FR7902477

LYCEN-7968
Octobre 1979

# A GEOMETRICAL APPROACH OF THE HADRONS AND THE BARYONIUM

E. ELBAZ

Institut de Physique Nucléaire (and IN2P3)
Université Claude Bernard Lyon-1
43, Bd du 11 Novembre 1918 - 69622 Villeurbanne Cedex, France

Abstract

The quark is defined as a flavour vector state in the $\mathcal{C}$ colour space and the antiquark as a vector in the $\mathcal{C}^*$ dual space. The observables become scalar in the colour-space built even with a vector and a covector (meson) or with a vector and a pseudo-vector (baryon). The pseudo-vector is responsible of the Pauli principle existence and allows a comprehension of the distinction between baryons and mesons. The investigation of the strong interaction as the exchange of gluons leads to a crossing rule in contradiction with the JOZI rule in the general case. The existence and allowed decay of different sort of baryonium is then considered.

Our starting point has been naive questions that everyone starting to learn about quarks can ask himself : are all hadrons constituted with three quarks ($qqq$) or with one quark and one antiquark ($q\bar{q}$) ? Why a certain class of particles (the fermions) obey to the Pauli principle and not the other (the bosons) ? Can we have states with more complicated quark configurations and what should be such particles, bosons or fermions ?

Many attempts to describe the colour symmetry and hadrodynamics have been done [1,2] and recently a great interest has been rised by the work on the topological aspects of the Q.C.D. by Chew et al. [3,4,5]. The string model [6] has been an interesting approach of the topological structure of the mesons or baryons, but all these diagrammatic approaches seem to be a technical support rather than a true description of the nature of particles.

We have first defined a quark as a flavour vector state in the colour space, and used the basis of the string model to get the diagrammatic representation of a meson and a baryon. It appeared then the very simple fact that an observable was a scalar in the colour space constructed with a vector of the $\mathcal{C}$ colour-space and a covector of the $\mathcal{C}^*$ dual space (the meson) or with a vector of $\mathcal{C}$ and a pseudo-vector of $\mathcal{C}^*$ obtained by the dot product of two vectors belonging to $\mathcal{C}$ (the baryon). Moreover, it appeared that the Pauli principle existence was linked to the presence of a pseudo-vector (antisymmetric) in the observable. The particles built with an even number of vectors or pseudo-vectors are bosons while and odd number of pseudo-vectors in an observable gives a fermion.

We have then tried to understand the interactions as a change in the topology of a system and introduced as a " basic interaction " the fusion of two mesons, or the inverse process, and we associated a strength $\lambda$ to the separation process [6], $\lambda$ being the Veneziano coupling constant. We thus obtained all the 5 " basic interactions " introduced in the string model by Artru [6] with the expected strength parameter. Those intriguing results mentioned by this author became thus clear. Using then the Graphical Spin Algebra [7] and its extension to the Graphical Vector Analysis [8, 9] we obtained a "crossing rule" which indicated how a complicated quark configuration could give by non leptonic decay a jet of mesons and baryons. Our rule is in complete disagreement with the JOZI rule introduced by Imachi et al. [10] which " forbid the breaking of a junction ". It seems that such a rule is only valid when quarks of the same flavour are involved. Recently Iguchi and Sugano [11] have pointed out some violation of the OZI rule, and this, perhaps have to do with our "crossing rule."

We have then studied what should be the baryonium and distinguished between the mesonium (formed of equal numbers of quarks and antiquarks) which behaves like a meson and decay into mesons, and the baryonium (formed of vectors and odd numbers of pseudo-vectors) which behaves like a baryon and always decay into at least one baryon. We have described the exotic particles thus obtained with $n \leq 4$ vertices. An interesting thing which appeared is that with $n < 3$ one obtains only one sort of exotic particle, mesonium or baryonium, but with $n > 3$ we can have with a definite number of vertices even a mesonium (equal number of vector or pseudo-vectors and covectors) or a baryonium. The obtention of the expected baryons and mesons with the given strength may give an indication of the existence of exotic particles.

## I. THE QUARK

The quark is a quantum state characterized by various quantum numbers :

spin $J = \frac{1}{2}$

baryon number $B = \frac{1}{3}$

electric charge $Q = -\frac{1}{3}$ or $\frac{2}{3}$

parity $P = +1$

flavour f

colour c (blue, red and green)

isospin I

strangeness S

charm C

beauty b

truth t

The four last quantum numbers have been introduced as to define without ambiguity a quark with a given flavour. Six different possible flavours have been stated with the characteristics given in Table 1 .

| Baryon number | 1/3 | | | | | |
|---|---|---|---|---|---|---|
| Spin J | 1/2 | | | | | |
| Charge Q | 2/3 | | | -1/3 | | |
| Flavour f | u | c | t | d | s | b |
| Isospin I | 1/2 | 0 | 0 | 1/2 | 0 | 0 |
| $I_3$ | 1/2 | 0 | 0 | -1/2 | 0 | 0 |
| Strangeness S | 0 | 0 | 0 | 0 | -1 | 0 |
| Charm C | 0 | 1 | 0 | 0 | 0 | 0 |
| Beauty b | 0 | 0 | 0 | 0 | 0 | 1 |
| Truth t | 0 | 0 | 1 | 0 | 0 | 0 |
| Mass (MeV) | $m_o$ | $m_o + 1500$ | $m_o + ?$ | $m_o$ | $m_o + 150$ | $m_o + ?$ |

Table 1

We note here that a specified flavour gives to a quark all its quantum numbers. If we use the d-flavour of the quark the $I$, $I_3$, $C$, $S$, $b$, $t$ numbers are specified as in Table 1. The pairs u and d form the basic representation of isospin SU(2). The triplet u, d, s form the basic representation of SU(3). When we add the C charm state one gets the basic representation of SU(4) and so on.

An antiquark of a given flavour f has the same mass, spin, and isospin that the quark of flavour f but all others quantum numbers ( B, Q, $I_3$, S, C, b, t and the parity P ) have the opposite sign.

Let us consider a quark as a vector (with three components) of flavour f in the colour-space $\mathcal{C}$

$$q^f_c = \langle f \mid 1c \rangle = \vec{f}_c = \hat{f} \vdash\!\xrightarrow{1c} = \hat{f} \vdash\!\xrightarrow{c} \in \mathcal{C} \qquad (I.1)$$

and the antiquark $\bar{q}$, its hermitian conjugate, as a vector in the dual colour space $\mathcal{C}^*$

$$\bar{q}^f_c = \langle 1c \mid f \rangle = \vec{f}^*_c = \hat{f} \vdash\!\xleftarrow{1c} = \hat{f} \vdash\!\xleftarrow{c} \in \mathcal{C}^* \qquad (I.2)$$

We have used a graphical representation of these vectors in the colour-space analog to that used in the Graphical Vector Analysis[7] in the coordinate space. The quark or antiquark kind of a state is thus graphically linked to the direction of the arrow with respect to the flavour index.

## II. THE MESONS

The fundamental hypothesis in the quark model is that the mesons are quark - antiquark systems. Since all quantum numbers previously defined for a quark are conserved when dealing with strong interactions one then gets a baryon number $B = 0$, a total spin $\vec{J} = \frac{\vec{1}}{2} + \frac{\vec{1}}{2} + \vec{\ell}$ and a parity $\pi_q \pi_{\bar{q}} (-)^{\ell}$. For simplicity sake we restrict our study to the $\ell = 0$ relative orbital momentum state. It appears that $J = 0$ or $1$ and $P = -1$. We thus get pseudo-scalar meson of state $J^P = 0^-$ and vectorial meson of state $J^P = 1^-$. It is then obvious to note that the mesons are bosons and do not obey to the Pauli principle.

Let us now consider the meson as a colour singlet flavour neutral $q\bar{q}$ state :

$$M = \frac{1}{\sqrt{3}} \sum_{c\,c'} \delta_{cc'}\, q^{c}_{f_1}\, \bar{q}^{c'}_{f_2} = \frac{1}{\sqrt{3}} \sum_{c}\, q^{c}_{f_1}\, \bar{q}^{c}_{f_2} \qquad \text{(II. 1)}$$

One obtains with the graphical representations of the quark and antiquark :

$$M = \frac{1}{\sqrt{3}} \sum_{c} \hat{f}_1 \vdash\!\overset{c}{\longrightarrow} \qquad \overset{c}{\longrightarrow\!\!\rightarrow}\!\dashv \hat{f}_2 \qquad \text{(II. 2)}$$

As usual in the Graphical Spin Algebra 7) one gets the summation on the c colour indices by linking the corresponding lines (lines with the same colour, one outgoing like a ket, the other doubly ingoing like a bra)

$$M = \frac{1}{\sqrt{3}}\ \hat{f}_1 \vdash\!\!\longrightarrow\!\!\dashv \hat{f}_2 \qquad \text{(II. 3)}$$

One can recognize in such a diagram (or directly on (II. 1) ) the scalar product of vectors $\vec{f}_1$ and $\vec{f}_2^{\,*}$ in the colour-space, or else, the tensorial product of zero order of these vectors :

$$M = \frac{1}{\sqrt{3}}\ \vec{f}_1 \cdot \vec{f}_2^{\,*} = (\vec{f}_1 \otimes \vec{f}_2^{\,*})_{oo} \qquad \text{(II. 4)}$$

In some cases it can be of great help to note the $J^P$ quantum numbers on a meson graphical representation as to completely define the involved particle. The $\frac{1}{\sqrt{3}}$ coefficient may be omitted for simplicity sake

$$M = \hat{f}_1 \underset{J^P}{\vdash\!\!\longrightarrow\!\!\dashv} \hat{f}_2 = \vec{f}_1 \cdot \vec{f}_2^{\,*} = (f_1\ \bar{f}_2) \qquad \text{(II. 5)}$$

When one changes the direction of the arrow in the meson diagram one changes the electric charge of this particle as well as the $I_3$ , S , C , b , t quantum numbers.

