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ASYMPTOTIC TIME DIPENDENT NEUTRON TRANSPORT IN MULTI-DIMENSTONAL SYSTEMS

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ABSTRACT

A model which predicts the asymptotic time behavior of the neutron distribution in mult-dimensional systems is presented. The model is based on the kernel factorization method used for stationary neutron transport in a rectangular parallelepiped. The accuracy of diffusion theory in predicting the asymptotic time dependence is assessed. The use of neutron pulse experiments for predicting the diffusion parameters is also investigated.

INTRODUCTION

The field of multi-dimensional transport theory is still largely unexplored [1]. Exact analytical solutions are available only for one speed neutron transport in one-dimensional systems [2,3]. The Fourier transform method was applied to the integral form of Boltzmann equation in multi-dimensional systems [4,5]. Fourier transform inversion is required to obtain the flux. Equivalence between the integral transform method and the spatial Legendre expansion of the flux was shown [6]. A method based on factorized kernel Legendre expansion is used to solve the stationary one speed integral transport equation in a rectangular parallelepiped [7,8]. The method has the advantage of converting the integral equation into a set of linear algebraic equations with the matrix elements being calculated precisely by virtue of analytical evaluation of integrals [7].

Exact solutions to time dependent neutron transport problems in multidimensional systems are extremely difficult [9,10]. The asymptotic time behavior of the flux in a reactor or in a pulsed (non multiplying or subcritical) assembly has been extensively investigated using diffusion theory and asymptotic reactor theory [10]. Experiments are suggested to measure thermal diffusion properties based on the diffusion theory predictions [11]. Transport theory models are used to investigate the asymptotic time behavior of the flux only in one-dimensional systems [10].

In this work, the kernel factorization is extended to give the asymptotic time behavior of the flux in a rectangular parallelepiped. The method is compared with available exact methods in the limiting case of stationary neutron transport in an infinite slab. The accuracy of diffusion theory in predicting the asymptotic time dependence of the neutron distribution is assessed. The validity of using diffusion theory for predicting the diffusion parameters in connection with neutron pulse experiments is also assessed.

II. THEORY

We present here briefly the model used in theis work based on the kernel factorization method used for solving steady state one speed neutron transport in a rectangular parallelepiped [8]. The time dependent one speed integrodifferential transport equation is given by

$$\left[\frac{1}{v}\frac{\partial}{\partial t} + \underline{\Omega}\cdot\underline{\nabla} + \Sigma_t\right]\psi(\underline{r},\underline{\Omega},t) = \frac{\Sigma_s + \nu\Sigma_f}{4\pi}\phi(\underline{r},t) + \frac{S(\underline{r},t)}{4\pi} \quad (1)$$

where $\psi(\underline{r},\underline{\Omega},t)$ and $\phi(\underline{r},t)$ are the angular and scalar fluxes, respectively. In Eq.(1); we assume scattering, fission and source to be isotropic and the system to be uniform. Since we are interested in the asymptotic time behavior where no external source is present, $S(\underline{r},t)$ is set equal to zero in Eq.(1).

Assuming the asymptotic solution to be seperable in space and time, we seek a solution to Eq.(1) of the form

$$\psi(\underline{r},\underline{\Omega},t) = \psi(\underline{r},\underline{\Omega})\exp(-\lambda t) \quad (2)$$

where λ is the asymptotic decay constant, if pulsed systems are considered, and the inverse of the asymptotic stable period, if the criticality of multiplying systems is investigated. Eq.(1) then reduces to

$$\left[\underline{\Omega}\cdot\underline{\nabla} + \left(\Sigma_t - \frac{\lambda}{v}\right)\right]\psi(\underline{r},\underline{\Omega}) = \frac{\Sigma_s + \nu\Sigma_f}{4\pi}\phi(\underline{r}) \quad (3)$$

The corresponding integral equation [1] is given by

$$\phi(\underline{r}) = \int_V \frac{d\underline{r}'}{4\pi|\underline{r}-\underline{r}'|^2} \exp(-\Sigma_\lambda|\underline{r}-\underline{r}'|)(\Sigma_s+\nu\Sigma_f)\phi(\underline{r}'), \quad (4)$$

where

$$\Sigma_\lambda = \Sigma_t - \frac{\lambda}{v} \quad (5)$$

Notice that assuming separability as given by Eq.(2), we are able to convert the time dependent problem into a pseudo stationary problem which contains the dynamic property of the system in Σ_λ which replaces Σ_t in the stationary problem. Notice, also, λ/v acts as a fictitious 1/v absorption. It is convenient to rewrite Eq.(4) with all distances being measured in units of Σ_λ^{-1}. In this case Eq.(4) reduces to

$$\phi(\underline{r}) = c_\lambda \int_V \frac{\exp(-|\underline{r}-\underline{r}'|)}{4\pi|\underline{r}-\underline{r}'|^2}\phi(\underline{r}')\,d\underline{r}', \quad (6)$$

where

$$c_\lambda = \frac{\Sigma_s+\nu\Sigma_f}{\Sigma_\lambda} \quad (7)$$

The integral kernel in Eq.(6) can be written in the form [8]

$$A(\underline{r},\underline{r}') = \frac{\exp(-|\underline{r}-\underline{r}'|)}{4\pi|\underline{r}-\underline{r}'|^2} = (8\pi)^{-1}\int_0^\infty u^{-3}\,\mathrm{erfc}(u)\exp(-|\underline{r}-\underline{r}|^2/4u^2)du, \quad (8)$$

where the complementary error function is defined as

$$\mathrm{erfc}(z) = \frac{2}{\sqrt{\pi}} \int_z^\infty \exp(-t^2)\,dt, \qquad (9)$$

we express the semi-infinite integral in Eq.(8) in terms of an N-points nonclassical Gaussian quadrature rule [8] associated with weight function erfc (u). The weights $w_n^{(N)}$ and abscissas $u_n^{(N)}$ are determined such that the Gaussian quadrature rule is exact for all polynomials of degree (2N-1) or less. This Gaussian quadrature rule was shown [8] to give very good results and to be superior to the usual Gauss - Laguerre quadrature rule. Using the Gaussian quadrature rule, Eq.(8) becomes

$$A(\underline{r},\underline{r}') = (8\pi)^{-1} \sum_{n=1}^{N} w_n^{(N)} \exp(-|\underline{r} - \underline{r}'|\, a/4\, [u_n^{(N)}]^2).\, [u_n^{(N)}]^{-3} \qquad (10)$$

We expand $\phi(\underline{r})$ and $A(r,r')$ in a complete set of orthogonal functions of position $\{h_i(\underline{r})\}$ as

$$\phi(\underline{r}) = \sum_{i=0}^{\infty} f_i h_i(r) \quad , \qquad (11)$$

and

$$A(r,r') = \sum_{i=0}^{\infty} A_i(r)\; h_i(\underline{r}') \quad , \qquad (12)$$

where

$$f_i = \int_V \phi(r)\, h_i(r)\, dr \qquad (13)$$

and

$$A_i(\underline{r}) = \int_V A(\underline{r},\underline{r}')\; h_i(\underline{r}')\; d\underline{r}' \qquad (14)$$

Substituting from Eqs.(10),(11), and (12) into Eq.(6), we get

$$\xi_k = c_\lambda \sum_{i=0}^{I} A_i^k \xi_i, \quad k=0,1,2,\ldots,I \tag{15}$$

where

$$A_i^k = \int_V d\underline{r} \; h_k(\underline{r}) \int_V d\underline{r}' \; A(\underline{r},\underline{r}')h_i(\underline{r}'), \tag{16}$$

and expansions in the set $\{h_i(r)\}$ are truncated to (I+1) terms.

