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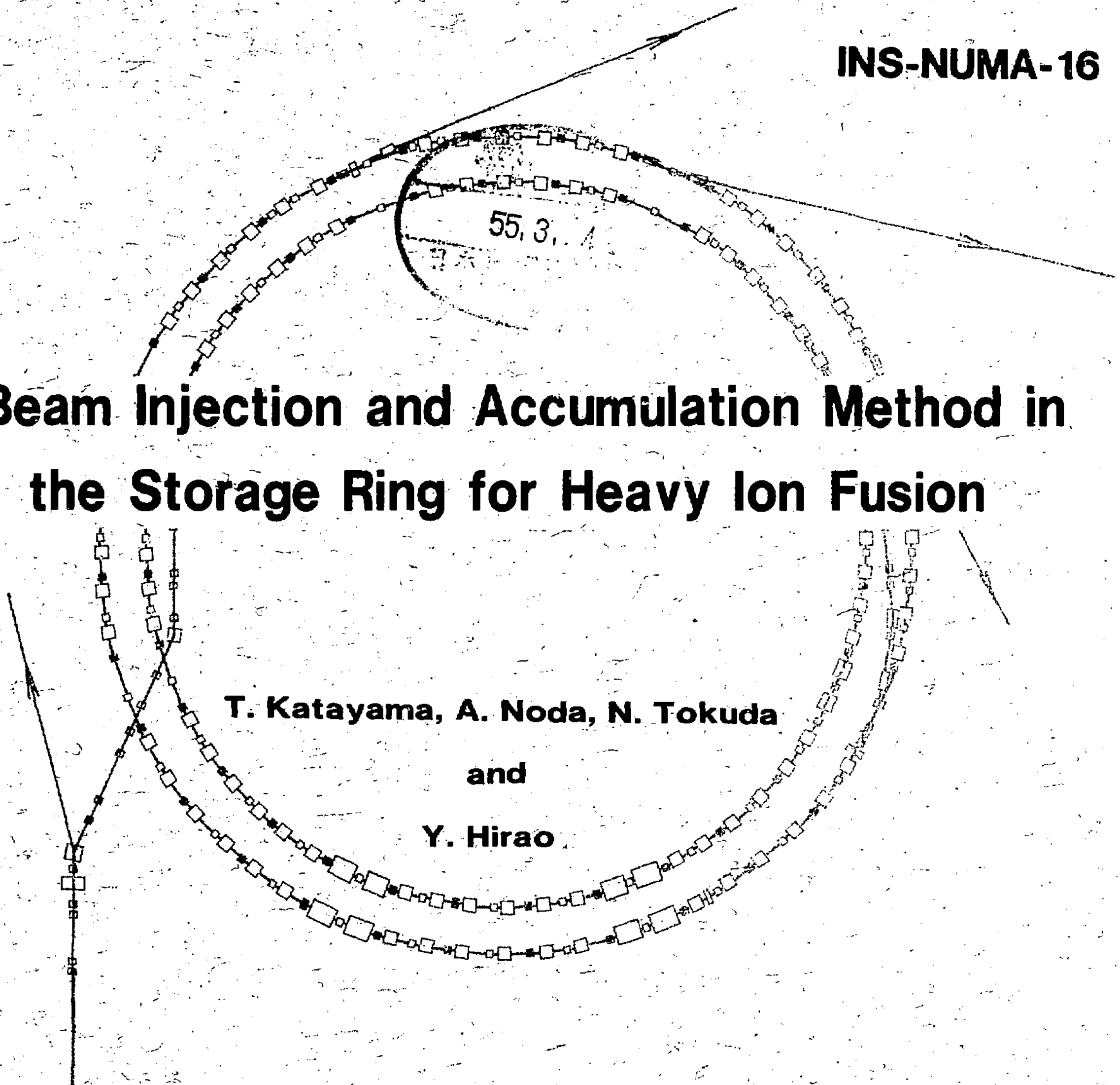

Beam Injection and Accumulation Method in the Storage Ring for Heavy Ion Fusion

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Beam Injection and Accumulation Method in the Storage Ring for Heavy Ion Fusion

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Abstract

A combination of multiturn injection and RF stacking is proposed as an efficient beam injection method in storage rings for heavy ion fusion program. Five turn injection in each transverse phase space and four RF stackings give total stacking turns of 100 which is a result of compromizing the tolerable emittances and momentum spread in the ring. Space charge limit and coherent beam instabilities are investigated and it is found that the most severe limit is transverse coherent instability but it will be managed by the use of sextupole and octupole magnetic fields. Assuming a charge exchange cross section as 1×10^{-15} cm^2, the e-folding life time is estimated at 180 ms, while the stacking time is 40 ms.

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1. Introduction.

Heavy-ion inertial fusion program has become more promising through the intense works[1~3] on the high-energy heavy-ion accelerator during the past three years. The heavy ion method is superior to those of the other particle beams because of its drastic reduction in the peak current requirement to the order of 1 kiloampere (particle current). This reduction of current is allowed by the high energy per particle considering its range and energy relations. At present it is a consensus among the accelerator physicists that such high current of heavy ions could be produced, handled, transported and focused on a pellet by the use of conventional high energy accelerator engineerings, especially RF linacs with storage rings or induction linacs. It is also true, however, that many kinds of research and development should be pursued, for example it is a serious problem that heavy ion beam should be accumulated by ∿ 100 turns in the limited emittances and momentum spread without any significant beam loss and should be compressed to the small bunches in the storage rings.