The $\pi^+$ is for example graphically represented by

$$\pi^+ = \hat{u} \overset{0^-}{\vdash\!\!\longrightarrow\!\!\dashv} \hat{d} \qquad \text{(II. 6)}$$

while the $\pi^-$ will be $\hat{u} \overset{0^-}{\vdash\!\!\longleftarrow\!\!\dashv} \hat{d}$ (II. 7)

One can for convenience denote by h the ( u , c , t ) states of the quark (electric charge 2/3) and $\ell$ the (d, s, b) states. One then obtains by including the spin :

$$M^+ = \hat{h} \overset{J}{\longmapsto} \hat{\ell} = (h\bar{\ell})$$

$$M^- = \hat{h} \overset{J}{\longleftarrow} \hat{\ell} = (\bar{h}\ell) \qquad \text{(II. 8)}$$

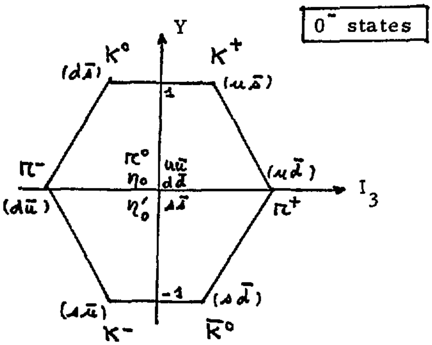
$$M^o = \hat{h} \longmapsto \hat{h} \text{ or } \hat{\ell} \longmapsto \hat{\ell} \text{ or } \hat{h} \longmapsto \hat{h} \pm \hat{\ell} \longmapsto \hat{\ell}$$

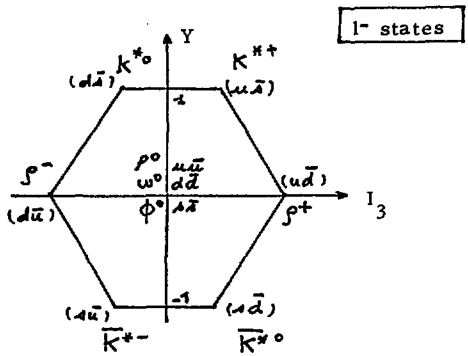
$$= (h\bar{h}) \text{ or } (\ell\bar{\ell}) \text{ or } (h\bar{h} \pm \ell\bar{\ell})$$

with h = u , c , t $\quad Q = \frac{2}{3}$ flavour states

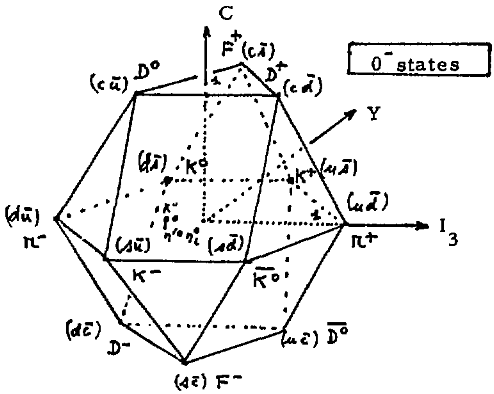
$\ell$ = d , s , b $\quad Q = -\frac{1}{3}$ flavour states .

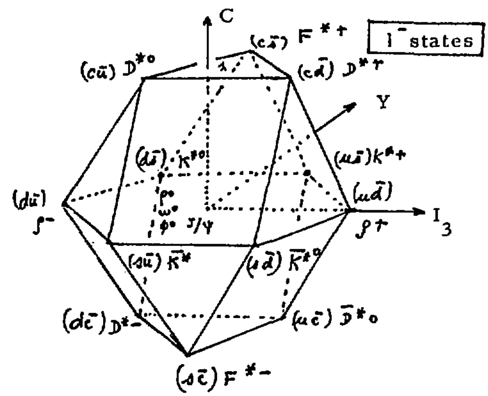
When one limits the quark flavours to the u , d , s states one obtains a nonet for the $0^-$ pseudo-scalar meson and one other nonet for the vectorial meson




When one introduces the charm as a supplementary flavour of the quark , the lowest meson states are 15 plets and singlets made up of the usual SU(3) octets and singlets plus three charmed mesons ( c $\bar{f}$ ) and three mesons ( f $\bar{c}$ )[12)]




In the above study we have only considered ground states of $(q\bar{q})$ systems in S relative orbital momentum states. One can have $0^+$ states mesons like the $\epsilon$ (0.7) or $\delta^*$(0.98). These states may come from relative P states of the $(q\bar{q})$ system or belong to more complicated exotic mesons like $(q\,q\,\bar{q}\,\bar{q})$ in a relative S-state, as we shall see later.

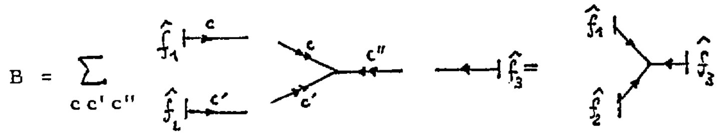
## III. THE BARYONS

The second fundamental hypothesis in the quark model is that baryons are constituted of three quarks. It explains the fact that the baryon number assigned to a quark is 1/3. The total spin of a baryon in its fundamental state is obtained by coupling the spin of the three quarks $\vec{J} = \frac{\vec{1}}{2} + \frac{\vec{1}}{2} + \frac{\vec{1}}{2}$ and one get states of spin $\frac{1}{2}$ or $\frac{2}{3}$ with a positive parity. The baryons are fermions.

Let us now consider the baryons as colour singlets with three quarks

$$B = \frac{1}{\sqrt{6}} \sum_{c c' c''} \epsilon_{c c' c''} \, q_{f_1}^{c} \, q_{f_2}^{c'} \, q_{f_3}^{c''} \qquad \text{(III. 1)}$$

We use the graphical representation of the $\epsilon_{c c' c''}$ Levi-Civita antisymmetric tensor as defined in the Graphical Vector Analysis[8, 9] and the graphical representation (I. 1) of the quarks to get after summation over the colour indices



(III. 2)

One can recognize on the above diagram, or directly on its analytical expression (III. 1) the triple scalar product of vectors $\vec{f}_1$, $\vec{f}_2$ and $\vec{f}_3$ in colour-space or else, the tensorial product of zero order of one of these vectors with the cross-product of the others

$$B = \frac{1}{\sqrt{6}} \vec{f}_1 . (\vec{f}_2 \wedge \vec{f}_3) = (\vec{f}_1 \otimes (\vec{f}_2 \otimes \vec{f}_3)^{*})_{oo} \qquad \text{(III. 3)}$$

We have first to note that $\vec{f}_2 \wedge \vec{f}_3$ must be a vector of the $\mathscr{C}^*$ dual colour space in order to give with the vector $\vec{f}_1 \in \mathscr{C}$ a scalar product. In fact $\vec{f}_2 \wedge \vec{f}_3$ is an axial vector or a pseudo-vector and we shall denote by $\vec{f}$ the vector belonging to $\mathscr{C}$ and $\vec{f}^*$ the vector belonging to $\mathscr{C}^*$, by $\vec{g}$ and $\vec{g}^*$ the pseudo-vectors constructed by dot products

$$\vec{f}_1 \wedge \vec{f}_2 = \vec{g}^* \qquad \in \mathscr{C}^*$$

$$\vec{f}_1^* \wedge \vec{f}_2^* = \vec{g} \qquad \in \mathscr{C}$$

The baryonic number of $\vec{g}^*$ is $\frac{2}{3}$ and that of $\vec{g}$ is $-\frac{2}{3}$ .

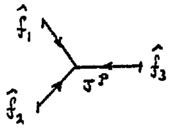
Contrarily to $\vec{f}$, $\vec{g}$ does not represent a quark (its baryonic number is $-\frac{2}{3}$ ), it represents an object which behaves like a quark since its representative vector belongs to $\mathscr{C}$. Hence with true vectors one may construct a scalar which is the observable meson while with a vector and a pseudo-vector one may construct an observable (their scalar product) which is the baryon

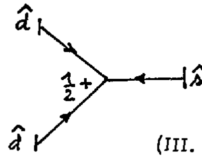
$$M = \vec{f} . \vec{f}^* = \vec{f}^* . \vec{f}$$

$$B = \vec{f} . \vec{g}^* \text{ and } \overline{B} = \vec{f}^* . \vec{g}$$

The presence of a dot product in the baryon state shows that when two flavour indices are permuted the baryon wave-function gets the opposite sign in the colour-space. This perhaps may explain why even if quarks do not obey Pauli statistics individually the baryons must obey to the Pauli principle. The fact that observables must be scalars in the colour-space may explain why $(qq)$ or $(qqqq)$ are not seen while $(q\bar{q})$ and $(qqq)$ are seen and why one may hope to observe exotic states like $(qq\bar{q}\bar{q})$ or $(qq\bar{q}qq)$ ....

As for the mesons it may be useful to note the J quantum numbers on a diagram and for simplicity sake omit the $\frac{1}{\sqrt{6}}$ coefficient, so that

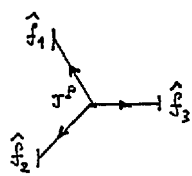
B =  $= \vec{f}_1 \cdot (\vec{f}_2 \wedge \vec{f}_3) = \vec{f}_1 \cdot \vec{g}_1^*$ (III. 4)

The $\Sigma^-$ is for example graphically represented by $\Sigma^-$ = 

(III. 5)

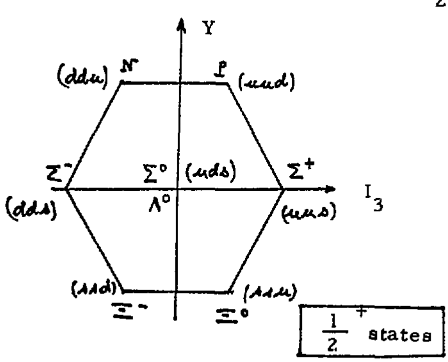
Let us note that the existence of the s flavour in the diagram implies that the $\Sigma^-$ has a strangeness $S = -1$ while the existence of two flavours d gives an isospin $I = 1$. The electric charge Q is determined by the electric charge of the quark components.

