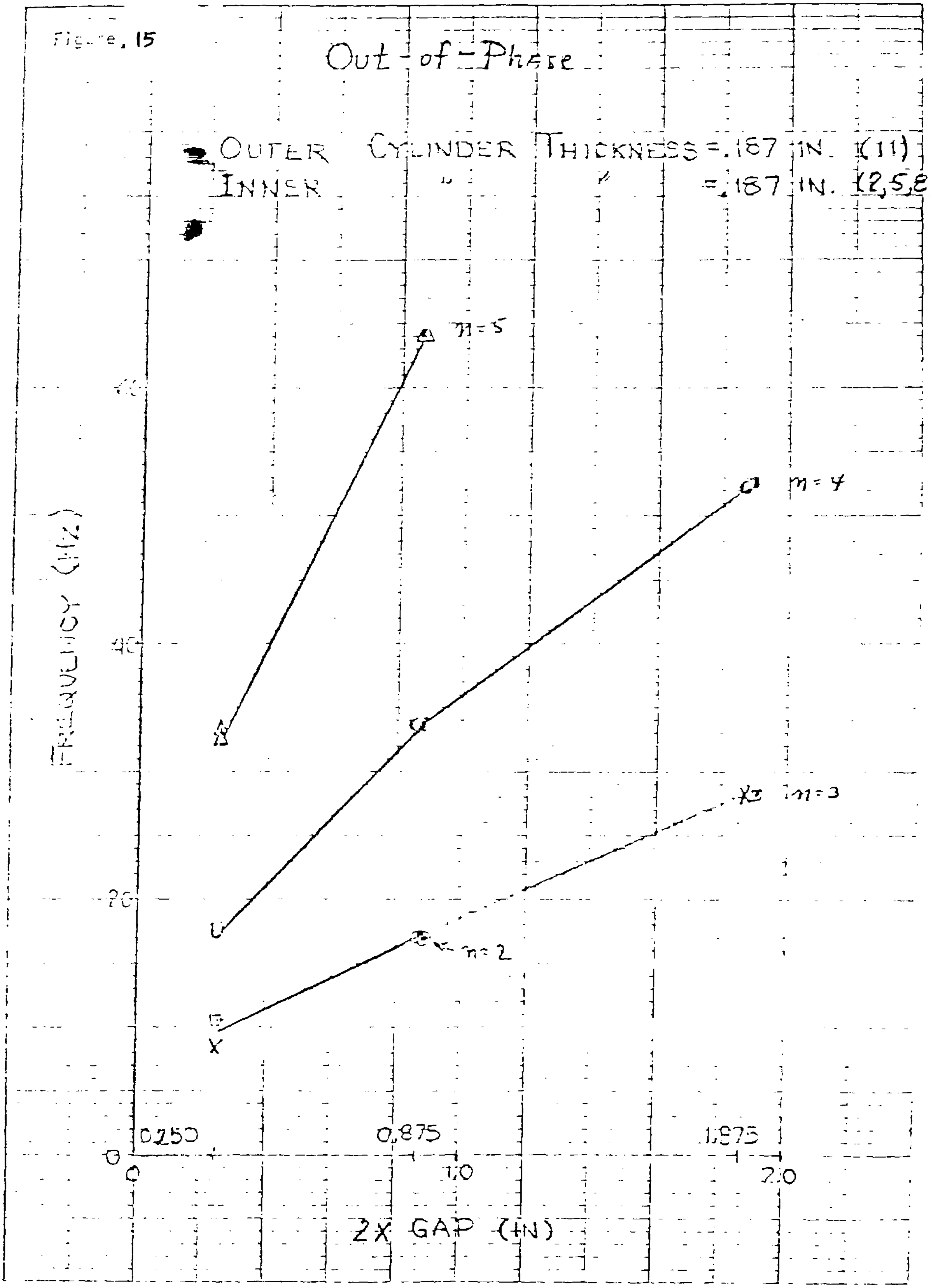
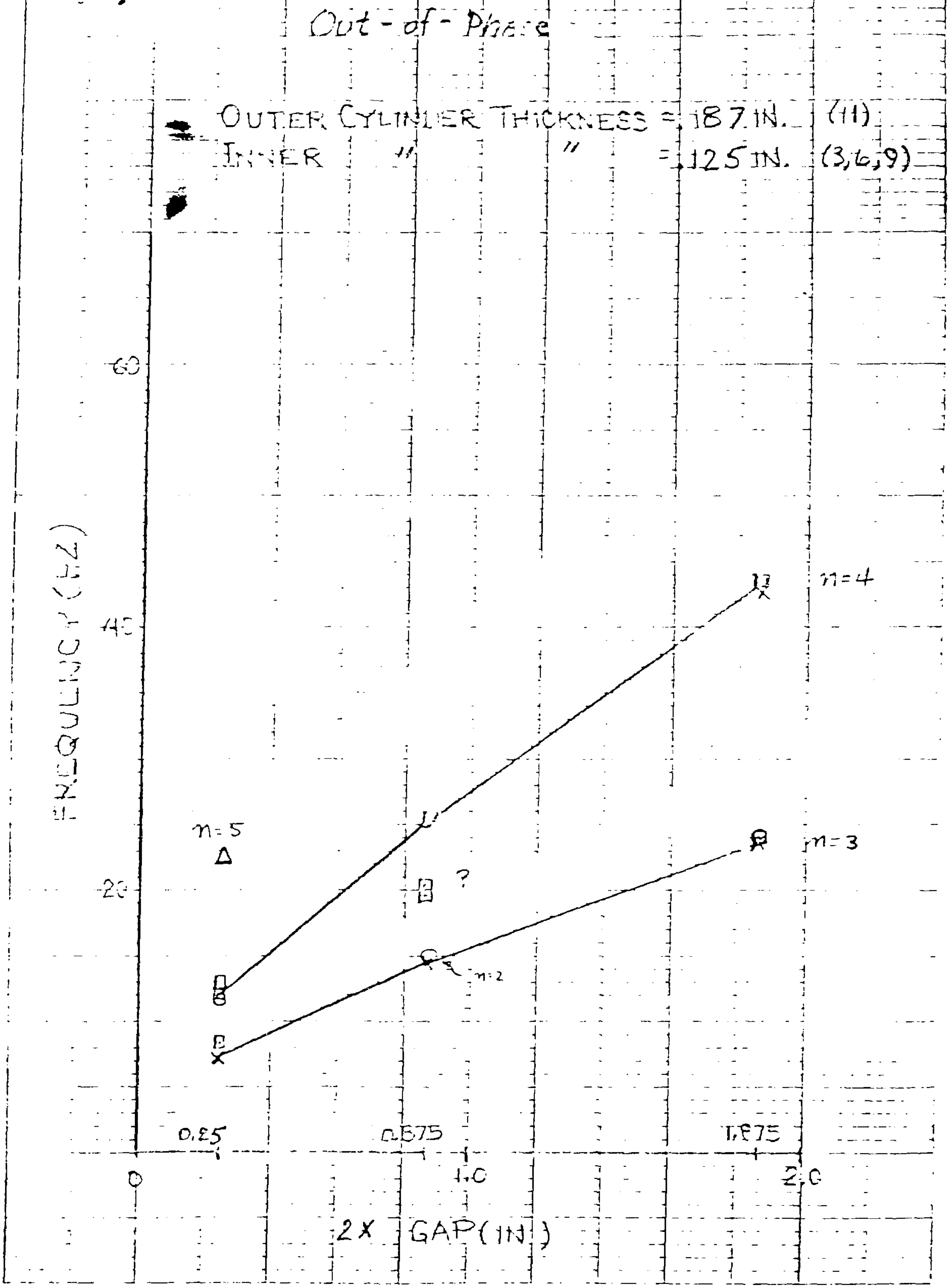

F. 14
Out-of-Phase
OUTER CYLINDER THICKNESS = .187 (11)
INNER " " .250 (1,4,7)
n=4
Δ n=5
n=3
n=2
2
FREQUENCY (HZ)
60
40
20
0
0.250
0.875
1.875
0
1.0
2.0
2X GAP (IN)

Figure. 15



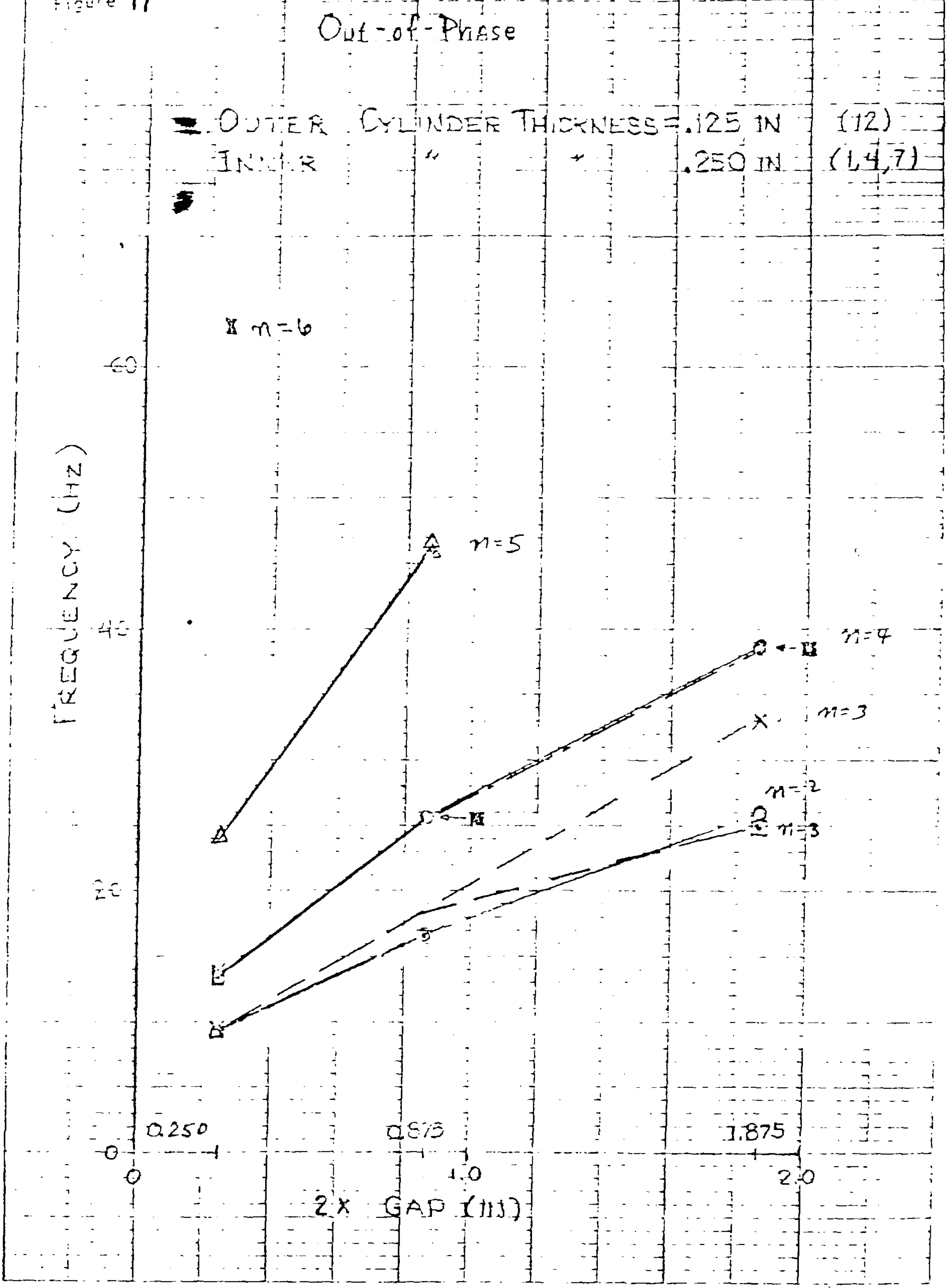
Out-of-Phase
OUTER CYLINDER THICKNESS = .187 IN. (11)
INNER " " = .187 IN. (2,5,8)
n=5
n=4
n=3
n=2
FREQUENCY (HZ)
20
40
0
0.250
0.875
1.875
0
1.0
2.0
2X GAP (IN)