The system of linear algebraic equations (11) can be arranged in a matrix form as

$$\underline{\xi} = c_\lambda \underline{\underline{A}} \underline{\xi} \tag{17}$$

where the $\underline{\underline{A}}$ matrix elements are defined by Eq.(16). Note that Eq.(17) describes an eigenvalue problem from which c_λ can be determined. However, the matrix elements depend on λ through their dependence on the dimensions of the system measured in units of Σ_λ^{-1} and λ cannot be determined in a straightforward manner. Before explaining the method of calculating λ , we give the expressions of the matrix elements.

For a three dimensional parallelepiped of dimensions 2a, 2b, and 2c, the set of orthogonal functions $\{h_i(\underline{r})\}$ is taken to be

$$\{h_{i_1 i_2 i_3}(\underline{r})\} = \left[\frac{2i_1+1}{2\alpha_\lambda} \cdot \frac{2i_2+1}{2\beta_\lambda} \cdot \frac{2i_3+1}{2\gamma_\lambda}\right]^{1/2} P_{i_1}\left(\frac{x}{\alpha_\lambda}\right) P_{i_2}\left(\frac{y}{\beta_\lambda}\right) P_{i_3}\left(\frac{z}{\gamma_\lambda}\right)$$

where $\alpha_\lambda = a\Sigma_\lambda$, $\beta_\lambda = b\Sigma_\lambda$, $\gamma_\lambda = c\Sigma_\lambda$, and $P_i(u)$ is the $i^{\underline{th}}$ Legendre polynomial.

The matrix elements are, therefore, given by

$$A^{k_1 k_2 k_3}_{i_1 i_2 i_3} = R^{k_1 k_2 k_3}_{i_1 i_2 i_3} \frac{1}{8\pi} \sum_{n=1}^{N} \frac{w_n^{(N)}}{[u_n^{(N)}]^s} \cdot I(\alpha_\lambda, i_1, k_1, 2u_n^{(N)}) \cdot I(\beta_\lambda, i_2, k_2, 2u_n^{(N)}) \cdot I(\gamma_\lambda, i_3, k_3, 2u_n^{(N)}), \quad (19)$$

where

$$R^{k_1 k_2 k_3}_{i_1 i_2 i_3} = \frac{2i_1+1}{2\alpha_\lambda} \frac{2i_2+1}{2\beta_\lambda} \frac{2k_1+1}{2\gamma_\lambda} \frac{2k_1+1}{2\alpha_\lambda} \frac{2k_2+1}{2\beta_\lambda} \frac{2k_3+1}{2\gamma_\lambda} \quad (20)$$

and

$$I(\epsilon, i, k, x) = \int_{-\epsilon}^{\epsilon} du\, P_k\left(\frac{u}{\epsilon}\right) \int_{-\epsilon}^{\epsilon} du'\, P_i\left(\frac{u'}{\epsilon}\right) \exp -(u-u')^2/x^2 \quad (21)$$

The integrals given by Eq. (21) are performed analytically and given by

$$I(\epsilon, i, k, x) = \begin{cases} x^2 \sum_{\nu=0}^{i+k+1} \frac{\nu+1}{\nu!} B^{k,i}_{\nu} (x/2\epsilon)^{\nu-1} H(2\epsilon/x, \nu), & (i+k)\text{even} \\ 0, & (i+k)\text{odd} \end{cases} \quad (22)$$

where

$$D^{k,i}_{\nu} = \begin{cases} 2\,\dfrac{\delta_{ki}}{2k+1} & , \nu = 0 \\ -1 & , \nu = 1 \\ \dfrac{2(-1)^{\nu}}{(\nu+1)(\nu-1)!} \displaystyle\prod_{j=1}^{\nu-1} (i+k+1+\nu-2j)(|k-i|+\nu-2j), & \nu > 1 \end{cases} \tag{23}$$

and

$$H(y,n) = \tfrac{1}{2}\,\gamma\left(\frac{n+1}{2}\,,\ y^2\right), \tag{24}$$

with $\gamma\left(\frac{n+1}{2}\,,\ y^2\right)$ being the incomplete gamma function of integer and half integer orders.

It is clear from Eq.(21) that interchanging any of the pairs i and k yields the same result implying that the matrix A is symmetric. If the summations over the indices $i_1 i_2$ and i_3 are truncated to values I_1, I_2, and I_3, respectively, the matrix $\underline{\underline{A}}$ is of order $(I_1+1)(I_2+1)(I_3+1)$. Considerable reduction in the order of the matrix can be obtained by noting that because of the symmetry of the problem, the flux $\phi(\underline{r})$ can be expanded only in Legendre polyomials of even order. The summations in Eqs.(11) and (12) have to be taken only over even values of i. Hence, the matrix order reduces to $(\frac{1}{2}I_1+1)(\frac{1}{2}I_2+1)(\frac{1}{2}I_3+1)$. The advantage of using the kernel factorization method is clear by noting that the 6-fold integral in Eq.(6) is converted into a finite sum of products of three 2-fold integrals which are evaluated analytically in terms of appropriate special functions.

In case of infinite two dimensional parallelepiped of rectangular cross sectional area of dimensions 2a and 2b, one can follow the same procedure and expressions for the matrix elements can be obtained from Eq.(19) by taking the limits as $\gamma_\lambda \to \infty$ and setting $i_3 = k_3 = 0$ because the flux is uniform in the z direction. The matrix elements for the infinite one dimensional slab of thickness 2a can be obtained by taking the limit as $\beta_\lambda \to \infty$ and setting $i_2 = k_2 = 0$ in the expressions for the two dimensional case.

The method adopted in this work to calculate λ is motivated by noting that

$$\alpha_\lambda C_\lambda = (\Sigma_a + \nu\Sigma_f)\, a = \alpha \left(\frac{\Sigma_a}{\Sigma_t} + \frac{\nu\Sigma_f}{\Sigma_t}\right) \qquad (25)$$

where $\alpha = a\Sigma_t$. Therefore, for a given geometry, i.e. for given ratios of b/a and c/a, one can obtain c_λ for different assumed values of using Eq.(19) and (15). The relation between the product $\alpha_\lambda C_\lambda$ and can, therefore, be determined and plotted for a given geometry. Given a system of certain dimensions and composition Eq.(25) can be used to calculate $\alpha_\lambda C_\lambda$ for the system. The corresponding value of α_λ can be obtained from the computed relation between $\alpha_\lambda C_\lambda$ and α_λ. Therefore, can be calculated from α_λ via

$$\lambda = v\left(\Sigma_t - \frac{\alpha_\lambda}{\alpha}\right) \qquad (26)$$

In a dimensionless form, Eq. (26) can be written as

$$\lambda t_d = \left(1 - \frac{\alpha_\lambda}{\alpha}\right)/\left(1 - \frac{\Sigma_a}{\Sigma_t}\right) \qquad (27)$$

The stable period can be determined from λ, via $\tau = -1/\lambda$.

It is interesting to consider the case of infinite system in which the flux is uniform and the matrix $\underline{\underline{A}}$ reduces to a single element which can beshown to be equal to unity. From Eq.(15) it is clear, therefore, that the eigenvalue c_λ for an infinite system is equal to unity implying that

$$\lambda = v\,\Sigma_a\left(1-\frac{\nu\Sigma_f}{\Sigma_a}\right) = \frac{1}{t_d}(1-k_\infty) \qquad (28)$$

where t_d is the diffusion time and k_∞ is the infinite multiplication factor. This result is identical with the well known result obtained by using diffusion theory for an infinite system.

In the limiting case of steady state, we set $\lambda=0$ and c_λ in Eq.(15) reduces to c ; and α_λ, β_λ and γ_λ used in the calculations of the matrix elements reduce to α, β and γ, respectively. The critical c value is, therefore, given by solving the eigenvalue problem.