In the present paper, the combination of multiturn injection in the two transverse phase spaces and RF stacking in the momentum space, is proposed as an efficient beam accumulation method which in principle brings about very small beam loss during the accumulation process. Details of the design of the accelerator are given for the ion, U^{1+} at the workshop, but the proposed method takes rather long period and might be generally favored for the long life ions such as Xe^{8+}.

2. Requirements on the storage rings

First we will start from R.O. Bangerter's three cases of target data at this workshop which is listed in Table 1 for convenience. Emittance consideration in beam lines gives an upper limit of the allowable transverse emittances in the storage ring as 30 π mm·mrad (unnormalized). The momentum spread at the ejection from the storage ring, should be lower than ± 0.4 % because the momentum spread at the target is assumed at ± 2 % and the bunch compression factor in the beam transport lines is designed at 5.

Table 1 3 cases of target data and beam parameters.

E(MJ)	P_p(TW)	T(GeV)	r(mm)	t(ns)	t_p(ns)	g
1	100	5	2	20	6	8
3	150	10	2.5	40	16	30
10	300	10	4	70	20	120

CASE	A	B	C
E(MJ)	1	3	10
N($\times 10^{15}$)	1.25	1.875	6.25
T(GeV)	5	10	10
E_p(MJ)	0.6	2.4	6
N_p($\times 10^{15}$)	0.75	1.5	3.75
I_p(A)	2×10^4	1.5×10^4	3×10^4
I_{av}(A)	10^4	0.75×10^4	1.43×10^4

Ions are U^{+1} and following notations are used. E; Beam stored energy, P; Peak power, T; Kinetic energy, r; Target radius, t; Pulse width, g; gain of the pellet, N; No. of ions. Subcript p refers to peak value at the end of pulse.

Table 2 Ring parameters

Case	A	B	C	
Number of rings (n_r)	7	4	9	
Harmonic number	6	3	2	
Particles / ring	1.79	4.69	6.94	($\times 10^{14}$)
Particles / bunch	0.30	1.56	3.47	($\times 10^{14}$)
Emittance	30 π	30 π	30 π	($\times 10^{-6}$ m·rad)
Average radius	59.8	83.1	97.1	m
Radius of curvature (B = 5T)	31.6	44.9	44.9	m
Circumference	375.6	522	610	m

Other parameters of the storage rings are given in Table 2 which are determined by the consideration of space charge power limit in beam lines (Courant-Maschke formula), limited tune shift in rings for accumulation ($\Delta\nu = 0.25$) and bunch lengths before and after the compression.

Momentum spread of the beam from the injector linac is assumed at $\pm 2 \times 10^{-4}$ after the debuncher, and the phase spread in the ring after the multiturn injection could be 2π, which means that the beam is completely debunched. The longitudinal emittance, ε_L, of the beam in the ring is

$$\varepsilon_L = \Delta\phi \cdot \Delta T = 105.6 \quad (\mathrm{keV \cdot rad}) \qquad (1)$$

where T denotes a kinetic energy of each nucleon in the ion. In the present paper numerical values are calculated for the case A, while the results for other two cases are also listed in Table 5.

3. Multiturn injection

Ions are first injected in the horizontal phase space of the injection ring by 5 turns, whose diameter is six times larger than that of storage rings. The reason why 5 turns are used is given in the following paragraph. Beam is ejected from the injection ring by the fast ejection method and its transverse phase spaces are interchanged with each other in the beam transport lines from the injection ring to the storage rings. Then beam is injected in each horizontal space of three storage rings by two turn injection method, whose tune values of betatron oscillation are adjusted to half integer at this period. This process is repeated two times and other four storage rings are filled with two-turn beams. After the two turn injection process, the tune value of betatron oscillation of each storage ring is adjusted to integer plus three quarters and beam is injected in each storage ring by three turns. Total layout of the injection ring and the storage rings is illustrated in Fig. 1.

In order to reduce the beam loss at the septum of the inflector during beam injection process and to minimize the dilution factor in each phase space, five turn injection is applied for the injection ring and storage rings, whose process is as follows.

1) The tune value of horizontal betatron oscillation should be adjusted to half integer and the beam is injected in the ring during the period of 2 τ_o, where τ_o is one revolution time in the ring.

2) After two turn injection, the position of the septum of the inflector should be moved by the distance of ∿ 10 mm in the transverse phase space within a time of 1/100 τ_o in order to reduce the beam loss at the septum less than 1 %. The horizontal tune value of the ring also should be changed from half integer to integer plus three quarters, when the tune shift due to the already injected two turn beams and one turn beam to be newly injected, is compensated.

3) Beam is subsequently injected in the horizontal phase space during the time interval of 3 τ_o instead of 4 τ_o, because the tune shift due to the space charge of successively injected beam, is significantly large and phase advance of the betatron oscillation per revolution is varied largely from $\pi/2$.

Details of the multiturn injection method are given in Appendix in the present paper.

Dilution factor of the emittance during the whole process of 5 turn injection is calculated at 2.4 in each phase space. The emittance of the linac beam is given by

$$5 \times \pi\varepsilon_{linac} \times 2.4 = 30\ \pi \times 10^{-6} \quad (m\cdot rad) \quad , \quad (2)$$

$$\pi\varepsilon_{linac} = 2.5\ \pi \times 10^{-6} \quad (m\cdot rad) \quad , \quad (3)$$

and normalized emittance is

$$\pi\varepsilon_n = \pi\varepsilon_{linac} \cdot \beta\gamma$$

$$= 0.534\ \pi \times 10^{-6} \quad (m\cdot rad) \quad , \quad (4)$$

which is smaller compared with the value estimated by the linac group at this workshop. But the peak current of the linac beam can be reduced to ∿ 50 mA in our method, then the such small emittance will be obtained.

