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A method for establishing the parameters of equal - reflection plasma

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In this paper is presented a method to find the conditions of existence for anisotropic different plasmas which present the same reflection coefficient. The analytical dependence of the complex reflection coefficient on the plasma parameters was examinated. Then the numerical calculations were made with the use of a computer. The results of these calculations have been plotted on a special diagram thus allowing the direct determination of the plasma parameters ω_p/ω, ω_b/ω, ν/ω with the reflection coefficient being known from measurements and on of the above mentioned parameters, or to establish what are the distinct sets of values for the plasma parameters ω_p, ω_b, ν for which the same reflection coefficient is obtained. The method also could be applied in two both cases of plasma explored by a means of TEM mode and by guided waves respectively.

1. INTRODUCTION

In the presence of an exterior magnetic field $\vec{B}_o$ the electromagnetic properties of a plasma are drastically modified. In this case the plasma becomes electrically anisotropic i.e. the permeability remains equal to μ_o but its permitivity becomes a tensor [1]. Plasma behaves as a doubly refracting medium and exhibits band-pass characteristics. That is, for a certain frequency ranges, depending upon the plasma properties and the strength and direction of the magnetic field relative to the incident wave, the plasma is transparent to radio waves, while the other frequency ranges it is opaque. In two peculiar cases of propagation namely perpendicular and parallel to $\vec{B}_o$ could be distinguished four principal or characteristic waves [2] whose polarisation does not change during the propagation. These waves exhibits resonance frequencies and cutt-off frequencies who delimit the propagation regions (pass-band) from the evanescent region (stop-band). This means that for specific conditions, radio energy at frequencies well below the plasma frequency can penetrate through the plasma.

In the case in which $\vec{B}_o$ is collineary to one of the coordinate axes say z direction of a rectangular system, the dielectric tensor from which the propagation characteristics can be determined is written:

$$\overleftrightarrow{\varepsilon}_p = \begin{vmatrix} \varepsilon_{11} & -j\varepsilon_{12} & 0 \\ j\varepsilon_{12} & \varepsilon_{11} & 0 \\ 0 & 0 & \varepsilon_{33} \end{vmatrix} \qquad (1)$$

where the element of the matrix are

$$\varepsilon_{11} = 1 - \frac{(\omega_p/\omega)^2\ (1 - j\nu/\omega)}{(1 - j\nu/\omega)^2 - (\omega_b/\omega)^2} \tag{2}$$

$$\varepsilon_{12} = - \frac{(\omega_p/\omega)^2\ (\omega_b/\omega)}{(1 - j\nu/\omega)^2 - (\omega_b/\omega)^2}, \tag{3}$$

$$\varepsilon_{33} = 1 - \frac{(\omega_p/\omega)^2}{1 - j\nu/\omega} \tag{4}$$

The introduced symbols in (2), (3) and (4) have all the well-known significations

$$\omega_p = (\frac{ne^2}{m\varepsilon_o})^{\frac{1}{2}} \tag{5}$$

s the plasma frequency,

n - is the free electron concentration,

e,m - are the electronic charge and mass respectively,

ε_o - is the permitivity of free space,

$\omega/2\pi$- is the wave frequency, ν is the collision frequency and respectively,

$$\omega_b = \frac{e}{m} B_o \tag{6}$$

denotes the electron cyclotron frequency.

If the plane wave sources is such oriented that the propagation vector is parallel to $\vec{B}_o$ (i.e. along the z-axis) the plan polarized E_x, E_y wave impinging on the plasma is splitted into two circularly polarized waves of opposite sense of rotation. The permittivity of the two circularly polarized waves $\varepsilon_{\pm} = \varepsilon_{11} \pm \varepsilon_{12}$ is given by:

$$\varepsilon_{\pm} = 1 - \frac{\omega_p^2(\omega \mp \omega_b)}{\omega[(\omega \mp \omega_b)^2 + \nu^2]} - j\,\frac{\omega_p^2\,\nu}{\omega[(\omega \mp \omega_b)^2 + \nu^2]} \qquad (7)$$

The $\pm$ signes are associated with the two opposite senses of rotation of the circularly polarized waves.