III. NUMERICAL RESULTS

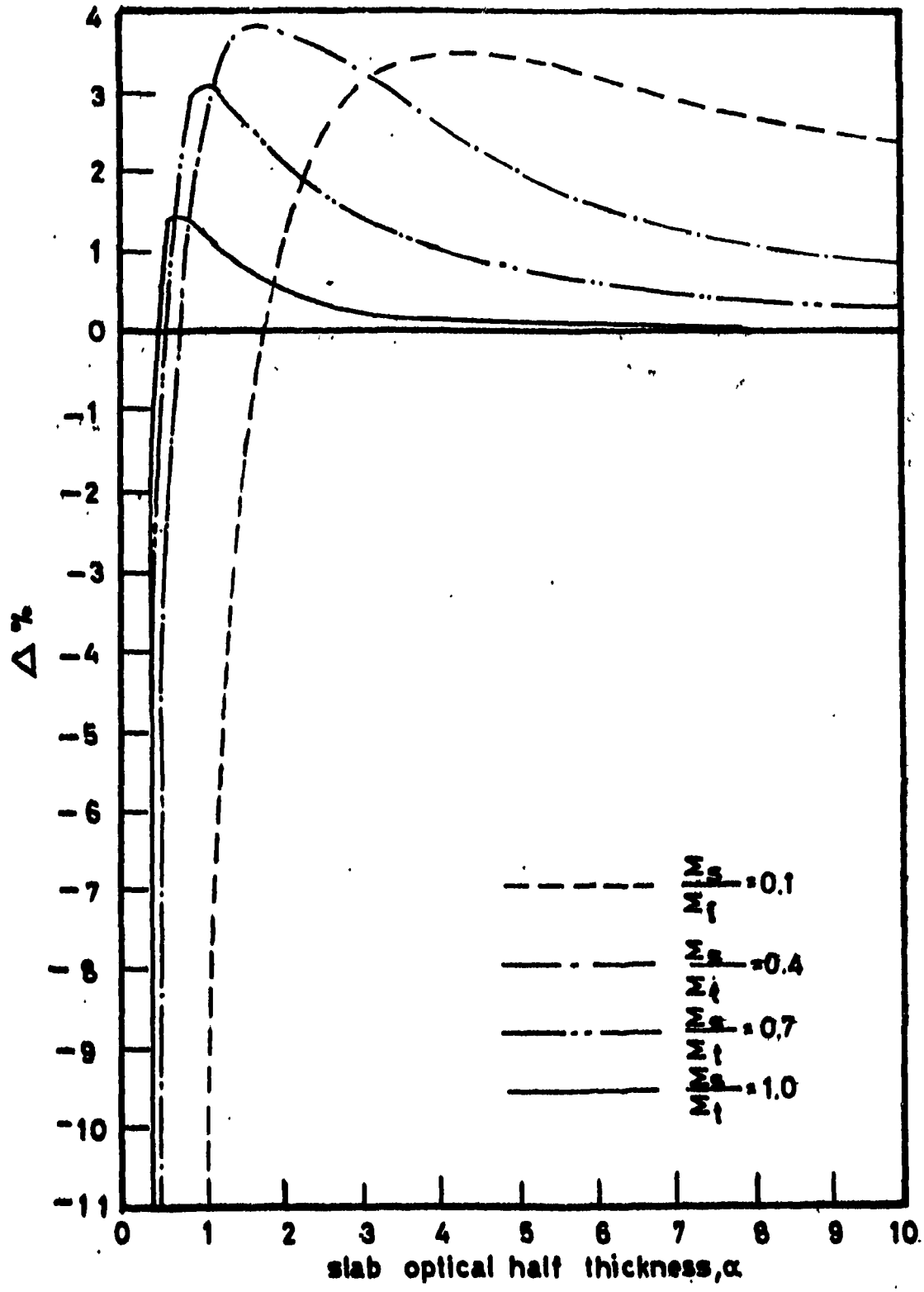
III.1 Criticality of Multiplying Systems

For a critical system, $\lambda=0$, and the critical c value is the lowest eigenvalue of the eigenvalue problem

$$\underline{\underline{A}}\,\underline{\xi} = \frac{1}{c}\,\underline{\xi} \quad . \qquad (29)$$

This value of c gives the mean number of secondary neutrons per collision for which a stationary neutron distribution can exist in the critical system. The eigenvector corresponding to the critical c is the coefficient vector from which the critical flux distribution is determined.

Table 1 gives the values of the critical calculated using the model developed here for different sizes of three dimensional rectangular parallelepiped and two dimensional infinite rectangular parallelepiped. The results are obtained using the Gaussian quadrature rule of order N = 20. In the two dimensional cases we use $I_1 = I_2 = 14$ and in the three dimensional case we use $I_1 = I_2 = 4$ with $I_3 = 14$. The critical c value obtained using diffusion theory are given also in the table with the percentage error for different values of Σ_s/Σ_t Fig.1 is a plot of the percentage error in the estimate of c resulting from using diffusion theory versus the optical half thickness a of an infinite one dimensional slab. It is clear from the results that the error in c calculated using diffusion theory decreases as Σ_s/Σ_t increases and the size increases with the same result ($c = 1$) obtained from this model and diffusion theory in the limiting case of an infinite system. The results show also that diffusion theory considerably under estimates the critical c for very small systems while it overestimates c for large systems.



FIG(1) PERCENTAGE ERROR IN C FROM DIFFUSION THEORY

Table.1

Comparison Between Critical c values Obtained
Using this Model and Diffusion Theory
for Two and three Dimonsional Rectangular Parallelepiped

α	β	γ	This Model	Diffusion Theory			
			C	$\varepsilon_s/\xi_t = 0.2$		$\varepsilon_s/\xi_t = 0.8$	
				C	Δ %	C	Δ %
0.5	0.30	"	2.20729022	1.5597130	- 30.28	2.156044	-3.61
1.0	1.00	"	1.54173800	1.4380003	- 6.73	1.6117524	+4.54
2.0	2.00	"	1.21151978	1.2891486	+ 6.41	1.2561316	+3.68
5.0	5.00	"	1.04917409	1.1184353	+ 6.60	1.0604262	+1.07
1.0	0.25	"	2.25304885	1.3402853	- 46.89	1.9307500	-23.4
3.0	0.25	"	2.29113441	1.3224926	- 42.28	1.8820833	-17.8
10.3	0.25	"	2.24156730	1.3204687	- 41.09	1.8765479	-16.29
10.0	1.00	"	1.88875220	1.5219880	- 19.42	1.8920334	+0.22

The Fourier transform method [5] was applied to the problem of critical one dimensional slab. Table 2 gives a comparison between the critical c values evaluated by this theory and by Fourier transform method for a slab with optical half thickness of (.3 and for different truncation orders i_1 . It is clear that our results agree very well with those of the Fourier transform method.

Table 2

Critical c Using this Theory and Fourier Transform Method for a Slab with α =0.3

I_1	c	
	This Theory	Fourier Transfrom
2	2.03603247	2.03602543
4	2.03599842	2.03599015
6	2.03599127	2.03598538
8	2.03598611	2.03598490

The singular eigenfunctions or Case's method [2] was developed to provide exact solutions for one-dimensional problems. Table 3 shows an excellent agreement between values of c calculated by case's method and those calculated by the theory used in this work. The ratio of the surface flux to the flux at the center of a slab of optical half thickness α is calculated using the model adopted in this work and diffusion theory. These results are compared with the results of Case's method [8] in Table 4. Very good agreement between the results of this model and Case's method is demonstrated. Diffusion theory is found to give higher estimates for the flux at the surface with the error being significantly high even for large Σ_s/Σ_t and large systems. We conclude that although diffusion theory might be useful for predicting integral properties such as the critical c value for relatively large systems with small absorption, it gives a bad estimate for the flux distribution.