Figure, 16



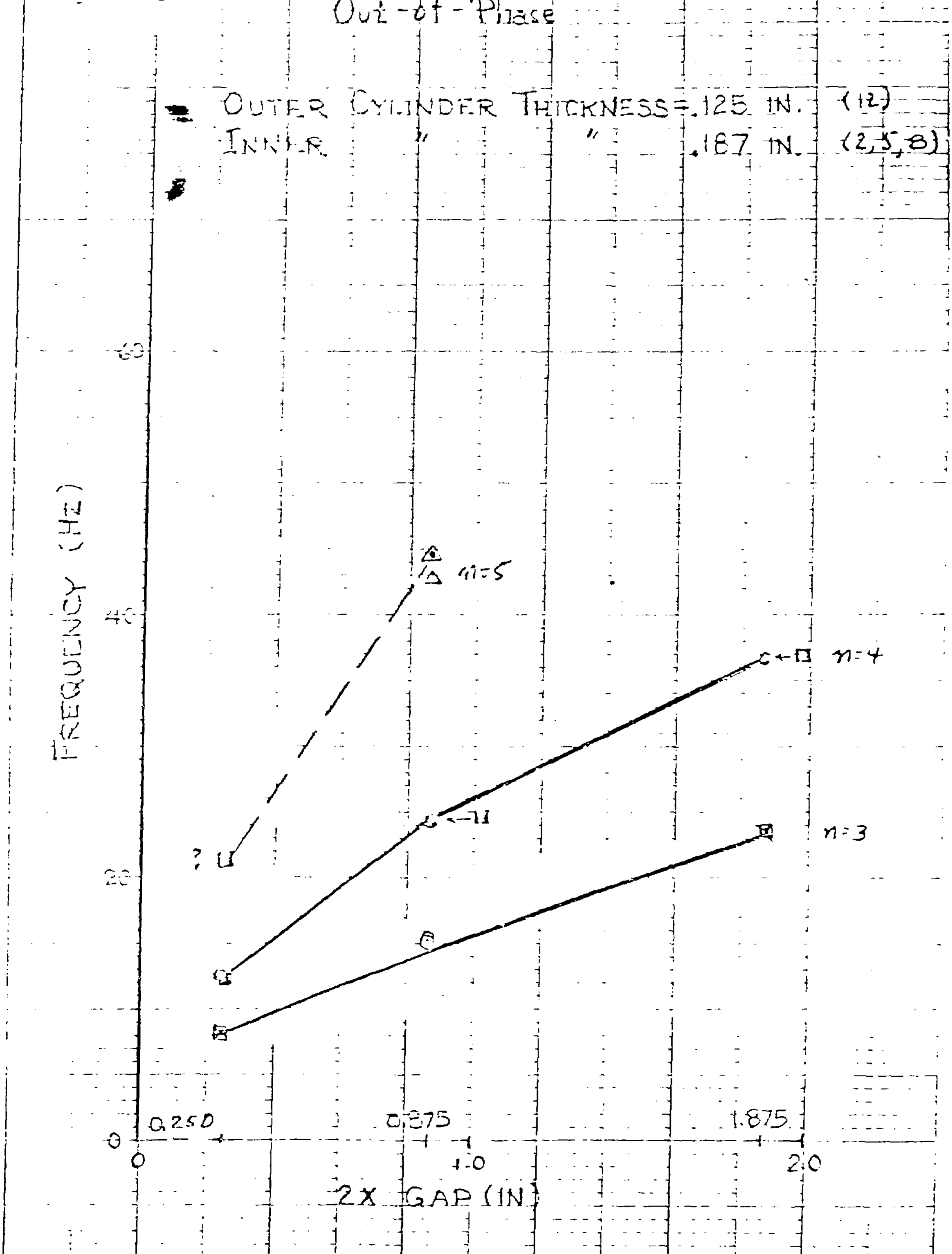
Out-of-Phase
OUTER CYLINDER THICKNESS = .187 IN. (11)
INNER " " = .125 IN. (3,6,9)
FREQUENCY (HZ)
60
40
20
n=4
n=5
n=3
?
n=2
0.25
0.875
1.875
0
1.0
2.0
2X GAP(IN.)

Figure 17



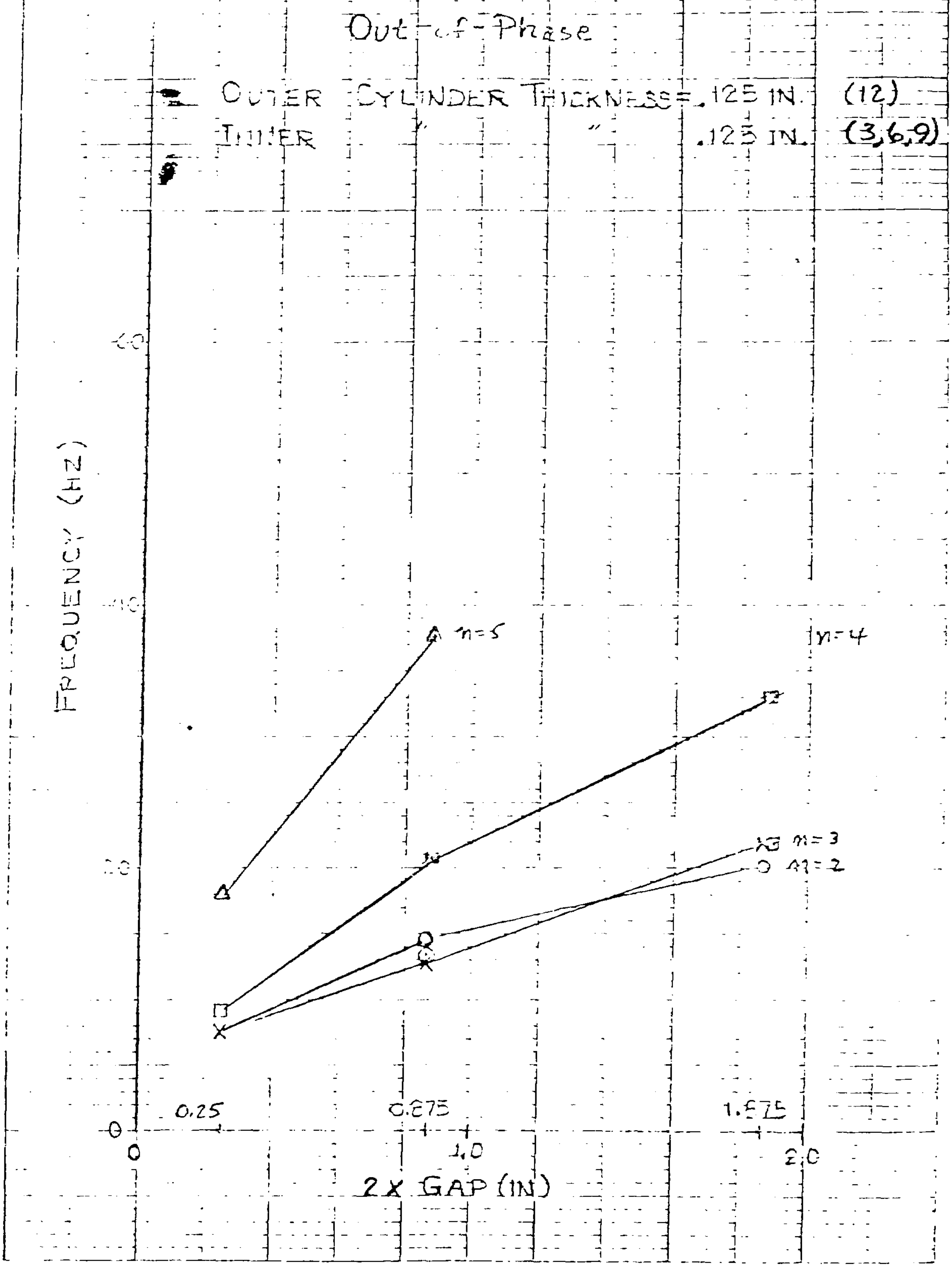
Out-of-Phase
OUTER CYLINDER THICKNESS = .125 IN (12)
INNER " " .250 IN (1,4,7)
n=6
n=5
n=4
n=3
n=2
n=3
FREQUENCY (HZ)
60
40
20
0
0.250
0.875
1.875
0
1.0
2.0
2 x GAP (IN)

Figure 18



Out-of-Phase
OUTER CYLINDER THICKNESS = .125 IN. (12)
INNER " " .187 IN. (2,5,8)
FREQUENCY (HZ)
60
40
20
0
n=5
n=4
n=3
?
0.250
0.875
1.875
0
1.0
2.0
2X GAP (IN)

Figure, 19



Out-of-Phase
OUTER CYLINDER THICKNESS = .125 IN. (12)
INNER " " .125 IN. (3,6,9)
FREQUENCY (HZ)
60
40
20
0
n=5
n=4
n=3
n=2
0.25
0.875
1.875
0
1.0
2.0
2 X GAP (IN)