In the cases in which $\vec{B}_o$ is perpendicular on the propagation direction (say along the x axis) the plasma permittivity is written:

$$\overleftrightarrow{\varepsilon}_p = \begin{vmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{22} & -j\varepsilon_{32} \\ 0 & j\varepsilon_{32} & \varepsilon_{22} \end{vmatrix} \qquad (8)$$

where the elements of the matrix are given by:

$$\varepsilon_{11} = 1 - \frac{(\omega_p/\omega)^2}{1 - j\nu/\omega} \qquad (9)$$

$$\varepsilon_{22} = 1 - \frac{(\omega_p/\omega)^2(1 - j\nu/\omega)}{(1 - j\nu/\omega)^2 - (\omega_b/\omega)^2} \qquad (10)$$

$$\varepsilon_{33} = \frac{(\omega_p/\omega)^2\,(\omega_b/\omega)}{(1 - j\nu/\omega)^2 - (\omega_b/\omega)^2} \qquad (11)$$

The permitivity of the ordinary and extraordinary wave respectively are given by :

$$\varepsilon_{or} = 1 - \frac{\omega_p^2}{\omega^2 + \nu^2} - j\,\frac{\omega_p^2 \cdot \nu}{\omega\,(\omega^2 + \nu^2)} \qquad (12)$$

$$\varepsilon_{ex} = \frac{\varepsilon_{11}^2 - \varepsilon_{12}^2}{\varepsilon_{11}} \qquad (13)$$

2. THE REFLECTION COEFFICIENT AND THE WAVE IMPEDANCE OF THE PLASMA REGION

The reflection coefficient R could be expressed in terms of the normalized input impedance of the plasma region by the relationship:

$$R = \frac{z_p - 1}{z_p + 1} \tag{14}$$

where

$$z_p = |z_p| e^{j\Psi} = \frac{1}{\sqrt{\varepsilon_p}} \tag{15}$$

and therefore $|z_p|, \Psi$ may be written as:

$$|z_p| = \frac{1}{[\sqrt{(R_e\varepsilon)^2 + (I_m\varepsilon)^2}]^{\frac{1}{2}}} \tag{16}$$

respectively

$$\Psi = \frac{1}{2} \text{ arc tg } \left(- \frac{I_m\varepsilon}{R_e\varepsilon}\right) \tag{17}$$

where ε could be either $\varepsilon_{\pm}$ or ε_o, ε_{ex} .

The following expression for modulus and argument of the reflection coefficient could be obtained:

$$|R| = \frac{[(|z_p|^2-1)^2 + 4|z_p|^2 \sin^2\Psi]^{1/2}}{|z_p|^2 + 2|z_p| \cos \Psi + 1} \tag{18}$$

$$\rho = \text{arc tg } \frac{2|z_p| \sin \Psi}{|z_p|^2 - 1} \tag{19}$$

3. THE ANALYTICAL AND GRAPHYCAL REPRESENTATIONS OF THE REFLECTION COEFFICIENT

From (14) it follows that waves impedance is expressed in terms of the reflection coefficient by the relationship:

$$|z_p| = \frac{1 - |R|^2}{[R_e(R)-1]^2 + [I_m(R)]^2} + j\,\frac{2I_m(R)}{[R_e(R)-1]^2 + [I_m(R)]^2} \quad (20)$$

By identifying the imaginary and realy terms of the equation (20) it follows that:

$$\frac{[R_e(R) - 1]^2 + [I_m(R)]^2}{2I_m(R)} = \frac{1}{I_m(z_p)} \quad (21)$$

$$\frac{1 - |R|^2}{[R_e(R) - 1]^2 + [I_m(R)]^2} = R_e(z_p) \quad (22)$$

From (19) results:

$$[R_e(R) - 1]^2 + [I_m(R) - \frac{1}{I_m(z_p)}]^2 = [\frac{1}{I_m(z_p)}]^2 \quad (23)$$

