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THE FIFTH FORCE

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ABSTRACT

We present a phenomenological description of the "fifth force" which focuses on the implications of the existing data from satellite and geophysical measurements of gravity, the Eötvös experiment, decays into hyperphotons, and the energy-dependence of the $K^0 - \overline{K}^0$ parameters.

I. INTRODUCTION

Among the many puzzling features of the four known interactions (strong, electromagnetic, weak, and gravitational) which remain to be fully understood, one of the most intriguing is the great disparity in their ranges, *i.e.*, the distances over which they are effective. The electromagnetic and gravitational interactions on the one hand are thought to have infinite range: If we write the potential $V(r)$ describing the interaction of two point objects a distance r apart in the form

$$V(r) \sim f^2 \frac{e^{-r/\lambda}}{r}, \tag{1.1}$$

where f^2 is a constant, then for electromagnetism and gravity $\lambda \to \infty$. By contrast λ is of the order 10^{-13} cm and 10^{-16} cm for the strong and weak interactions respectively. Although forces with infinite range can be naturally accommodated in modern gauge theories, since they arise from the exchange of the massless quanta (photons or gravitons) of the corresponding gauge fields, finite range forces present additional problems. These have to do with the fact that we have as yet no fundamental theory of the elementary particle mass spectrum, and hence no deep understanding, for example, of why the range of the weak interaction, $\lambda_W = \hbar/m_W c = 2.4 \times 10^{-16}$cm has the value that it does. In the absence of any argument which rules out massive but extremely light bosons, we must therefore be open to the possibility that new forces could exist mediated by the exchange of ultralight bosons, whose ranges could be sufficiently large as to produce observable effects at the macroscopic level. Searches for such forces, which have been underway for many years, have recently been given an additional impetus by a reanalysis[1] of the classic experiments of Eötvös, Pekár, and Fekete[2] (EPF) which uncovered in the EPF data evidence for a new intermediate range force. Although the implications of the EPF data are supported by indications of anomalies in geophysical measurements[3] of the Newtonian constant of gravity G and, to a lesser extent, by high-energy data from the $K^0 - \overline{K}^0$ system,[4] the overall evidence for the putative "fifth force" is still tenuous. A detailed review[5] of the present status of the various arguments in favor of the fifth force has been given recently, which includes a summary of the relevant literature. It is clear, however, that the question of whether a fifth force in fact exists can only be settled by the various experiments presently in progress, a compilation of which is also presented in Ref. 5. In anticipation of the arrival of the first data from some of these experiments, we present here a phenomenological description of the fifth force in which we focus on the information that can be obtained from various types of experiments.[6] Irrespective of whether or not these experiments confirm the presence of the fifth force, they will close a "window" which up to recently has been left open by our failure to carry out experiments of sufficient precision at intermediate distance scales ($1\,\text{m} \lesssim \lambda \lesssim 10^3\,\text{m}$). Our objective here is to collect together a number of useful results which explicitly exhibit the constraints on possible new forces implied by the current round of experiments. To aid the reader, we have enclosed the most important of these results in boxes.

II. GENERAL FORMALISM

The experimental data that we are seeking to understand in terms of the fifth force are obviously quite

* Dedicated to the memory of Idella Marx whose love of science was an inspiration to all.

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limited at the present time, and hence it would be premature to attempt a detailed field-theoretic description of the fifth force at this stage. Nonetheless the data do provide a number of clues which can be used to develop a phenomenological framework within which various experiments can be related to one another. The most important of these clues are the following:

1) We learn from the geophysical data (see Sec. IV below) that the new interaction has a *repulsive* component for like charges, and also that its intrinsic strength is approximately 1% that of gravity. Since a repulsive force naturally arises form the exchange of a *vector* ($J = 1$) particle, we assume that at the very least such a field should exist in the theory, with couplings to protons and neutrons.

2) The existence of a vector coupling is also suggested by the EPF data, which point to a field whose source is predominantly baryon number (B). Since the eigenvalues of B (or charge Q, or lepton number L) are opposite for particles and antiparticles, any field which couples to B, Q, or L must be odd under charge conjugation. Since a vector field has this property, but scalar fields and symmetric tensor fields do not, a vector field emerges as the natural candidate to account for the EPF results (see Sec. V below). Because the interactions of baryons or leptons with themselves are necessarily repulsive, if mediated by a vector field, it follows that the EPF results must also be consistent with a repulsive force. This now appears to be the case,[5,7] despite an earlier suggestion to the contrary.[8]

