NIKHEF NATIONAAL INSTITUUT VOOR KERNFYSICA EN HOGE-ENERGIEFYSICA

January 1986 NIKHEF-H/86-5

SUPERMULTIPLETS IN HILBERT SPACE FOR THE SUPERSYMMETRIC BAG MODEL

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ABSTRACT:

It is shown in the quasiclassical approximation that the off-shell supermultiplets of the Supersymmetric Bag Model can be constructed merely according to the 'intuitive rule' (changing a creation operator into its supersymmetric partner). We also demonstrate how to form the candidate on-shell supermultiplets to be used as the trial state vectors for the variational method. The energy expression for an on-shell supermultiplet (a set of composite particles) of the model is given.

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Supermultiplets in Hilbert Space for the Supersymmetric Bag Model

The lagrangian density of the field theoretical model to be discussed is

$$
\begin{aligned}
L(x) = \Big\{ & \Big[\tfrac{1}{2}(\partial_0 A^{\dagger}\partial^0 A + F^{\dagger}F) + {}^{1}/_{8}\,(i\,\xi \partial\!\!\!/ \xi - i\,\xi \overleftarrow{\partial\!\!\!/} \xi) \Big] \\
& + \big[A\to B,\ F\to G,\ \xi\to\chi \big] + 2\big[A\to\sigma,\ F\to K,\ \xi\to\eta \big] \Big\} \\
& + \Big\{ \tfrac{1}{2}\big[(m+f\sigma)(GA+FB) + fKAB + 2MK\sigma + \text{h.c.} \big] \\
& - {}^{1}/_{4}\big[(m+f\sigma^{\dagger})\xi\chi^{c} + M\bar{\eta}\eta^{c} + f\bar{\eta}^{c}(\xi B+\chi A) + \text{h.c.} \big] \Big\}. \qquad (1)
\end{aligned}
$$

It is nothing but a simple case of the supersymmetric model proposed previously which is renormalizable merely by rescaling the field operators[3]. One may look upon it as a supersymmetric extension of the "field theoretical bag Model" in hadron theory which appeared in the early seventies[4]. The left-handed chiral superfield (A,ξ,F) together with (B,χ,G) may be considered as a "matter field" of mass m and (σ,η,K) as a "Bag field" of mass M. They are coupled to each other through non-gauge interaction terms in which f is the coupling constant. Therefore we give it the name "Supersymmetric Bag Model" (SBM).

In this note, we discuss the problem of solving an SBM to obtain the low-lying bound-state solutions which are composed of a number of matter field particles.

The variational method we adopted follows T.D. Lee's works[1], especially in the aspect of separating the bag-field operators into a classical part and a quantum part. Our discussion is approximate on the quasiclassical level, neglecting the bag-field quantum excitation. But it gives us good results at least when the bag field mass is large and the coupling constant is small.[1]

The main problem we encounter is how to construct a representation of supersymmetry on the Hilbert space of state vectors.

We find that the representation problem on Hilbert space can be solved to a certain extent for an SBM, because its supersymmetry charge operator Q can be brought into a form consisting of monomials with products of only two field operators which are separable with respect to the matter field and the bag field.

According to Noether's theorem, one obtains from (1) the conserved supersymmetric current of the model

$$S^{\mu} = \tfrac{1}{2}\{[i\gamma^{\mu}(\xi^{c}F+\xi F^{\dagger}) - \gamma^{\nu}\gamma^{\mu}(\xi^{c}\partial_{\nu}A+\xi\partial_{\nu}A^{\dagger})] + [\xi\to\chi,\ F\to G,\ A\to B] + 2[A\to\sigma,\ \xi\to\eta,\ F\to K]\},$$

which is consistent with the results of [5]. By a repeated application of the equations of motion, the supersymmetry charge operator $Q\equiv\int S^{0}D^{3}x$ of an SBM can be put into the form

$$Q = \frac{i}{2}\int d^{3}x\ \{[\frac{\gamma^{0}}{2}(A^{\dagger}i\partial\xi^{c}) - \frac{m_{0}}{2}\gamma^{0}(\chi A+\chi^{c}A^{\dagger}) - i\gamma^{0}\gamma^{\kappa}\partial_{\kappa}(\xi^{c}A+\xi A^{\dagger}) + i(\xi\overleftrightarrow{\partial}_{0}A+\xi\overleftrightarrow{\partial}_{0}A^{\dagger})] + [A\to B,\xi\to\chi] + 2[A\to\sigma,\xi\to\eta,m_{0}\to M]\}$$