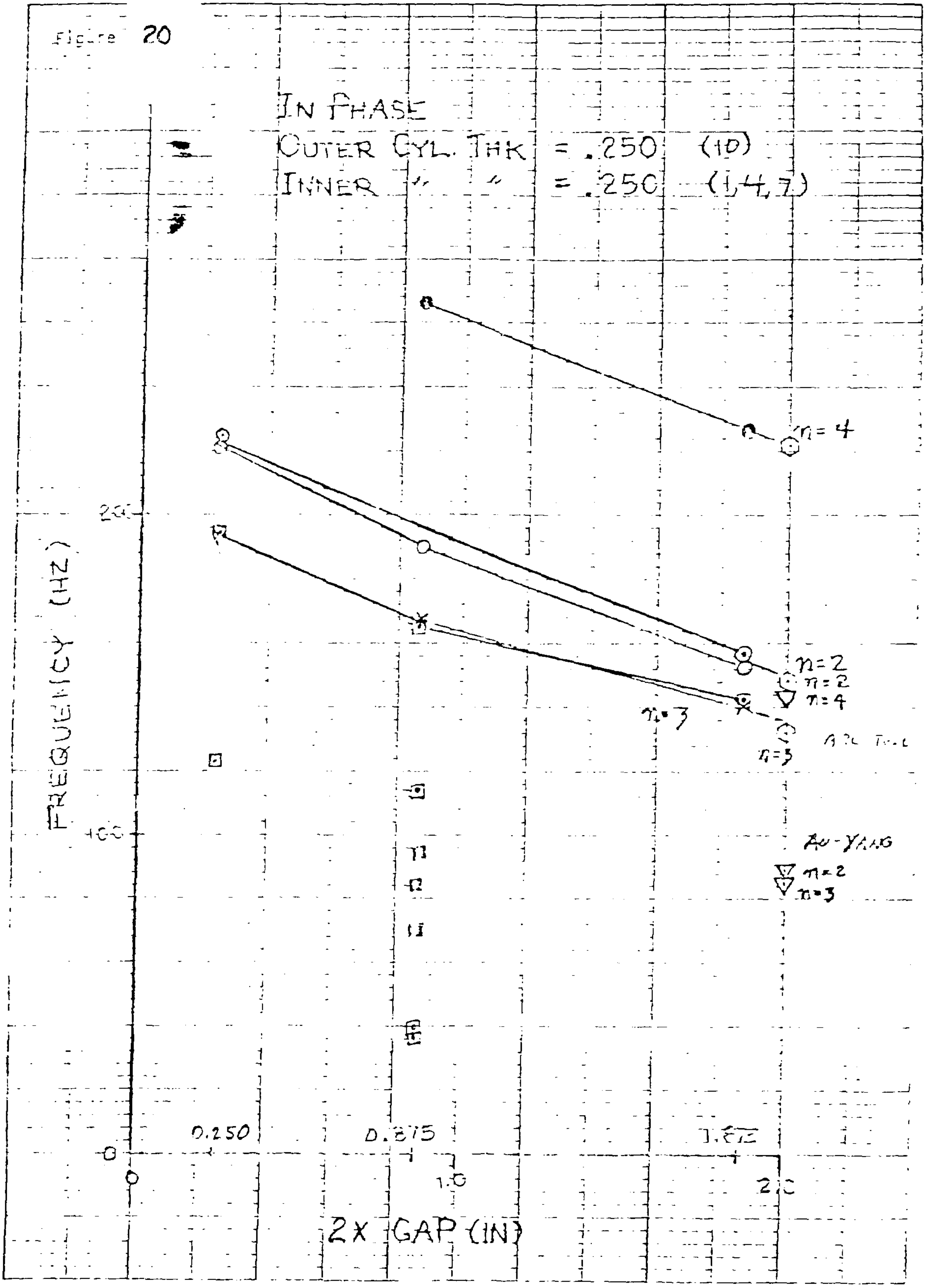
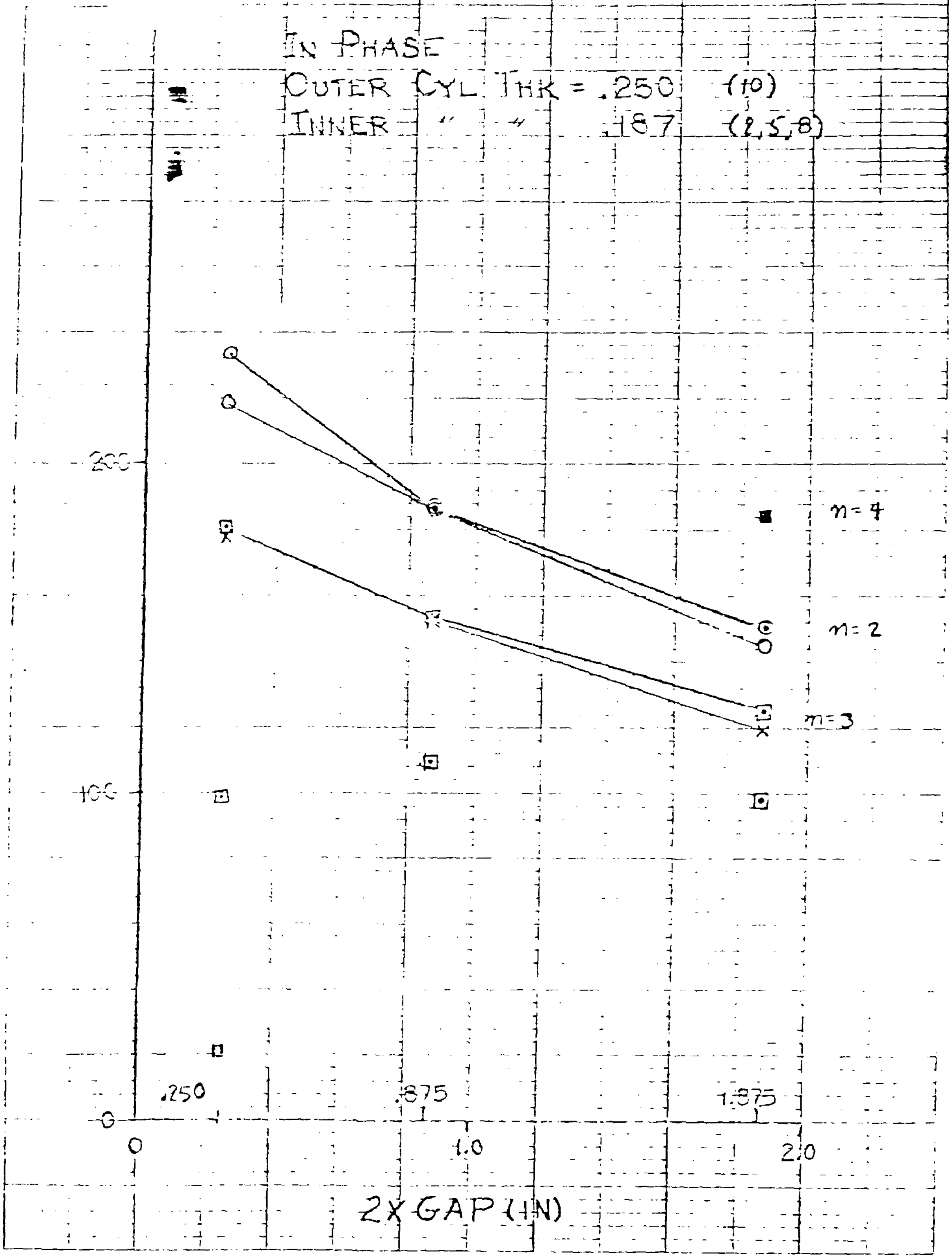


Figure 20
IN PHASE
OUTER CYL. THK = .250 (10)
INNER " " = .250 (1,4,7)
FREQUENCY (HZ)
200
100
0
n = 4
n = 2
n = 2
n = 4
n = 3
n = 3
AU-YANG
n = 2
n = 3
0.250
0.875
1.0
2.0
2X GAP (IN)

Figure. 21


IN PHASE
OUTER CYL THK = .250 (10)
INNER " " .187 (2,5,8)
n = 4
n = 2
n = 3
200
100
0
.250
.875
1.875
0
1.0
2.0
2X GAP (IN)

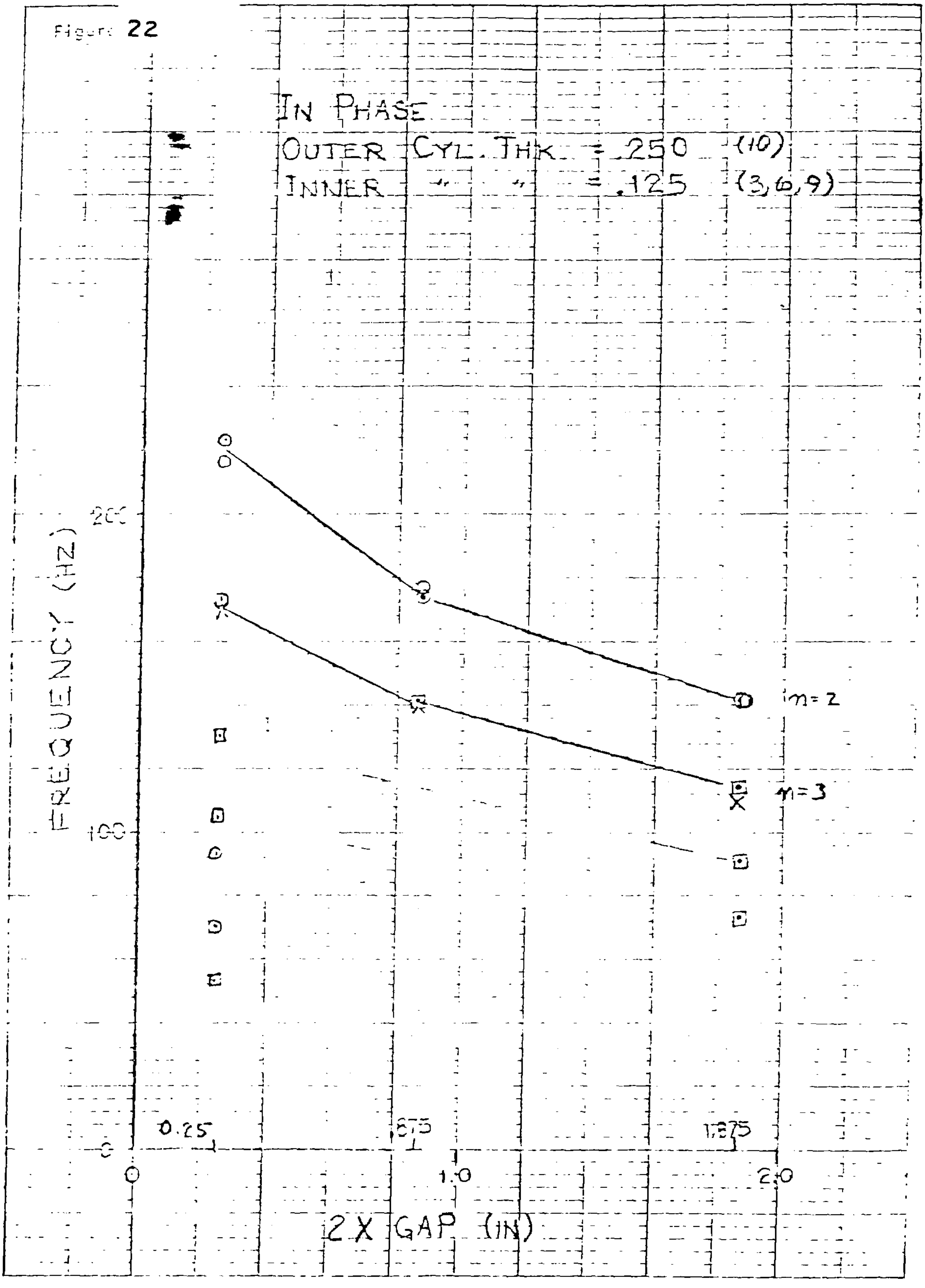


Figure 22
IN PHASE
OUTER CYL. THK. = .250 (10)
INNER " " = .125 (3,6,9)
FREQUENCY (HZ)
200
100
0
0.25
.875
1.875
0
1.0
2.0
2 X GAP (IN)
n=2
n=3

