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DESIGN OF A HYPERBOLIC MICROWAVE METALLIC LENS

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ABSTRACT

Due to problems caused by multiple reflections in the cavity walls of the EBT fusion research device, the use of a horn becomes important for the directivity of waves in the millimetric range. An ordinary dielectric lens cannot be used because of plasma-wall interactions. Microwave metallic lenses, designed to focus the energy into a plane wave, can improve the directivity considerably. By implementing a 70-GHz standard-gain horn with a delay-type hyperbolic lens, which consists of a solid metallic disk with a number of equal size small holes has indicated a gain of 15 dB over the no lens case.

1. INTRODUCTION

The design of a multichannel microwave interferometer at 70 GHz for the density profile measurements in EBT necessitates the use of standard-gain horns. In this frequency range, directivity of the horn is not adequate and as a result multiple reflections in the cavity are a severe problem. The reflection coefficient of a metallic wall may be given by[1]

$$R = 1 - \frac{3}{\sqrt{2}} \delta_s \left(\frac{\omega}{c}\right) , \tag{1}$$

where

$$\delta_s = \frac{c}{\sqrt{2\pi\sigma\omega}}$$

is the skin depth at the frequency ω, and σ is the conductivity of the material. For aluminum walls, as in EBT, we have $\delta_s \simeq 3 \times 10^{-5}$ cm at 70 GHz and $R \approx 100\%$. Hence, the multiple reflections from the wall may distort the information at the receiving horn. Fortunately, the directivity of the horn can be improved by means of a microwave lens that produces a narrow beam, reducing multiple reflections considerably.

As we know, a microwave waveguide lens makes use of the optical properties of the rays; that is, it produces a constant-phase (a plane wave) wavefront from a spherical waveform. Figure 1 illustrates the basic construction principle of the lens that is discussed in the following sections. With proper surface selection, the ray phases can be adjusted to produce a plane wave.

An alternative approach[2] to the design of the lens is to decrease the diameter of the holes rather than to change the shape of the surface, as a function of distance from the center of the lens. In practice, the tolerances of the hole sizes become important because a large number of holes of various sizes on the surface is required.

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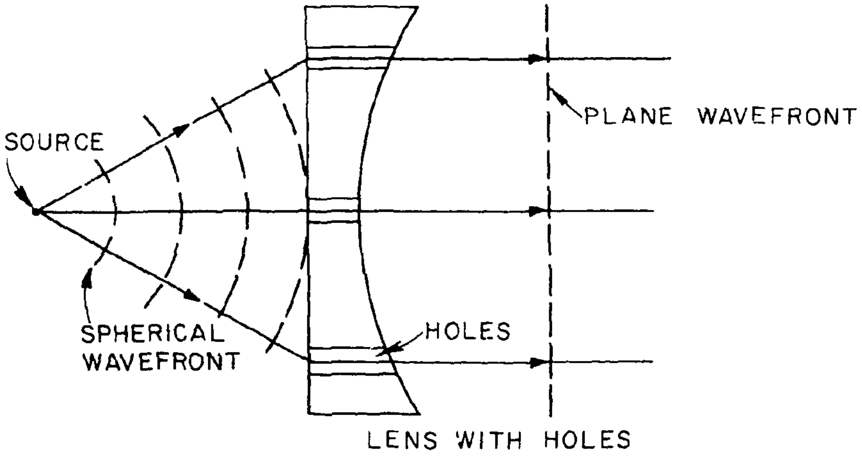

Fig. 1. Wavefronts before and after the lens.

2. BASIC EQUATIONS

The phase velocity, v_p, of a propagating wave in a waveguide is[3]

$$v_p = \frac{\omega}{2\pi} \lambda_g \ , \tag{2}$$

where

$$\lambda_g = \frac{\lambda}{\sqrt{1 - (\lambda/\lambda_c)^2}} \ , \tag{3}$$

and λ_c is the cutoff wavelength.