Taking into account that in (22), $|R|^2 = [R_e(R)]^2 + [I_m(R)]^2$ and using (23) the following expression can be derived

$$[I_m(R) + \frac{R_e(z_p)}{I_m(z_p)}]^2 + [R_e(R)]^2 = [1 + \frac{R_e(z_p)}{I_m(z_p)}]^2 \quad (24)$$

The expressions (23) and (24) are the analytical forms of two families of circles with radii $r_1 = \frac{1}{I_m(z_p)}$ and $r_2 = \sqrt{1 + [\frac{R_e(z_p)}{I_m(z_p)}]^2}$ respectively and having the centres in $1, \frac{1}{I_m(z_p)}$, and $0, -\frac{R_e(z_p)}{I_m(z_p)}$

In the cartesian representation of the complex plane $R_e(R)$, $I_m(R)$ the two families are represented in fig. 1. The z variation is similar to a shift of the two centres along the line x=0, x=1. The circles of minimum radius are obtained from the (21) and (22) when $I_m(z_p) = 0$ or $R_e(z_p) = 0$ and they have the values 0 and 1 respectively. From the expressions (16), (17) it could be seen that z_p is a function of four variables: ω, ω_p, ν, ω_b. So a graphycal family could be obtained by varying one of the four parameters while the other three remain unchanged.

4. WAVEGUIDE COMPLETELY FILLED WITH PLASMA (AXIAL $\vec{B}_0$)

The method previously presented could be applied both to free space plasma and to a plasma contented in microwave circuit elements.

Opposite to the propagation of the plane wave TEM in unbounded plasma some constraints appear during the propagation of certain types of polarization in a plasma contented in a waveguide due to the presence of the metallic walls.

For instance the shift of the polarizing plane for a rectangular waveguide filled with plasma cannot occur in the presence of a static magnetic field $\vec{B}_0$ parallel to the propagation [1] and if B_0 is parallel to the great wall or to the small wall, only the extraordinary mode or the ordinary mode could be propagated respectively.

If the waveguide is filled with a homogeneous plasma, its wave impedence normalized to the value in the absence of the plasma is

$$z_p = \left[\frac{1 - (\omega_c/\omega)^2}{\epsilon_p - (\omega_c/\omega)^2}\right]^{\frac{1}{2}} \qquad (25)$$

where ω_c denotes the critical frequency of the waveguide without plasma, ε_p - the equivalent dielectric constant of the plasma.

To give a few examples for the manner in which the above method could be applied, some situations have been considered in which a circular waveguide ($\omega_c/\omega = 0.5$) filled with plasma imersed in a magnetic field $\vec{B}_o$ could be found.

1. What value are to be given to the waveguide reflection coefficient (in both module and phase) if the plasma parameters and B_o are known. To solve the problem (25) has been introduced in (23) and (24); in which ν/ω and ω_p/ω have been fixed to the values $\nu/\omega = 0.7$, $\omega_p/\omega = 0.5$ and ω_b/ω has been taken to generate the family of circles. In fig. 2 the centres and the radii of the curves in the families given by (23) and (24) obtained by a computer calculation have been presented. In the figure it can be noticed that the vector in $|R|$ - plane whose end is given by the intersection of the circles for the same $\vec{B}_o$ in the two different families indicates by its length the module of the reflection coefficient and by the angle to the abscissa axis - the phase. Pecularly if $\omega_b/\omega = 1.5$ then $|R| = 0.85$ $\varphi = 120^o$

2. The parameters ν/ω and ω_p/ω for the plasma waveguide are known; what value is to be given to the magnetic field in which plasma is immersed to obtain a desired reflection coefficient with respect to a certain frequency of the wave in the guide. The above procedure could be applied once again, obtaining the two circle families upon which the vector corresponding to the desired reflection coefficient is superposed. If the end of the vector reaches the intersection of two curves for the same B_o in the two families, than this B_o is the one needed; if not the problem is solutionless.