3) Another important clue concerns the range of the new force. Assuming the existence of a single $V(r)$ as in (1.1), the geophysical data suggest a "best value" of $\lambda \approx 200m$. In reality, however, the main constraints on λ come from elsewhere: For values of f^2 consistent with the geophysical results, laboratory experiments[9,10] imply that $\lambda \gtrsim 1\,\mathrm{m}$, whereas satellite data[11] require that $\lambda \lesssim 10^3\,\mathrm{m}$. The allowed range of λ in the "geophysical window" ($1\,\mathrm{m} \lesssim \lambda \lesssim 10^3\,\mathrm{m}$) also delineates the allowed masses $m_Y = \lambda^{-1}$ of the quanta the exchange of which give rise to the fifth force:

$$2 \times 10^{-7}\,\mathrm{eV} \gtrsim m_Y \gtrsim 2 \times 10^{-10}\,\mathrm{eV}. \qquad (2.1)$$

It is interesting to note[5] that the energy scale set by m_Y is as far removed from the ordinary hadron scale, as the latter is from the presumed grand unification (GUT) scale:

$$m_Y : m_\pi : m_{\mathrm{GUT}} \approx 10^{-8}\,\mathrm{eV} : 10^{8}\,\mathrm{eV} : 10^{24}\,\mathrm{eV} \approx 1 : 10^{16} : (10^{16})^2 \qquad (2.2)$$

Whether this observation proves to have a deep significance remains to be seen. However, (2.2) does suggest that we might anticipate the possibility of a rich structure at the scale of 10^{-8} eV, similar to what is observed at the familiar hadron scale but perhaps governed by a different set of symmetries.[12]

4) There is one additional numerical result which, although not essential for our present phenomenological picture, may be important in framing a more detailed theory. Nussinov,[13] and also Bars and Visser[14] have noted that if we describe the geophysical data by the potential in (1.1), then the values of f and m_Y implied by these data satisfy

$$\frac{f^2}{m_Y^2} \cong \frac{1}{(131\mathrm{GeV})^2}. \qquad (2.3)$$

This means that if we assume that the quantum γ_Y which gives rise to (1.1) obtains its mass through some Higgs mechanism [so that $m_Y = f\langle\phi\rangle$] then $\langle\phi\rangle \cong 240$ GeV, which is appropriate to the standard model, comes within a factor of ~ 2 of satisfying (2.3). As noted in Ref. 5, this may be an important relation in pointing to an ultimate unification of the fifth force with other known forces. In a similar vein De Rújula[15] has noted that

$$\frac{f}{e} \approx \frac{M_{\mathrm{hadron}}}{M_{\mathrm{Planck}}} \approx 10^{-19}, \qquad (2.4)$$

where e is the electric charge and $M_{\mathrm{hadron}} \cong 1$ GeV is a typical hadronic mass.

Collecting together the previous observations we assume that the static interaction of two elementary particles a and b mediated by the putative fifth force can be written in the form

$$V_5(r) = \sum_{k=1}^{N} Q_{ka} Q_{kb} \frac{e^{-r/\lambda_k}}{r}, \qquad (2.5)$$

where the summation extends over all N of the scalar, vector, and tensor fields (denoted by ϕ_k) which collectively represent the fifth force. It should be emphasized that we are limiting the present discussion to models in which $V_5(r)$ can be represented by a sum of Yukawa terms, as in (2.5). For those theories[5] in which $V_5(r)$ decreases as a power of r, rather than as an exponential, our formalism should be appropriately modified. We assume that the ϕ_k have only diagonal couplings, so that the "charge" Q_{ka} denotes the strength of the $\phi_k aa$ vertex, etc. For the present we also ignore magnetic effects, which will be discussed elsewhere.[16] Given the limited experimental data now at our disposal, we can safely simplify (2.5) by assuming that the only ϕ_k present in the theory are a single scalar, vector, and tensor field whose couplings to particles a will be denoted by Q_{0a}, Q_{1a}, and Q_{2a} respectively. We must now find the quantum numbers upon which the Q_k depend. In the simplest theory Q_0 and Q_2 will simply be proportional to the mass, and will thereby lead to a universal (*i.e.*, composition-independent) modification of Newton's law, as in Eq. (2.16) below. However, there will be in addition a composition-dependent force which arises from the dependence of Q_1 on the various charges which are the sources of the vector field. In the electromagnetic case one can write

$$Q = Ze, \tag{2.6}$$

where Z is an integer denoting the number of charges, and $e^2/\hbar c \cong 1/137$. Hence by analogy we assume that the vector coupling of ordinary matter is described by

$$Q_1 = (c'_Z Z + c'_N N + c'_L L)\, f, \tag{2.7}$$

where Z, N, and L denote the numbers of protons, neutrons, and electrons respectively, f is a fundamental constant, and $c'_Z, \ldots$ are constants. For ordinary matter $Z = L$ and $Z + N = B$, where B is baryon number, and hence Q_1 can be written in the form

$$Q_1 = (c_B B + c_L L)\, f, \tag{2.8}$$

where c_B and c_L are new constants. Without loss of generality we can define $c_B \equiv 1$ so that for the (presumably dominant) vector coupling