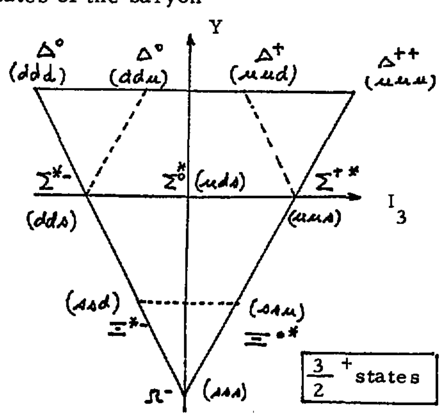
The change of direction of the three arrows of a baryon diagram leads to the graphical representation of the antibaryon

$\overline{B}$ =  $= \vec{f}_1^{\,*} \cdot (\vec{f}_2^{\,*} \wedge \vec{f}_3^{\,*}) = \vec{f}_1^{\,*} \cdot \vec{g}_1$ (III. 6)

The antibaryon $\overline{B}$ has the same spin and isospin that the B baryon but all other quantum numbers (B, Q, I, S, C, b, t, P) change in signs as can be seen by replacing the quarks by antiquarks with the same flavour.

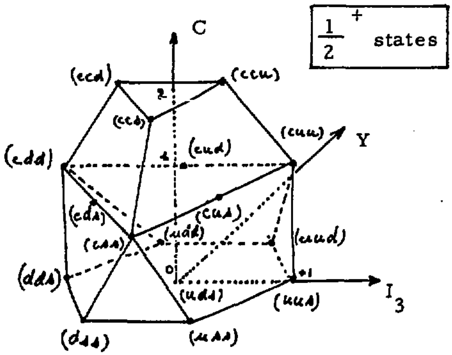
When one limits the study to the (u, d, s) flavours of the quark one obtains an SU(3) internal symmetry and an octet for the fundamental $\frac{1}{2}^+$ states and a decuplet for the fundamental $\frac{3}{2}^+$ states of the baryon

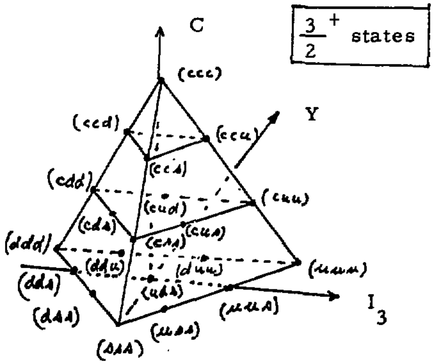


It seems moreover that the (u d s) state with $J^P = \frac{1}{2}^-$ may be assigned to the $\Lambda$ (1405) particle and constitutes the singlet state of the SU(3) internal symmetry.

When one introduces the charm in the possible flavours of the quark one gets a 20 plet for the baryon states that form an irreducible representation of SU(4) [12].




Here too, we have only considered the ground states of the baryons and not taken into account a relative orbital angular momentum.

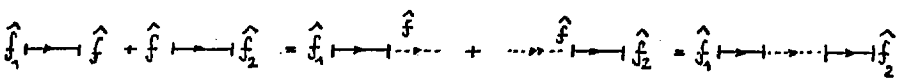
## IV. THE INTERACTIONS

In the following we only consider the strong interaction between the quarks. Such an interaction is known to be independent of the flavour, due to the exchange of colored gluons and to conserve the descriptive quantum numbers S, C, b, t.

We shall consider two " basic " interactions, the first one corresponding to a fusion process by exchange of one gluon and annihilation of a $(q\bar{q})$ pair of quarks with identical flavour and the inverse process ( a separation) . We shall symbolically denote it by

$$M + M \rightleftarrows M$$

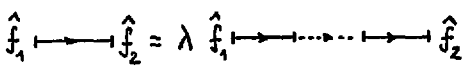
If we use the graphical representation of the mesons we get



(IV. 1)

The exchange of a gluon is thus here represented by a link between the quark lines.

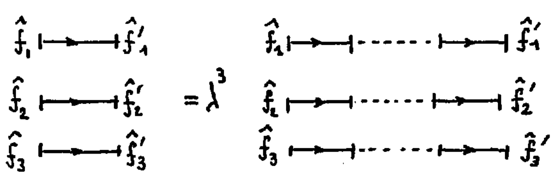
We can associate a strength $\lambda$ to a separation process, where $\lambda$ is the Veneziano coupling constant [6)]



(IV. 2)

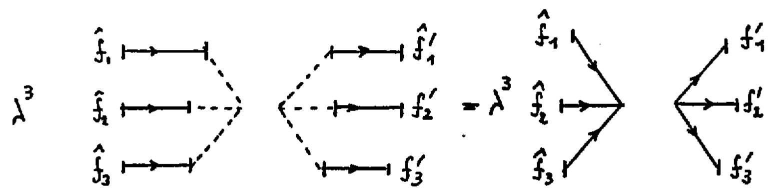
The second " basic interaction " will be the triple rearrangement between three quark lines symbolically written as $M + M + M \rightleftarrows B + \bar{B}$ .

We begin with the use of the separation process on the three involved mesons



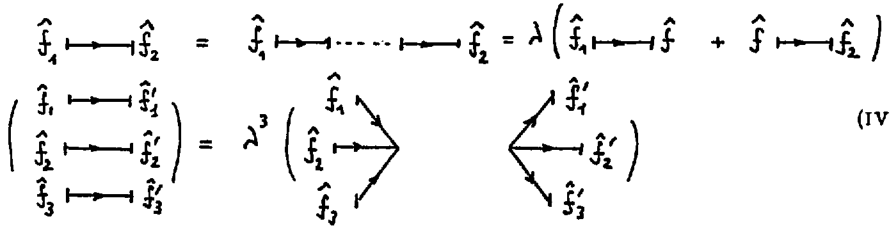
(IV. 3)

We then use the usual pinching rule of the graphical technique to get



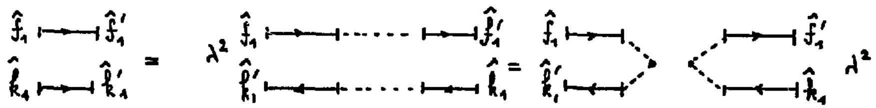
(IV. 4)

We have introduced as a basic hypothesis the pinching rule on three gluon lines and its validity will be shown by the consequences thus obtained. We shall write the two basic interactions in the following way :

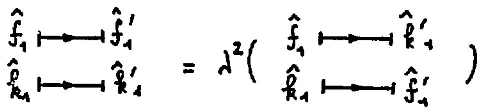


(IV. 5)

We have to note that the relative position of the quark $q^{f_1}$, $q^{f_2}$, $q^{f_3}$ in the baryon and $\bar{q}^{k_1}$ $\bar{q}^{k_2}$ $\bar{q}^{k_3}$ in the antibaryon is not important since as soon as a choice is maid in the baryon the order is correspondingly fixed in the antibaryon. We can deduce from (IV. 5) the strength of a double rearrangement process



and we write it as

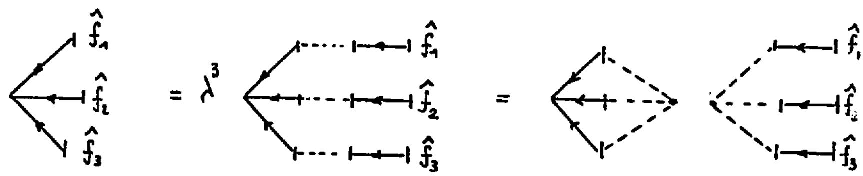


(IV. 6)

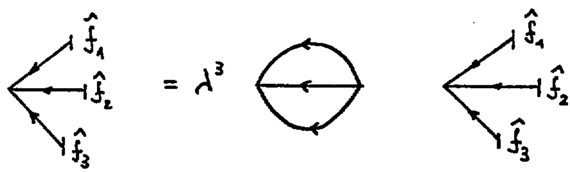
We note here that the pinching rule has been used as to pinch corresponding quark-lines and allow the formation of scalars in the colour-space

in

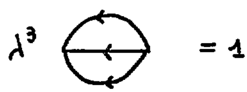
The virtual processes in which quark-lines disappear (or are created of) the vacuum may be deduced for our fundamental hypothesis (IV. 5) . Starting from a baryon diagram we get



or eliminating the three convergent gluon lines it comes that

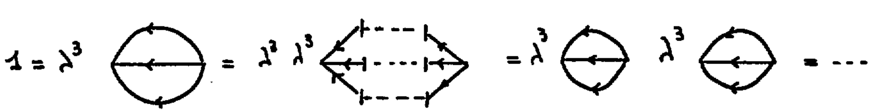


which gives the identity

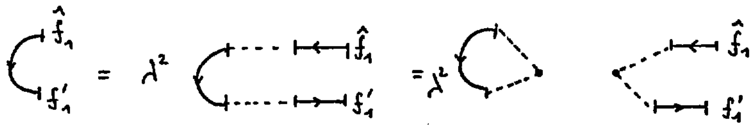


(IV. 7)

which is coherent with the triple rearrangement virtual process since



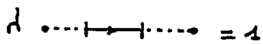
An analog procedure gives the double virtual rearrangement process and the graphical representation of a pomeron



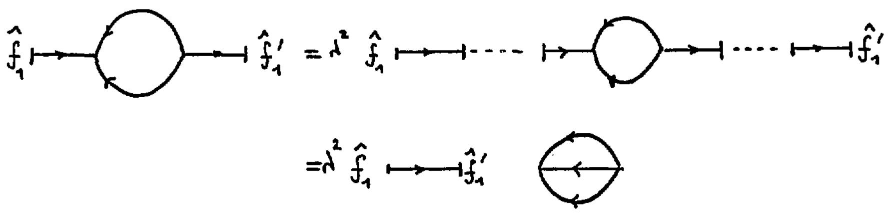
which gives the identity

(IV.8)

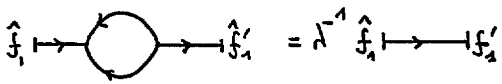
With one single interaction one must get too



Let us now try to identify a special type of diagram with a loop on a quark-antiquark line :

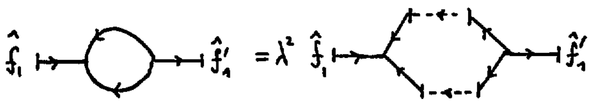


We find (with IV.7) that



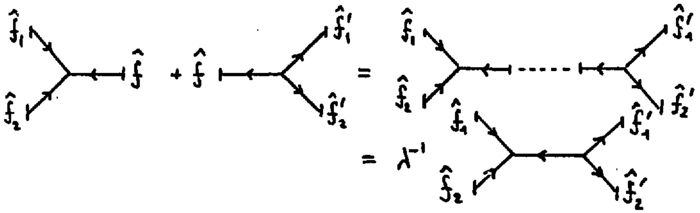
(IV.9)

A question may arise. Why do we not apply a separation process on the loop ?