8. Let two circular waveguides filled with two plasmas having $\omega_p/\omega = 2$, $\nu/\omega = 0.7$ and $\omega_p/\omega = 2$, $\nu/\omega = 0.5$ respectively are given what are the magnetic fields B_{01} and B_{02} respectively in which the plasmas have to be immersed to obtain:

a) the same module of reflection coefficient and

b) the same phase of reflexion coefficient.

The procedure above has been done by computer, for each plasma, obtaining families of curves (23) and (24) respectively (fig. 2 and fig. 3). From these figures the points having $\omega_b/\omega = 0.1$; 1.0; 1.5; 2.0 have been chosen and plotted on a $I_m(z_p)/R_e(z_p)$ diagram (fig. 4). The condition of equal module are given by the points to be found on the same circumference centered in the origin of the coordinates. As fig. 4 shows, these points are 1($\omega_p/\omega = 2$, $\nu/\omega = 0.5$, $\omega_b/\omega = 0.5$) and 8($\omega_p/\omega = 2$, $\nu/\omega = 0.7$ $\omega_b/\omega = 2$). To get the parameters of equal phase reflexion coefficient plasmas we have to look for the point layed on the same line which passed through the origin of the coordinates. As fig. 4 shows, these points are 3($\omega_p/\omega = 2$, $\nu/\omega = 0.5$, $\omega_b/\omega = 1.0$) and 6($\omega_p/\omega = 2$, $\nu/\omega = 0.7$, $\omega_b/\omega = 1.5$).

5. CONCLUSIONS

Certain simplifying assumptions were considered in order to perform this study. Namely we assumed a uniform homogeneous and isotropic plasma; the B_o field in which the plasma is immersed was also assumed to be uniform and homogeneous; the influence of the metallic vessel with holding the plasma was assumed to be negligible.

We have also to mention that we have neglected the energetic balance for the interaction plasma-immersion field, being known that there is an increase in concentration and electronic temperature once $\vec{B}_o$ is increased. The laws to which these changes obey are rather complicated depending on the nature and pressure of the gas the design of the vessel and the gradient of magnetic field. We note that the above method can be equally applied without neglecting the changes in electron concentration with respect to $\vec{B}_o$ once the variation law $n = n(B_o)$ is known.

REFERENCES

[1] HEALD, M.A., WHARTON, C.B.: Plasma diagnostics with microwaves (N.Y., Wiley) 1965

[2] QUEMADA, D.: Ondes dans les plasmas (Paris, Herman) 1968.

FIGURE CAPTIONS

Fig. 1. Typical dependence of curves families (23) and (24)

Fig. 2. Superposition of curve families (23) and (24) for $\omega_p/\omega = 2 \quad \nu/\omega = 0.7$

Fig. 3. Superposition of curve families (23) and (24) for $\omega_p/\omega = 2, \quad \nu/\omega = 0.5$

Fig. 4. Equal modulus points $1(\omega_p/\omega = 2,\ \nu/\omega = 0.5,\ \omega_b/\omega = 0.5)$, $8(\omega_p/\omega = 2,\ \nu/\omega = 0.7,\ \omega_b/\omega = 2)$, and equal phase points $2(\omega_p/\omega = 2,\ \nu/\omega = 0.5,\ \omega_b/\omega = 1.0)$ and $6(\omega_p/\omega = 2,\ \nu/\omega = 0.7,\ \omega_b/\omega = 1.5)$ obtained from fig. 2 and fig. 3.

