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A.Szanto de Toledo and Mahir S. Hussein

Instituto de Física - Universidade de São Paulo

UNIVERSIDADE DE SÃO PAULO
INSTITUTO DE FÍSICA
Caixa Postal - 20.516
Cidade Universitária
São Paulo - BRASIL

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Instituto de Física da Universidade de São Paulo

Lab. Pelletron - Caixa Postal 20516 - São Paulo, SP - Brasil

ABSTRACT

Effects arising from the existence of lower-ℓ cutoff (ℓ_{min}) in Heavy-Ion fusion processes are discussed within the framework of the statistical theory. Both the fluctuation cross-section as well as the angular cross-correlation function are seen to be qualitatively affected by ℓ_{min}.

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The existence of a lower-ℓ cutoff in angular momentum that limits the complete fusion cross-section of heavy-ions has been extensively discussed in the last few years[1-5]. Both the Time Dependent Hartree-Fock (TDHF) calculations[2] as well as the semiclassical coupled channel model calculations of the Copenhagen group[3a] have persistently predicted the inhibition of fusion for low impact parameters. The usual interpretation of the effect is that these low partial waves contribute to quasi-fusion processes. A classical picture of this has been put forward by Wong[6], according to which the nuclei will not fuse for head-on collisions whenever the kinetic energy of the fastest nucleons exceeds the average nucleon binding energy $B \sim 8$ MeV. Thus $\ell_{min}(E) \neq 0$ whenever $\frac{1}{2}m\ (V_F + V_{rel})^2 \geq B$ where V_F is the Fermi velocity and V_{rel} is the relative ion-ion velocity at the top of the barrier. Consequently one is led to a picture of fusion in ℓ-space as a window cut at low-ℓ by ℓ_{min} and at higher-ℓ by a higher, critical, value ℓ_{crit}. Although the theoretical predictions of Ref. 2 and Ref. 3a were based on simplified models for the heavy-ion reaction process, the fact that these models are different in details, makes their final common predictions rather hard <u>not</u> to accept. Further support for the existence of ℓ_{min} can be cited in connection with "fusion" of water droplets[7]. It becomes clear, therefore, that an unambiguous experimental verification of the existence of ℓ_{min} in the fusion process of heavy-ions could be a very stringent test of the theoretical models for these processes.

Several experiments were designed with the final aim of observing such effects, but no clear evidence was found[1,5]. These experiments were based on the measurement of the non-fusion cross-section that presumably was attributed to the quasi-fusion or deep-inelastic processes. Since the expected magnitude of the effect influences a small fraction of the reaction cross-section, analysis based on absolute cross-section measurements do not lead to unambiguous results.

In the present Letter, we show that the existence of low-angular-momentum-cutoff must sensitively affect the main characteristics of statistical quantities such as angular cross-correlation functions and, in particular, the angular distribution of 0^+ -final state transitions in zero channel-spin reactions.

In so far as the angular cross-correlation function[8] is concerned, the recent work of Braun-Munzinger and Barrette[9] has demonstrated that the coherence angle, θ_c, for heavy-ion reactions is larger than commonly assumed. Rather than being related to the geometrical extension of the system, the authors of ref. 9 presented evidence that θ_c is directly related to the width of the distribution of the partial cross-sections σ_J. As we show below, the presence of ℓ_{min} further increases the value of θ_c and would qualitatively change the shape of the diffraction pattern of the angular cross-correlation function.