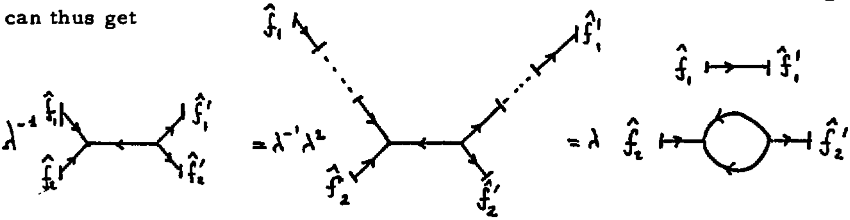
It appears that such a process will separate the quark and the antiquark-lines and lead to non-scalar objects in the colour space. Such a separation is thus forbidden. We can now determine the strength of the interaction between a baryon and anti-baryon with exchange of one , two or three gluons.

i) <u>One gluon exchange</u> :

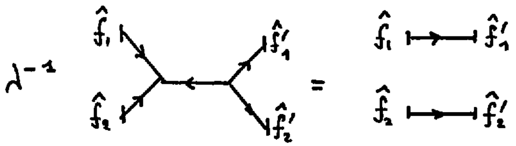


(IV. 10)

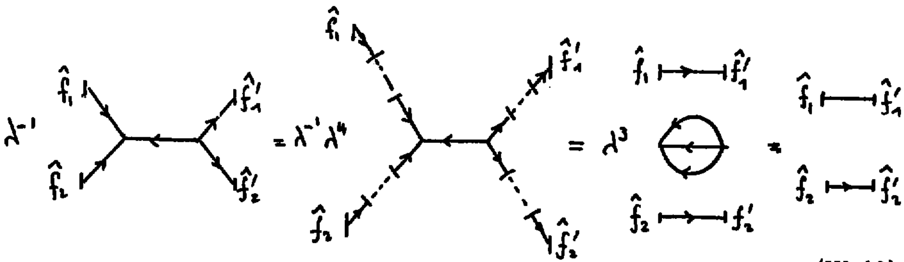
A separation process may now occur on the different quark lines. As previously indicated a n y separation must lead to a scalar in the colour space. It is thus impossible to consider a separation on $f_1$ and $f_2$ without a separation on $f'_1$ and $f'_2$. We can thus get



and with (IV. 9) we find that

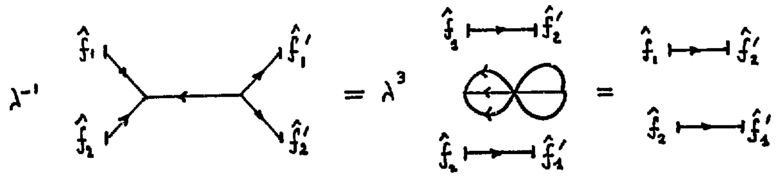


A same result should have been obtained by considering directly a separation of the four quark lines



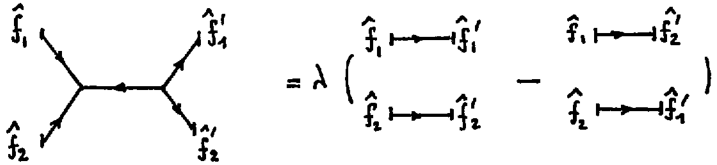
(IV. 11)

A pinch on $(f_1 \overline{f'_2})$ and $(f_2 \overline{f'_1})$ is also possible leading to the value



(IV. 12)

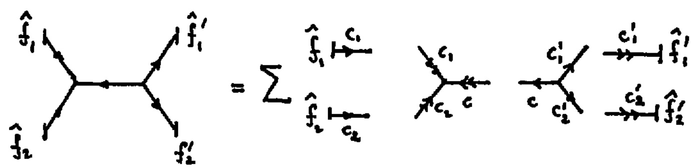
The two processes described in (IV. 11) et (IV. 12) are in competition and we shall write it as



(IV. 13)

If the involved quarks have the same flavour one gets zero in the right hand part. This may perhaps explain why some authors have given this result as an extension of the OZI rule and called it the JOZI rule[10] which " forbids a breaking of a junction ".[13] Our crossing rule is in complete disagreement with this statement and is based on a mathematical transcription of the above result.

Let us first recall that the $\lambda$ coefficient only compares the strength of such a process to the fusion process. On the other hand, the graphical equivalence (IV. 13) is simply a diagrammatic expression of the crossing rule as defined in the Graphical Vector Analysis[8]. We can recall it by an analytical transcription of the diagram involved



(IV. 14)

$$= \sum_{c_1 c_2 c'_1 c'_2} q^{f_1}_{c_1} \; q^{f_2}_{c_2} \; \sum_{c} \varepsilon^{c_1 c_2 c} \, \varepsilon_{c'_1 c'_2 c'} \; \bar{q}^{f'_1}_{c'_1} \; \bar{q}^{f'_2}_{c'_2}$$

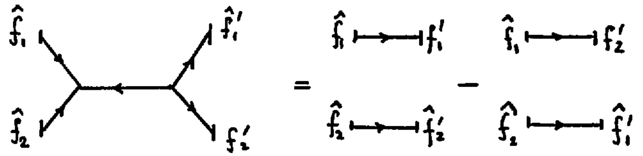
We use the well known relation

$$\sum_{c} \epsilon^{c_1 c_2 c} \, \epsilon_{c'_1 c'_2 c'} = \delta_{c_1 c'_1} \delta_{c_2 c'_2} - \delta_{c_1 c'_2} \delta_{c_2 c'_1} \tag{IV. 15}$$

to get

$$\sum_{c_1} q^{f_1}_{c_1} \bar{q}^{f'_1}_{c_1} \sum_{c_2} q^{f_2}_{c_2} \bar{q}^{f'_2}_{c_2} - \sum_{c_1} q^{f_1}_{c_1} \bar{q}^{f'_2}_{c_1} \sum_{c_2} q^{f_2}_{c_2} \bar{q}^{f'_1}_{c_2}$$

and we refind the graphical equivalence (without the strength parameter)

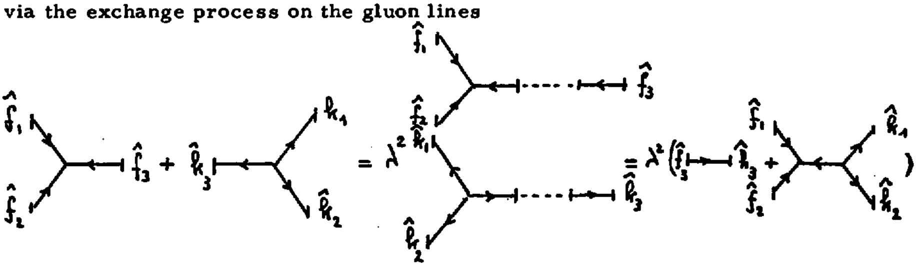



(IV. 16)

$$(\vec{f}_1 \wedge \vec{f}_2) \cdot (\vec{f'}^{*}_1 \wedge \vec{f'}^{*}_2) = (\vec{f}_1 \cdot \vec{f'}^{*}_1)(\vec{f}_2 \cdot \vec{f'}^{*}_2) - (\vec{f}_1 \cdot \vec{f'}^{*}_2)(\vec{f}_2 \cdot \vec{f'}^{*}_1)$$

This equivalence will be of great importance when dealing with the baryonium.