4. RF stacking

The injected beam in the storage ring by the five turn injection method, is completely debunched. It is captured adiabatically by the RF separatrix and is accelerated to the stacking orbit, when the rate of change of momentum for the sychronous particle is given by

$$\frac{dp/dt}{p} = \frac{f_{rev}}{E_N \beta^2} \frac{q}{A} \cdot eV \cdot \Gamma \quad (5)$$

where f_{rev} is a revolution frequency around the ring, E_N is a total energy of each nucleon, $\beta = v/c$, q/A is a charge to mass ratio and

$\Gamma = \sin\phi_s$. The fractional momentum difference between the injected orbit and the bottom of the stacked region is designed at 1.5 %, the acceleration period is 5 ms, and the required RF voltage is 356 kV. The period of phase oscillation during the acceleration is 1.48 ms.

During a period of acceleration from the bottom to the top of the stacked region, the RF voltage should be reduced to avoid an undesirable energy spread of the stacked beam in the stacked region. Final RF voltage is determine as the area of the separatrix is just equal to the longitudinal phase space area of the injected beam, 105.6 keV•rad. In order to cover the longitudinal phase space area, S, of the injected beam by the separatrix, minimum RF voltage is given by the following relation.

$$S = \left(\frac{hqeV}{A}\right)^{1/2} \alpha(\Gamma) \frac{16\beta}{h} \left(\frac{E_N}{2\pi|\tilde{\eta}|}\right)^{1/2} , \qquad (6)$$

where h is a harmonic number and $\tilde{\eta}$ is defined as

$$\tilde{\eta} = \frac{1}{\gamma_{tr}{}^2} - \frac{1}{\gamma^2} . \qquad (7)$$

Other notations concerning the synchrotron oscillation can be found in reference 4. Substituting numerical values in the relation, minimum voltage 81.6 keV is obtained. Phase oscillation period at the final voltage is 3.09 ms and the necessary time to change adiabatically from the initial bucket to the final one is given by

$$T = \frac{1 + \kappa}{2(1 - \kappa)} \left(\frac{1}{\omega_2} - \frac{1}{\omega_1}\right) , \qquad (8)$$

where ω_1 is an angular frequency of phase oscillations associated with the initial bucket, ω_2 is that of the final bucket and κ is a quantity related to the phase space efficiency of the process. Substituting numerical values and κ is assumed to be 0.9, T is 2.43 ms. Thus the shape of the envelope of the RF voltage is shown in Fig. 2.

Next we must consider the relations among the number of RF stackings, the compression voltage and the final momentum spread in the storage ring. For simplicity, we assume that the momentum spread after n times RF stacking is

$$n \times \left(\frac{\Delta p}{p}\right)_i \quad , \qquad (9)$$

where $(\Delta p/p)_i$ represents a initial momentum spread of $\pm\ 2 \times 10^{-4}$. The compression voltage including the effect of space charge and momentum spread is given by[5)]

$$\frac{eV}{2\pi\gamma A mc^2} = \frac{3N_b q h^2 r_o g}{A\gamma^3 R\cdot\Delta\phi_o\cdot\Delta\phi_{MIN}\ (\Delta\phi_o + \Delta\phi_{MIN})} + \frac{1}{q\cdot\Delta\phi^2_{MIN}}\ h|\tilde{\eta}|\beta^2\left(\frac{\Delta p}{p}\right)^2 \qquad (10)$$

where

$mc^2 = 931.5$ MeV, $r_o = 1.547 \times 10^{-18}$ m

q = charge state (= 1)

g = geometrical factor (= 1.5)

N_b = number of particles/bunch

$\Delta\phi_o$ = initial phase spread (= 2π)

$\Delta\phi_{MIN}$ = final phase spread (= $0.2\ \pi$).

The final phase spread is determined so as the phase compression factor in the ring is 10, when the tune shift, $\Delta\nu$, during the compression is assumed to be 2.5. The compression voltage is

$$eV = 0.1424\ (n^2 + 0.1798) \quad \text{(MeV)} \qquad (11)$$

Next we should calculate the separatrix height, H, related to the compression voltage and the final momentum spread, $\Delta p/p$, by using the following formulae,

$$H = \left(\frac{hqeV}{A}\right)^{1/2} \gamma \frac{\beta}{h} \left(\frac{E_N}{\pi|\tilde{\eta}|}\right)^{1/2} \text{(keV)} \quad , \qquad (12)$$

$$\frac{\Delta p}{p} = \frac{1}{\beta^2} \frac{2H}{E_N} \quad . \qquad (13)$$

Numerical results are given in Table 3.

Table 3

n	eV (MeV)	H (keV)	$\Delta p/p$ (%)
0	0.025	22.29	0.107
2	0.595	107.48	0.517
4	2.304	211.46	1.016
6	5.152	316.21	1.520
8	9.139	421.15	2.024
10	14.266	526.17	2.529

n: Number of RF stacking
eV: Compression voltage
H: Separatrix half height
$\Delta p/p$: Momentum spread (full width) after the compression

If we assume that the final momentum spread in the storage ring should be less than 1 %, the maximum number of RF stacking is 4.