which shows explicitly the characteristics just mentioned above and can be put further into the form more convenient for our discussion:

$$Q = (U_{LS'}^{(n')*}L + U_{RS'}^{(n')*}R)\{a_{s'n'}\Gamma_{s'n',sn}\, b^{\dagger}_{sn} + \bar{a}_{s'n'}\Gamma_{s'n',sn}\, b_{sn} + c.c.\}$$

$$+ (U_{LS'}^{(n')*}R + U_{RS'}^{(n')*}L)\{\bar{a}_{s'n'}\Gamma_{s'n',sn}\, \bar{b}^{\dagger}_{sn} + a_{s'n'}\Gamma_{s'n',sn}\, \bar{b}_{sn} + c.c.\}$$

$$+ \text{similar terms for Bag fields,} \qquad (2)$$

where

$$\Gamma_{s'n',sn} \equiv \{iC^{(0)}{}_{s'n',sn}[(-i\vec{\partial}_0+2i\overleftarrow{\partial}_0) + m_0\gamma^0] + iC^{(\kappa)}{}_{s'n',sn}\gamma^0\gamma^\kappa\}\phi_{sn},$$

$$C^{(0)}{}_{s'n',sn} \equiv \frac{-1}{4\sqrt{2\omega_{n'}}} \int W_{n'}W_n d^3x,$$

$$C^{(\kappa)}{}_{s'n'sn} \equiv \frac{-1}{4\sqrt{2\omega_{n'}}} \int W_{n'}(i\vec{\partial}_\kappa+2i\overleftarrow{\partial}_\kappa)\ W_n d^3x,$$

$$W_n \equiv W_n^* \ ,\ \ L \equiv (1-\gamma_5)/2\ ,\ \ R \equiv (1+\gamma_5)/2\ ,$$

$$a^\dagger{}_{sn} = U_{s\lambda}{}^{(n)}a_{\lambda n}\ ,\ \ \lambda=L,R;\ s=\tfrac{1}{2},-\tfrac{1}{2}\ ,\ \ [\{U_{s\lambda}{}^{(n)}\} \text{ - a unitary matrix}],$$

and $a^\dagger{}_{\lambda n}(b^\dagger{}_{sn})$ is the bosonic (fermionic) creation operator of a matter field corresponding to a complete set of orthogonal wave functions $\{W_n\}$ $(\{\phi_{sn}W_n\}$ with ϕ_{sn} as a constant spinor) in the expression

$$\begin{pmatrix} A(x) \\ \\ B^\dagger(x) \end{pmatrix} = \sum_n \frac{1}{\sqrt{2\omega_n}} \begin{pmatrix} W_n(\vec{x})\ a_{Ln}(t) + W_n^*(\vec{x})\ \bar{a}^\dagger{}_{Rn} \\ \\ W_n(\vec{x})e^{i\Theta'}a_{Rn}(t) + W_n^*(\vec{x})e^{-i\Theta'}\bar{a}^\dagger{}_{Ln} \end{pmatrix}$$

$$\left(\ \psi(\sim) \equiv \xi + \chi^0 = \sum_{s,n} \{\phi_{sn}\ W_n(\vec{x})b_{sn}(t) + \phi^c_{sn}\ W_n^*(\vec{x})\bar{b}^\dagger{}_{sn}(t)\}\ \right).$$

[We put a bar on an operator to indicate that for an antiparticle.]

For the bag field we make the expansion

$$\sigma(x) = \sigma_{cl}(\vec{x}) + \sum_j \frac{1}{\sqrt{2\omega_j}} \{\sigma_j(\vec{x})c_j(t) + \sigma_j^*(\vec{x})\bar{c}_j^\dagger(t),$$

$$\eta(x) = \eta_{cl}(\vec{x}) + \sum_{j,s} \{\zeta_{jL}\sigma_j(\vec{x})d_j(t) + \zeta^c{}_{jL}\sigma_j^*(\vec{x})\bar{d}_j^\dagger\}$$

in which a classical part $\sigma_{cl}(\eta_{cl})$ is separated out. This kind of separation technique is effective in finding low-lying bound-state solutions[1] and is equivalent to the coherent state technique[4].