Table 3

Comparison Between Values of c Calculated by The Theory Used in this Work and Case's Method for Slabs

α	c	
	Case's Method	This Model
0.5	1.6153785	1.6153806
1.0	1.2771018	1.2771018
2.0	1.1084678	1.1084679
3.0	1.0582959	1.0582959
5.0	1.0364020	1.0364021
6.0	1.0180722	1.0487942
8.0	1.0107663	1.0180722
10.0	1.0071357	1.0071358

Table 4

ϕ_s/ϕ_o Calculated by Case's Method the Theory Used in This Work, and Diffusion Theory

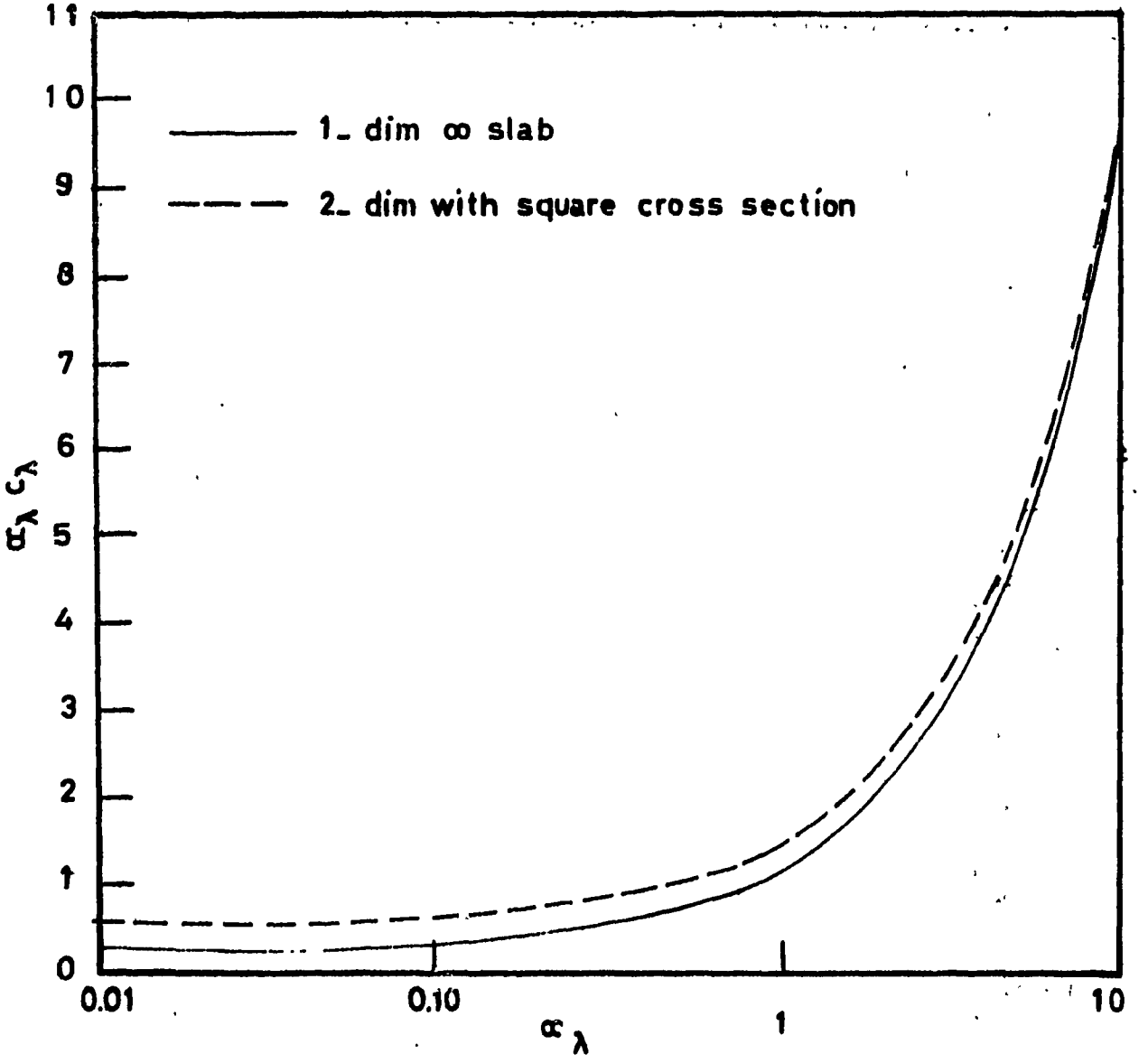
α	Case's Method	This Model		Diffusion Theory			
				$\Sigma_s/\Sigma_t=0.4$		$\Sigma_s/\Sigma_t=1.0$	
	ϕ_s/ϕ_o	ϕ_s/ϕ_o	Δ %	ϕ_s/ϕ_o	Δ %	ϕ_s/ϕ_o	Δ %
0.5	0.55555	0.55394	-0.29	0.93502	68.30	0.78183	40.73
1.0	0.43412	0.43529	0.27	0.83147	91.53	0.58778	35.40
2.0	0.30166	0.30425	0.86	0.65486	117.08	0.38268	26.86
3.0	0.22973	0.23248	1.19	0.53203	131.59	0.28173	22.63
4.0	0.18483	0.18765	1.53	0.44574	141.16	0.22252	20.39
5.0	0.15430	0.15715	1.84	0.38268	148.01	0.18375	19.09
6.0	0.13229	0.13514	2.15	0.33488	153.13	0.15643	18.25
8.0	0.10277	0.10577	2.72	0.26753	160.32	0.12054	17.29
10.0	0.08394	0.08668	3.27	0.22252	165.12	0.09802	16.76

III.2. Asymptotic Period in a Multiplying System.

We have shown that the kernel factorization method gives results which agree very well with the exact Case's method for slab geometry in the steady state case. In this section we use the theory developed in section II to calculate the asymptotic period of multiplying system. Since the time dependent problem is transformed into a pseudo stationary problem in which the dynamic properties of the system under consideration are contained, we expect the results to be highly accurate. The accuracy of the non-steady state diffusion equation in predicting the asymptotic reactor period is assessed by cimparing the results of diffusiontheory with those of the exact transport model used in this work.

As explained in Section II, the starting point in determining the asymptotic time behavior, using the kernel factorization method, is to determine the relation between the product $\alpha_\lambda c_\lambda$ and α_λ for a given geometry. The relation between c_λ and α_λ is obtained by solving the eigenvalue problem given by Eq.(19). From this relation the required $\alpha_\lambda c_\lambda - \alpha_\lambda$ relation is obtained. Fig.2 gives this relation for an infinite slab and a two dimensional parallelepiped with square cross section. Given a reactor with certain composition and dimensions, the product $\alpha_\lambda c_\lambda$ is calculated using Eq.(25) and α_λ is obtained from Fig.2. The asymptotic λ is then calculated using Eq.(25) from which the stable period can be calculated via $T = -1/\lambda$ The period is calculated also using diffusion theory and compared with that obtained using the transport model adopted in this work to assess the validity of using diffusion theory in non-steady state calculations We have shown in Section II that both methods give the same value for and infinite system.