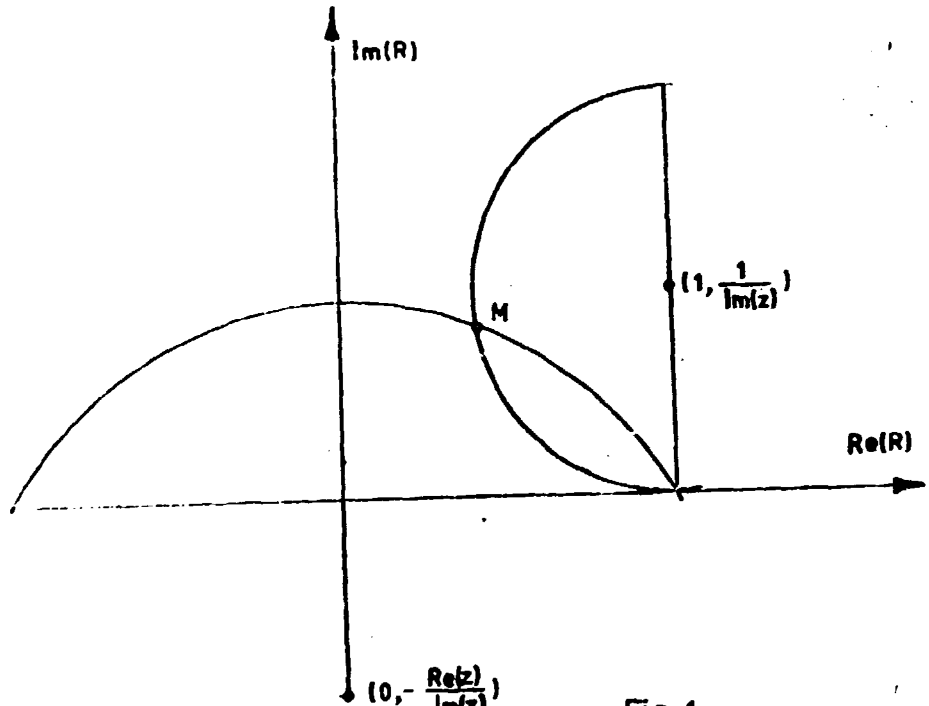
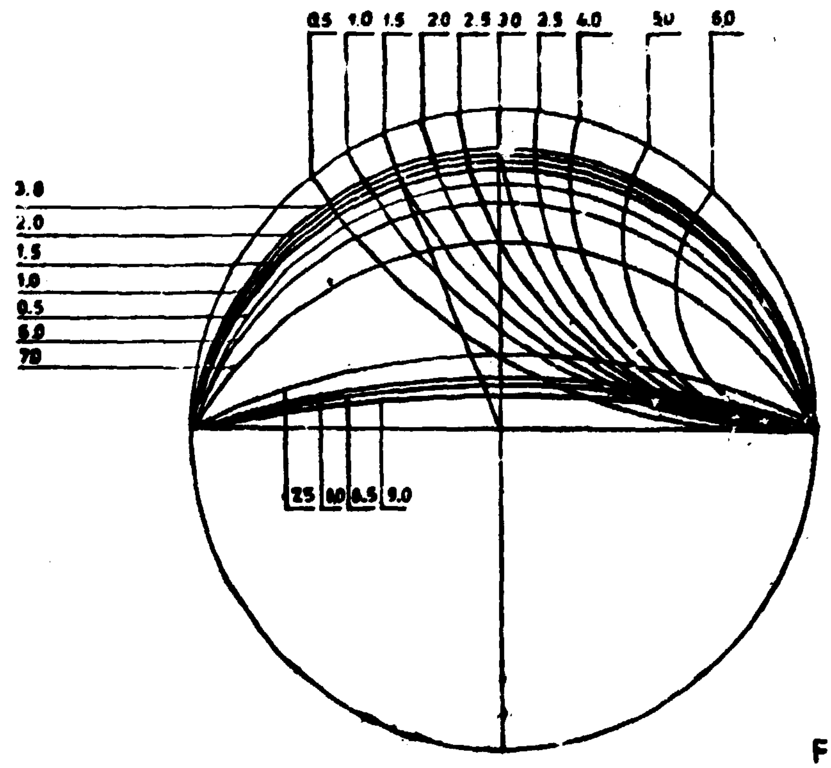