In most cases the interaction between a baryon and an antibaryon occurs via the exchange process on the gluon lines

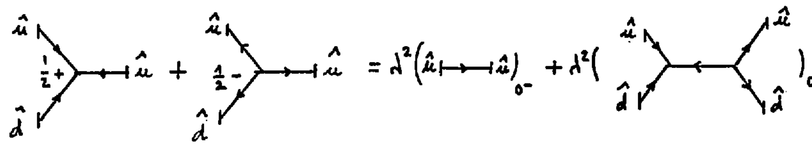



(IV. 17)

Now the last diagram may decay through the formation of an other baryon - antibaryon system or through the formation of mesons with (IV. 13)

$$(f_1 f_2 f_3) + (\bar{k}_1 \bar{k}_2 \bar{k}_3) = \begin{cases} \lambda^2 (f_3 \bar{k}_3) + \lambda^3 (f_1 f_2 \bar{k}_3) + \lambda^3 (\bar{k}_1 \bar{k}_2 \bar{k}_3) \\ \lambda^2 (f_3 \bar{k}_3) + \lambda^3 (f_1 \bar{k}_1) + \lambda^3 (f_2 \bar{k}_2) \\ \lambda^2 (f_3 \bar{k}_3) + \lambda^3 (f_1 \bar{k}_2) + \lambda^3 (f_2 \bar{k}_3) \end{cases} \tag{IV. 18}$$

With the $p\bar{p}$ system one may obtain for instance (among the many opened processes)



(IV.19)

$$p + \bar{p} = \lambda^2 (\pi^o) + \lambda^3 (n + \bar{n}) \qquad \text{(IV.20)}$$

$$= \lambda^2 (\pi^o) + \lambda^3 (\pi^o + \eta^o) \qquad \text{(IV.21)}$$

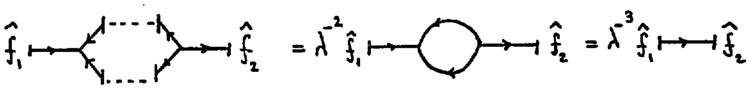
$$= \lambda^2 (\pi^o) + \lambda^3 (\pi^+ + \pi^-) \qquad \text{(IV.22)}$$

With (IV.20) one gets $\dfrac{\sigma(n)}{\sigma(\pi^o)} = \lambda^2$

With (IV.21) one gets $\dfrac{\sigma(\eta^o)}{\sigma(\pi^o)} = \dfrac{1}{\lambda^2 (1+\lambda)^2}$ (IV.23)

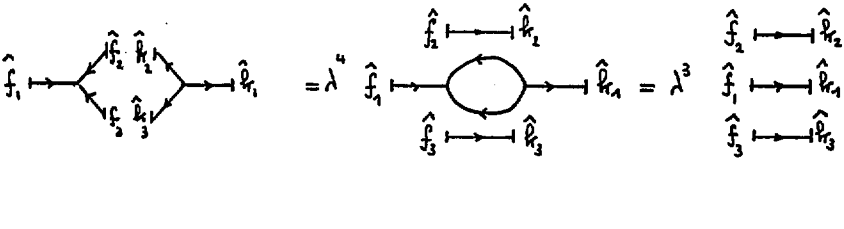
With (IV.22) one gets $\dfrac{\sigma(\pi^+)}{\sigma(\pi^o)} = \dfrac{\sigma(\pi^-)}{\sigma(\pi^o)} = \lambda^2$

ii) Two gluons exchange :



(IV.24)

If one looks at a reaction $B\bar{B}$ in which two gluons are exchanged one gets



(IV.25)

One can have an exchange between $k_2$ and $k_3$ getting thus two possibilities

$$(f_1 f_2 f_3) + (\bar{k}_1 \bar{k}_2 \bar{k}_3) = \begin{cases} \lambda^3 (f_2 \bar{k}_2) + \lambda^3 (f_1 \bar{k}_1) + \lambda^3 (f_3 \bar{k}_3) & \text{or} \\ \lambda^3 (f_2 \bar{k}_3) + \lambda^3 (f_1 \bar{k}_1) + \lambda^3 (f_3 \bar{k}_2) \end{cases} \qquad \text{(IV. 26)}$$

With the $p\bar{p}$ reaction one may obtain

$$\begin{aligned} p + \bar{p} &= \lambda^3 (\pi^o) + \lambda^3 (\pi^o) + \lambda^3 (\eta^o) \\ &= \lambda^3 (\pi^+) + \lambda^3 (\pi^o) + \lambda^3 (\pi^-) \end{aligned} \qquad \text{(IV. 27)}$$

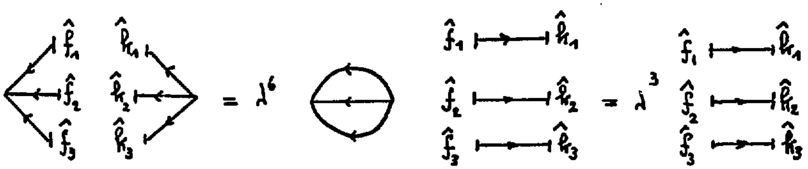
One gets the same mesons than in (IV. 22) and (IV. 23) but

$$\frac{\sigma(\eta_o)}{\sigma(\pi^o)} = 1 \quad \text{and} \quad \frac{\sigma(\pi^+)}{\sigma(\pi^o)} = 1 \qquad \text{(IV. 28)}$$

If such a ratio is obtained it supposes that the $p\bar{p}$ reaction does not produce the intermediate state as described in (IV. 19).

iii) <u>Three gluons exchange</u> :

If we introduce a separation on the three quark and the three antiquark lines one gets




(IV. 29)

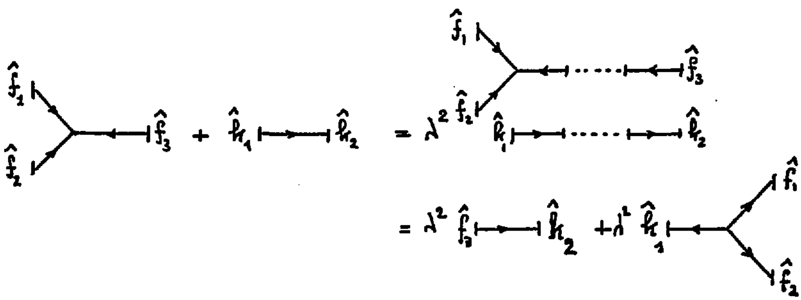
One may think that there is not any difference between that result and (IV. 26) . In fact, one sees on (IV. 26) that in any case on must obtain the meson $(f_1 \bar{k}_1)$ if there is an exchange of two gluons , but with the exchange of three gluons one may not detect the production of the meson $(f_1 \bar{k}_1)$ and get for instance

$$(f_1 f_2 f_3) + (\bar{k}_1 \bar{k}_2 \bar{k}_3) = \begin{cases} \lambda^3 (f_1 \bar{k}_2) + \lambda^3 (f_2 \bar{k}_3) + \lambda^3 (f_3 \bar{k}_1) \\ \lambda^3 (f_1 \bar{k}_2) + \lambda^3 (f_2 \bar{k}_1) + \lambda^3 (f_3 \bar{k}_3) \\ \lambda^3 (f_1 \bar{k}_3) + \lambda^3 (f_2 \bar{k}_1) + \lambda^3 (f_3 \bar{k}_2) \end{cases} \qquad \text{(IV. 30)}$$

With the proton - antiproton system where four of the six flavours involved are identical to u , it will be difficult to distinguish between the two last processes with exchange of three or two gluons but it seems possible to distinguish between the first process and the others.

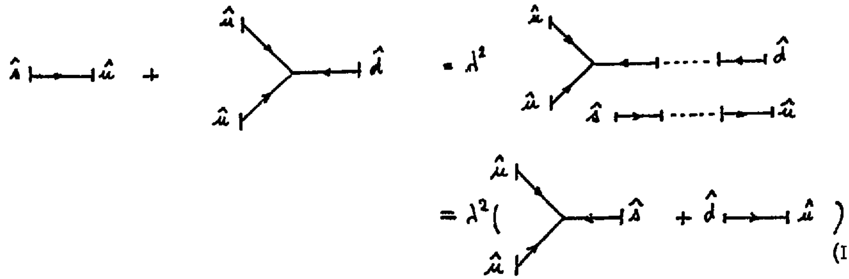
iv) Examples of application :

As an application of the separation and pinching procedure one may consider a reaction of a meson on a baryon. In such a case one must have only one gluon exchange



(IV. 31)

This process may for instance explain the reaction $K^- p \rightarrow \pi^- \Sigma^+$ since [14)]

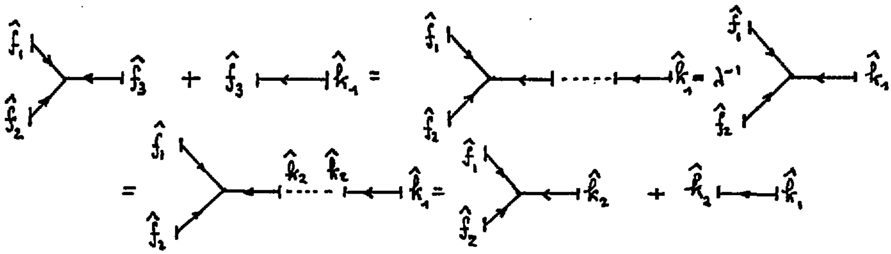


(IV. 32)

$$(s\,\bar{u}) + (u\,u\,d) \;\rightarrow\; \lambda^2\,(u\,u\,s) + \lambda^2\,(d\,\bar{u})$$

$$K^- + p \;\rightarrow\; (\Sigma^+) + \pi^-$$

A second example will concern an exchange of a gluon between a quark - antiquark of same flavour [14)]



Such a process may explain the $\pi^+ p \rightarrow K^+ \Sigma^+$ reaction if one sets

$$\begin{cases} f_1 = f_2 = k_1 = u \\ f_3 = d \\ k_2 = s \end{cases}$$

$$(u\bar{d}) + (u\,u\,d) \;\rightarrow\; (u\bar{s}) + (u\,u\,s)$$

$$\pi^+ + p \;\rightarrow\; K^+ + \Sigma^+$$

We note that such a mechanism allows the change of one flavour by exchange of one gluon.

## V. THE BARYONIUM

As previously mentioned the physical observablesare scalars, built with flavour vectors in the colour space. One can then imagine more complicated systems than the mesons and baryons usually detected and such expected objects have been denominated baryonium in the literature. In fact as we shall see later, one should have two classes of exotic particles. The first sort of such particles are bosons and thus do not obey to the Pauli principle, behave like mesons and may be called exotic mesons or mesonium. A mesonium will be always constituted of equal numbers of quarks and antiquarks, so that its baryonic number will be zero and its decomposition by strong interaction will produce baryon-antibaryon pairs or mesons. The second class of object will call baryonium are fermions and thus have to obey to the Pauli principle or bosons, and do not obey to the Pauli principle. A baryonium may be constituted of quarks only or of quarks and antiquarks in non equal proportions. A non leptonic desintegration of a baryonium will always give at least one baryon (it corresponds in fact to the conservation law of the baryon quantum number).

