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ABSTRACT

The decay properties of overlapping compound nucleus resonances ~~is~~ are discussed. It is argued that the correlation width of Agassi, Weidenmüller and Mantzouranis is to be identified with the average total width of Kawai, Kerman and McVoy. It is also shown that in the very strong coupling limit, the compound system behaves like a system with isolated resonances [illegible] absorption).

Properties of the compound nucleus in the overlapping, $\Gamma >> D$, limit have been discussed extensively in recent years in two quite different formalisms[1,2], which unfortunately employ the same symbols for "obvious" quantities such as the average width, $\bar{\Gamma}$, and spacing, $\bar{D}$, whose definitions may seem at first sight to be unambiguous. However, the widths, spacings and partial widths of strongly overlapping compound resonances are not only not unambigously defined, they are not in any sense physically measurable, and so may be given different definitions for different purposes. Precisely this seems to have occurred, with the consequence that the same symbol (D) has been used to describe different quantities, and different symbols, e.g., the correlation width, Γ_{corr} and $\bar{\Gamma}$ appear to have been applied to the same quantity, thus generating the maximum confusion possible.

We offer the present comments in an attempt to alleviate this confusion, and do so by considering an important physical characteristic of a system with overlapping resonances, namely its time-decay properties, which are mathematically equivalent[3] to the properties of the S-matrix energy-autocorrelation function,

$$C^{(s)}_{ab}(\varepsilon) \equiv \left\langle S^{fl}_{ab}(E)\, S^{fl\,*}_{ab}(E+\varepsilon)\right\rangle_{\Delta E} \qquad (1)$$

Just as the fluctuation cross section

$$\langle \sigma^{fl}_{ab} \rangle = C^{(s)}_{ab}(0) \qquad (2)$$

provides one measurable parameter to describe the compound system, the ε-dependence of $C_{ab}^{(s)}(\varepsilon)$ may, under certain circumstances, provide another, Γ_{corr} or the decay rate of the system.

In order to define notation carefully in the KKM approach[2], we note that the arguments employed to calculate $C_{ab}^{(s)}(\varepsilon)$ can be immediately extended to provide an expression for $C_{ab}^{(s)}(\varepsilon)$ [4]:

$$C_{ab}^{(s)}(\varepsilon) = 2\pi \left\langle \frac{|g_{qa}|^2 \, |g_{qb}|^2}{D_q \, (\Gamma_q - i\varepsilon)} \right\rangle_q \tag{3}$$

It is essential to recognize that all the symbols carrying the resonance index q are <u>not</u> immediate properties of the "true" (but unobservable, and in that sense unphysical) overlapping poles of the S-matrix. Rather, they refer to poles of an approximation to that S-matrix, re-written via the KKM optical background techniques. They have been tailored to free their residues g_{qc} from the constraints of analytic unitarity, without large violation of unitarity for E near the real axis and thus make them as random as possible in their distribution over q. What we here designate as Γ_q, D_q and g_{qc} are to be understood as part of the KKM parametrization of the compound system. They are certainly not observable quantities, nor are even their averages necessarily so, but (as usual) the physically observable quantities are parametrized in terms of them.

Equation (3) is true provided that these parameters satisfy

$$\overline{\Gamma}_q \gg \overline{D}_q \tag{4}$$

If no further conditions are imposed, nothing more specific can be said about the ε-dependence of $C_{ab}^{(s)}(\varepsilon)$; if the q-average (a sum over q) is approximated by an integral, one would clearly expect $C_{ab}^{(s)}(\varepsilon)$ to have a branch cut along the negative imaginary axis, and the time-decay of the system of overlapping resonances (given by the Fourier transform of $C_{ab}^{(s)}(\varepsilon)$ if the arrival-time of the incident pulse is very short) would have the form

$$P(t) = \int_0^\infty \rho(\Gamma)\, e^{-\Gamma t}\, d\Gamma \tag{5}$$

which in general is certainly not exponential.

Consider, however, the implication of assuming that the number N of open channels is very large, $N \gg 1$. Because of their maximum randomness, the $|g_{qc}|^2$ are ideally suited to describe this limit, for they can be expected to behave like independent random variables, whose sums $\sum_{c=1}^{N} |g_{qc}|^2$, in the large - N limit, become random Gaussian variables by the Central limit theorem, with distributions whose widths are proportional to $N^{-1/2}$. Although the Γ_q are not exactly equal to these sums, this clearly implies that the width of the Γ_q-distribution $\rho(\Gamma)$ will become very small in the large - N limit. In this limit (with no further conditions on transmission coefficients), the Γ_q's can be removed from the average in Eq. (3) and replaced by their average. Defining the average of the KKM Γ_q's to be

$$\langle \Gamma_q^{KKM} \rangle \equiv \bar{\Gamma}_K \tag{6}$$

we thus have

$$C_{ab}^{(s)}(\varepsilon) = \langle \sigma_{ab}^{fl} \rangle \frac{\bar{\Gamma}_K}{\bar{\Gamma}_K - i\varepsilon} \quad , \tag{7}$$

$$N \gg 1 \ , \ \bar{\Gamma}_K \gg \bar{D}_K$$

and correspondingly the occupation probability of the compound nucleus becomes

$$P(t) \sim e^{-\bar{\Gamma}_K t} \qquad , \ N \gg 1 \ , \ \bar{\Gamma}_K \gg \bar{D}_K \tag{8}$$