In Fig.3, the dimensionless stable period T/t_d is plotted versus slab optical half thickness for $\Sigma_s/\Sigma_t = 0.1$ & $\Sigma_f/\Sigma_t = 0.5$



FIG(2) α_λ c_λ-α_λ RELATION FOR A ONE DIMENSIONAL SLAB AND A TWO DIMENSIONAL INFINITE PARALLELEPIPED WITH SQUARE CROSS SECTION

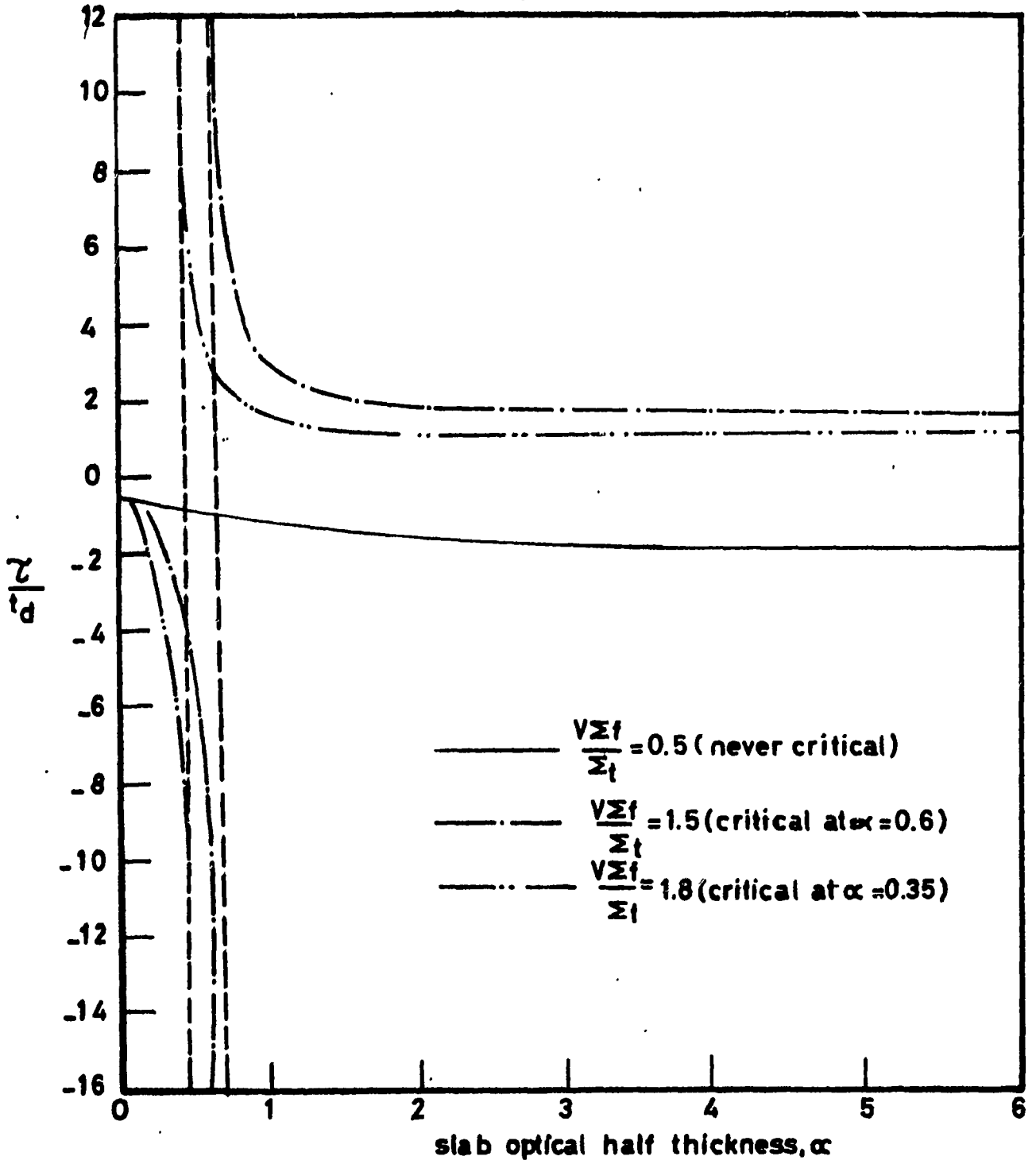


FIG (3) $\frac{\tau}{t_d}$ FROM TRANSPORT THEORY VERSUS α OF SLABS WITH $\frac{\Sigma_s}{\Sigma_t} = 0.1$

1.5, and 1.8 with the corresponding c values being 0.0, 1.6 and 1.9; respectively. It is clear that as Σ_f/Σ_t increases, the critical α decreases. Tables 5 and 6 give τ/t_d for different values of α with $\Sigma_s/\Sigma_t = 0.4$ and $\nu\Sigma_f/\Sigma_t = 0.5$ and 1.0 . The results are given for the cases of diffusion theory and the transport model used here with the percentage error of diffusion theory given also. In Fig.4, the results of τ/t_d calculated from transport and diffusion theories are plotted versus α for $\Sigma_s/\Sigma_t = 0.7$.

Table 5

Dimensionless Stable Period From Transport and Diffusion Theories for Different Slab of Optical Halfthickness α and c = 0.9 With $\Sigma_s/\Sigma_t = 0.4$ and $\nu\Sigma_f/\Sigma_t = 0.5$

α	τ/t_d(transport)	τ/t_d(diffusion)	Δ %
0.2389	- 0.6262098	- 0.9005593	- 43.86
0.6209	- 1.0044672	- 1.2172208	- 21.19
0.8974	- 1.3548429	- 1.4536579	- 7.30
1.4190	- 2.0319741	- 1.8990129	+ 6.59
3.5280	- 4.0113328	- 3.4052615	+ 15.10
5.6938	- 4.9241720	- 4.3493027	+ 11.67
8.9850	- 5.4751290	- 5.0794631	+ 9.99
∞	- 6.0	- 6.0	0.0