Figure 23

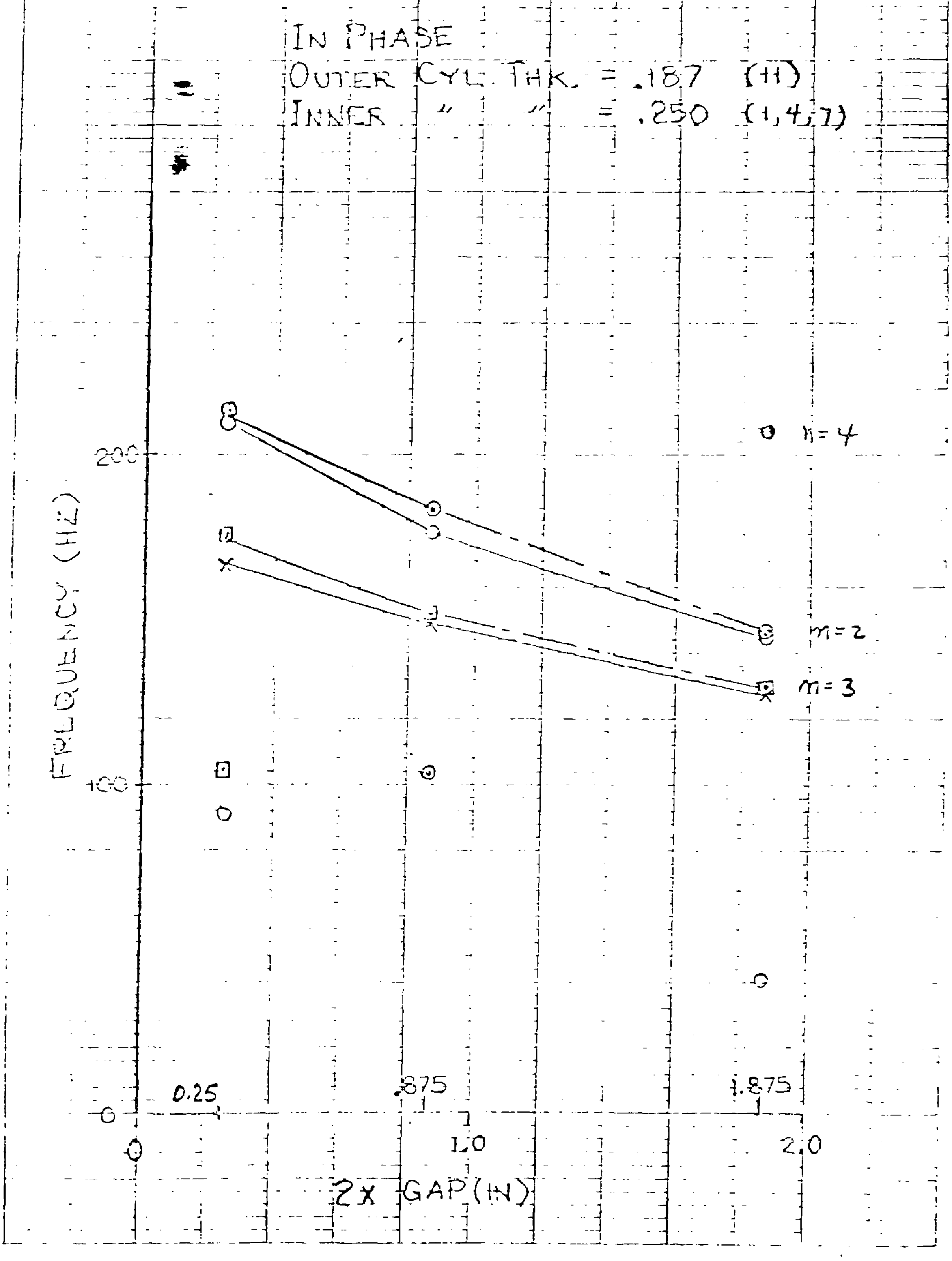
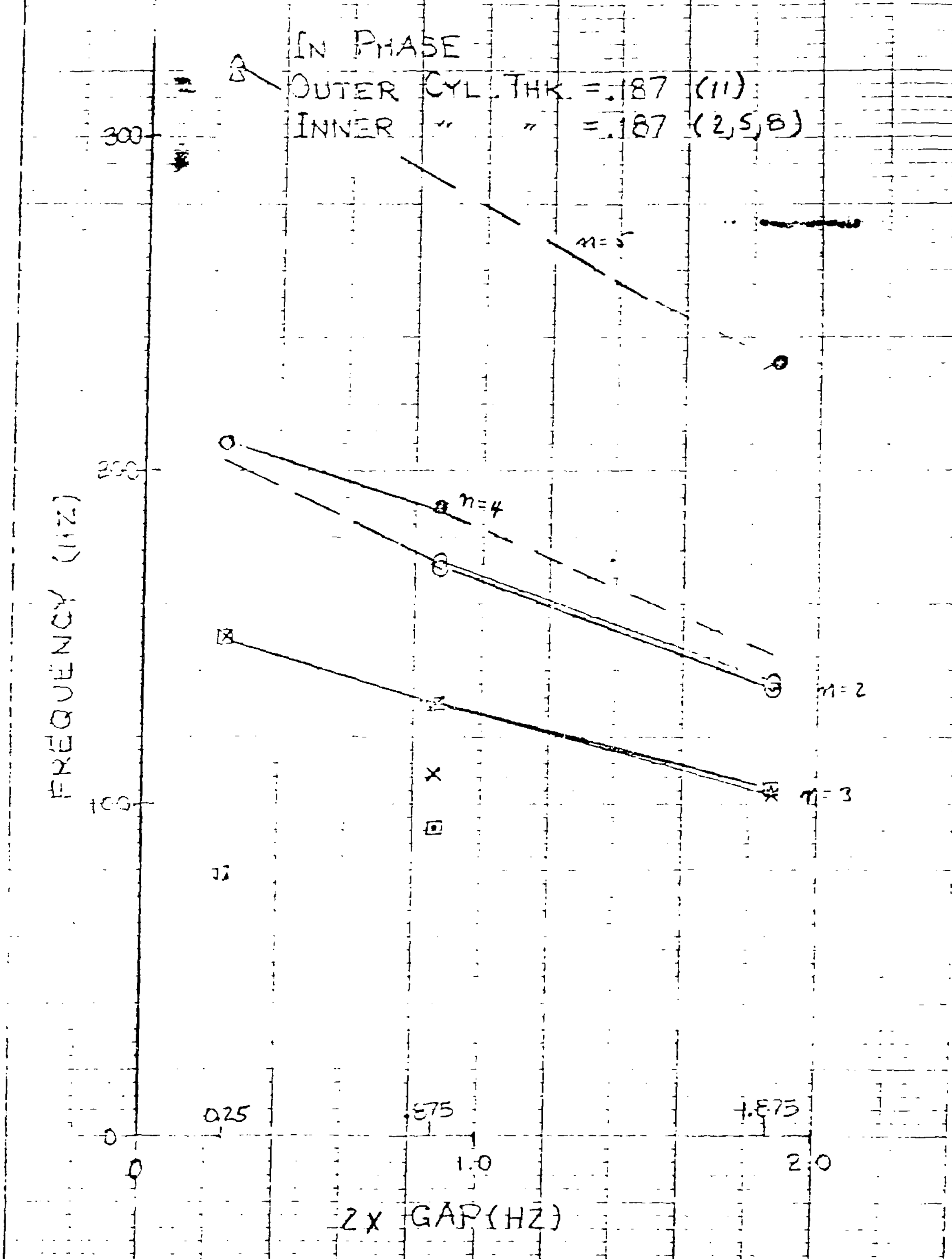
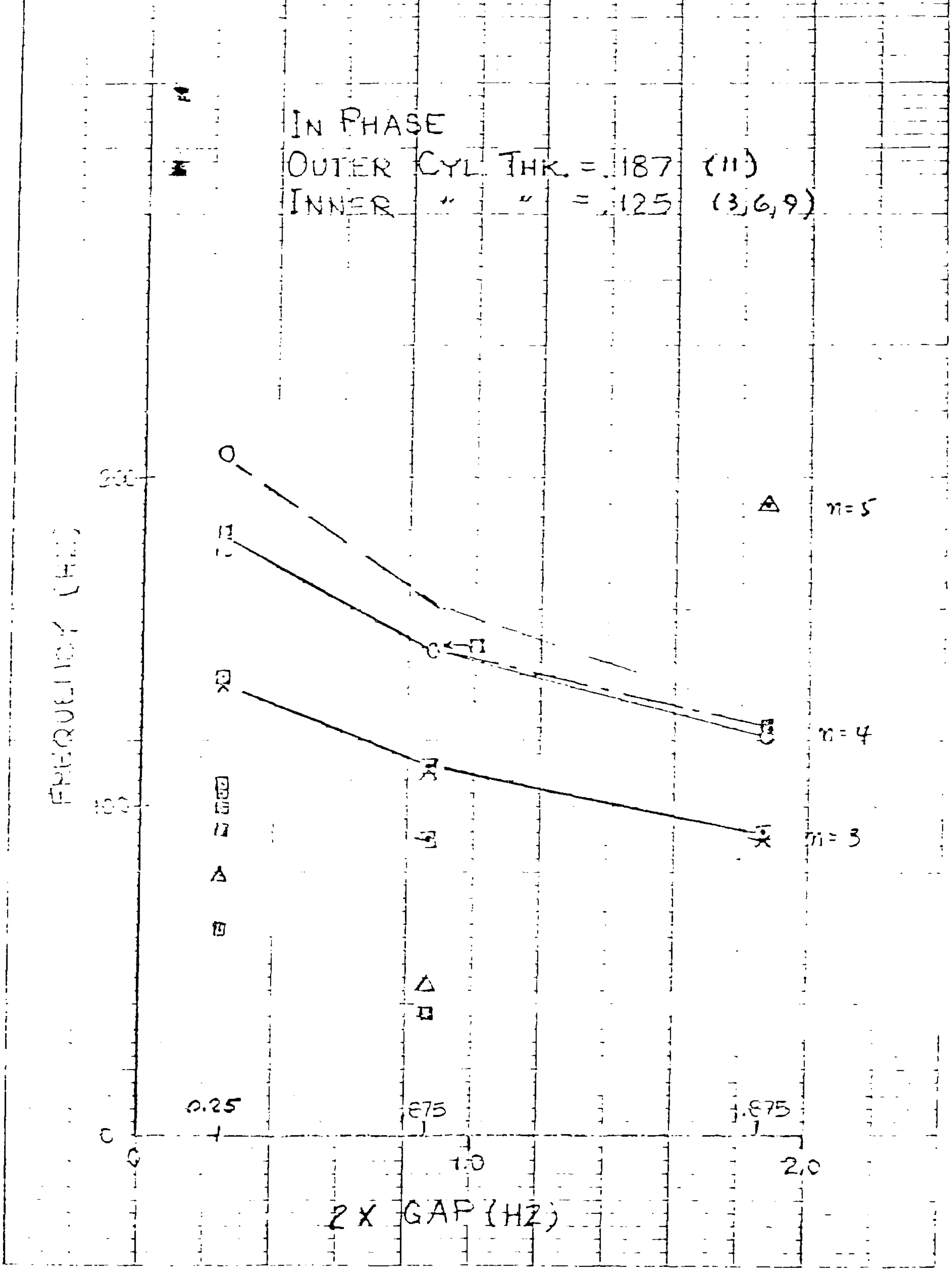

IN PHASE
OUTER CYL. THK. = .187 (H)
INNER " " = .250 (1,4,7)
n=4
m=2
m=3
200
100
0
0.25
.875
1.875
0
1.0
2.0
FREQUENCY (HZ)
2X GAP (IN)

Figure 24



IN PHASE
OUTER CYL. THK. = .187 (11)
INNER " " = .187 (2,5,8)
n=5
n=4
n=2
n=3
300
200
100
0
FREQUENCY (HZ)
0.25
.875
1.875
0
1.0
2.0
2X GAP (HZ)

Figure 25



IN PHASE
OUTER CYL. THK. = .187 (11)
INNER " " = .125 (3, 6, 9)
FREQUENCY
200
100
0
n = 5
n = 4
n = 3
0.25
.875
1.875
0
1.0
2.0
2 X GAP (HZ)