Let us examine now how one can get the mesonium and baryonium. We have first to recall the two fundamental rules of our graphical theory of the particles

1. One may sum only over corresponding lines ,( contravariant like a quark line and covariant like an antiquark line ).

2. At each vertex the three quark lines must have the same direction , (ingoing or outgoing).

With these two fundamental rules one may ask ourselves what sort of scalars one can build with the flavour vectors in the colour-space and thus what sort of observables one may expect. We shall examine the question with respect to the number of vertices involved.

1) <u>n = 0</u> :

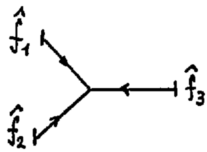
We obtain the usual meson (baryonic number 0)

$$M_o = (f_1 \bar{f}_2) = \hat{f}_1 \longmapsto\!\!\!\!\dashv \hat{f}_2 = (\vec{f}_1 \cdot \vec{f}_2^{\,*}) \qquad (V.1)$$

The first rule indicates that $(f_1 f_1)$ or $(\bar{f}_2 \bar{f}_2)$ cannot exist since they are not scalars in the coulour-space.

2) <u>n = 1</u> :

One gets the baryon or the antibaryon in virtue of our second fundamental rule

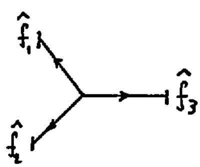
$B_1 = (f_1 f_2 f_3) =$ 

$= \vec{f}_1 \cdot (\vec{f}_2 \wedge \vec{f}_3) = \vec{f}_1 \cdot \vec{g}_1^{\,*} = \vec{f}_2 \cdot \vec{g}_2^{\,*} = \vec{f}_3 \cdot \vec{g}_3^{\,*}$

with

$$\left.\begin{aligned} \vec{f}_2 \wedge \vec{f}_3 &= \vec{g}_1^{\,*} \\ \vec{f}_3 \wedge \vec{f}_1 &= \vec{g}_2^{\,*} \\ \vec{f}_1 \wedge \vec{f}_2 &= \vec{g}_1^{\,*} \end{aligned}\right\} \quad \text{pseudo-vectors} \in \mathcal{C}^*$$

With the scalar product of a pseudo-vector of $\mathcal{C}$ and a vector of $\mathcal{C}^*$, one obtains the antibaryon

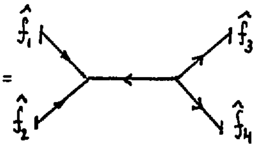
$\bar{B}_1 = (\bar{f}_1 \bar{f}_2 \bar{f}_3) =$ 

$= \vec{f}_1^{\,*} \cdot \vec{g}_1 = \vec{f}_2^{\,*} \cdot \vec{g}_2 = \vec{f}_3^{\,*} \cdot \vec{g}_3$

One may link the existence of the Pauli principle to the existence of an axial vector $\vec{g}$ (or $\vec{g}^{\,*}$) in the detected observable.

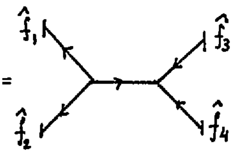
3) <u>n = 2</u> :

With two vertices the only possible diagram according to the fundamental rules is the following

$M_2 =$ 


$$= (f_1 f_2 \bar{f}_3 \bar{f}_4) = (\vec{f}_1 \wedge \vec{f}_2) . (\vec{f}_3^{\,*} \wedge \vec{f}_4^{\,*}) = \vec{g}_3^{\,*} . \vec{g}_5 \qquad \text{(V.2)}$$

Its structure is analog to that of a meson since its baryon number is zero but the vectors are replaced by pseudo-vectors. Such an object should then be a mesonium $M_2$ . The antiparticle $\overline{M}_2$ is easily obtained by inversing all the arrows

$\overline{M}_2 =$ 


$$= (\bar{f}_1 \bar{f}_2 f_3 f_4) = (\vec{f}_1^{\,*} \wedge \vec{f}_2^{\,*}) . (\vec{f}_3 \wedge \vec{f}_4) = \vec{g}_3 . \vec{g}_5^{\,*} \qquad \text{(V.3)}$$

The presence of $\vec{g}$ or $\vec{g}^{\,*}$ axial vectors in a particle leads to the Pauli principle existence as previously mentioned but when two (or more generally an even number) of pseudo-vectors are involved the corresponding particle becomes a boson and the Pauli principle is no more fulfilled.

With the use of the graphical technique (G. V. A.) (or analytically) one may express $M_2$ in a different way. For instance

$$M_2 = \vec{f}_1 . (\vec{f}_2 \wedge (\vec{f}_3^{\,*} \wedge \vec{f}_4^{\,*})) = \vec{f}_1 . (\vec{f}_2 \wedge \vec{g}_5) \qquad \text{(V.4)}$$

One sees that $\vec{f}_2 \wedge \vec{g}_5$ becomes a vector of the $\mathcal{C}^*$ colour space

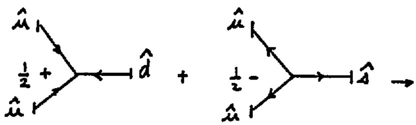
$$\vec{f} \wedge \vec{g} = \vec{f}^{\,*} \qquad \text{(V.5)}$$

The mesonium $M_2$ has a baryon number equal to zero, a positive parity and a spin equal to 0, 1, or 2 .

The remarkable thing is that it is impossible to form an other kind of observable with two vertices and our fundamental rules. A diagram like $M_2$ has already been considered in the previous section and we have seen that through exchange of gluons in non leptonic desintegration such an object may separate into mesons or a baryon-antibaryon pair

$$M_2 = (f_1 f_2 \bar{f}_3 \bar{f}_4) = \begin{cases} \lambda\,(f_1 f_2 g) + \lambda\,(\bar{g} \bar{f}_3 \bar{f}_4) \\ \lambda\,(f_1 \bar{f}_3) + \lambda\,(f_2 \bar{f}_4) \\ \lambda\,(f_1 \bar{f}_4) + \lambda\,(f_2 \bar{f}_3) \end{cases} \qquad (V.6)$$

If one considers for instance the reaction $p + \Sigma^+$ one should get as indicated in the preceding section




$$\rightarrow \begin{cases} \lambda^2 (u\bar{u}) + \lambda^2 (ud\bar{u}\bar{s}) \\ \lambda^2 (u\bar{s}) + \lambda^2 (ud\bar{u}\bar{u}) \\ \lambda^2 (d\bar{u}) + \lambda^2 (uu\bar{u}\bar{s}) \\ \lambda^2 (d\bar{s}) + \lambda^2 (uu\bar{u}\bar{u}) \end{cases} \qquad (V.7)$$

If we consider only the mesons produced in a zero orbital relative state $0^-$ or $1^-$ the mesonium should be in a state $0^+$, $1^+$ or $2^+$. Let us consider for simplicity sake the meson in the $0^-$ state and the mesonium in the $0^+$ state. In (V.7) we have considered as equiprobable all the interaction processes, even if one may suspect that due to the fact that the u flavour is more often presen, the first process should be more favorized than the others. If we only look at the meson production of such a reaction one should get with application of rule (III. 13)

$$(uud)_{\frac{1}{2}+} + (\bar{u}\bar{u}\bar{s})_{\frac{1}{2}-} \longrightarrow \begin{cases} \begin{cases} \lambda^2 (u\bar{u}) + \lambda^3 (u\bar{u}) + \lambda^3 (d\bar{s}) \\ \lambda^2 (u\bar{u}) + \lambda^3 (u\bar{s}) + \lambda^3 (d\bar{u}) \end{cases} \\ \lambda^2 (u\bar{s}) + \lambda^3 (u\bar{u}) + \lambda^3 (d\bar{u}) \\ \lambda^2 (d\bar{u}) + \lambda^3 (u\bar{s}) + \lambda^3 (u\bar{u}) \\ \lambda^2 (d\bar{s}) + \lambda^3 (u\bar{u}) + \lambda^3 (u\bar{u}) \end{cases} \qquad (V.8)$$

Even if the same mesons seem to be produced in many of these processes their branching ratios appear to be different .

In the first case

$$\frac{\sigma(d\bar{s})}{\sigma(u\bar{u})} = \left(\frac{\lambda^3}{\lambda^3 + \lambda^2}\right)^2 = \left(\frac{\lambda}{\lambda + 1}\right)^2 \qquad \text{(V. 9)}$$

while in the last case

$$\frac{\sigma(d\bar{s})}{\sigma(u\bar{u})} = \left(\frac{\lambda^2}{\lambda^3 + \lambda^3}\right)^2 = \frac{1}{4\lambda^2} \qquad \text{(V. 10)}$$

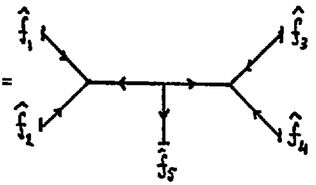
The same remark applies for the production of the $(d\bar{u})$ meson. In the first case

$$\frac{\sigma(d\bar{u})}{\sigma(u\bar{u})} = \frac{\sigma(u\bar{s})}{\sigma(u\bar{u})} = \left(\frac{\lambda^3}{\lambda^2}\right)^2 = \lambda^2 \qquad \text{(V. 11)}$$

In the second process $\frac{\sigma(d\bar{u})}{\sigma(u\bar{u})} = 1$ and $\frac{\sigma(u\bar{s})}{\sigma(u\bar{u})} = \frac{1}{\lambda^2}$ (V. 12)

In the third case $\frac{\sigma(d\bar{u})}{\sigma(u\bar{u})} = \frac{1}{\lambda^2}$ and $\frac{\sigma(u\bar{s})}{\sigma(u\bar{u})} = \lambda^2$ (V. 13)

4) n = 3 :

$B_3 =$ 

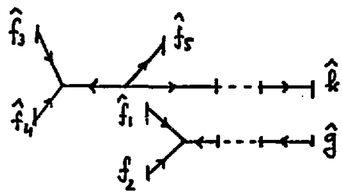
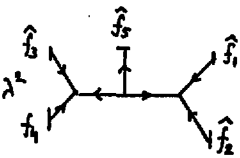

$$= (f_1 f_2 \bar{f}_5 f_3 f_4)$$
$$= \vec{f}_1 \cdot (\vec{f}_2 \wedge (\vec{f}_5^{\,*} \wedge (\vec{f}_3 \wedge \vec{f}_4))) \qquad \text{(V. 14)}$$

Such a baryonium has the structure $\vec{f} \cdot \vec{g}^{\,*}$ of a baryon. One may determine its baryon number and one gets as expected $B = 1$ . Its spin will be in the ground state $\frac{1}{2}$, or $\frac{3}{2}$ , or $\frac{5}{2}$ and its parity negative.