To be specific, we consider the reaction $^{12}C(^{16}O,\alpha)^{24}Mg$ at E_{CM} = 30 MeV. This system has been extensively studied and it exhibits a high degree of selectivity in populating discrete states. This choice of incident energy is low enough to allow contributions from compound processes, but is high enough to surpass the estimated threshold energy (in the center of mass system)[6), $E_0 \sim 27$ MeV, above which low-impact parameters events do not fuse. Therefore at $E_{CM} = 30$ MeV, $\ell_{min} \neq 0$ is expected. In this connection TDHF calculation predicts[2) a value of $E_0 \sim 27$ MeV for the symmetrical system $^{16}O + ^{16}O$. The relative contributions of the partial cross-sections $\sigma_J(E^*,I)$ for the excitation of final states (E^*,I) of the residual nucleus are depicted in figure 1. Calculations were performed for the ground-state and $E^* = 15$ MeV transitions using the code STATIS[10). Among all the 0^+ - final state transitions, the ground-state has the largest cross-section . As it can be seen in figure 1a, at higher excitation energy, e.g. $E^* = 15$ MeV, the magnitude of the cross-section for 0^+ - transition is reduced, but, on the other hand, the $\sigma_J(I=0)$ distribution is centered at a lower J-value, resulting in a higher degree of sensitivity on the J_{min} - truncation as will be shown below. In figure 1, the limitation imposed by the critical angular momentum for fusion has not been taken into account. Assuming $\ell_{min} = 0$, the $\sigma_J(I)$ distribution should be truncated at $J_{max} = 16$ in order to reproduce the measured fusion cross-section for this system[11). As a result of this truncation, the cross-sections for high spin final states shown in fig. 1b decrease significantly.

When the low-ℓ cutoff is applied[6), the mean value of the compound nucleus angular momentum that contributes

in the reaction process (fig. 1a) is increased, i.e. the center of gravity $\langle J\rangle$ of the σ_J distribution is shifted to higher J-values, and consequently a higher degree of alignment between the compound nucleus angular momentum and the orbital angular momentum of the emitted particle occurs. This implies that: a) the mean angular distribution of the emitted light particle approaches the classical "flywheel" $1/\sin\theta$ - limit; b) angular distributions of transitions to zero-spin final states oscillate more rapidly, with a period $\Delta\theta_{cm} \sim \pi/\langle J\rangle$, and display a larger "peak-to-valley" ratio. These features are clearly illustrated in figure 2.[12)]
It can be seen from figure 1b that transitions to low spin final states are the ones mainly affected, partly due to the low values of angular momentum carried by the evaporated particles. Notice that ratios of cross-sections to different spin final states, $\frac{\sigma(I\neq 0)}{\sigma(I=0)}$, are sensitive to the value of J_{min} (as they are also sensitive to J_{max}[13)]). Furthermore, the J_{max}-value was not adjusted, in order to conserve the value of the fusion cross-section, each time J_{min} was changed. However, the conclusions of this Letter depend very little on how the transmission coefficients are selected.[14)]

In addition, a detailed study of angular cross-correlation functions, $C(\theta,\theta')$[8)], also furnishes information about J_{min}. This is clear from the fact that the presence of J_{min} results in a reduction of the effective number of independent channels that contribute to a given transition. A consequence of this fact is the increase in magnitude of the oscillations in the angular cross-correlation function, and an increase in the value of the coherence angle, θ_c. As was shown in reference 9, the coherence angle for heavy-ion compound reactions is related to the angular momentum width ΔL of the partial-cross section distribution σ_J. An empirical relationship between θ_c and ΔL was deduced[9] as being

$$\theta_c \sim \frac{1.4}{\Delta L} \tag{1}$$

It is clear from figure 1a, that the presence of J_{min} would effectively decrease the width, ΔL, of the J-window, for low-spin final states, leaving, however, the high-spin final states practically unaffected.

To exhibit the dependence of $C(\theta,\theta')$ on the width, ΔL, and the position, L_0, of the peak of the J-windows, we have worked out a simple formula for the angular cross-correlation function using the defining relation given by Brink et al.[8]. In the absence of ℓ_{min}, our result for $C(\theta,\theta')$ is[12] (for zero channel-spins, i.e. $L = J$)

$$C(\theta,\theta') = \left[\cos\left[L_0(\theta-\theta')\right] \frac{\pi(\frac{\Delta L}{2.9})(\theta-\theta')}{\sinh\pi\ (\frac{\Delta L}{2.9})(\theta-\theta')}\right]^2 \tag{2}$$