Figure 26

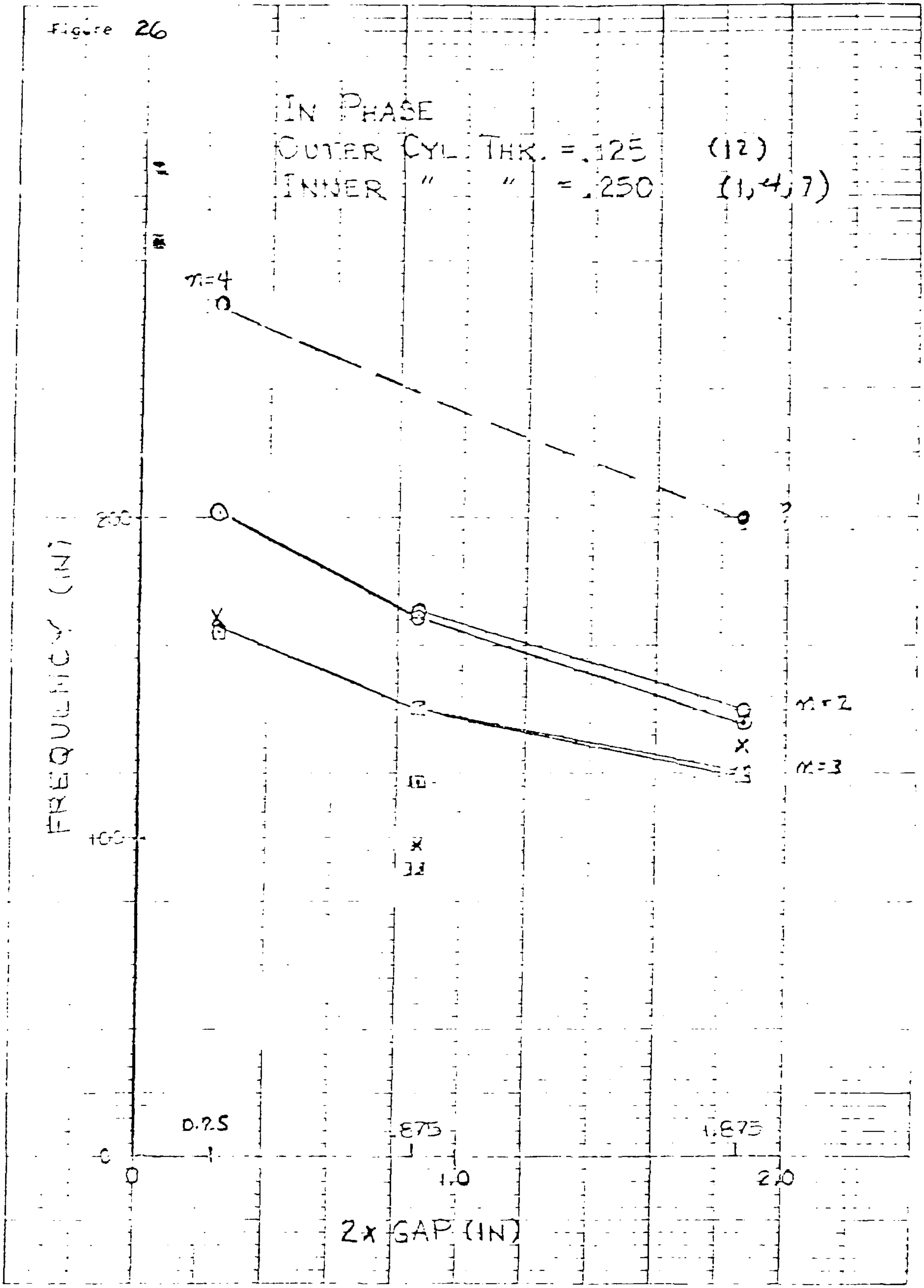

IN PHASE
OUTER CYL. THK. = .125 (12)
INNER " " = .250 (1,4,7)
n=4
n=2
n=3
FREQUENCY (HZ)
200
100
0
0.25
.875
1.875
0
1.0
2.0
2x GAP (IN)

Figure 27

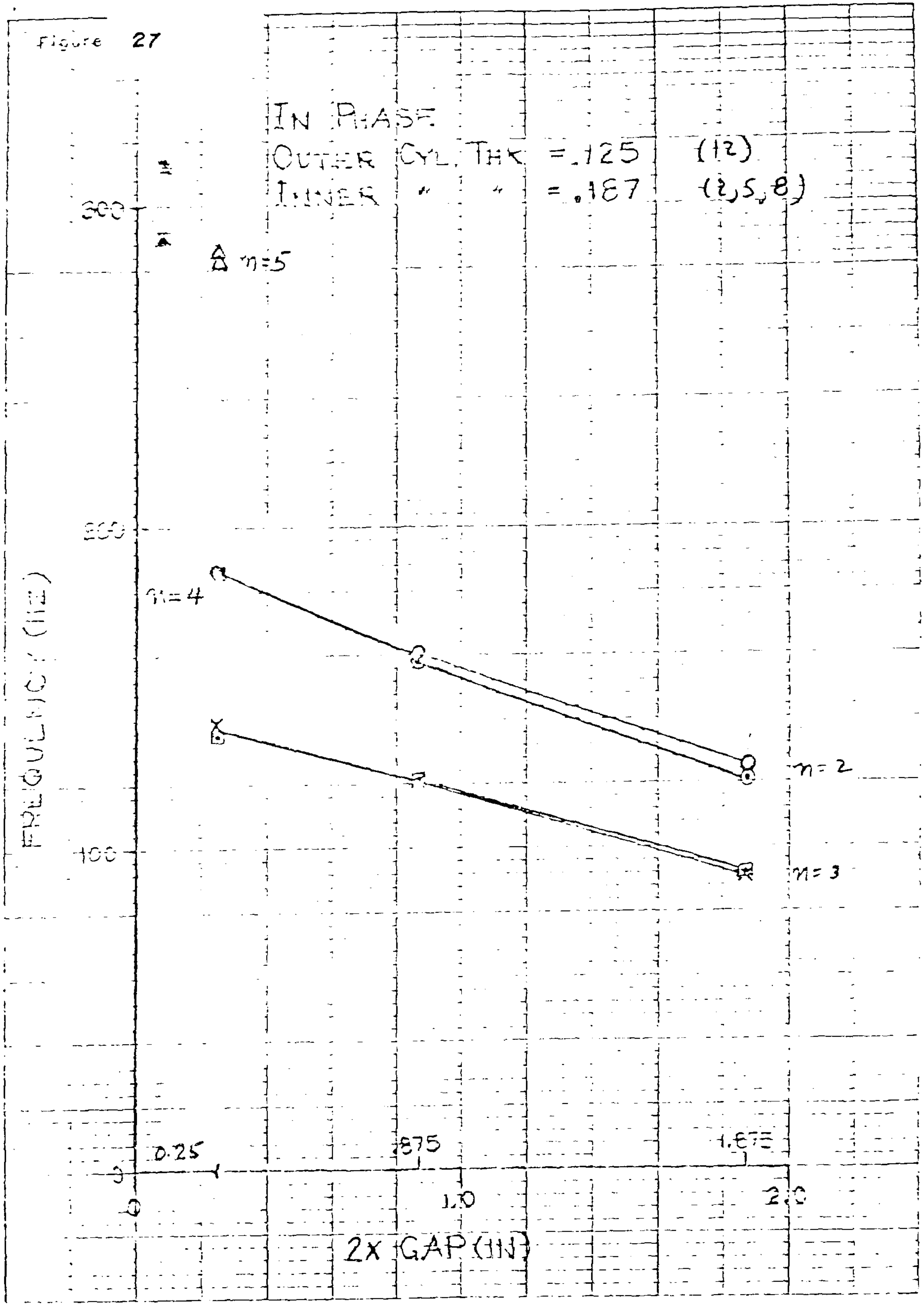
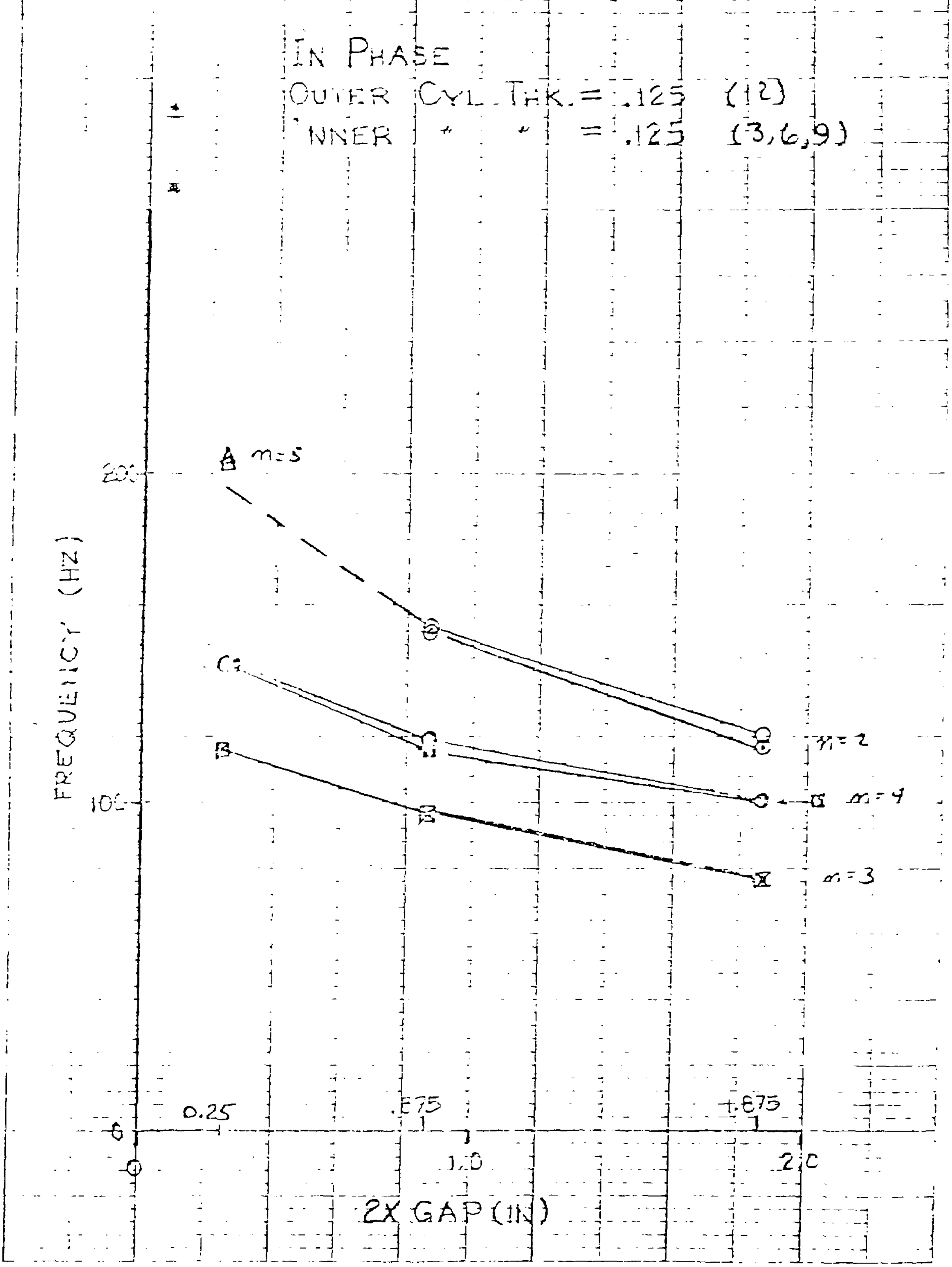
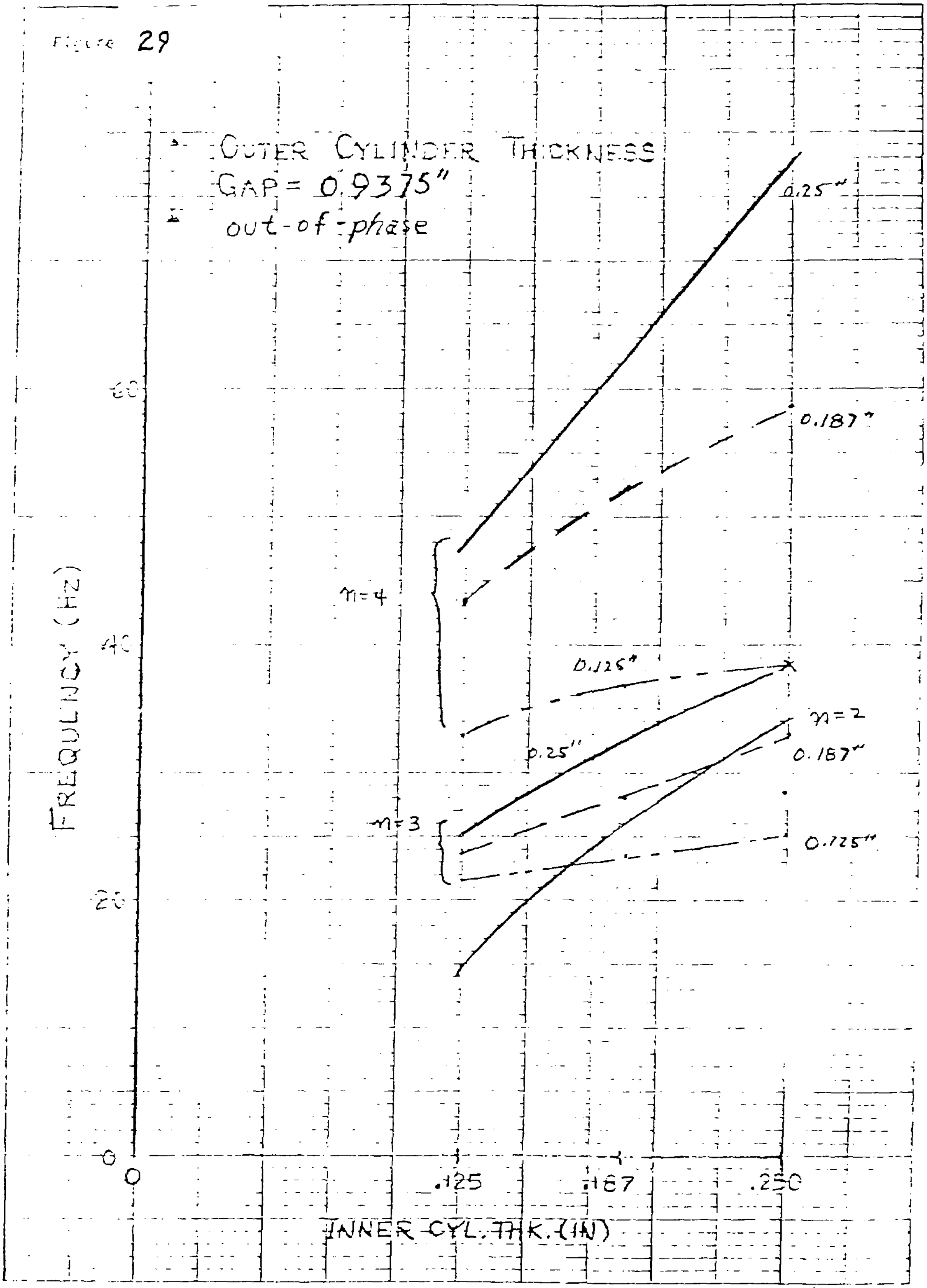