The global U(1) invariance of an SBM enables one to separate the Hilbert space (constructed by means of the creation operators and the corresponding bare vacuum) into a series of sectors characterized by the net number N of the matter field particles (the bag field is 'neutral') and to consider only the representations of supersymmetry on a sector of given U(1)-charge N (later on, we denote the vectors belonging to sector N as $|N\rangle$, $|N'\rangle$ ... etc.).

The closed set of independent states obtained from a $|N\rangle$ by successive application of $\bar{\varepsilon}Q$ provides us with an off-shell supermultiplet in the N-sector. If the members of the set are at the same time the eigenstates of the Hamiltonian with the same energy, the supermultiplet will become an off-shell one.

Every solution of an SBM must belong to a certain on-shell supermultiplet of the model. But as the first step for finding a solution, one has to know how to construct an off-shell supermultiplet.

The possible resultant state vectors obtained during the process of a successive application of $\bar{\varepsilon}Q$ on $|N\rangle$ are wholly determined by the structure of Q (see (2)). The first type of monomial term ($a^\dagger b$, $ab^\dagger$, etc.) changes a bosonic (fermionic) creation operator in $|N\rangle$ into a fermionic (bosonic) one without altering the total number of operators in $|N\rangle$. The second type of term ($\bar{a}b$, $\bar{a}^\dagger b^\dagger$, etc.) alters the number of particle-antiparticle pairs, hence, the total number of operators in $|N\rangle$.
Moreover, the time derivative $\dot{a}_{sn}$ for instance is determined by

$$\dot{a}_{sn} = i\,[H,\ a_{sn}] \tag{3}$$

[for H, see (9)] and in general depends not only on the matter field operators of modes other than n but also on the bag-field operators.

Therefore, it is most desirable for the aim of constructing an off-shell supermultiplet of finite and minimum dimension to make the Γ's in (2) to satisfy

$$\text{(i)}\quad \Gamma_{s'n',s\dot{n}} \propto \delta_{n'n}\ , \qquad \text{for all } s,s',n,n',$$

$$\text{(ii)}\quad \Gamma s'n',sn = 0\ , \qquad \text{for all } s,s',n,n', \tag{4}$$

by an appropriate choice of wave functions. In fact, if it were the case, all members of an off-shell supermultiplet to which a given state $|N\rangle$ belongs could be constructed from $|N\rangle$ merely by changing some creation operators in $|N\rangle$ (in all possible combinations) each into its supersymmetric partner, i.e., according to the 'intuitive rule'

$$a^\dagger{}_{sn}(\bar{a}^\dagger{}_{sn}) \to b^\dagger{}_{s'n}(\bar{b}^\dagger{}_{s',n})\ ,\quad b^\dagger{}_{sn}(\bar{b}^\dagger{}_{sn}) \to a^\dagger{}_{s'n}(\bar{a}^\dagger{}_{s'n}),$$
$$s,s' = \tfrac{1}{2}, -\tfrac{1}{2}\ . \tag{5}$$

For the non-interacting case (f=0), all requirements of (4) can be fulfilled exactly by simply taking plane waves as $W_n(x)$ together with a suitable choice of constant 4-spinors ϕ_{sn}. And the off-shell supermultiplet constructed from a $|N\rangle$ according to the 'intuitive rule' is also an on-shell one, i.e., an exact solution of the model.

For the interacting case ($f\neq 0$), (4) implies a series of constraints on the wave functions which are too strict to be fulfilled for all modes.

As for the low-lying bound states, however, one can confine oneself within a subspace $\mathcal{H}^{(N)}$ of the N-sector in Hilbert space, which consists of state vectors containing only the lowest mode (n=1) matter-field creation operators. It has been proved that for a model given in [3] the state vectors from $\mathcal{H}^{(N)}$ give good approximations to energy eigenstates ('quasiclassically approximate solution') at least in the case of weak coupling ($f \ll 1$) together with negligible bag-field quantum excitations[3] and one can easily check that the proof is also available for the SBM.