$$Q_1 = (B + c_L L)\, f. \tag{2.9}$$

The parametrization in (2.9) is convenient because our reanalysis of the EPF data indicates that if only the vector coupling is retained then[1,5] $c_L \lesssim O\left(10^{-3}\right)$. Hence to a good approximation we can assume that for ordinary matter the fifth force couples dominantly to B, as suggested in Ref. 1. However, there is nothing which precludes contributions from other flavors in (2.9), such as strangeness (S), charm, truth, beauty, etc. Indeed the implication of the curves in Fig. 1 of Ref. 5 is that if the $K^0 - \overline{K}^0$ data are taken at face value, then there is in fact a coupling to S. Hence (2.9) should be generalized to

$$Q_1 = (B + c_L L + c_S S + \cdots) f, \tag{2.10}$$

where the ... denote possible contributions from other flavors. In Ref. 1 it was assumed that the fifth force coupled to hypercharge, $Y = B + S$, but since there is no fundamental reason to assume that $c_S \equiv 1$ and $c_L \equiv 0$, the expression in (2.10) is a more appropriate starting point to describe experiments. The constants c_L, c_S, ... can in principle be disentangled from one another, since they appear in different combinations in the various processes that we are considering.

The primary evidence for the claimed fifth force comes from the EPF and geophysical experiments, in which anomalies are seen in what would have been thought otherwise to be a pure gravitational interaction. It is thus natural to ask how the presence of V_5 in (2.5) would modify the usual Newtonian gravitational interaction. Consider the potential energy between two point objects a and b located a distance r apart, in the presence of the Newtonian potential $V_N(r)$ and the *vector* contribution to $V_5(r)$. (We will return to include the scalar and tensor contributions below.) For simplicity we use the results of Refs. 1 and 5 to set $c_L \cong 0$ for present purposes, bearing in mind that the contribution $\propto L$ can easily be reinstated at any stage via the substitution $B \to B + c_L L$. The total potential $V(r)$ is then given by

$$\begin{aligned} V(r) &= V_N(r) + V_5(r) \\ &= -G_\infty \frac{m_a m_b}{r} + f^2 \frac{B_a B_b}{r} e^{-r/\lambda_1} \\ &= -G_\infty \frac{m_a m_b}{r}\left(1 - \frac{f^2 B_a B_b}{G_\infty m_a m_b} e^{-r/\lambda_1}\right) \\ &\equiv -G_\infty \frac{m_a m_b}{r}\left(1 + \alpha_{ab} e^{-r/\lambda_1}\right). \end{aligned} \tag{2.11}$$

It is convenient to simplify α_{ab} by expressing all masses in terms of $m_H \equiv m({}_1H^1) = 1.00782519(8)u$ so that $m_a = \mu_a m_H$, etc. This gives

$$\alpha_{ab} = -\frac{B_a}{\mu_a}\frac{B_b}{\mu_b}\xi; \quad \xi = \frac{f^2}{G_\infty m_H^2} \cong 10^{-2}, \tag{2.12}$$

where the value of ξ is inferred from the geophysical data as we describe in Sec. IV below. In the form of Eq. (2.11) the effect of $V_5(r)$ appears as a modification of the Newtonian interaction. However, one must be careful to note that *the modified coupling destroys the universality of the effective gravitational interaction*, by introducing terms proportional to α_{ab} which explicitly depend on the chemical compositions of the interacting objects. For those experiments in which the chemical compositions of various bodies are not a significant factor, which include the geophysical and satellite analyses to be discussed below, we can write

$$B_a/\mu_a \cong B_b/\mu_b \cong 1; \quad \alpha_{ab} \cong -\xi = \text{constant}. \tag{2.13}$$

However, for the EPF experiment the entire effect depends on the difference from unity of the various B/μ, and hence the complete expression for $V(r)$ in (2.11)–(2.12) must be used.

In addition to the modification of the Newtonian interaction introduced by the vector couplings, there will be contributions to $V(r)$ from scalar and tensor exchanges as well. As noted previously, in simple models these typically give rise to effects which are proportional to the masses of the interacting bodies and, if this is the case, their contributions will preserve the universality of the gravitational interaction. Eq. (2.11) thus generalizes to

$$\begin{aligned} V(r) = -G_\infty \frac{m_a m_b}{r}\Big(1 &+ \alpha_{ab} e^{-r/\lambda_1} \\ &+ \alpha_0 e^{-r/\lambda_0} + \alpha_2 e^{-r/\lambda_2}\Big), \end{aligned} \tag{2.14}$$

where α_0 and α_2 are strictly constant (*i.e.*, composition-independent), and where α_{ab} can be approximated (for some purposes only!) by a constant, which we call α_1. From (2.12) we see that $\alpha_1 \cong -\xi$, and we presume that