As is given in preceding sections, the number of multiturn injection is 5 in each transverse phase space, then the required peak current, I_p, for the linac is

$$5^2 \times I_p \tau_o n = eN \qquad (14)$$

where N is a number of the particle in each storage ring, and

$$I_p = \frac{1.6 \times 10^{-19} \times 1.79 \times 10^{14}}{5^2 \times 4 \times 6 \times 10^{-6}} = 47.7 \quad \text{mA} . \qquad (15)$$

5. Beam instabilities

5-1) Space charge limit

Space charge limit in a circular ring is given by

$$N = \frac{2\pi\Delta\nu}{B \cdot r_p} \left(\frac{A}{q^2}\right) \varepsilon\beta^2\gamma^3 \quad , \qquad (16)$$

where B is a bunching factor, r_p is a classical proton radius 1.547×10^{-18} m and ε is an unnormalized emittance. In the injection ring, emittance should be averaged over horizontal and vertical phase spaces, each of which has a numerical values of 30 π mm·mrad and 2.5 π mm·mrad. The averaged emittance is

$$\pi\varepsilon = \pi\sqrt{\varepsilon_x \cdot \varepsilon_y} = 8.66\,\pi \times 10^{-6} \quad (\text{m}\cdot\text{rad}) . \qquad (17)$$

If we take a bunching factor as 1.0, space charge limit in the injection ring is 9.8×10^{13} particles. In the storage ring, emittance is 30π mm·mrad both in the horizontal and vertical phase spaces and the space charge limit is 3.38×10^{14} particles. In both rings space charge limit exceeds the designed circulating currents.

5-2) Resistive wall instability

Next we will consider the longitudinal and transverse coherent resistive wall instabilities. Longitudinal coherent limit is given by Keil-Schnell criterion[6] of

$$\left|\frac{Z_L}{n}\right| < F \frac{\beta^2 \gamma E_o}{e} \frac{|\tilde{\eta}|}{I} \left(\frac{\Delta p}{p}\right)^2 \qquad (18)$$

where Z_L/n is a longitudinal coupling impedance and its numerical value should be examined further for heavy ion machine. However we will adopt here the value of 25 Ω which is scaled from the experimental values at ISR & CPS[7]. Thus longitudinal coherent limit is 59 A for the momentum spread of $\pm 2 \times 10^{-4}$ and there could be no problem related to the longitudinal coherent instability in the injection ring and the storage rings.

On the other hand the transverse coherent instability limit is given by

$$\left|\frac{Z_\perp}{n}\right| < 4F \frac{AE_o}{qe} \frac{\nu\beta\gamma}{I_o R} \left(\left|(n - \nu)\tilde{\eta} + \xi\right| \frac{\Delta p}{p} + \frac{\partial \nu}{\partial a^2} \Delta a^2\right) \qquad (19)$$

where $Z_\perp/n$ is a transverse coupling impedance and ξ is a chromaticity. The first term in the bracket shows the effects of sextupole fields and second term the octupole fields. In the storage ring the momentum spread is fairly large $\sim$1 % and the correction due to the sextupole fields is much efficient than that of octupole fields. When we introduce chromaticity of −10, the intensity limit is 0.54 A or 2.0×10^{13} particles, which is much smaller than the space charge limit. The e-folding growth time of this instability is given by [8)]

$$\tau = \frac{4\pi\nu\cdot\gamma A E_o/qe}{cI\ \mathrm{Re}(Z_\perp)} \simeq 46\ \mathrm{ms}\ , \qquad (20)$$

if we assume the radius of the vacuum chamber as 5 cm and the stored current as 4.78 A. This formula, however, holds under the condition that there is no sextupole and no octupole corrections. Then we expect that TCI can be managed by their corrections during a total accumulation time of $\sim$40 ms.

6. Life time of the beam in the storage ring.

In the high-intensity heavy-ion storage ring, a beam loss according to an electron transfer proc 's between ions in the beam, $A^{n+} + A^{n+} \rightarrow A^{(n+1)+} + A^{(n-1)+}$, may be a severe problem. The loss rate is estimated as follows.

The loss rate is given by

$$\alpha \equiv \frac{1}{N}\frac{dN}{dt} \qquad (21)$$

$$= n_{lab} v_{cm} \sigma_{cm}\ . \qquad (22)$$

The meanings of symbols are referred to Table 4, where machine parameters are also listed. The density of ions in the ring is

$$n_{lab} = \frac{N}{2\pi RS} , \tag{23}$$

on the assumption that the beam is completely debunched. The beam is to be stored in the ring as shown in Fig. 3. Then the cross section of the beam is

$$S = \pi ab + b\Delta x_p , \tag{24}$$

where a and b are obtained from the beam emittance ε and the average betatron amplitude function, $\bar{\beta}$,

$$a = \sqrt{\varepsilon_x \bar{\beta}} , \tag{25}$$

$$b = \sqrt{\varepsilon_y \bar{\beta}} , \tag{26}$$

The beam spread due to a momentum dispersion is

$$\Delta x_p = \eta \frac{\Delta p}{p} . \tag{27}$$

The dispersion function is approximately

$$\eta = \bar{\beta}^2/R . \tag{28}$$

$$= 1.81 \quad (m)$$

Then the beam cross section is numerically calculated with values listed in Table 4, and the density is

$$n_{lab} = 4.62 \times 10^{14} \quad (m^{-3}) . \tag{29}$$

The speed of the ion in the center of mass frame is given by

$$\beta_{cm}^{\,2} = \left[\frac{\beta}{2}\frac{\delta' p}{p}\right]^2 + \left(\beta\gamma \sin\frac{\theta}{2}\right)^2 . \qquad (30)$$