For large values of the reference angle θ, Equation (2) gives practically identical results as those obtained with the Brink formula[8]. Furthermore, the coherence angle obtained from Equation (2) agrees with that given in Equation (1). Results for $C(90^o,\theta')$ are presented in fig. 3 taking, e.g. $\ell_{min} = 12$. For comparison, we also show the results for $\ell_{min} = 0$. It is clear that the effect of low-ℓ cutoff is pronounced not only in shifting the position of the minima in $C(\theta,\theta')$ from the original ones, $(\theta-\theta') = (n + 1/2)\pi/L_0$; $n = 0, 1, \ldots,$ but also in increasing the value of the coherence angle θ_c, from the original ($\ell_{min} = 0$) value $\theta_c = 23^o$ to about $\theta_c \simeq 28^o$ (for $\ell_{min} \sim 12$). This corresponds to an effective reduction of ΔL from 3.4 to $\sim$ 2.8. It should be noted that such behaviour becomes more pronounced in transitions to excited states (see fig. 3), due to the lower J-value of the peak of the corresponding partial wave distributions (fig. 1a). In other words, the sensitivity to J_{min} is strongly concentrated around the position, L_0, of the peak of the partial wave distribution. As a consequence, the coherence angle, θ_c, is further increased.

This suggests that experimentally one should reach a compromise for the bombarding energy, high enough to allow larger J_{min}-values, but low enough to have measurable compound cross-sections and small L_0-value. In either case transitions to highly excited 0^+ states should be looked for.

In all of the above discussion we have neglected effects due to non-statistical processes. Although at the energies we are considering, direct reactions become important and therefore require a modification of the statistical theory[15], we have, however, accounted roughly for direct processes by fixing the entrance channel transmission coefficients so as to reproduce the experimental total fusion cross-section.

The results of this Letter suggest that accurate measurement of compound differential cross-sections involving spinless particles in the entrance and exit channels and subsequent statistical analysis, involving fluctuation cross-sections and angular cross-correlation functions, should furnish information about low ℓ-cutoff in heavy-ion fusion processes.

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$$\ell^2_{min} \sim \frac{2\mu R_B}{\hbar^2}\left[E_{CM} - \frac{Z_1 Z_2 e^2}{R_B} - \frac{A_2(A_1+A_2)}{A_1}\left[(1+B/\varepsilon_E)^{1/2} - 1\right]^2 \varepsilon_F\right]$$

where R_B is the contact separation and ε_F ($\sim$ 35 MeV) is the Fermi energy.

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$$\frac{d\sigma}{d\Omega} \propto \frac{1}{\sin\theta}\left\{1 + \frac{1}{2}\left[(\sin 2L_0\theta)\,\frac{2\pi\,\frac{\Delta L}{2.9}\,\theta}{\sinh 2\pi\,\frac{\Delta L}{2.9}\,\theta} + (\sin 2L_0(\pi-\theta))\,\frac{2\pi\,\frac{\Delta L}{2.9}\,(\pi-\theta)}{\sinh 2\pi\,\frac{\Delta L}{2.9}\,(\pi-\theta)}\right]\right\};$$

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FIGURE CAPTIONS

Figure 1a:

Partial cross-section distribution for the excitation of residual states of different spins I at E^* (^{24}Mg) = 15 MeV and ^{24}Mg - ground state - (J represents the compound nucleus angular momentum). The arrow indicates the value of J_{max} that reproduces the measured fusion cross-section assuming $J_{min} = 0$.

Figure 1b:

Total cross-section $\sigma_{J_{min}}$ ($E^* = 15$ MeV, I) = $= \sum_{J_{min}}^{J_{max}} \sigma_J$ as a function of the low ℓ-cutoff in the compound nucleus, showing that the dependence of the fluctuation cross-section on J_{min} can be approximated by a Fermi function. (Notice that no J_{max} cutoff has been applied.)