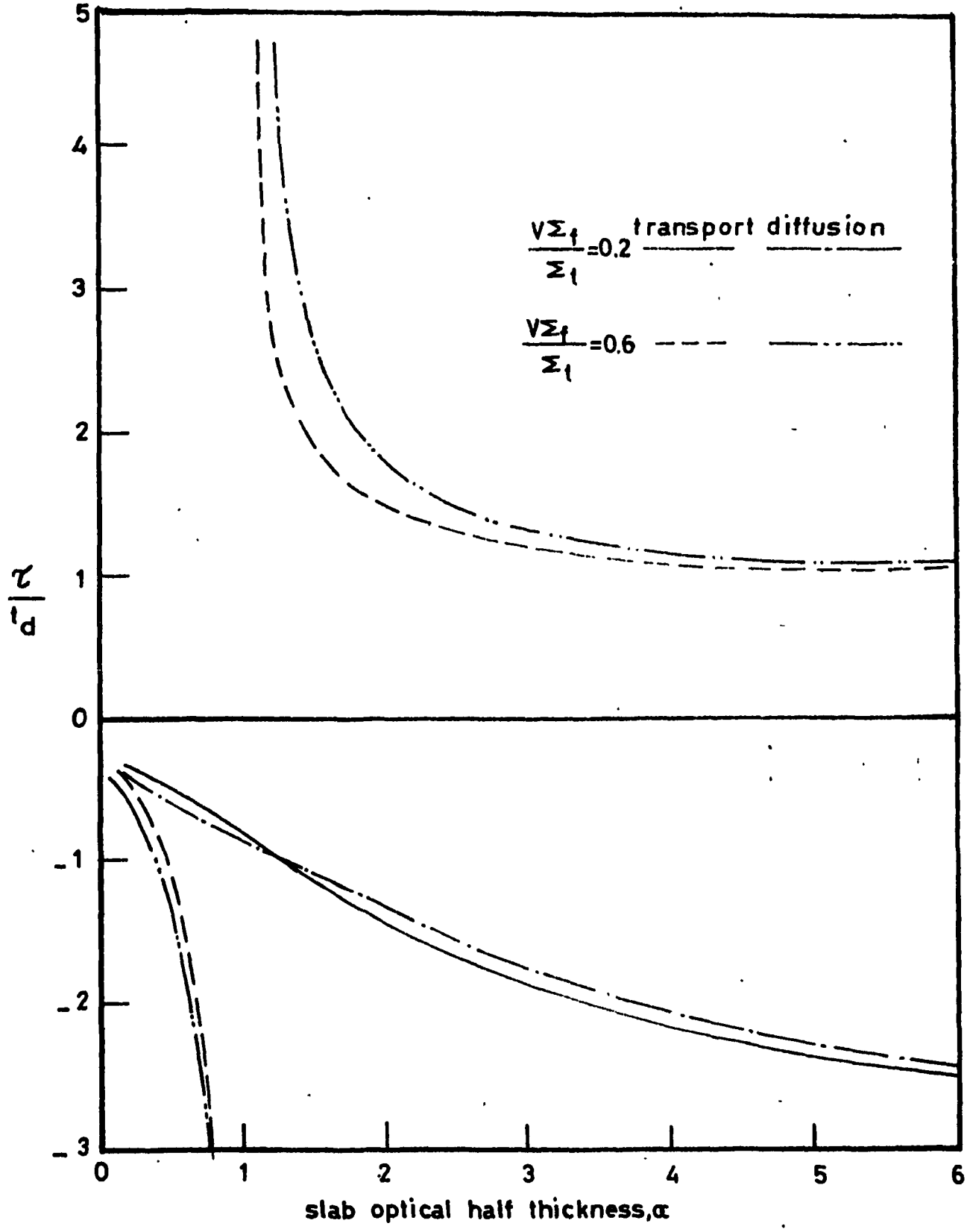


FIG.(4) $\frac{\tau}{t_d}$ VERSUS α FROM TRANSPORT AND DIFFUSION THEORIES FOR $\frac{\Sigma_s}{\Sigma_t}$ =0.7

Table 6

Dimensionless Stable Period from Transport and Diffusion Theories for differents Slabs of Optical Halfthickness α and c=1.4 with Σ_s/Σ_t=0.4 and

$$\nu \Sigma_f/\Sigma_t = 1.0$$

α	τ/t_d (transport)	τ/t_d (diffusion)	Δ %
0.1536	- 0.6417849	- 2.7202654	- 325.0
0.3091	- 1.6058801	- 7.3320614	- 356.0
0.5769	- 4.5000727	- 70.778262	- 1470.
0.9182	6.2849252	6.6059222	5.95
2.2680	1.8582673	2.2456235	20.84
3.6600	1.6302708	1.8218832	11.75
5.7760	1.5580665	1.6534966	6.12
7.1940	1.5381429	1.6050910	4.35
∞	1.5	1.5	0.0

It is clear from the results that diffusion theory gives criticality at values of α smaller than those predicted by transport t theory. We notice also that diffusion theory gives a period which is smaller than that given by transport theroy for very small systems. For larger systems, diffusion theory is found to overestimate the period. It is observed that the largest error in τ occurs in the range where the system is nearly critical with the error decreasing a as α increases reaching zero as .

The stable period may be rewritten as

$$\tau = 1/\nu \, (\Sigma_{af} - \Sigma_a) \quad , \qquad (80)$$

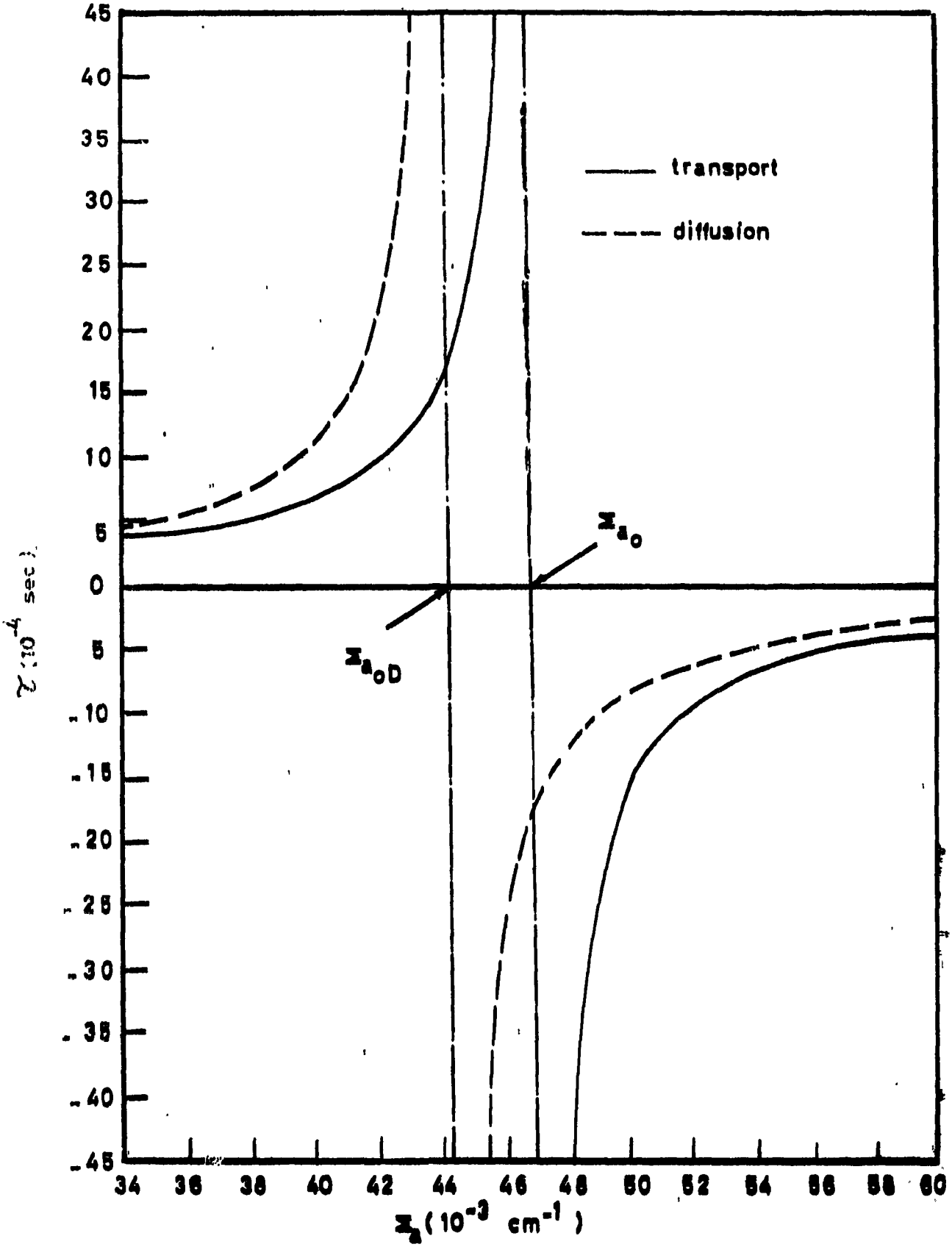


FIG (5) τ VERSUS Σ_a FOR A TWO DIMENSIONAL PARALLELEPIPED OF a = b = 20 cm.

where $\Sigma_{ao}=(\alpha_\lambda/a)-\Sigma_a$ is the absorption cross section required for criticality ($\tau=\infty$). Using diffusion theory, one can easily show that the period is given by

$$\tau = 1/v(\Sigma_{aoD}-\Sigma_a) \quad ,$$

with

$$\Sigma_{aoD}=\nu\Sigma_f - DB^2 \quad ,$$

where B^2 is the bucking. The relation between τ and Σ_a, for a two dimensional infinite rectangular parallelepiped with square cross section or a given size, is obtained using transport and diffusion theories.