As the momentum of ions are considered to distribute as in Fig. 4, the typical momentum difference between the ions which will collide with each other, is

$$\frac{\delta' p}{p} = \frac{\Delta p}{p} \frac{2a}{2a + \Delta x_p} , \qquad (31)$$

where $\delta' p/p$ is determined so that areas of the parallelogram and the rectangle are equal. Then the first term of eq.(30) is 1.65×10^{-4}. The maximum collision angle in the laboratory is evaluated by

$$\theta = 2\sqrt{\epsilon_x/\bar{\beta}} , \qquad (32)$$

and the second term is numerically 3.61×10^{-4}. Then the velocity in the c.m. frame is

$$v_{cm} = 1.19 \times 10^5 \text{ (m/s)} . \qquad (33)$$

which corresponds to the kinetic energy of 75 eV.

According to papers[9,10] the cross sections for the electron transfer process of various ions are estimated to be of the order of 10^{-15} cm^2. Therefore a value of 1×10^{-15} cm^2 is appropriately adopted here for U^{1+}.

Now the loss rate can be numerically calculated, and

$$\alpha = 5.50 \text{ (s}^{-1}\text{)} . \qquad (34)$$

The life time, the inverse of the loss rate, is

$$\tau = 0.182 \quad (s) \quad . \tag{35}$$

which means that the beam will be lost by the amount of 20 % during stacking process of $\sim$40 ms. Therefore if such a amount of beam loss is serious, even though it does not occur at the localized position such as septum of the inflector but could be uniformly lost around the ring, another kind of ion of low intrabeam charge exchange cross section such as Xe^{8+} should be used.

Table 4 List of symbols and machine parameters for case A

N	number of ions in the ring	1.79×10^{14}
n_{lab}	density of ions	
R	mean radius of the ring	59.8 m
S	cross section of the beam	
v_{cm}	velocity of ions in the center of mass frame	
α	loss rate	
β	ratio of ion velocity to that of light	0.208 (21 MeV/u)
$\bar{\beta}$	average betatron amplitude function	10.4 m
γ	$1/\sqrt{1 - \beta^2}$	1.0224
$\frac{\delta' p}{p}$	momentum difference between colliding ions	
$\frac{\Delta p}{p}$	total momentum spread	1.6×10^{3}
ε_x	emittance in the horizontal direction	30×10^{-6} m·rad
ε_y	emittance in the vertical direction	30×10^{-6} m·rad
η	dispersion function	
θ	collision angle in the laboratory frame	
σ_{cm}	cross section of the electron transfer process	1×10^{-19} m^2
τ	life time	

Table 5 Summary of the calculations for three cases.

Case	A	B	C
No of multiturn	5 × 5 = 25	25	25
Dilution factor for multiturn	2.4	2.4	2.4
Normalized emittance of the injected beam from the linac	$0.534\pi\times10^{-6}$ (m·rad)	$0.759\pi\times10^{-6}$ (m·rad)	$0.759\pi\times10^{-6}$ (m·rad)
LINAC peak current	48 mA	125 mA	158 mA
RF stacking number	4	4	4
Momentum spread after the compression in the ring	1 %	1 %	0.899 %
required period for one RF stacking	10 ms	28 ms	47 ms
Total injection period for each ring	40 ms	112 ms	188 ms
Total accumulation period for n_r rings	∿40 ms	∿112 ms	∿200 ms
Fusion repetition rate	20 Hz	8 Hz	5 Hz
Space charge limit in the stacking ring	8.6×10^{13}	1.8×10^{14}	5.7×10^{14}
storage ring	3.3×10^{14}	8.7×10^{14}	1.4×10^{15}
Longitudinal coherent limit	300 A	more safe than in case A	more safe than in case A
Transverse coherent limit	0.54 A	"	"
Growth time of transverse instability	46 ms	"	"
Compression voltage	2.3 MV	2.2 MV	1.5 MV
Compression time	102.4 μs (17 turns)	∿100 μs	∿100 μs

Appendix

Scheme of multiturn injection into transverse phase space

For the purpose of reducing the beam loss due to the collision with an inflector septum, the following process was studied.

1st) Before the beam injection the tune value of the betatron oscillation should be adjusted to be half integer taking account of the space charge effect due to the intensity of single turn.

2nd) The beam from the linac is injected by a two turn injection method during the time interval of 2 τ_o, where τ_o is the revolution time of the beam.

3rd) The position of the septum in the phase space should be moved in a time interval of $\frac{1}{100}\tau_o$ in order to reduce the beam loss to around 1 %.

4th) The horizontal tune value is shifted to integer +3/4 taking account of the effect of the space charge force due to the two turn beam already stacked in the ring and the first one turn beam to be injected in the next step.

5th) The beam from the linac should be three turn injected during the time interval 3 τ_o and just before the three turn the position of the septum is moved from x = 11 mm to x = 21 mm in a time interval of $\frac{1}{100}\tau_o$.

The acceptance of the ring and the emittance of the beam from the linac were assumed to be $30\pi \times 10^{-6}$ and $2.5\pi \times 10^{-6}$ m·rad (unnormalized), respectively.