Its non leptonic desintegration will give at least one baryon as one can see it by isolating the baryon-antibaryon or using the (III. 13) crossing rule. One obtains

$$B_3 = (f_1 f_2 \bar{f}_5 f_3 f_4) \longrightarrow \left\{ \begin{array}{l} \lambda^2 (f_1 f_2\, g) + \lambda^2 (\bar{g} \bar{f}_5 \bar{k}) + \lambda^2 (k\, f_3 f_4) \\ \left\{ \begin{array}{l} \lambda (f_1 f_3 f_4) + \lambda (f_2 \bar{f}_5) \\ \lambda (f_2 f_3 f_4) + \lambda (f_2 \bar{f}_5) \end{array} \right. \\ \left\{ \begin{array}{l} \lambda (f_1 f_2 f_3) + \lambda (f_4 \bar{f}_5) \\ \lambda (f_1 f_2 f_4) + \lambda (f_3 \bar{f}_5) \end{array} \right. \end{array} \right. \qquad (V.15)$$

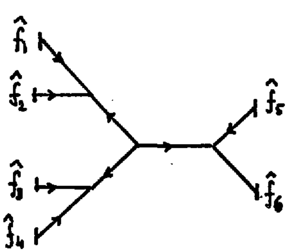
A baryonium may be obtained by interaction of a baryon with a $M_2$ mesonium (if exists ) since

$M_2 + B_1 = \lambda^2$  $= \lambda^2 (g \bar{k}) + \lambda^2$  (V. 16)

In this case too, it appears that the above diagram is the only possible. This is no more the case when $n > 3$ .

5) n = 4 :

The first diagram we can obtain is the following

$B_4 =$ 

$$= (f_1 f_2 f_3 f_4 f_5 f_6)$$

$$= (\vec{f}_1 \wedge \vec{f}_2) \cdot ((\vec{f}_3 \wedge \vec{f}_4) \wedge (\vec{f}_5 \wedge \vec{f}_6)) \qquad (V. 17)$$

It has the structure of a baryon in which the vectors are now replaced by pseudo-vectors. It is effectively a baryonium since its baryon number is different of zero equal to B = 2 . However it is not a fermion but a boson since its spin may be 0, 1, 2 or 3 (its parity will be positive). Such a particle will not obey to the Pauli principle, but this is understable since appears in its structure four dot products or in other words four pseudo-vectors

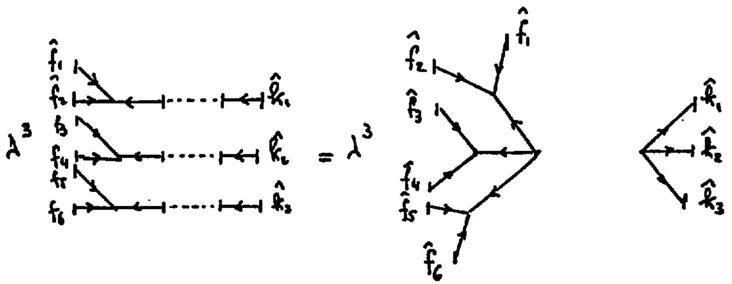
$$B_4 = (f_1f_2f_3f_4f_5f_6) = \lambda^3(f_1f_2g_1) + \lambda^3(f_3f_4g_2) + \lambda^3(f_5f_6g_3) + \lambda^3(\bar{g}_1\bar{g}_2\bar{g}_3)$$

$$= \begin{cases} \lambda(f_1f_3f_4) + \lambda(f_2f_5f_6) \\ \lambda(f_1f_5f_6) + \lambda(f_2f_3f_4) \end{cases}$$

$$= \begin{cases} \lambda(f_1f_2f_3) + \lambda(f_5f_6f_4) \\ \lambda(f_1f_2f_4) + \lambda(f_5f_6f_3) \end{cases} \qquad (V.18)$$

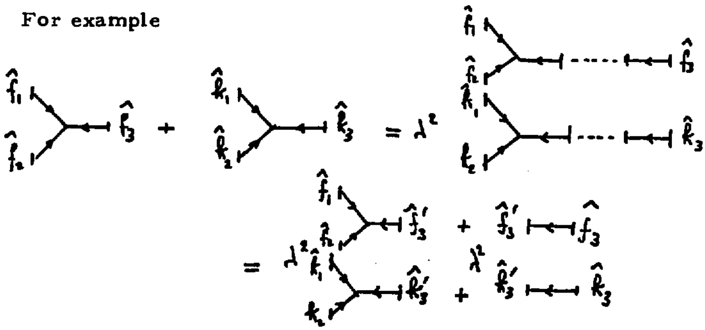
$$= \begin{cases} \lambda(f_6f_1f_2) + \lambda(f_5f_3f_4) \\ \lambda(f_5f_1f_2) + \lambda(f_6f_3f_4) \end{cases}$$

Such a baryonium may be obtained by interaction of three baryons with a strength $\lambda^3$ since

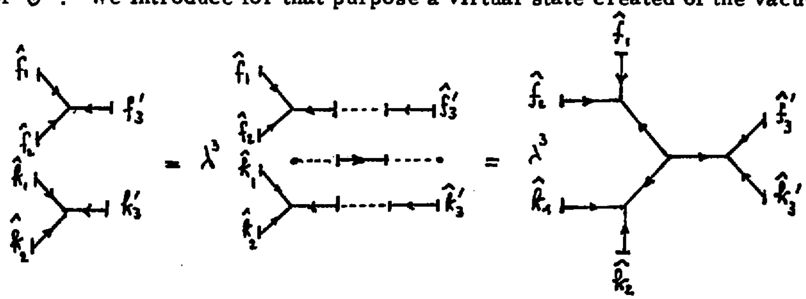


(V. 19)

Let us consider an example of reaction in which a covector (an antiquark) is not immediately available to fusion with a contravector (a quark) and form a scalar (an observable). In such a case the only way to get a covector is to form with two contravectors a dot product (a covector) and then use it to obtain an observable. For example



If such a reaction occurs one does not see the necessity of each baryon to wait the presence of the other baryon to decay into a baryon and a meson. One must thus think that in such a reaction the remaining baryons must interact. How can they do since there is not any covector available ? One must then take two vectors (and why not $\vec{f}'_3$ and $\vec{k}'_3$ before they combine to the baryon ?) to form a pseudo-vector of $\mathcal{C}^*$. We introduce for that purpose a virtual state created of the vacuum to get



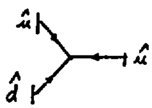
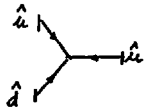
We have thus obtained a $B_4$ baryonium through the formation of a pseudo-vector with two vectors. Hence one may write

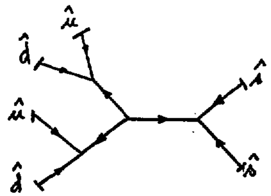
$$B_1 + B'_1 = \lambda^2 M_o + \lambda^2 M'_o + \lambda^5 B_4$$

or with the flavour vectors

$$(f_1f_2f_3) + (k_1k_2k_3) = \lambda^2 (f_3\bar{f}'_3) + \lambda^2 (k_3\bar{k}'_3) + \lambda^5 (f_1f_2k_1k_2f'_3k'_3)$$

The search of such a baryonium has been made with the reaction $pp \to K^+K^+X$ [15)]

 +  $= \lambda^2 (s\bar{u}) + \lambda^2 (s\bar{u}) + \lambda^5 B_4^o$

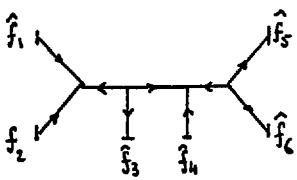
where $B_4^o$ = 

$$\left\{ \begin{array}{l} B = 2 \\ J = 0,\ 1,\ 2,\ 3 \\ Q = 0 \\ I = 0 \quad I_3 = 0 \\ c = b = t = 0 \\ S = -2 \\ P = + \end{array} \right.$$

We note that the $\lambda^3$ strength should have been obtained for the formation of the $B_4^o$ baryonium by interaction of pp $\Xi^-$ or pp $\Xi^o$ but the triple rearrangement is certainly less probable that a double rearrangement

$$(pp\,\Xi^-) \rightarrow \lambda^3 (B_4^o) + \lambda^3 (p)$$
$$(pp\,\Xi^o) \rightarrow \lambda^3 (B_4^o) + \lambda^3 (\Delta^{++})$$
$$(pp) \rightarrow \lambda^5 (B_4^o) + \lambda^2 (K^+) + \lambda^2 (K^+)$$

One can get also a diagram with four vertices which represents a mesonium $M_4$ since

$M_4$ = 

$$= (f_1 f_2 \bar{f}_3 f_4 \bar{f}_5 \bar{f}_6)$$
$$= ((\vec{f}_1 \wedge \vec{f}_2) \wedge \vec{f}_3^{\,*}) . (\vec{f}_4 \wedge (\vec{f}_5^{\,*} \wedge \vec{f}_6^{\,*}))$$
$$= (\vec{g}_3^{\,*} \wedge \vec{f}_3^{\,*}) . (\vec{f}_4 \wedge \vec{g}_7) \qquad \text{(V.20)}$$

and we have seen above that $\vec{g}^* \wedge \vec{f}^*$ is a vector belonging to $\mathscr{C}$ and $\vec{f} \wedge \vec{g}$ a vector belonging to $\mathscr{C}^*$.