Consider two systems with a=b=20 cm and a=b=10 cm. The data used are $\Sigma_a=0.203$ cm^{-1}, $\nu\Sigma_f=0.0593$ cm^{-1}, and $v=2.2\times10^5$ cm/sec. For fully enriched Uranium fuel, it is important to notice that Σ_a can take only values greater than $\nu\Sigma_f/\tau=0.0286$ cm^{-1}. Σ_{ao} for the two systems are calculated; they are 0.047 and 0.01845 cm^{-1}, respectively. It is clear therefore, that the second system is always suboritical and cannot be made critical. Relations between τ and Σ_a from transport and diffusion theories are plotted in Fig.5 for the first system. The results show that diffusion theory underestimates the absorption cross section required for criticality. The error in τ is found to be pronounced in the range where the period is large, i.e., when the system is nearly critical. The error decreases as the system goes away from criticality. Moreover, for supercritical systems, diffusion theory gives higher values for τ, while for suboritical systems, diffusion theory gives smaller values for τ. The values of Σ_a required to give certain period τ is calculated using transport and diffusion theories. The percentage error in Σ_a is plotted versus $1/\tau$, for the two systems under consideration, in Fig.6. It is clear that diffusion theory underestimates the value of Σ_a required to give certain value of τ. For the same period, the error in Σ_a decreases as the size increases. Moreover, for the same size, $\Delta\Sigma_a\%$ decreases as the system becomes more

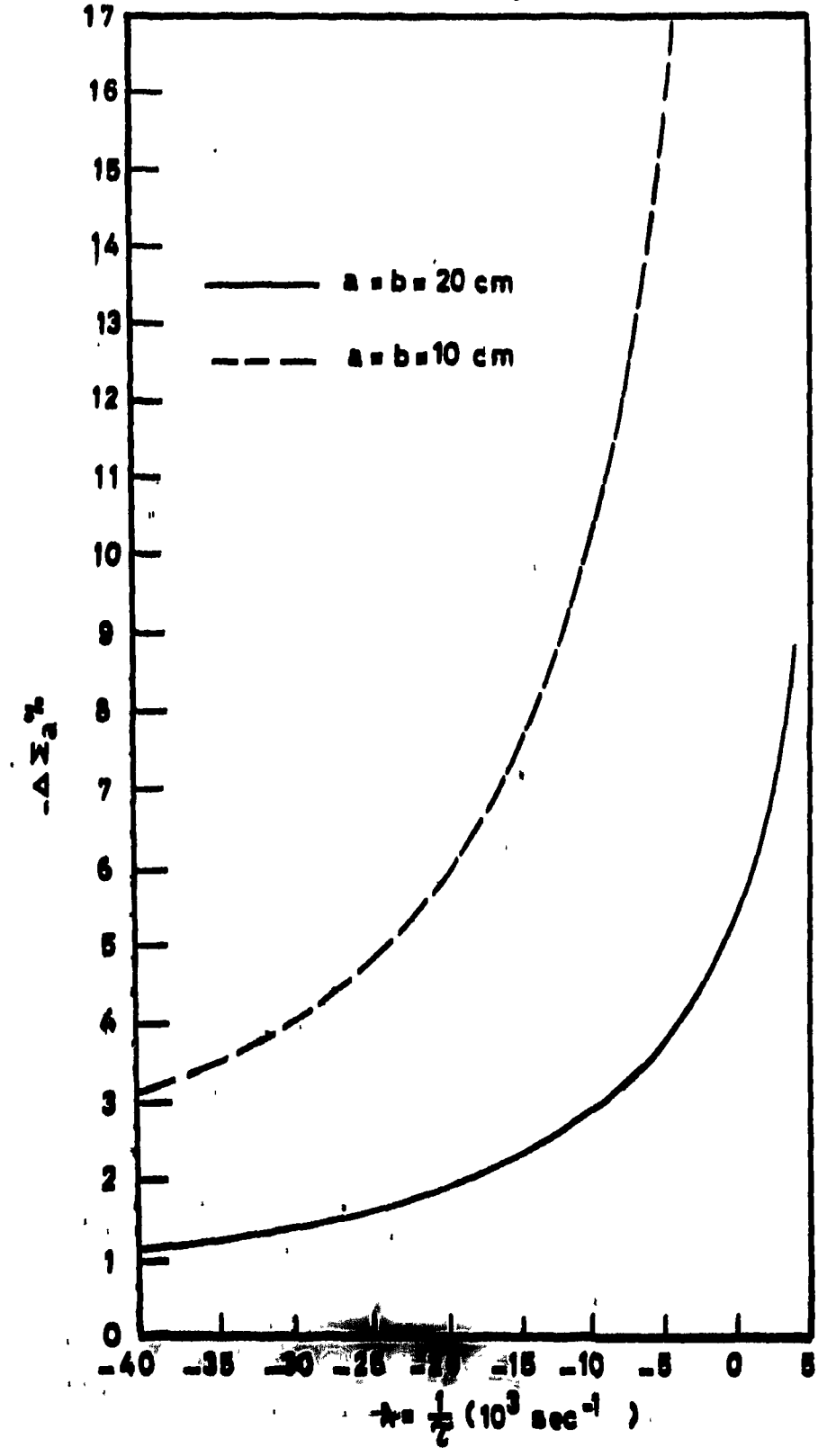


FIG (6) $\Delta\Sigma_a\%$ VERSUS $\frac{1}{\tau}$ FOR TWO DIMENSIONAL CASES

subcritical. The reason is that $\Delta\Sigma_a$ is independent of τ and is equal to $\Delta\Sigma_{ao}$. Therefore, $\Delta\Sigma_a/\Sigma_a$ decreases as Σ_a increases.

11.8. Decay of a Neutron Pulse in an Assembly:

When an intense neutron pulse of brief duration is injected into an assembly (either nonmultiplying or subcritical), the subsequent of the neutron distribution can be followed by detectors and the results may be used to infer many properties of the assembly. Assuming space and time seperability and using one speed diffusion theory, the asymptotic decay constant in a pulsed nonmultiplying assembly is found to be [11]

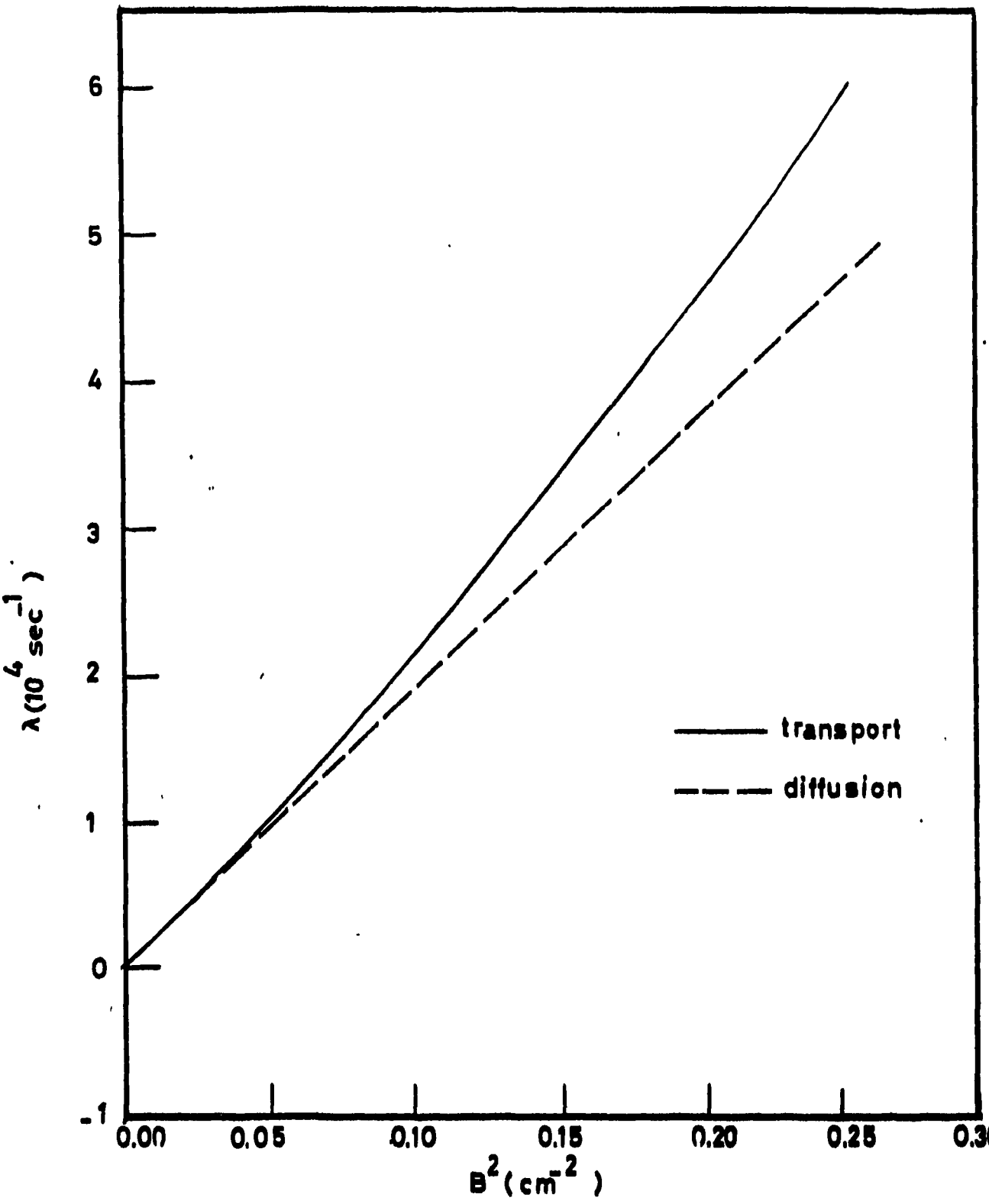
$$\lambda = v\Sigma_a + vDB^2 = v\Sigma_a (1 + L^2B^2) \qquad (81)$$