It should be noted from Eqs. (2) and (3) that the waveguide wavelength, λ_g, is greater than the free-space wavelength, λ, and thus the corresponding phase velocity is greater than the free-space velocity. A path of ray that comes out of a focal point, O, is shown in Fig. 2. Considering the requirement of having a plane wavefront at AA′, we may write

$$L + \varepsilon = \sqrt{y^2 + L^2} + \eta\varepsilon \ , \tag{4}$$

where $\eta = \lambda/\lambda_g$ is the refractive index of the waveguide. Here we have made use of the fact that the time of flight of the wave must be the same for all paths from the source to the AA′ plane. Since $\varepsilon = x - (L + s)$, then the above relation reduces to an equation of hyperbola, which is

$$\frac{(x - \alpha)^2}{a^2} - \frac{y^2}{b^2} = 1 \ , \tag{5}$$

where

$$\alpha = \frac{\ell}{\eta - 1} \ ,$$

$$a = \frac{L}{1 - \eta} \ , \tag{6}$$

$$b = L \ ,$$

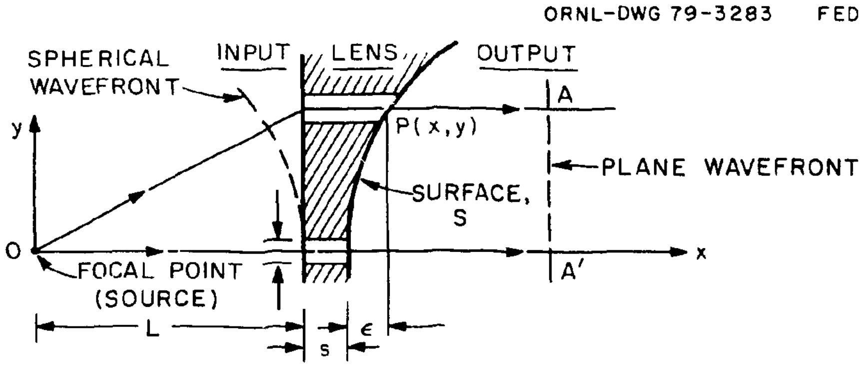

Fig. 2. Geometry of the problem.

and

$$\ell = \eta(L + s) - s \ . \qquad (7)$$

Consequently, the surface shape of the lens must be a hyperbola in order to have a plane wavefront.

3. DESIGN PARAMETERS OF THE LENS

The lens will be used with a microwave horn whose focal length, L, is known. The remaining parameter to be chosen is the refractive index, η, of the lens. We would like to choose η to minimize reflections from the lens. The power reflection coefficient from the lens is given by[4]

$$R_L \simeq \left(\frac{1 - \eta}{1 + \eta}\right)^2 \ . \qquad (8)$$

On the other hand, η depends on the cutoff frequency of the holes. For the circular holes of diameter d, the cutoff wavelength is given by[3]

$$\lambda_{c_{nm}} = \frac{\pi d}{\nu_{nm}} \ , \qquad \text{for } TM_{nm} \text{ modes}$$

and

$$\lambda_{c_{nm}} = \frac{\pi d}{\nu'_{nm}} \ , \qquad \text{for } TE_{nm} \text{ modes} \ .$$

Here, ν_{nm} and ν'_{nm} are the mth roots of $J_n(x) = 0$ and $J'_n(x) = 0$, respectively.

The $\lambda_{c_{nm}}$ of the typical modes that we are interested in are given in Table 1.

Table 1. Modes of TM_{n1} and TE_{n1}

n (m = 1)	$\lambda_{c_{nm}}/d$	
	TM_{n1}	TE_{n1}
0	1.306	0.8198
1	0.8198	1.706
2	0.6118	1.028

The condition for microwave propagation in a metallic guide is

$$1 - \left(\frac{\lambda}{\lambda_c}\right)^2 > 0$$

or $\lambda < \lambda_c$. Therefore, the dominant mode, TE_{11}, propagation is

$$\frac{d}{\lambda} > 0.586 \ . \qquad (9)$$

But, on the other hand, the higher order modes must be avoided. In this case the second higher order mode is TM_{01}; thus,

$$\frac{d}{\lambda} < 0.765 \ . \qquad (10)$$

Combining Eqs. (9) and (10), we write

$$0.586 < \frac{d}{\lambda} < 0.765 \ , \qquad (11)$$

which is the condition for the dominant TE mode propagation in the circular waveguide. Furthermore, making use of the definition of η, we obtain

$$0 < \eta < 0.642 \ . \qquad (12)$$

4. THE DESIGN OF 70-GHz LENS FOR THE STANDARD-GAIN HORN

A 70-GHz microwave interferometer utilizes a standard-gain horn, which is shown in Fig. 3. This pyramidal microwave horn is made by flaring the waveguide in both planes. The directivity of this horn is poor, causing intolerable multiple reflections in the cavity. The lenses, which are used to focus the energy into a plane wave, are necessary.