$\alpha_{0,2} = O(\xi)$ as well. Collecting the preceding results together we can finally write

$$V(r) = -G_\infty \frac{m_a m_b}{r}\left(1 + \sum_{k=0}^{2} \alpha_k e^{-r/\lambda_k}\right). \qquad (2.15)$$

Differentiating (2.15), the force $\vec{F}(r) = -\vec{\nabla}V(r)$ is

$$\boxed{\begin{aligned}\vec{F}(r) &= -G_\infty \frac{m_a m_b}{r^2}\hat{r}\left[1 + \sum_k \alpha_k e^{-r/\lambda_k}\left(1 + r/\lambda_k\right)\right] \\ &\equiv -G(r)\frac{m_a m_b}{r^2}\hat{r}\end{aligned}} \qquad (2.16)$$

with

$$G(r) = G_\infty\left[1 + \sum_k \alpha_k e^{-r/\lambda_k}\left(1 + r/\lambda_k\right)\right]. \qquad (2.17)$$

For laboratory experiments, where $r/\lambda_k \ll 1$ can be presumed to hold,

$$G(r) \cong G(0) \equiv G_0 = G_\infty\left[1 + \sum_k \alpha_k\right], \qquad (2.18)$$

so that G_0 is the usual laboratory value. By contrast for satellite measurements or planetary motion, where $r/\lambda_k \gg 1$,

$$G(r) \cong G(\infty) = G_\infty. \qquad (2.19)$$

It follows from Eqs. (2.16)–(2.19) that the usual $1/r^2$ force law pertains over distance scales that are either much smaller or much larger than λ_k, albeit with different values of $G(r)$. Observing such a difference between G_0 and G_∞ would, of course, be evidence for a new coupling. However, G_∞ always appears multiplied by some mass M which must itself be independently determined. One therefore requires an independent measurement of M, and the Airy technique — recently revived by Stacey and coworkers[3] — is a practical method of doing just that. Bearing in mind that the geophysical results have in fact indicated that G_0 is about 1% smaller than G_∞, we conclude that the masses of various celestial bodies as determined by measuring $G_\infty M$ (and then identifying G_∞ with G_0), are approximately 1% too high. Combining Eqs. (2.18) and (2.12) we can write for the case where only $\alpha_1 \cong -\xi$ contributes,

$$\frac{f^2}{G_0 m_H^2} \cong \frac{\xi}{1-\xi} \cong \frac{-\alpha_1}{1+\alpha_1}. \qquad (2.20)$$

Equations (2.15) and (2.16), which describe the coupling of point particles, can be integrated to give the appropriate expressions for the interaction of an extended source (such as the Earth) with a point mass. If we assume for simplicity that the Earth is a rigid sphere with mass $M_\oplus$, radius $R_\oplus$, and density $\rho_\oplus$ then with

$$\Phi(x) = \frac{3}{x^3}\left(x\cosh x - \sinh x\right) \qquad (2.21)$$

we have

$\underline{r \geq R_\oplus}$

$$\begin{aligned}V_{5a}(r) &= \frac{f^2 B_\oplus B_a}{r}\left\{\frac{\bar{\rho}}{\rho_\oplus} e^{-r/\lambda}\Phi\left(\frac{R_\oplus}{\lambda}\right)\right\} \\ \vec{F}_{5a}(r) &= \hat{r}_5 \frac{f^2 B_\oplus B_a}{r^2}\left\{\frac{\bar{\rho}}{\rho_\oplus}(1 + \frac{r}{\lambda})e^{-r/\lambda}\Phi\left(\frac{R_\oplus}{\lambda}\right)\right\}\end{aligned} \qquad (2.22)$$

$\underline{r \leq R_\oplus}$

$$\begin{aligned}V_{5a}(r) &= \frac{f^2 B_\oplus B_a r^2}{2R_\oplus^3}\left\{\frac{\bar{\rho}}{\rho_\oplus}\frac{6\lambda^2}{r^2}\right. \\ &\quad\left.\times\left[1 - \left(1 + \frac{R_\oplus}{\lambda}\right)e^{-R_\oplus/\lambda}\frac{\sinh(r/\lambda)}{r/\lambda}\right]\right\} \qquad (2.23) \\ \vec{F}_{5a}(r) &= \hat{r}_5 \frac{f^2 B_\oplus B_a r}{R_\oplus^3}\left\{\frac{\bar{\rho}}{\rho_\oplus}\left(1 + \frac{R_\oplus}{\lambda}\right)e^{-R_\oplus/\lambda}\Phi\left(\frac{r}{\lambda}\right)\right\}.\end{aligned}$$