In the second process, the transfer matrix of one turn, M_o, can be written as,

$$M_o = \begin{pmatrix} \cos\{2\pi(N+\frac{1}{2}+\Delta\nu)\}, & \beta\sin\{2\pi(N+\frac{1}{2}+\Delta\nu)\} \\ -\frac{1}{\beta}\sin\{2\pi(N+\frac{1}{2}+\Delta\nu)\}, & \cos\{2\pi(N+\frac{1}{2}+\Delta\nu)\} \end{pmatrix}, \qquad \text{(A-1)}$$

where N is an integer and $\Delta\nu$ is a tune shift due to the space charge effect of the beam and α $\left(= -\frac{1}{2}\beta'\right)$ is assumed to be zero. We represent the beam ellipse in the phase space just one turn after the injection as ($a \cos\theta + x_c$, $b \sin\theta$), where a and b are the length of horizontal and vertical axises of the beam ellipse and x_c the position of the center of the beam as shown in Fig. A-1. Using

$$M_o = \begin{pmatrix} -\cos\Delta\mu, & -\beta\sin\Delta\mu \\ \frac{1}{\beta}\sin\Delta\mu, & -\cos\Delta\mu \end{pmatrix}, \qquad \text{(A-1)'}$$

where $\Delta\mu = 2\pi\cdot\Delta\nu$, the position of the beam from the closed orbit after another one revolution is given by

$$x = -\cos\Delta\mu(a\cos\theta + x_c) - \beta\, b\sin\theta\cdot\sin\Delta\mu\ . \qquad \text{(A-2)}$$

The maximum value of x is obtained for the value of θ which satisfies

$$\frac{dx}{dx'} = 0 \qquad \text{(A-3)}$$

and

$$\tan\theta = \frac{\beta b}{a}\tan\Delta\mu\ . \qquad \text{(A-4)}$$

For such a value of θ, the next relation holds

$$x = -x_c\cos\Delta\mu \pm \sqrt{a^2\cos^2\Delta\mu + \beta^2 b^2\sin^2\Delta\mu}\ . \qquad \text{(A-5)}$$

The maximum value of x in the equation (A-5) when $\Delta\mu$ is varied, is obtained as

$$x_{max} = \beta \cdot b \sqrt{\left| 1 - \frac{x_c^2}{a^2 - b^2\beta^2} \right|} \qquad (a \neq b\beta) \qquad \text{(A-6)}$$

$$= |x_c| + a \qquad (a = b\beta)$$

In our case the numerical values are as follows;

$$\begin{aligned} \beta &\simeq 15 \text{ m}, \\ a &= 4.0 \times 10^{-3} \text{ m} \\ b &= 6.25 \times 10^{-4} \\ x_c &= -4.5 \times 10^{-3} \text{ m} \ . \end{aligned} \qquad \text{(A-7)}$$

and the maximum value is

$$x_{max} = 0.0106 \text{ m} \ . \qquad \text{(A-8)}$$

Therefore when the position of the inflector septum is shifted outward as large as 11 mm after 2 turn injection in $\frac{1}{100}\ \tau_o$ interval, then no further beam collision with the septum is expected.

In the third process, it is necessary to estimate the required high voltage of the pulsing system for bump magnets. The bump magnets should be located 90° up and down stream of the inflector. In case A, the required deflection angle of the bump magnet is estimated to be 6.733×10^{-4} rad in order to distort the closed orbit by the distance of 11 mm at the position of the inflector. If each bump magnet is devided into 6 units which are excited in parallel, then the necessary deflection angle for each unit is $\frac{1}{6} \times 6.733 \times 10^{-4} = 1.122 \times 10^{-4}$ rad. The field strength of the bump magnet is calculated at 177 G for the case A, where the total momentum of U^{14+} is 47349 MeV. The required current for each bump magnet unit is calculated at 704.8 A if one turn coil is used.

If we assume a critical damping, the rise time t_r from 5 % to 95 % of the maximum value is represented as

$$t_r = 1.14 \frac{L}{Z_o} \quad , \tag{A-9}$$

where L and Z_o are the inductanse of the coil and the characteristic impedance of the circuit system, respectively. The inductance of the magnet is given by the relation

$$L = N^2 \cdot \mu_o \frac{w \cdot \ell}{d} \cdot F \quad , \tag{A-10}$$

where μ_o is a permeability of the air, w, ℓ, d are width of the pole, the length of the magnet, the gap height of the magnet, respectively, and F is the ratio of the leakage flux defined by

$$F = \frac{\phi}{\phi_1} \quad , \tag{A-11}$$

where ϕ and ϕ_1 are the total flux in the iron yoke and the total flux which goes through the pole face, respectively. Assuming the following values

$$\begin{aligned} w &\simeq 0.07 \text{ m} \\ \ell &\simeq 1.0 \text{ m} \\ d &= 0.05 \text{ m} \\ F &\simeq 2.0 \quad , \end{aligned} \tag{A-12}$$

L is 3.52 μH and the characteristic impedance Z_o should be 67 Ω so as to make the rise time t_r as short as 60 μs. The required high voltage V_o is given by

$$V_o = 2 Z_o \cdot I$$

and is 94.44 kV, which is a manageable value.

Then the horizontal tune of the betatron oscillation is moved to an integer +3/4 including the space charge effect due to the beam already injected into the ring and the beam which will be injected in the next one turn.