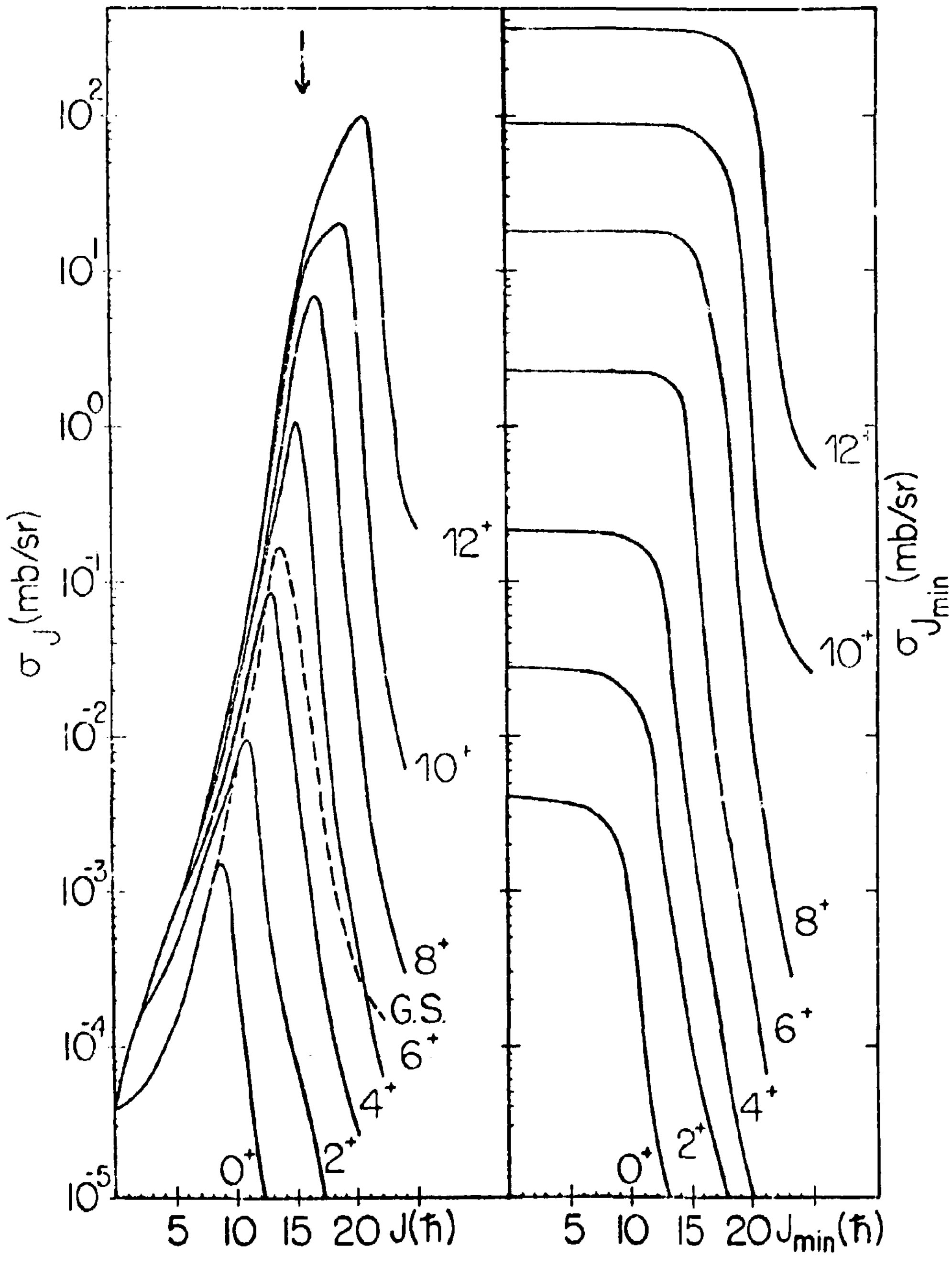
Figure 2:

Differential cross-section for the transition $^{12}C(^{16}O,\alpha)^{24}Mg$ ($E^* = 15$ MeV, I=0) calculated by STATIS, for $\ell_{min} = 0$, 9 and 14 ($J_{max} = 30$). (Curves were normalized to unity at zero degree.)

Figure 3:

Angular cross-correlation functions [dashed

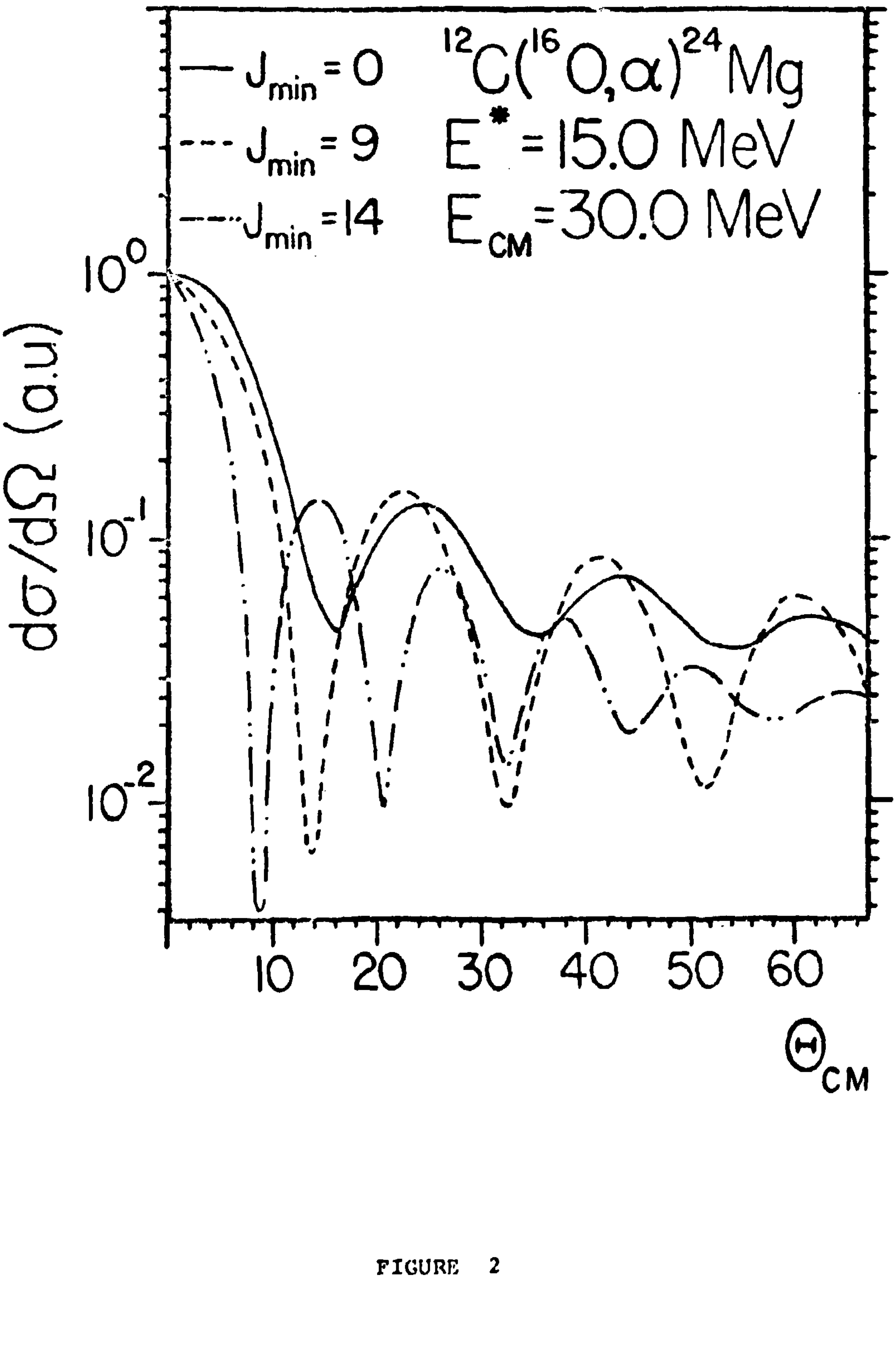
(ℓ_{min} = 12) and dashed-doted (ℓ_{min} = 0) curves for the $^{12}C(^{16}O,\alpha)^{24}Mg$ (g.s.) at E_{CM} = 30 MeV. The envelopes for ℓ_{min} = 0, g.s. (full curve I), ℓ_{min} = 12, g.s. (full curve II) and ℓ_{min} = 0, E^* = 15 MeV (full curve III) are also shown. The coherence angles obtained for curves I, II and III are θ_c = 23°, 28° and 40° respectively.