IN PHASE
OUTER CYL. THK = .125 (12)
INNER " " = .187 (2,5,8)
n=5
n=4
n=2
n=3
FREQUENCY (HZ)
300
200
100
0
0.25
.875
1.675
1.0
2.0
2X GAP (IN)

Figure 28



IN PHASE
OUTER CYL THK. = .125 (12)
INNER " " = .125 (3,6,9)
m=5
m=2
m=4
m=3
200
100
0
FREQUENCY (HZ)
0.25
.875
1.875
1.0
2.0
2X GAP (IN)

Figure 29


OUTER CYLINDER THICKNESS
GAP = 0.9375"
out-of-phase
FREQUENCY (Hz)
INNER CYL. THK. (IN)
0
20
40
60
0
.125
.187
.250
0.25"
0.187"
0.125"
0.25"
0.187"
0.125"
n=4
n=3
n=2

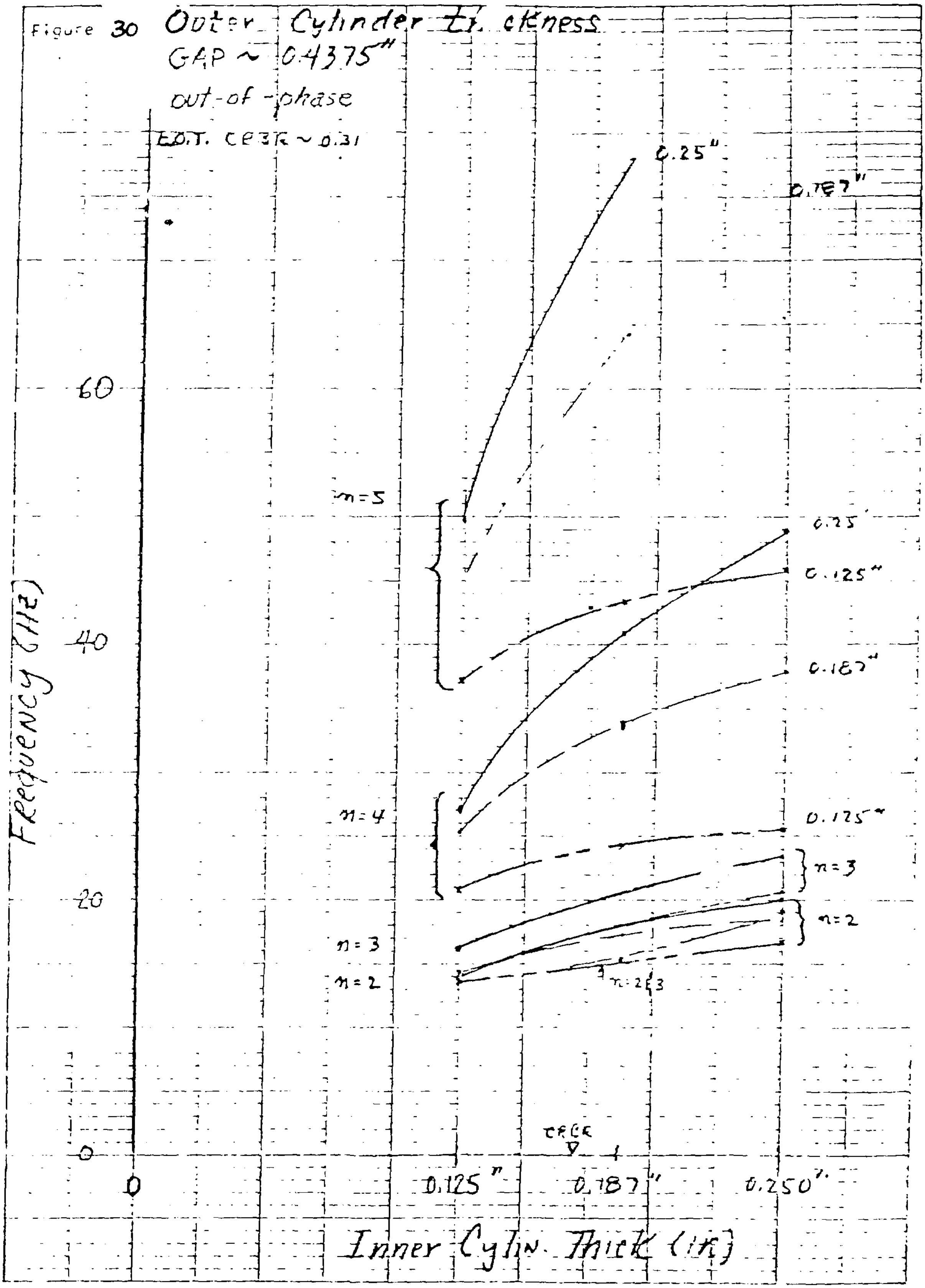
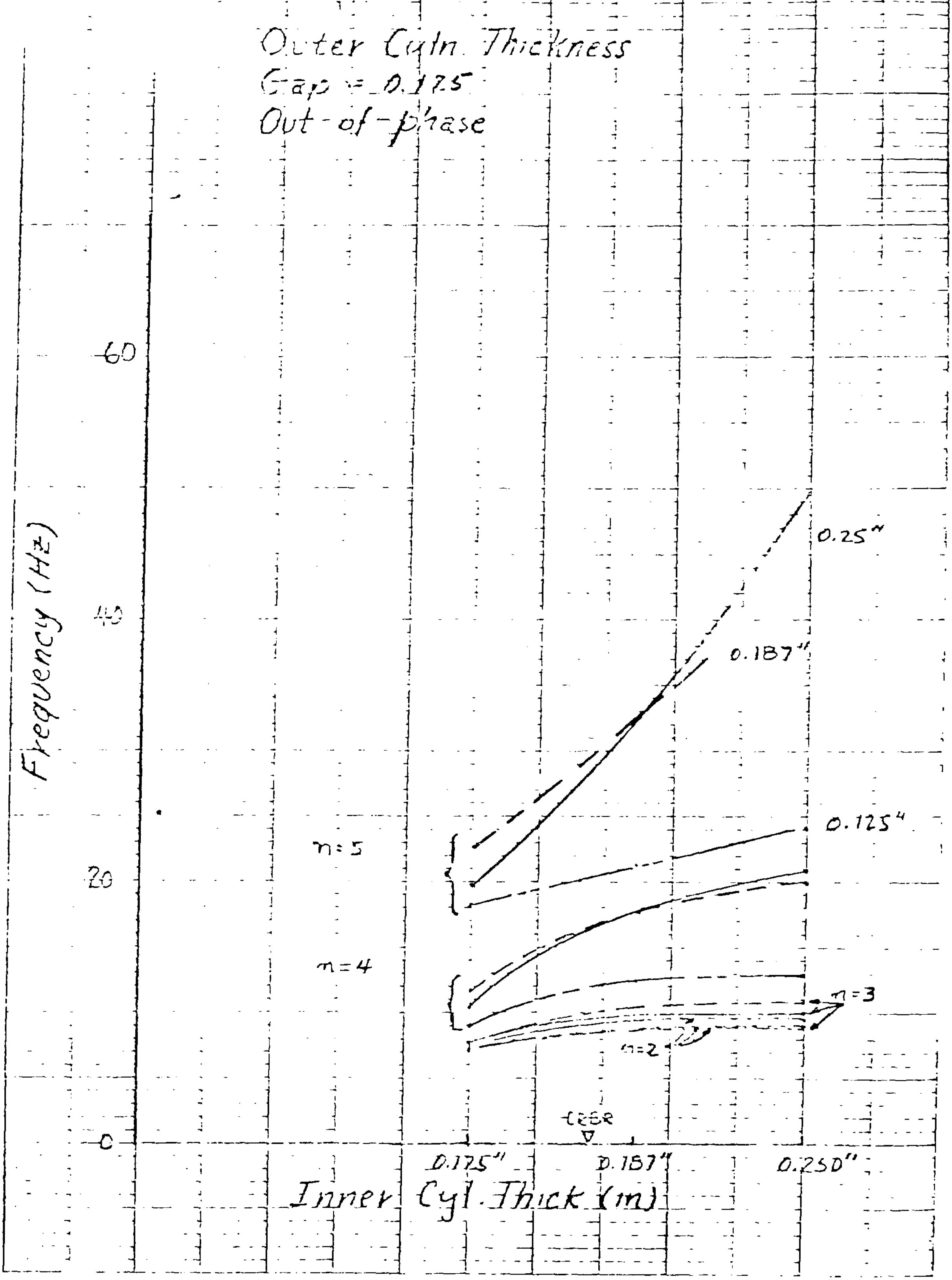

Outer Cylinder Thickness
GAP ~ 0.4375"
out-of-phase
60
40
20
0
Frequency (Hz)
n=5
n=4
n=3
n=2
0.25"
0.187"
0.25
0.125"
0.187"
0.125"
n=3
n=2
0
0.125"
0.187"
0.250"
Inner Cylin. Thick (in)

Figure 30

Figure 31



Outer Cyln. Thickness
Gap = 0.175
Out-of-phase
Frequency (Hz)
60
40
20
0
0.25"
0.187"
0.125"
n=5
n=4
n=3
n=2
0.175"
0.187"
0.250"
Inner Cyl. Thick (in)