On $\mathcal{H}^{(N)}$, the time derivatives $\dot{a}_s$ and $\dot{b}_s$ (index n is dropped for n=1) will take the effective forms

$$\dot{a}_s = -i(\omega' a_s + \omega'' \bar{a}_s^{\dagger}) \ , \quad \dot{b}_s = -i(\varepsilon' b_s + \varepsilon'' \bar{b}_s^{\dagger}) \ , \tag{6}$$

with constant $\omega' - \omega'' \equiv \omega$ and $\varepsilon' - \varepsilon'' \equiv \varepsilon$ independent of specific state vectors of $\mathcal{H}^{(N)}$. And the matrix elements of Q between two states $|N\rangle, |N'\rangle$ of $\mathcal{H}^{(N)}$ become

$$\langle N'|Q|N\rangle = C_{ss'}^{(+)} \langle N'|a_s^{\dagger} b_{s'}|N\rangle + C_{ss'}^{(-)} \langle N'|\bar{a}_s b_{s'}|N\rangle + \ldots$$

with

$$C_{ss'}^{(\pm)}(t) \propto \left[-(\varepsilon \pm 2\omega) + m\gamma^0 + p_\kappa^{(1)}\gamma^\kappa\right] \phi_{s1} \ ,$$

$$p_\kappa^{(1)} \equiv \int W_1^*(i\overleftarrow{\partial}_\kappa + 2i\overrightarrow{\partial}_\kappa) W_1 d^3x \ / \int W_1^* W_1 d^3x \ . \tag{7}$$

Hence, one can make $C_{ss'}^{(-)} = 0$ and $C_{ss'}^{(+)} \neq 0$ (for all s,s') by choosing n=1 mode matter-field wave functions as follows:

(i) same spatial wave function for both fermionic and bosonic matter field;

(ii) ϕ_{s1} and $|2\omega - \varepsilon|$ satisfying
$$[(\varepsilon - 2\omega) + \gamma^0 m + \gamma^0\gamma^\kappa p_\kappa^{(1)}]\phi_{s1} = 0 \ .$$
$$|\varepsilon - 2\omega| = [m^2 + (p_\kappa^{(1)})^2]^{\frac{1}{2}};$$

iii) $\varepsilon = \omega$ (to satisfy $\langle N'|\frac{\partial Q}{\partial t}|N\rangle = 0$). (8)

Then, the validity of (4) will be established for $\mathcal{H}^{(N)}$ on the quasi-classical level and one can construct the off-shell supermultiplets in $\mathcal{H}^{(N)}$ merely according to the 'intuitive rule' (5). The outstanding peculiarity of a supermultiplet thus constructed is that all its member state-vectors have the same total number of creation operators ($\because \Gamma_{s'1,s1}$ has been made zero by $C_{s's}^{(-)} = 0$).

To check the validity of (5) on $\mathcal{H}^{(N)}$, one inserts (5) into (3) and finds that the three-operator terms of a(b) containing a particle-anti-particle pair (coming from the four-scalar interaction term of H) do not contribute to the matrix element of Q between any two member state vectors of the supermultiplet constructed as above. Moreover, from the structure of $[H, a_s]$, one sees that the difference $\omega'-\omega'$ is a quantity not dependent on $|N\rangle$ or $|N'\rangle$ of $\langle N'|Q|N\rangle$. Of course, (6) is valid only when the bag-field operators in H are approximated by their classical parts.

From (1), the Hamiltonian of an SBM with one matter field can be put into the form