The terms in { } are modifications to $V_{5a}(r)$ and $\vec{F}_{5a}(r)$ arising from the short-range nature of the force, and $\bar{\rho}/\rho_\oplus$ accounts for fact that only the local matter distribution (with average density $\bar{\rho}$) contributes to the net coupling of the non-Newtonian term. Finally $\hat{r}_5$ is the unit vector pointing in the direction of $\vec{F}_5(r)$, so that $\hat{r}_5 = \hat{r}$ for the case a spherical Earth. The results in (2.22) and (2.23) describe the contribution from a single vector component of the fifth force, whose strength is characterized by f^2 or α_1, using (2.20). We can generalize these results in an obvious way to include an arbitrary number of scalar, vector, and tensor fields with strengths f_k or α_k. It is important to bear in mind that in the model we are considering α_0 and α_2 will be composition-independent, but α_1 will not. However, in the satellite and geophysical determinations of $g(z)$, no attempt has been made to date to search for composition-dependent effects, and so for these systems *all* of the α_k can be taken to be composition-independent. (Of course, if one were to make a deliberate effort to measure $g(z)$ with different test masses, then by looking for small differences in the results one could in principle sort out the composition-dependent contribution from α_1.) Eqs. (2.22) and (2.23) form the basis for the ensuing discussion.

III. THE SATELLITE DATA

Satellite data, which measure the local acceleration of gravity $g(z)$ at a height z above the surface of the Earth, can be used to derive constraints on intermediate-range forces.[11,17] A satellite orbiting at ≈ 5900 km above the surface of the Earth would be beyond the range of a force for which $\lambda \approx (10 - 10^3)$ m, and hence its motion would be governed solely by Newtonian gravity. Thus if satellite data for $g(z)$ are extrapolated to the surface of the Earth, these determine the quantity $g_{sat}(0)$ which is a direct measure of the pure Newtonian contribution. However, if there are additional non-Newtonian terms present in $V(r)$, then a terrestrial determination of the acceleration $g_{terr}(0) = g_0 = |\vec{g}_0| \cong 978\,\mathrm{cm\,s^{-2}}$ at the surface of the Earth will differ from $g_{sat}(0)$. Hence the relevant experimental quantity is

$$\frac{\Delta g}{g_{sat}(0)} = \frac{g_{terr}(0) - g_{sat}(0)}{g_{sat}(0)}. \qquad (3.1)$$

From Eq. (2.22) we see that in the presence of the fifth force, the acceleration $g(z)$ at a height z above the Earth is given by

$$g(z) = \frac{G_\infty M_\oplus}{(R_\oplus + z)^2}\left\{1 + \sum_k \frac{\bar{\rho}_k}{\rho_\oplus}\alpha_k \Phi(x_k) e^{-x_k} e^{-z/\lambda_k} \times \left[1 + (R_\oplus + z)/\lambda_k\right]\right\}, \qquad (3.2)$$

where $x_k = R_\oplus/\lambda_k$. The factors of $\bar{\rho}_k/\rho_\oplus$ account for the fact that the Newtonian acceleration which multiplies $\{\ldots\}$ depends on the mean density $\rho_\oplus$ of the Earth, whereas each of the intermediate-range non-Newtonian terms depends instead of on the average local density $\bar{\rho}_k$ within a distance $\approx \lambda_k$ of the surface. We see from (3.2) that for a satellite measurement (where $z \gg \lambda_k$), $g(z)$ goes over to the usual Newtonian expression

$$g_{sat}(z) \xrightarrow{z \gg \lambda_k} \frac{G_\infty M_\oplus}{(R_\oplus + z)^2}. \qquad (3.3)$$

It follows that if the satellite measurements are extrapolated to the surface of the Earth, the apparent acceleration will be

$$g_{sat}(0) = \frac{G_\infty M_\oplus}{R_\oplus^2}. \qquad (3.4)$$

However, the actual terrestrial value of the acceleration $g_0 \equiv g_{terr}(0)$ (as measured at the surface) is

$$g_0 = \frac{G_\infty M_\oplus}{R_\oplus^2}\left[1 + \sum_k \frac{\bar{\rho}_k}{\rho_\oplus}\alpha_k \Phi(x_k) e^{-x_k}(1 + x_k)\right] \cong \frac{G_\infty M_\oplus}{R_\oplus^2} + 2\pi G_\infty \bar{\rho} \sum_k \alpha_k \lambda_k, \qquad (3.5)$$

where we have assumed that $\bar{\rho}_k \cong \bar{\rho} =$ constant. Combining the previous results we find immediately,

$$\boxed{\frac{\Delta g}{g_{sat}(0)} \cong \frac{3}{2R_\oplus}\frac{\bar{\rho}}{\rho_\oplus}\sum_k \alpha_k \lambda_k} \qquad (3.6)$$

(A slightly different variant of (3.6) is quoted by Stacey, *et al.*[18]) Measuring $\Delta g/g_{sat}(0)$ thus constrains $\sum_k \alpha_k \lambda_k$ but, unlike the geophysical case to be discussed below, α_k or λ_k cannot be extracted separately.