Note that diffusion cooling effect is absent in Eq. [81] because it is based on one speed theory. Eq. (81) is the basis for neutron pulse experiments in which the asymptotic decay constant is measured for assemblies of different sizes and plotted versus the buckling B^2. The data are then fitted to a straight line from which the diffusion time $1/v\Sigma_a$ and the diffusion length (or the diffusion coefficient) are determine. In this section, we calculate λ for assemblies of different sizes using the kernel factorization mehtod presented in Section 11 to assess the validity of diffusion theory in studying the neutron pulse decay. The calculations are performed for nonmultiplying assemblies. Note that the asymptotic decay constant in a pulsed subcritical assembly is obtained from the stable period, given in Section 11.2, via $\lambda = -1/\tau$.

Consider the decay of a neutron pulse in a graphite assembly. The thermal properties are : $\Sigma_t = 0.37741\ cm^{-1}$, $\Sigma_a = 0.0002409\ cm^{-1}$, $\Sigma_s = 0.3771691\ cm^{-1}$, and $D = 0.84$ cm. The asymptotic decay constant is calculated for an infinite slab and a two dimensional parallelepiped of square cross section. The results of the transport model presented

in this work together with the results of diffusion theory are given for different slab sizes in Table 7. λ is plotted versus B^2 for a two dimensional parallelepiped with square cross section in Fig.7. The results of diffusion are also included for comparison.

It is clear from the results that if small assemblies are used in the pulse neutron experiments, diffusion theory underestimates the asymptotic decay constant λ . If the results of transport theory, considered to be exact, are fitted to a straight line, a slope which is higher than that predicted by diffusion theory is obtained. Therefore, if experiments are performed on relatively small systems of sizes investigted here, they will overestimate the diffusion length and the diffusion coefficient and underestimate $v\Sigma_a$. For example, if the λ versus B^2 relation obtained by transport theory in Fig.7 is fitted to a straight line, we get D = 1.008 cm as compared with the exact value of 0.84 cm. It also gives $v\Sigma_a$ = -950 sec^{-1} which is much smaller than the exact value of 58 sec^{-1}. We conclude that for a good estimate of diffusion parameters, very large assemblies must be used in neutron pulse experiments.



FIG(7) λ VERSUS B^2 FOR TWO DIMENSIONAL PARALLELEPIPED WITH SQUARE CROSS SECTION.

Table. 7

λ vs. B^2 for an Infinite Slab of Graphite

a(cm)	$B^2(cm^{-2})$	Transport $\lambda(sec^{-1})$	Diffusion $\lambda(sec^{-1})$	$\Delta\lambda\%$
0.570	0.3760840	79115.53	69524.9	- 12.12
1.015	0.2729897	61358.20	50480.9	- 17.73
1.481	0.2043352,	45881.50	37854.1	- 17.50
1.618	0.1893955	42249.00	35038.9	- 17.07
2.140	0.1445590	31630.40	26756.6	- 15.41
3.384	0.0853922	18010.40	15827.0	- 12.12
5.874	0.0339883	8123.60	7420.5	- 8.65
8.412	0.0227976	4573.60	4264.2	- 6.76
10.980	0.0146645	2916.30	2761.8	- 5.29
13.580	0.0101762	2028.70	1932.7	- 4.73
16.150	0.0074972	1476.10	1437.9	- 2.59
18.800	0.0057078	1128.20	1107.3	- 1.42,
21.420	0.0045018	883.00	884.5	+ 0.17
24.050	0.0036384	701.70	725.1	+ 3.33
26.680	0.0030015	585.30	607.4	± 3.77
∞	0.0	53.00	53.0	0.00

An extensive work is now in progress to improve the model used in this work by treating the energy dependence in the context of the multi-group method to examine the diffusion cooling effect. The space time seperability assumption is also being removed. The model will be capable of predicting not only the asymptotic time behavior, but also the system transients.

IV. SUMMARY AND CONCLUDING REMARKS

A model, based on the kernel factorization mehtod, is presented to predict the asymptotic time behavior of the neutron distribution in a rectangular parallelepiped. The kernel factorization method converts the one speed integral transport equation into a set of homogeneous linear algebraic equations in which the matrix elements are accurately determined by analytically evaluating the integrals in terms of appropriate special functions.

The limiting steady state case is examined and values of the critical c value obtained here are copared with those obtained using Case's method and Fourier transform method in the limting case of slab geometry. A very good agreement is obtained. The critical c value is also calculated using diffusion theory and compared with the results of the transport model used here. Diffusion theory is found to give good results only in the case of very small absorption and for large systems. On the other hand, diffusion theory is found to give a bad estimate for the flux distribution even for relatively large systems with small absorption.

The model is used to assess the accuracy of diffusion theory in predicting the asymptotic reactor period. It is found that diffusion theory gives a stable period which is smaller than that given by transport theory for subcritical systems. For supercritical systems, diffusion theory is found to overestimate the period. The largest error in is found ro occur in the range where the system is nearly critical with the error decreasing as the size increases reaching zero an infinite system. Diffusion theory is found to underestimate the value of absorption cross section required to give certain value of τ

The relation between the asymptotic decay constant λ and the buckling B^2 in pulsed nonmultiplying assemblies is obtained using the model presented in this work.

The results are compared with the diffusion theory prediction which is used to measure thermal diffusion parameters in neutron pulse experiments. It is found that diffusion theory underestimates particularly for small systems. Using relatively small assemblies in neutron pulse experiments is found to give values of D and t_d that are higher than their actual values. It is concluded that for a good estimate of diffusion parameters, ver large assemblies must be used in neutron pulse experiments.

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FIGURE CAPTIONS

Fig.1 Percentage Error in C from Doffusion Theory.

Fig.2 $\alpha_\lambda C_\lambda - \alpha_\lambda$ Relation for a one Dimensional Slab a Two Dimensional Infinite Parallelepiped with Square Cross Section

Fig.3 τ/t_d from Transport Theory Versus α of Slabs with $\varepsilon_s/\varepsilon_t = 0.1$.

Fig.4 τ/t_d Versus from Transport and Diffusion Theories for $\varepsilon_s/\varepsilon_t = 0.7$.

Fig.5 τ Versus ε_a for Two Dimensional Parallelepiped of a=b=20 cm.

Fig.6 $\Delta\varepsilon_a/\varepsilon_a$ Versus $1/\tau$ for Two Dimensional Cases.

Fig.7 λ vs. B^2 for Two Dimensional Parallelpiped with Square Cross Section.