Let us compute the design parameters of a waveguide-type hyperbolic lens that we have discussed. The dimensions of a TRG series V861 standard-gain horn are $L = 9.164$ cm, $A = 4.064$ cm, and $B = 3.309$ cm. We may begin by choosing the diameter of the holes to be $d = 0.125$ in. = 3.175 mm. Then from Eq. (3) and $\lambda_c = 1.706\ d$, we find $\eta = 0.6115 < 0.642$ so that the criterion, Eq. (12), is satisfied. Using the value of η in Eq. (8), we find $R_L = 6\%$ from the lens surface.

The rest of the design parameters of the lens are easily obtained from Eq. (6) after having chosen $s = 0.5$ cm (see Fig. 4). They are $\alpha = -13.91$ cm, $a = 23.59$ cm, $b = 9.16$ cm, $C = 1.39$ cm, and $D = 5.08$ cm.

The lens contains a total of 123 holes, which are placed on a brass disk in a triangular pattern. The web between the holes is about $w = 0.127$ mm. With this horn-lens combination, we are able to get a gain of 15 dB over the no lens case on the EBT cavity, which has a diameter of 22 in.

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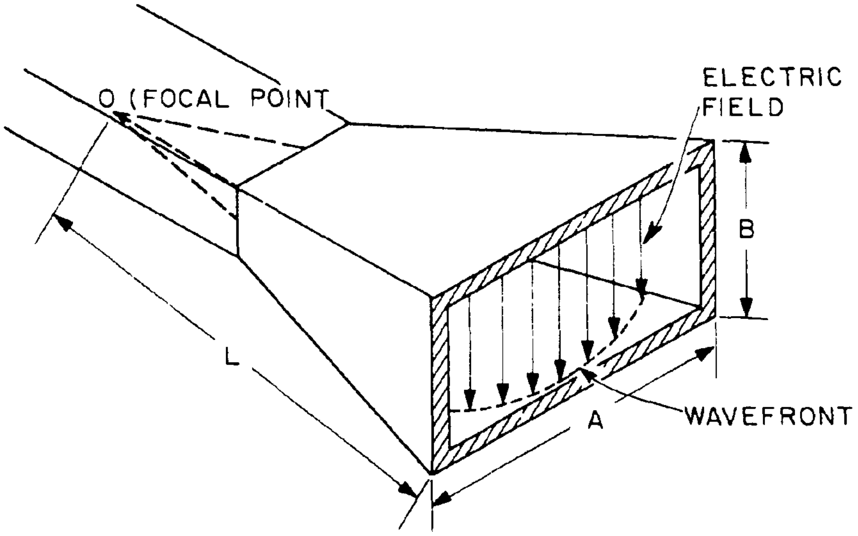

Fig. 3. Typical pyramidal horn.

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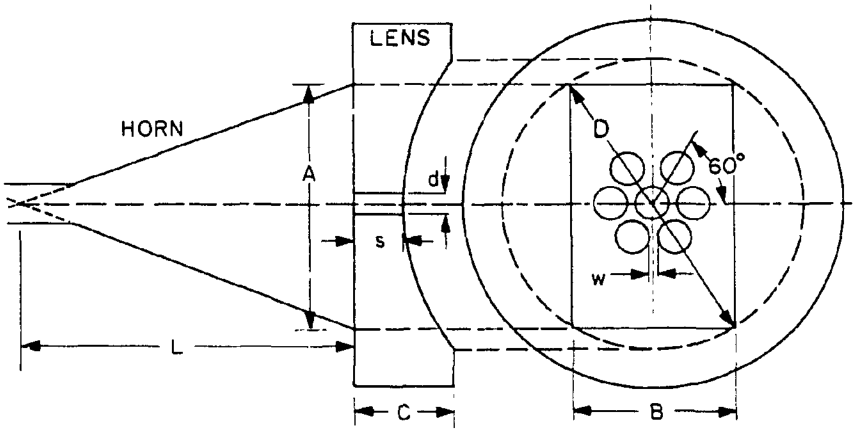

Fig. 4. The geometry of 70-GHz horn-lens combination and the hole pattern. The dimensions are: L = 9.164 cm, A = 4.064 cm, B = 3.309 cm, C = 1.39 cm, D = 5.08 cm, d = 3.175 mm, s = 5 mm, and w = 0.127 mm. There are 123 holes on the lens surface.

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