In practice the modeling and experimental determination of $\Delta g/g$ are somewhat involved, as has been discussed recently by Stacey, *et al.*,[18] and by Rapp.[11] The best experimental value is

$$\left(\frac{\Delta g}{g}\right)_{expt} \cong (2 \pm 10) \times 10^{-7}, \qquad (3.7)$$

where the central value is from Rapp and the 1σ error is taken from the more recent analysis of Stacey, *et al.*,[18] from whom we infer the constraint

$$\left|\sum_k \alpha_k \lambda_k\right| \lesssim 14\,\mathrm{m}. \qquad (3.8)$$

By way of comparison, the best values of HST, $\alpha = -0.0075$ and $\lambda = 200\,\mathrm{m}$, correspond to

$$|\alpha\lambda| \cong 1.5\,\mathrm{m}. \qquad (3.9)$$

We thus see that the comparison of satellite and terrestrial measurements would have to be improved by roughly an order of magnitude in order to detect an effect at the level suggested by the nominal geophysical parameters. However, even at the present level of precision, the satellite data provide a strong constraint on possible values of α_k and λ_k. In particular if we combine (3.8) with the best HST value of α we find $\lambda \lesssim 2 \times 10^3\,\mathrm{m}$, which gives the upper end of the "geophysical window".

IV. THE GEOPHYSICAL DATA

The geophysical support for the fifth force arises from evidence that the value of the Newtonian constant measured over distance scales of order $(100 - 1000)\,\mathrm{m}$ differs from the laboratory value[19] G_0,

$$G_0 = 6.6726(5) \times 10^{-8}\,\mathrm{cm^3 g^{-1} s^{-2}}. \qquad (4.1)$$

To understand how the geophysical value G_1 is determined, we begin by noting that the experimental input consists of data for the accelerations $g(z)$ at various depths z below the surface, along with information on the local density $\bar{\rho}$. Consider the function $g(z, \alpha_k)$, which is the acceleration that would be measured at a depth z below the surface of the Earth in the combined presence of gravity and the fifth force (characterized by the α_k). One can then examine the functions

$$\Delta g(z, \alpha_k) = g(z, \alpha_k) - g(0, \alpha_k), \qquad (4.2)$$

$$\Delta g(z, 0) = g(z, 0) - g(0, 0), \qquad (4.3)$$

which describe the difference in the local acceleration at the surface and at a depth z, as would hold in the presence of the fifth force [Eq. (4.2)], or if only Newtonian gravity were present [Eq. (4.3)]. The relevant experimental quantity is the difference of these two functions, which is defined as the *gravity residual* $\delta\Delta g(z)$

$$\delta\Delta g(z) \equiv \Delta g(z,\alpha_k) - \Delta g(z,0). \qquad (4.4)$$

Physically $\delta\Delta g(z)$ is just the difference of the local acceleration measured at the surface and at a depth z, corrected for what one expects from Newtonian gravity. If we consider the simplified model of a spherical non-rotating Earth with constant local density $\bar{\rho}$, then if there were no fifth force ($\alpha_k = 0$), G_∞ in Newton's law would have the value measured in the laboratory G_0, and the expected value for $g(z) - g(0)$ would be

$$\Delta g(z,0) = 2g_0\frac{z}{R_\oplus} - 4\pi G_0\bar{\rho}z, \qquad (4.5)$$

where we assume that $z \ll R_\oplus$. The first term on the right hand side of (4.5) is known as the *free air gradient*, and describes the *increase* of $g(z,0)$ with depth as one comes closer to the center of mass of the Earth. The second term is known as the *double Bouguer term*, and represents the *decrease* in $g(z,0)$ owing to the fact that in going from 0 to z one crosses a layer of matter with average density $\bar{\rho}$ and thickness z. [It is an interesting fact that these two terms very nearly cancel in practice, so that $g(z,0)$ varies relatively slowly with z.] In the presence of the fifth force, which acts independently of gravity, there is an additional contribution proportional to the α_k so that the value of g_0 is given by

$$g_0 = \frac{G_0M_\oplus}{\left(1+\sum_j\alpha_j\right)R_\oplus^2} + \frac{2\pi G_0\bar{\rho}}{\left(1+\sum_j\alpha_j\right)}\sum_k\alpha_k\lambda_k, \qquad (4.6)$$

where we have used (3.5) and (2.18). Hence when $\alpha_k \neq 0$, $\Delta g(z,\alpha_k)$ is given by

$$\Delta g(z,\alpha_k) = \left[\frac{2G_0M_\oplus z}{\left(1+\sum_j\alpha_j\right)R_\oplus^3} - \frac{4\pi G_0\bar{\rho}z}{\left(1+\sum_j\alpha_j\right)}\right] + \frac{2\pi G_0\bar{\rho}}{\left(1+\sum_j\alpha_j\right)}\sum_k\alpha_k\lambda_k\left(e^{-z/\lambda_k}-1\right). \qquad (4.7)$$