$$
\begin{aligned}
H = \int d^3x \, : \{ & \tfrac{1}{2}(\partial_0 A^\dagger \partial^0 A + \partial_0 B^\dagger \partial^0 B + 2\partial_0 \sigma^\dagger \partial_0{}^0 \sigma + \\
& + \nabla A^\dagger \cdot \nabla A + \nabla B^\dagger \cdot \nabla B + 2\nabla\sigma^\dagger \cdot \nabla\sigma \\
& + \tfrac{1}{2} |m_0 + f\sigma|^2 \left(A^\dagger A + B^\dagger B\right) + M^2 \sigma^\dagger \sigma \\
& + \frac{fM}{2}\left(\sigma^\dagger AB + \sigma A^\dagger B^\dagger\right) + \frac{f^2}{4} A^\dagger B^\dagger AB \\
& + {}^1/_8 \left(i\xi^\dagger \partial_0 \xi + i\xi^{c\dagger} \partial_0 \xi^c + i\chi \partial_0 \chi + i\chi^{c\dagger} \partial_0 \chi^c\right. \\
& + 2\, i\eta^\dagger \partial_0 \eta + 2\, i\eta^{c\dagger} \partial_0 \eta^c)\} \; : + \text{counter-terms.} \qquad (9)
\end{aligned}
$$

The counter-terms are introduced for the renormalization of the model and contribute only to the quantum corrections of the solutions. They therefore have nothing to do with the quasiclassical approximation, provided that the free parameters appearing in H have been set to the renormalized values[1].

As for low-lying bound states in the case when the quasiclassical approximation is good enough, one can consider only $\{\mathcal{H}^{(N)}\}$ in the Hilbert space. The independent members of an off-shell supermultiplet of dimension N_D in a $\mathcal{H}^{(N)}$ can be taken as

$$|t_i\rangle = \prod_{s,\bar{s},s',\bar{s}'} (a_s{}^\dagger)^{N_{BS}^{(i)}} (\bar{a}_{\bar{s}}{}^\dagger)^{\bar{N}_{BS}^{(i)}} (b_{s'}{}^\dagger)^{N_{FS'}^{(i)}} (\bar{b}_{\bar{s}'}{}^\dagger)^{\bar{N}_{FS'}^{(i)}} |0\rangle, \quad s,\bar{s},s',\bar{s}'=\tfrac{1}{2},-\tfrac{1}{2},$$

with

$$\sum_s (N_{BS}^{(i)} + N_{FS}^{(i)}) - \sum_{\bar{s}} (\bar{N}_{BS}^{(i)} + \bar{N}_{FS}^{(i)}) = N , \quad i = 1, 2, \ldots, N_D \,. \tag{10}$$

$[N_{BS}{}^{(i)}$ and $\bar{N}_{BS}{}^{(i)}$ $(N_{FS'}{}^{(i)}$ and $\bar{N}_{FS}{}^{(i)})$ denote the numbers of bosonic (fermionic) n=1 mode creation operators of matter-field particle and anti-particle respectively$]$. But the average energy $\langle t_i|H|t_i\rangle \equiv E^{(i)}$ of $|t_i\rangle$ is generally not of the same value for various values of i. So, in order to get a candidate for the on-shell supermultiplet, one has to form N_D independent linear combinations of (10)

$$|t_i'\rangle = \sum_j C_{ij} |t_j\rangle , \quad i,j = 1, \ldots, N_D, \tag{11}$$

with the constraints

$$|C_{ij}| = |C_j| , \qquad \text{for all } i = 1, \ldots, N_D, \tag{12}$$

such that the average energy of $|t_i'\rangle$ is the same

$$\langle t_i'|H|t_i'\rangle = \sum_{j=1}^{N_D} |C_{ij}|^2 E^{(j)} = \sum_{j=1}^{N_D} |C_j|^2 E^{(j)} \equiv E \text{ for all } i=1, \ldots, N_D.$$

And E can be evaluated straightforwardly from (9) and (11) as

$$E = \frac{\tilde{N}_B\, m_0}{2} \int d^3\rho \,\{[v^2(1 + 2\frac{\tilde{N}_F}{\tilde{N}_B}\frac{E}{|v|}) + |1+X|^2 + 2\mu k \cos\tilde{\theta}\, X + \frac{k^2\tilde{h}}{2\tilde{N}_B} Y^2] Y^2$$

$$+ (\nabla Y)^2 + \frac{2}{\tilde{N}_B}[(\nabla X)^2 + \mu^2 X] + \frac{1}{2\tilde{N}_B m_0{}^3}(i\eta^\dagger{}_{c\ell}\partial_0\eta_{c\ell} + i\eta_{c\ell}{}^{c\dagger}\partial_0\eta^c{}_{c\ell})\}, \tag{13}$$