After tuning, the beam from the linac is injected by three-turn injection as is illustrated in Fig. A-2 (a) ∿ (c). In the calculation, the additional effect of space charge due to newly injected into the ring is taken into account. The tune shift is given by the formula

$$\Delta\nu = \frac{-NB \cdot r_p \; q^2}{2\pi\varepsilon\beta^2\gamma^3 A}, \qquad \text{(A-14)}$$

where B, r_p and ε are the bunching factor, the classical radius of proton and the unnormalized emittance of the beam. In the case A, this effect is estimated to be $\Delta\nu_1 = -0.07$ and the transfer matrices of the 1-st, 2-nd and 3-rd turns, M_1, M_2 and M_3, are given by

$$M_i = \begin{pmatrix} \cos 2\pi\left(N + \frac{3}{4} + (i-1)\Delta\nu_1\right), & \beta \sin 2\pi\left(N + \frac{3}{4} + (i-1)\Delta\nu_1\right) \\ -\frac{1}{\beta}\sin 2\pi\left(N + \frac{3}{4} + (i-1)\Delta\nu_1\right), & \cos 2\pi\left(N + \frac{3}{4} + (i-1)\Delta\nu_1\right) \end{pmatrix}. \qquad \text{(A-15)}$$

$i = 1, 2, 3.$

Due to space charge tune shift, the beam will come back to the septum position after three turns as is shown in Fig. A-2. Hence it is needed to shift the septum position to x = 21 mm in the time interval $(2 + \frac{99}{100})\,\tau_o < t < 3\tau_o$. The required current for the bump magnet is 576.7 A, and the high voltage is calculated at 77.3 kV.

During the multiturn injection process, the beam with the emittance of $2.5\,\pi \times 10^{-6}$ m·rad (unnormalized) is injected into the ring with the acceptance of $30\,\pi \times 10^{-6}$ m.rad by the amount of $\{(2+3) - \frac{1}{100} \times 2\}$ turns. Then the dilution factor due to this multiturn injection is

$$D = \frac{30}{2.5 \times \{(2+3) - \frac{1}{100} \times 2\}} = 2.41 \quad . \qquad \text{(A-16)}$$

This factor is close to the value of the usual multiturn injection, but in this scheme beam loss due to collision with the inflector septum is reduced to 2 % of the total beam.

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Figure captions

Fig. A-1. The beam injection is executed with the beam which is shaped to an ellipse as is shown in the figure. The number of betatron oscillation per resolution is tuned to half integral value including the space charge effect due to the one-turn injected beam. During the first turn, the beam ellipse revolves by 180° in a phase space but during the second turn the beam ellipse rotates in the manner as is illustrated in the figure due to the space charge effect of another one-turn beam. In order to avoid the beam loss, the septum is shifted to x = 11 mm in the time interval $(1 + \frac{99}{100})\tau_o < t < 2\tau_o$, where τ_o is a revolution time of the beam.

Fig. A-2. In this process the tune value of betatron oscillation is adjusted to integer plus three quarters including the space charge effect due to one-turn beam.

(a) In the first turn, the beam ellipse rotates in the phase space by 90°.

(b) In the second turn, the tune is shifted by the space charge force due to another one-turn beam and beam ellipses rotate as is shown in the figure.

(c) After three turns, the first beam comes back to the septum position as is illustrated in the figure because of the tune shift due to additional space charge effect. In order to reduce beam loss, the septum is shifted from x = 11 mm to x = 21 mm in the time interval $(2 + \frac{99}{100})\tau_o < t < 3\tau_o$.

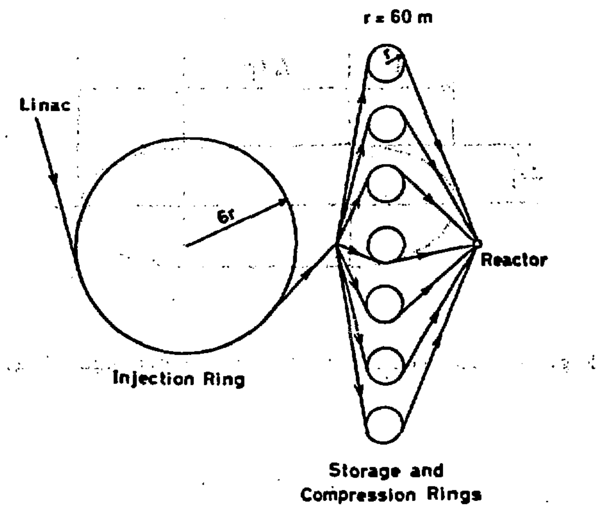

Fig.1 Schematic layout of the injection ring and storage rings for case A.

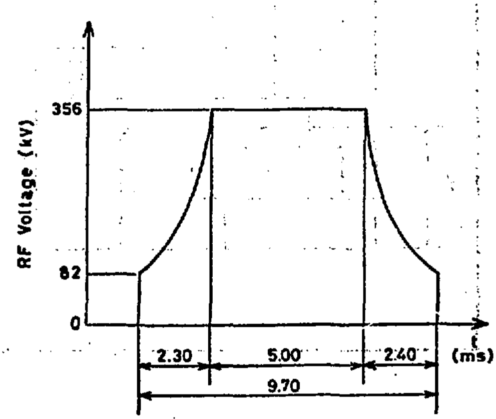

Fig.2 An envelope of the RF field for the momentum stacking.

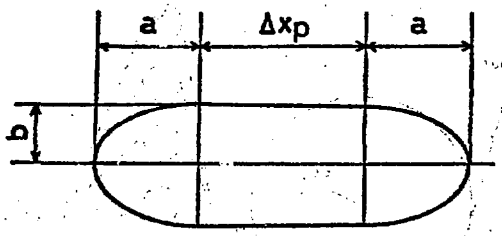

Fig. 3. The beam profile in the storage ring.