10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
σ_J(mb/sr)
$\sigma_{J_{min}}$(mb/sr)
12+
10+
8+
G.S.
6+
4+
2+
0+
5 10 15 20 J(ħ)
5 10 15 20 J_{min}(ħ)

(a) (b)

FIGURE 1



— J_{min} = 0
--- J_{min} = 9
—·—·J_{min} = 14
$^{12}C(^{16}O,\alpha)^{24}Mg$
$E^* = 15.0$ MeV
$E_{CM} = 30.0$ MeV
10^0
10^{-1}
10^{-2}
$d\sigma/d\Omega$ (a.u)
10
20
30
40
50
60
Θ_{CM}

FIGURE 2

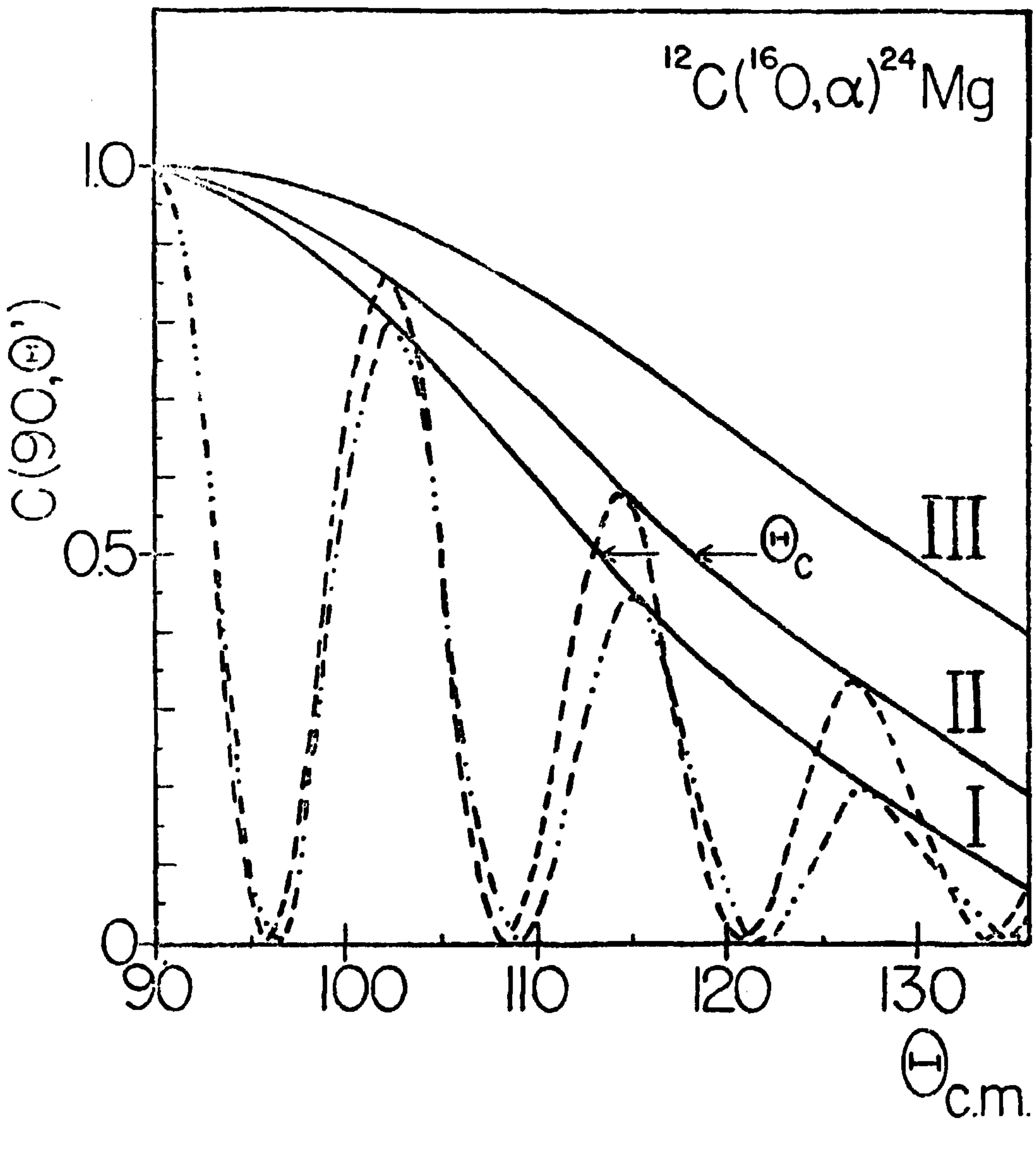

$^{12}C(^{16}O,\alpha)^{24}Mg$
1.0
0.5
0
$C(90,\Theta')$
III
II
I
Θ_c
90
100
110
120
130
$\Theta_{c.m.}$

FIGURE 3