The expression in [ ] is the Newtonian contribution, which from (2.18) now depends on α_k when expressed in terms of G_0, and the remaining term is the direct contribution from $\vec{F}_5$ in (2.23). Combining Eqs. (4.4)–(4.7) we obtain the expression of HST[3] for the gravity residual $\delta\Delta g(z)$, for the case $\lambda_k \ll R_\oplus$:

$$\boxed{\delta\Delta g(z) = +\frac{4\pi G_0\bar{\rho}}{\left(1+\sum_j\alpha_j\right)}\sum_k\alpha_k \times\left[z+\frac{\lambda_k}{2}\left(e^{-z/\lambda_k}-1\right)\right].} \qquad (4.8)$$

It should be emphasized that the left-hand side of Eq. (4.8) is a directly measurable quantity. It is arrived at by first determining $[g(z) - g_0]$, and then correcting the values so obtained by subtracting the full Newtonian contribution [as given in Eqs. (2)-(5) of HST], which includes the effects of the Earth's rotation, variable ellipticity, variable density, etc.

In principle the experimental values $\delta\Delta g(z)$ and $\bar{\rho}$ can be used to infer the α_k and λ_k. In practice the information that can be extracted is limited by the lack of sensitivity to λ_k: For $z \ll \lambda_k$ the expression in [ ] in (4.8) is just $z/2$, whereas for $z \gg \lambda_k$ it is z. In either case only $\sum_k\alpha_k/(1+\sum_j\alpha_j)$ can be determined from the slope $d(\delta\Delta g(z))/dz$, and if we assume that $\sum_k$ is dominated by a single constant α, then the slope directly determines α. This is the origin of one of the most interesting features of the geophysical data, namely, that $\alpha/(1+\alpha)$ [or $\sum_k\alpha_k/(1+\sum_j\alpha_j)$] can be determined separately from λ. Hence the "compensation" for not being able to determine λ, is that the decoupling of α and λ in the geophysical data allows the characteristic strength f^2 of the fifth force to be established with some confidence through Eq. (4.8).

Since the decoupling of α and λ is so important a part of the geophysical analysis, it is useful to have a physical picture of how this comes about. It is easy to see that an observer located at a depth $z \gg \lambda$ experiences no effects due to the fifth force. This is because such an observer is effectively at the center of a sphere of radius $r \approx \lambda$, and so the effects of the matter with which he interacts via the fifth force cancel by spherical symmetry. Only when the observer is at a depth $z \lesssim \lambda$ is the spherical symmetry of the surrounding matter destroyed, and the effects of the fifth force start to show up. In particular an observer at the surface of the Earth interacts with a hemisphere of matter below him, and since the cancelling upper hemisphere is absent, a net effect due to the fifth force results. Although measurements of $g(z)$ at depths $z \lesssim \lambda$ could thus tell us about λ, these are hard to carry out because weathering of the surface layers of the Earth leads to unreliable values for the densities $\bar{\rho}(z)$. For $z \gg \lambda$ the observer sees only the usual Newtonian gravitational interaction, whose strength is characterized by the constant $G_\infty = G_0/(1+\alpha)$. In this case a measurement of $\Delta g(z)$ will give a result different from that expected only because G_∞ is different from G_0. Hence

$$\begin{aligned}\delta\Delta g(z) &= \Delta g(z)|_{G=G_\infty} - \Delta g(z)|_{G=G_0}\\ &\cong -4\pi\bar{\rho}z\left\{\frac{G_0}{1+\alpha} - G_0\right\}\\ &= +4\pi G_0\bar{\rho}z\frac{\alpha}{1+\alpha}.\end{aligned} \qquad (4.9)$$

We have thus rederived the gravity residual (for $z \gg \lambda$), and have shown that it is approximately independent of λ.

We conclude this section with a brief discussion of the results of the geophysical analysis. Expressed in terms of α and λ, HST find

$$\alpha \approx \begin{cases} -0.0075 & \text{if } \lambda \leq 200\,\text{m} \\ \vdots & \\ -0.014 & \text{if } \lambda \geq 10^4\,\text{m} \end{cases} \qquad (4.10)$$

where the dots indicate a set of intermediate values of α for $200\,\text{m} < \lambda < 10^4\,\text{m}$. Alternatively one can simply assume that Newtonian gravity works, and use a more complete version of (4.5) [Eqs. (2)-(5) of HST] to infer the Newtonian constant G. If there are additional non-Newtonian forces present, the geophysical value of G obtained this way (G_1) will differ from the laboratory value G_0, and this is exactly what HST find. Their best value, obtained from the Hilton mine data, is[3]