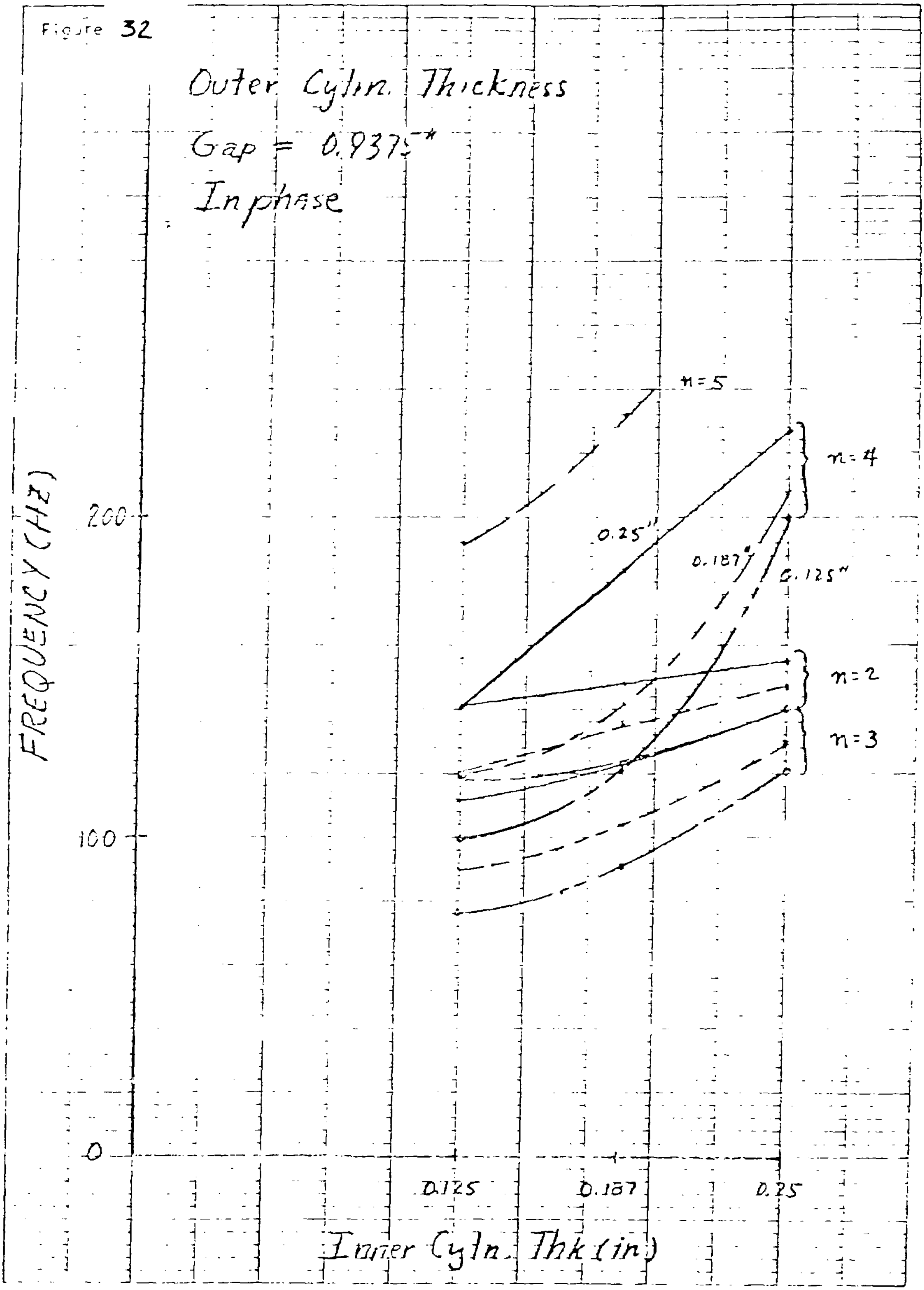


Figure 32
Outer Cylin. Thickness
Gap = 0.9375"
Inphase
FREQUENCY (HZ)
200
100
0
n=5
n=4
n=2
n=3
0.25"
0.187"
0.125"
0.125
0.187
0.25
Inner Cyln. Thk (in)

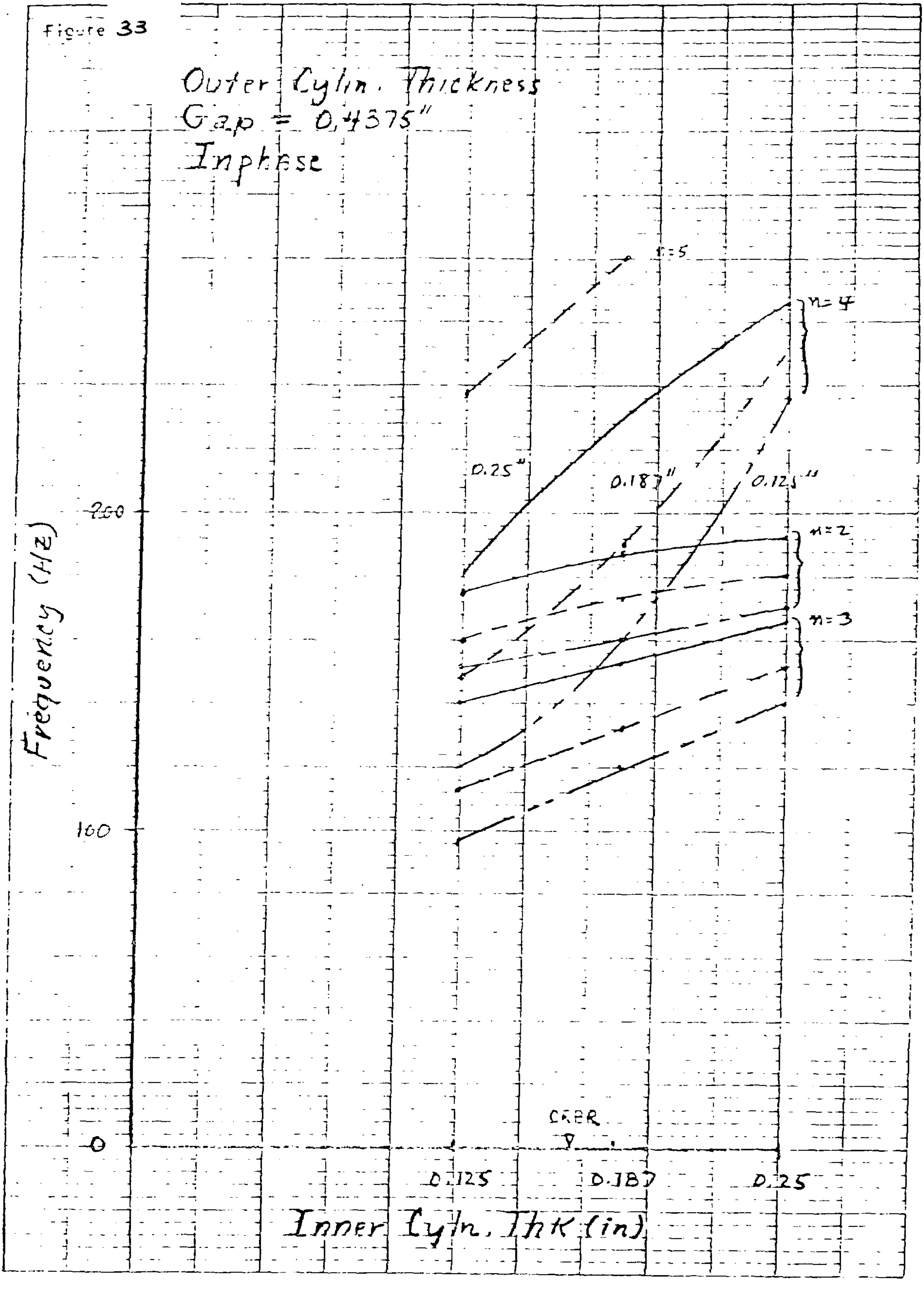
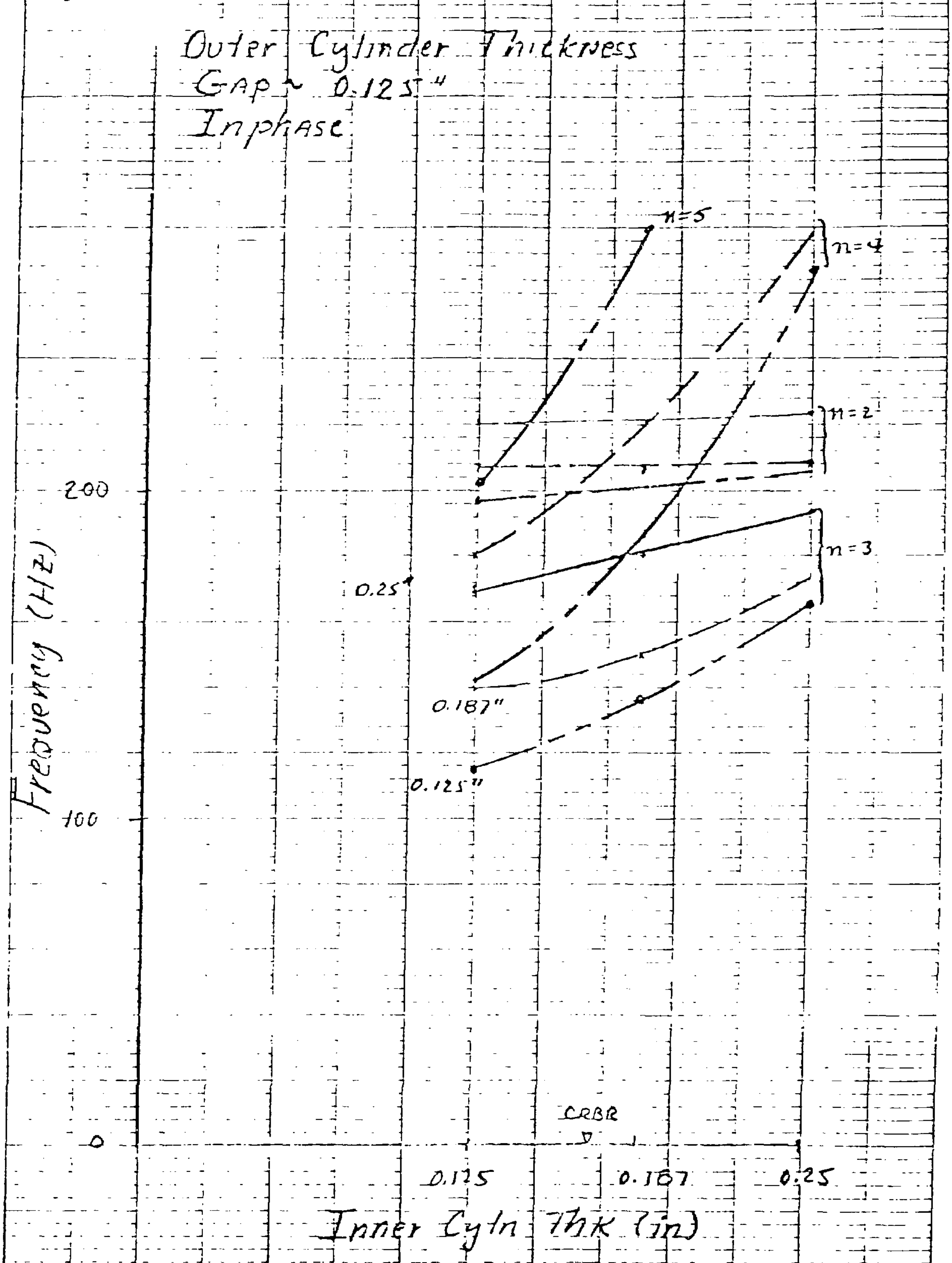


Figure 33
Outer Cylin. Thickness
Gap = 0.4375"
Inphase
n=5
n=4
0.25"
0.187"
0.125"
n=2
n=3
200
100
0
Frequency (Hz)
CRBR
0.125
0.187
0.25
Inner Cyl. Thk (in)

Figure 34



Outer Cylinder Thickness
GAP ~ 0.125"
Inphase
n=5
n=4
n=2
n=3
0.25"
0.187"
0.125"
200
100
0
Frequency (Hz)
CRBR
0.125
0.187
0.25
Inner Cyln Thk (in)