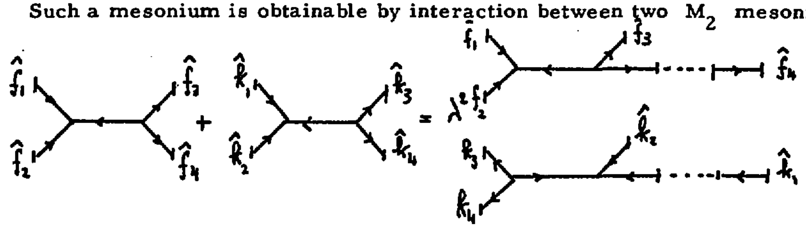
We note that a mesonium is always constructed with the scalar product of pseudo-vectors or vectors of the same structure.

The baryon number of $M_4$ is zero, its spin 0 or 1 or 2 and its parity negative.

The non leptonic decomposition of $M_4$ will involve the exchange of one, two or three gluons and one gets

$$
\begin{aligned}
M_4 \rightarrow\ & \lambda^3 (f_1 f_2 g) + \lambda^3 (\bar{g} \bar{f}_3 \bar{k}) + \lambda^3 (k f_4 m) + \lambda^3 (\bar{m} \bar{f}_5 \bar{f}_6) \\
& \rightarrow \left\{ \begin{array}{l} \lambda (f_2 \bar{f}_3) + \lambda^2 (f_1 \bar{f}_5) + \lambda^2 (f_4 \bar{f}_6) \\ \lambda (f_2 \bar{f}_3) + \lambda^2 (f_1 \bar{f}_6) + \lambda^2 (f_4 \bar{f}_5) \\ \lambda (f_1 \bar{f}_3) + \lambda^2 (f_2 \bar{f}_5) + \lambda^2 (f_4 \bar{f}_6) \\ \lambda (f_1 \bar{f}_3) + \lambda^2 (f_2 \bar{f}_6) + \lambda^2 (f_4 \bar{f}_5) \end{array} \right. \\
& \rightarrow \left\{ \begin{array}{l} \lambda (\bar{f}_3 f_4) + \lambda^2 (f_1 \bar{f}_5) + \lambda^2 (f_2 \bar{f}_6) \\ \lambda (\bar{f}_3 f_4) + \lambda^2 (f_1 \bar{f}_6) + \lambda^2 (f_2 \bar{f}_5) \\ \lambda (f_1 f_2 f_4) + \lambda (\bar{f}_3 \bar{f}_5 \bar{f}_6) \end{array} \right. \\
& \rightarrow \left\{ \begin{array}{l} \lambda (f_4 \bar{f}_6) + \lambda^2 (f_1 \bar{f}_5) + \lambda^2 (f_2 \bar{f}_3) \\ \lambda (f_4 \bar{f}_6) + \lambda^2 (f_1 \bar{f}_3) + \lambda^2 (f_2 \bar{f}_5) \\ \lambda (f_4 \bar{f}_5) + \lambda^2 (f_1 \bar{f}_6) + \lambda^2 (f_2 \bar{f}_3) \\ \lambda (f_4 \bar{f}_5) + \lambda^2 (f_1 \bar{f}_3) + \lambda^2 (f_2 \bar{f}_6) \end{array} \right.
\end{aligned}
\qquad (V.21)
$$

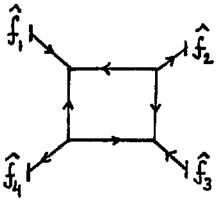
Such a mesonium is obtainable by interaction between two $M_2$ mesonium since




(V.22)

$$
(f_1 f_2 \bar{f}_3 \bar{f}_4) + (k_1 k_2 \bar{k}_3 \bar{k}_4) = \lambda^2 (k_1 \bar{f}_4) + \lambda^2 (f_1 f_2 \bar{f}_3 k_2 \bar{k}_3 \bar{k}_4)
$$

One can now imagine an other kind of diagram with four vertices on a square [16)]




$$= (f_1 \bar{f}_2 f_3 \bar{f}_4) = M'_4 \qquad (V.23)$$

The baryon number of such an object is zero, the spin 0, 1 or 2 and the parity positive. It is thus a mesonium like $M_4$. However such a diagram is not easily expressible in term of scalar and dot products of the flavour vectors as for $M_4$. Using the graphical technique for the vector analysis [7)] one can see that such a diagram is expressible in term of the scalar product, the dot product and a rank 2 tensor of the vectors $\vec{f}_1 \vec{f}_2^{\,*} \vec{f}_3 \vec{f}_4^{\,*}$. Since one has not yet defined the tensor of higher rank than 1 in the colour space, one cannot give an analytic expression of $M'_4$ analog to that of $M_4$ but one may write it with the Levi-Civita tensors

$$M'_4 = \sum_{\substack{c_1 c_2 c_3 c_4 \\ c'_1 c'_2 c'_3 c'_4}} q_{c_1}^{f_1} \bar{q}_{c_2}^{f_2} q_{c_3}^{f_3} \bar{q}_{c_4}^{f_4} \, \varepsilon_{c_1 c'_1 c'_2} \, \varepsilon_{c_2 c'_2 c'_3} \, \varepsilon_{c_3 c'_3 c'_4} \, \varepsilon_{c_4 c'_4 c'_1} \qquad (V.24)$$

The non-leptonic decomposition of $M'_4$ may lead to a baryon-antibaryon pair or with the crossing rule (III. 12) to a pair of mesons

$$M'_4 = (f_1 \bar{f}_2 f_3 \bar{f}_4) = \begin{cases} \lambda^4 (f_1 k_1 k_2) + \lambda^4 (\bar{f}_2 \bar{k}_2 \bar{k}_3) + \lambda^4 (f_3 k_3 k_4) + \lambda^4 (\bar{f}_4 \bar{k}_4 \bar{k}_1) \\ \lambda^2 (f_1 \bar{f}_2) + (\bar{f}_4 f_3) \\ \lambda \; M_2 (f_1 f_3 \bar{f}_2 \bar{f}_4) = \begin{cases} \lambda^2 (f_1 \bar{f}_2) + \lambda^2 (f_3 \bar{f}_4) \\ \lambda^2 (f_1 \bar{f}_4) + \lambda^2 (f_3 \bar{f}_2) \end{cases} \end{cases} \qquad (V.25)$$

and the same result by a cyclic permutation of the indices.

## Conclusion

We have shown that the observed meson and baryon may be considered as rank zero tensors (scalars) in the colour space. The existence of the Pauli principle is thus linked to the fact that such scalars are often obtained by contraction of a vector with a pseudo-vector (axial vector) which is nothing but the dot product of two vectors of the $\mathcal{C}$ colour space. Such a geometrical insight gives an easy comprehension of the possible existence of meson and mesonium, baryon and baryonium. A rule, corresponding to the crossing rule of the Graphical Vector Analysis [8] has been established. It defines the non leptonic decay of elaborated quark configurations into usual mesons and baryons. Such a rule is in disagreement, in the more general case, with the J OZI rule stated by Imachi et al. [10] " which forbids the breaking of a junction ". It means that the J OZI rule predicts that any mesonium or baryonium will decay into mesons, baryons and antibaryons while our rule predicts that mesonium may decay into mesons only while baryonium have always to decay into at least one baryon and mesons. It should be interesting to verify experimentally such a statement. One has finally to note that we have introduced in the fusion process a $\lambda$ strength independent of the quarks involved. It seems however that such a statement is somewhat crude and that one has to introduce a different strength for each flavour involved. But such a refinement does not alter essentially our results.

I would like to thank my colleagues for many helpful discussions and especially Professor Lambert and Professor Giffon. I am indebted to my collaborator, Dr. J. Meyer, for many comments and a careful reading of the manuscript.

References


1) G. P. Canning, Phys. Rev. D 8 (1973) 1151

2) G. P. Canning, Colour symmetry and hadrodynamics, preprint NBI (1974) 74-2

3) G. Chew, J. Finkelstein, J. P. Sursock and G. Weissmann, Nucl. Phys. B 136 (1978) 493

4) G. F. Chew, B. Nicolescu, J. Uschersohn and R. Vinh Mau, preprint CERN TH-2635 (1979)

5) G. F. Chew and C. Rosenweig, Physics Reports 41C n°5 (1978)

6) C. Rebbi, Physics Reports 12 C (1974) 1
G. Veneziano, Proceedings of the 12th Rencontre de Moriond (1977), J. Tran Thanh Van Ed.
X. Artru, Nucl. Phys. B 85 (1975) 442

7) E. Elbaz and B. Castel, Graphical Methods of Spin Algebras, Marcel Dekker (New York) 1972

8) E. Elbaz, Traitement Graphique de l'Algèbre de Racah et de l'Analyse Vectorielle, Université Lyon, 1979

9) E. Elbaz et J. Meyer, A graphical method of the vector analysis (to be published)

10) M. Imachi, S. Otsuki and F. Toyoda, Prog. Theor. Phys. 57 (1977) 517

11) K. Iguchi and R. Sugano, Prog. Theor. Phys. 61 (1979) 1725

12) E. Predazzi, The quark parton model, Lectures given at the University of Lyon, Lycen 7835 (C)
R. Vinh Mau, IVth European Antiproton Symposium, Strasbourg (1978), IPNO, TH 78-27

13) H. Högaasen and P. Sorba, CERN TH-2586 (1978)

14) H. M. Chan, Proc. 19th Int. Conf. High Energy Physics, Tokyo (1978)

15) T. F. Kycia, Proc. 19th Int. Conf. High Energy Physics, Tokyo (1978)

16) M. Uehara, Proc. 19th Int. Conf. High Energy Physics, Tokyo (1978)