where

$$\vec{\rho} \equiv m_0 \vec{x}\ , \quad \nu \equiv \frac{\omega_1}{m_0}\ , \quad \varepsilon \equiv \varepsilon_1/m_0\ , \quad \mu \equiv M/m_0\ , \quad k \equiv |U^*_{L1}U_{R1}|$$

$$X(\vec{\rho}) \equiv f\ \sigma_{c\ell}(\vec{x})/m_0, \quad Y(\vec{\rho}) \equiv f\ W_1(\vec{x})/\omega^{\frac{1}{2}} m_0;$$

$$\tilde{N}_B \equiv \sum_j |C_j|^2 N_B^{(j)}\ , \qquad N_B^{(j)} = \sum_s \left(N_{BS}^{(j)} + \bar{N}_{BS}^{(j)}\right), \qquad (B \to F);$$

$$\cos\tilde{\Theta} \equiv \sum_j |C_j|^2 \cos\Theta^{(i)}, \qquad \cos\Theta^{(i)} \equiv \sum_s \left(N_{BS}^{(i)} + \bar{N}_{BS}^{(i)}\right)\cos(\Theta_s + \Theta')/N_B^{(i)};$$

$$h^{(i)} \equiv \sum_s \left\{\left(N_{BB}^{(i)} + \bar{N}_{BS}^{(i)}\right)^2 + 2N_{BS}^{(i)}\bar{N}_{BS}^{(i)}\cos 2\Theta_s\right\};$$

$$k \equiv |U^*_{L1}U_{R1}|\ , \qquad \Theta_s \equiv \text{Arg}\ U_{SR} - \text{Arg}\ U_{SL}\ ,\ \theta' \text{ - see (2)}\ .$$

One can easily check that the coefficients $\{C_{ij}\}$ of (11) are thoroughly determined by (12) and the orthonormal relations $\langle t_i' | t_j' \rangle = \delta_{ij}$, $i,j = 1, \ldots, N_D$ (hold up to some phase factors which can be absorbed into $|t_i'\rangle$ and $\{|t_i\rangle\}$). Namely, for a given off-shell supermultiplet, there is only one set of N_D linear combinations which has the same average energy among its members and one has to choose this unique set of state vectors as the trial state vectors to perform the variational procedure.

To find the energy eigenstate vectors, one should vary $E \equiv \langle t_i' | H | t_i' \rangle$ to reach its minimum with respect to $|t_i'\rangle$, i.e., the wave functions and all other parameters characterizing the creation operators and the bare vacuum, such as, $X(\rho)$, $Y(\rho)$, Θ , Θ_s, k and even ρ_0 (the position of the surface S_0 across which X, Y may have spatial discontinuities). Then, from $\delta E = 0$, $\delta^2 E > 0$, one obtains the equations (together with appropriate boundary conditions) determining wave functions X, Y and all other parameters. Substituting all these results into (13), one obtains the formula for E, i.e., a mass formula for a massive supermultiplet of composite particles ($\eta_{c\ell}$ does not contribute to E, since $\delta E = 0$ gives $\partial_0 \eta_{c\ell} = 0$).

The approach described briefly above can be applied also to similar models obtained by other suitable choices of parameters appearing in [3]

$$L(x) = \phi_i^\dagger \phi_i |_D + \left(\lambda_i \phi_i + \tfrac{1}{2} m_{ij} \phi_i \phi_j + {}^1/_3\ g_{ijk} \phi_i \phi_j \phi_k + \text{h.c.}\right) |_F \ .$$

A model with matter field of zero mass (m=0) has no mixing of left- and right-handed matter-fields and gives rise to an 'intuitive rule' $a_L^\dagger \rightleftarrows b_L^\dagger$, $a_R^\dagger \rightleftarrows b_R^\dagger$. The contents of supermultiplets will change radically in comparison with that of the m≠0 case. For example, there can exist the supermultiplet containing only one ½-spin composite fermion in a number of sectors with different U(1)-charges.

One may also consider the possibility of introducing gauge interactions (including gravitation) into the model.

Acknowledgements

The author is grateful for the warm hospitalities of W. Hoogland and the theory group of NIKHEF-H. He wishes to thank B. de Wit, J.W. van Holten and D. Petcher for helpful discussions and comments.

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