Im(R)
M
(1, $\frac{1}{Im(z)}$)
Re(R)
(0, $-\frac{Re(z)}{Im(z)}$)

Fig. 1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1.5
1.0
0.5
8.5 9.0

Fig. 2

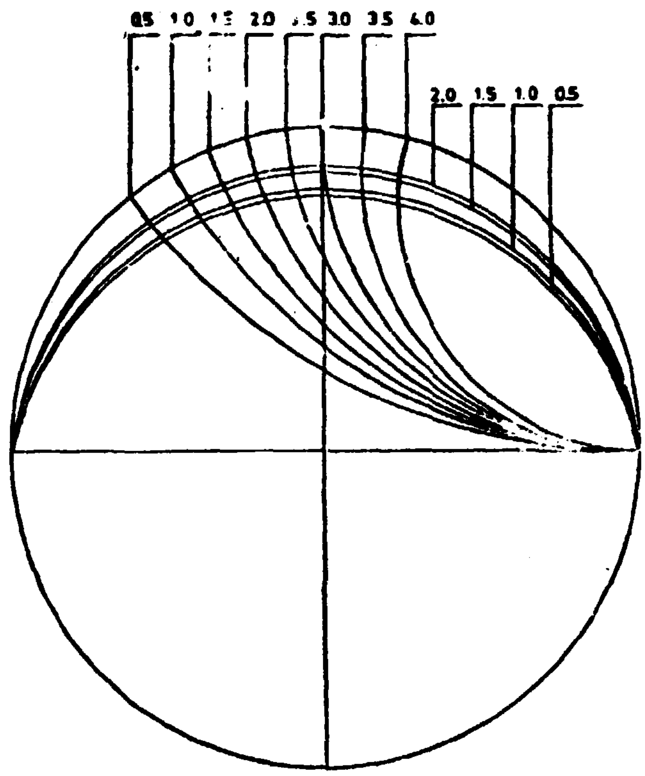

Fig.3

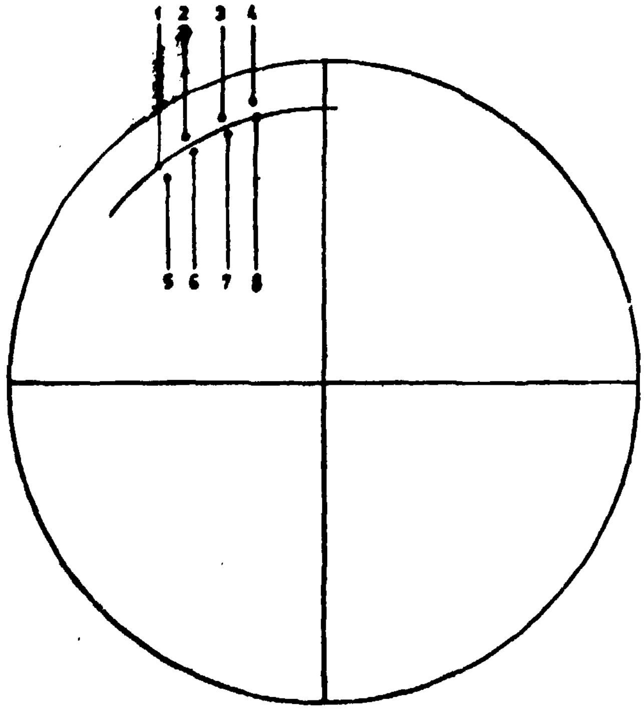

Fig.4