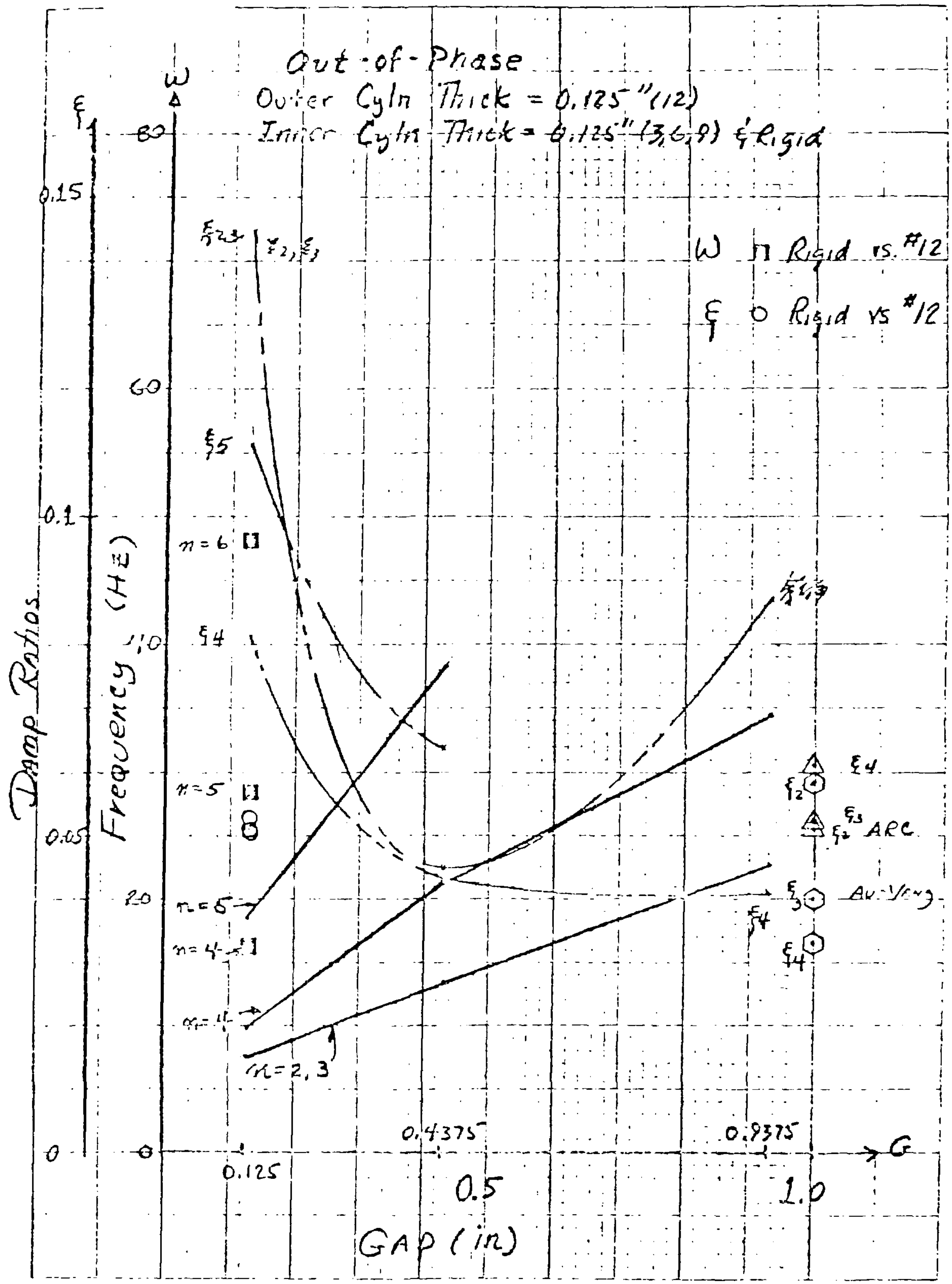
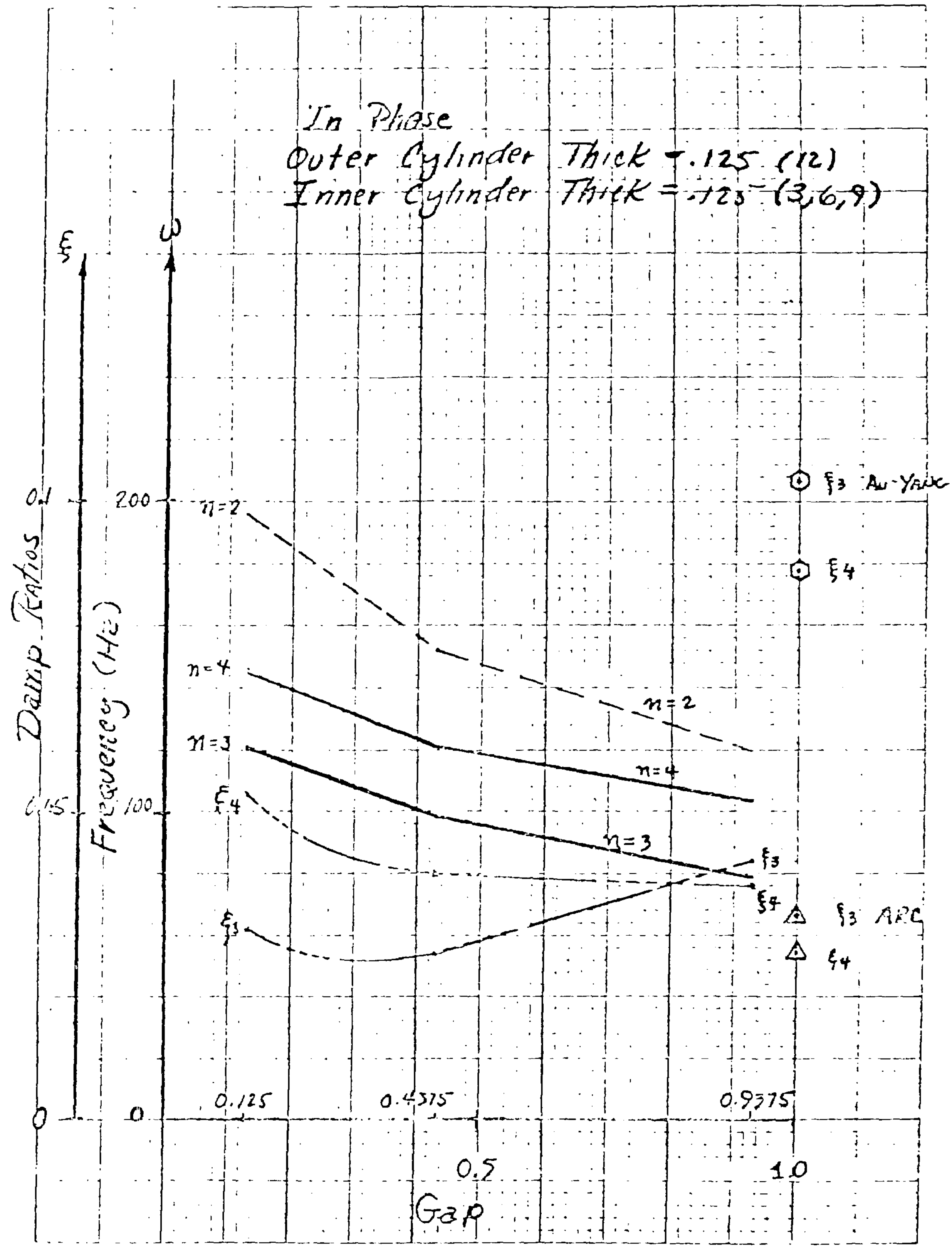

Out-of-Phase
Outer Cyln Thick = 0.125"(12)
Inner Cyln Thick = 0.125" (3,6,9) & Rigid
ω ◻ Rigid vs. #12
ξ ○ Rigid vs #12
Damp Ratios
Frequency (Hz)
n = 6
n = 5
n = 4
n = 2, 3
ARC
0.125
0.4375
0.9375
0.5
1.0
GAP (in)
G

Figure 35



In Phase
Outer Cylinder Thick = .125 (12)
Inner Cylinder Thick = .125 (3,6,9)
ξ
ω
Damp Ratios
Frequency (Hz)
0.1
0.05
0
200
100
0
n=2
n=4
n=3
ξ4
ξ3
ξ3 Au-Yang
ξ4
ξ3 ARC
ξ4
0.125
0.4375
0.9375
0.5
1.0
Gap

Figure 36

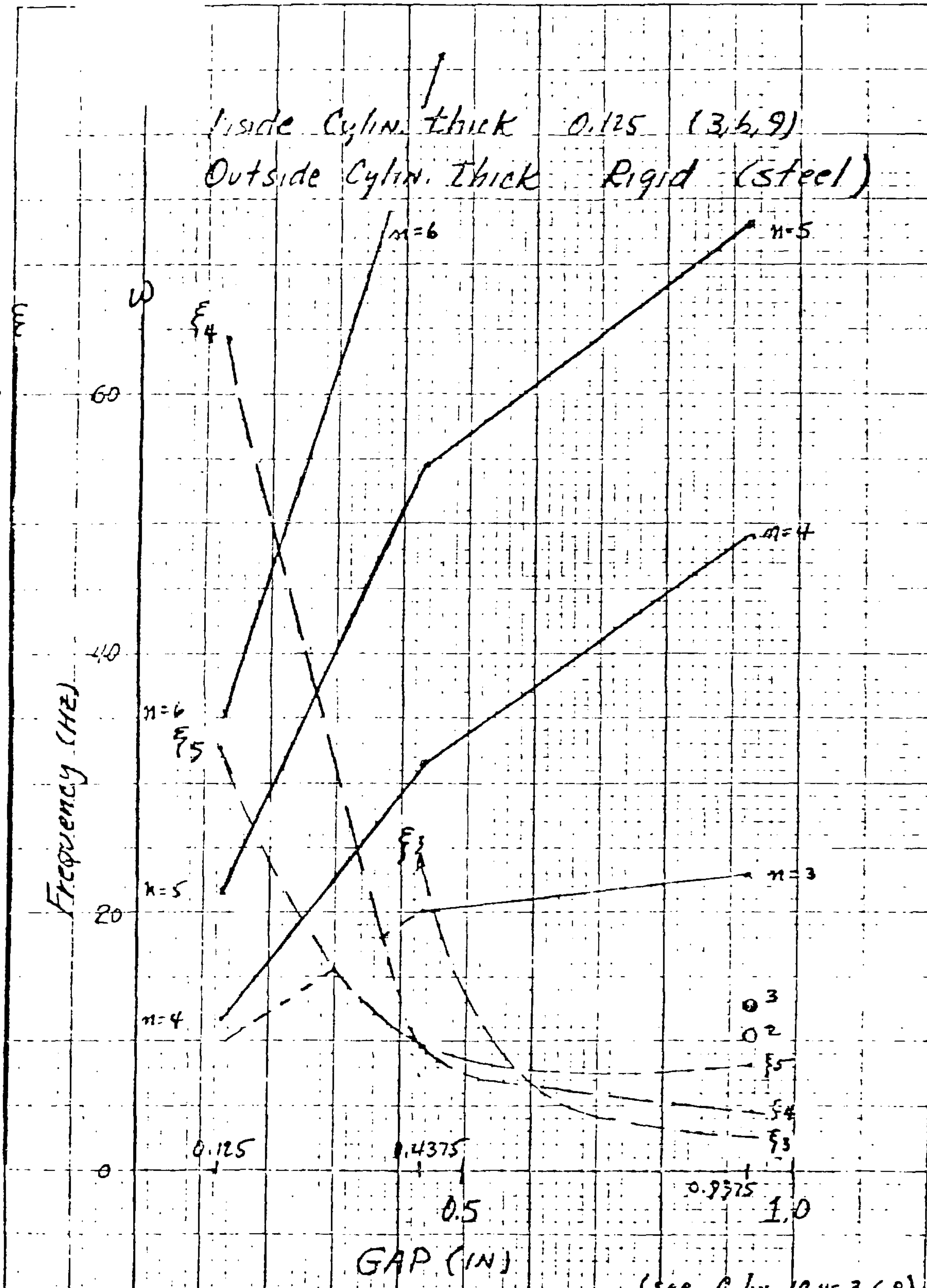

Inside Cylin. thick 0.125 (3,6,9)
Outside Cylin. Thick Rigid (steel)
Damp Ratio
Frequency (Hz)
0.3
0.2
0.1
0
60
40
20
0
n=6
n=5
n=4
n=3
0.125
0.4375
0.9375
0.5
1.0
GAP (IN)
(see Cylin 10 vs 3,6,9)

Figure 37