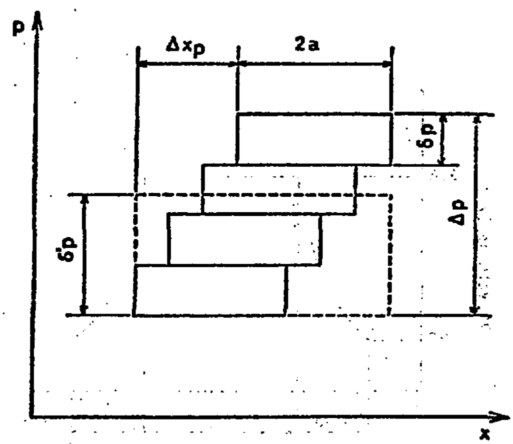

Fig. 4. Four beam pulses of different momenta are stacked in the storage ring. The typical momentum spread $\delta' p$ is determined so that the area of the ractangle (dashed line) and that of the four pulses are equal.

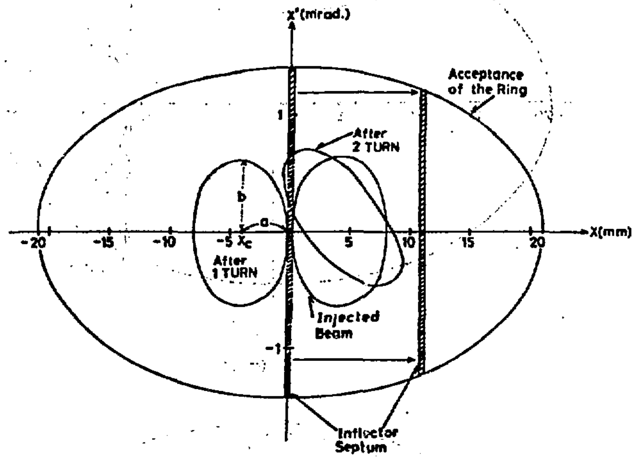

Fig. A-1 Schematic diagram of the first 2-turn-injection with the half integral tune value.

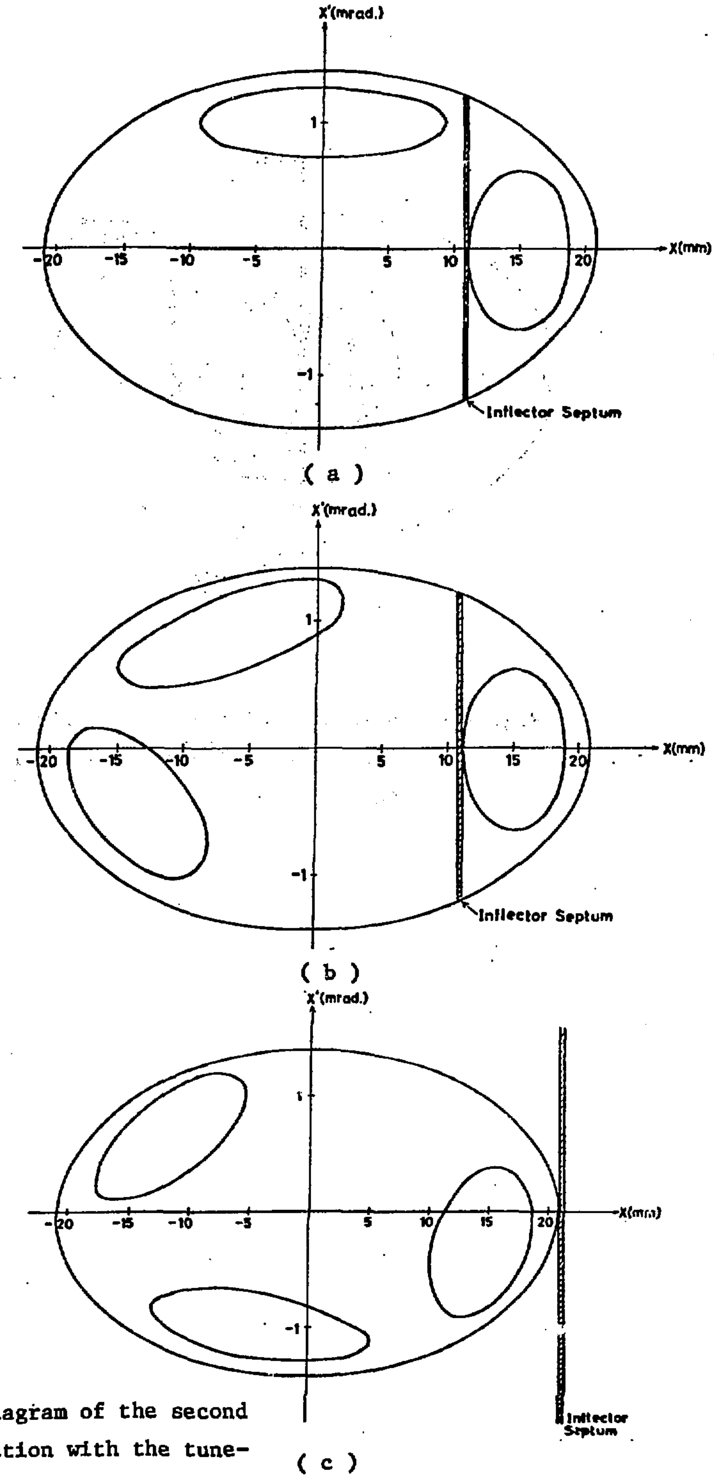

Fig. A-2

Schematic diagram of the second 3-turn-injection with the tune-value of integer plus three quarters.