I.e., requiring nothing but the condition for Eq. (8) on the KKM parameters, the system of overlapping resonances decays exponentially, with a decay rate given by $\bar{\Gamma}_K$; because of the statistical independence of the KKM Γ_q's , their average is precisely the (exponential) decay rate of the overlapping resonances in the limit of many open channels.

By comparison, AWM obtain, in the same $N \gg 1$ limit (expressed as $\mathrm{Tr}\, p \gg 1$),

$$C_{ab}^{(s)}(\varepsilon) = \langle \sigma_{ab}^{fl} \rangle \frac{\Gamma_{corr.}}{\Gamma_{corr.} - i\varepsilon} \tag{9}$$

which implies that

$$\Gamma_{corr.}^{AWM} = \bar{\Gamma}_{KKM} \tag{10}$$

This may at first sight seem strange, since AWM observe that in general $\Gamma_{corr.}^{AWM} > \bar{\Gamma}$, but by $\bar{\Gamma}$ they mean twice the average of the "true pole" distances from the real energy axis. This is no contradiction, but merely implies that $\bar{\Gamma}_{KKM} \geq \bar{\Gamma}_{\text{"true"}}$, which is indeed born out by a numerical calculation[5] of $\bar{\Gamma}_{KKM}$ carried out some years ago.

But what of the AWM unitarity sum rule,

$$\mathrm{Tr}\,\underset{\sim}{P} = 2\pi \frac{\Gamma^{AWM}_{corr.}}{\bar{D}_{AWM}} \left(1 + \mathcal{O}(N^{-1})\right) \quad ? \tag{11}$$

For camparison we recall that average unitarity is expressed in KKM long ago by

$$\mathrm{Tr}\,\underset{\sim}{P} = 2\pi \frac{(\Gamma^{\uparrow}_{KKM})^2}{\bar{D}_{KKM}\,\bar{\Gamma}_{KKM}} \left(1 + \mathcal{O}(N^{-1})\right) , \tag{12}$$

where

$$\Gamma^{\uparrow}_{KKM} = \sum_c \langle |g_{qc}|^2 \rangle_q \tag{13}$$

and we have assumed $\langle |g_{qc}\, g_{qc'}|^2 \rangle_q = \langle |g_{qc}|^2 \rangle \langle |g_{qc'}|^2 \rangle$

(here and throughout, we do not consider direct coupled channels effects). Eqs. (10), (11) and (12), rather than contradicting each other, simply imply that

$$\frac{\bar{D}_{KKM}}{\bar{D}_{AWM}} = \left(\frac{\Gamma^{\uparrow}_{KKM}}{\bar{\Gamma}_{KKM}} \right)^2 \tag{14}$$

We are not able to provide an estimate of the right-hand side of Eq. (14), but by analogy with what is known[5] about "true pole" parameters, we suspect that in the $\Gamma \gg D$ case considered here, it is greater than unity. Its actual value does not seem to be of great significance, however, for we recall that neither of the D's is physically observable. $\bar{D}_{KKM}$ is the spacing of "KKM poles" and D^{AWM} is the spacing of the bound (q-space) states of H_o, the "model hamiltonian" employed in their $H = H_o + V$ approach.

Finally, we note that the ratio of Eq. (14) can be evaluated in the limit of very strong coupling of the AWM model states $|\mu\rangle$ to the continuum, since, amusingly enough, this limit implies very weak absorption from the optical space (P). I.e., as $V \to \infty$, the resonances approach the weak-absorption limit $\Gamma \ll D$ and $g_{qc} \to 0$, (for any definition of Γ and D), in which very little flux penetrates into the compound-nucleus. The argument is immediate: if $V \to \infty$, then the imaginary part of the optical potential,

$$W = Im \sum_{\mu} \frac{|\langle c | V | \mu \rangle|^2}{(E - E_{\mu} + iI)} \tag{15}$$

also becomes infinite. But this implies an infinite change in momentum as a wave attempts to cross the boundary - and this in turn makes the reflection from the boundary perfect. No flux enters the interior of the nucleus, so the compound states are never excited. Hence the conclusion: strong coupling implies weak absorption. The resonances in this limit are non-overlapping, $\Gamma^{\uparrow} = \bar{\Gamma}$ and Eq. (14) (which obviously obtains in this limit) implies that $D^{AWM} = D_{KKM}$.

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