$$G_1 = (6.720 \pm 0.002 \pm 0.024) \times 10^{-8}\,\text{cm}^3\,\text{g}^{-1}\,\text{s}^{-2}, \quad (4.11)$$

where the first error term is statistical and the second is systematic. The latter arises from the lack of precision in the density determinations, and the indicated error is an estimate of the maximum possible contribution that the density uncertainty can have on G_1. There is, however, an additional source of uncertainty: This arises from the possibility of a bias in the gravity gradient due to a mass irregularity located far below the surface of the Earth, which has up to this point gone undetected. The interested reader is referred to the excellent papers by Stacey and collaborators[3] which discuss this and other related questions. For purposes of describing decays into hyperphotons it is useful to express the geophysical results in terms of (f^2/e^2). Using the approximate value $-\alpha \cong (1.0 \pm 0.4) \times 10^{-2}$ we find

$$(f^2/e^2) \cong (8 \pm 3) \times 10^{-39}. \qquad (4.12)$$

V. THE EÖTVÖS EXPERIMENT

The Eötvös experiment as carried out by EPF[2] is designed to compare the local acceleration of substances with differing chemical composition. In principle such a comparison can be carried out by simply dropping two objects simultaneously, as was allegedly done by Galileo, but until very recently the sensitivity of such an experiment was not very great. In the EPF experiment the test masses, rather than being dropped, are placed at opposite ends of a rod which is suspended by a fibre. Any difference in the local acceleration of the masses leads to a torque about the fibre axis, which is what the EPF experiment measures.

In terms of the phenomenological picture of Sec. II, the EPF experiment is a direct measure of α_{ab} in (2.12) and (2.14). Since α_0 and α_2 do not give rise to composition-dependent effects, they can contribute to the satellite and geophysical data, but not to the EPF results. For this reason care must be exercised in comparing these experiments quantitatively. More specifically if a in Eq. (2.12) denotes the attracting body (*e.g.*, the Earth) and b, b' represent the test masses being compared, then the EPF experiment determines the quantity

$$\alpha_{ab} - \alpha_{ab'} \cong -\xi \left(\frac{B_b}{\mu_b} - \frac{B_{b'}}{\mu_{b'}} \right). \qquad (5.1)$$

Considerable attention has been devoted recently to the question of what the dominant attracting source in the EPF experiment actually is, and the answer appears to depend on what the range λ_1 in (2.14) is. To understand the various possibilities we begin with the simplified picture in Ref. 7 in which the dominant matter sources are taken to be the Earth itself (viewed as a rotating sphere), along with the building (and its basement) which housed the EPF experiment. EPF measured the quantity $\Delta a_\perp / g_0$, where $\Delta a_\perp = (a_1 - a_2)_\perp$ is the acceleration difference of two objects in a direction perpendicular to the net acceleration field $\vec{g}_0 \equiv (\vec{a}_c + \vec{g}_N)$, and $\vec{a}_c$ is the centrifugal acceleration due to the Earth's rotation. $\Delta\vec{a} = \vec{a}_1 - \vec{a}_2$ is given by

$$\Delta\vec{a} = \Delta(B/\mu)\vec{y}, \qquad (5.2)$$

where $\vec{y} = \vec{F}_{5a}/(B_a m_H)$ is the force field of the presumed source, and $\Delta(B/\mu) = B_1/\mu_1 - B_2/\mu_2$. If we denote the contributions from the building (basement) by $y'(y'')$, and assume that these are located at an azimuthal angle $\phi'(\phi'')$ relative to the apparatus, then the experimental quantity $\Delta\kappa$ quoted by EPF is,[5,7]

$$\Delta\kappa \equiv \frac{\Delta a_\perp}{-g_0 \sin\beta} = \Delta\left(\frac{B}{\mu}\right) \left\{ -\frac{y}{g_0} - \frac{y'}{g_0} \frac{\cos(\phi' + \beta)}{\sin\beta} + \frac{y''}{g_0} \frac{\cos(\phi'' + \beta)}{\sin\beta} \right\}, \qquad (5.3)$$

where $\beta \cong \beta_1 \cong \beta_2$ is the angle that a plumb line makes (for mass 1,2) with the vertical. Numerically β is given by

$$\beta \cong \tan\beta = \frac{a_c \sin\theta}{g_0} \cong \frac{1}{581}, \qquad (5.4)$$

with $\theta \cong 45°$ being the latitude at which the EPF experiment was performed. The presence of the factor $\sin\beta$ in (5.3) enhances the contributions from y' and y'' relative to that from y, with the result that y' and y'' largely determine both the sign and magnitude of the expression in { } in (5.3). It is important to point out that the signs of the second and third terms in { } depend on the location of the building and its basement relative to the apparatus, but since this information is provided in the EPF paper, we have evaluated (5.3) for the appropriate configuration. Note that the relative sign of y'' and y' is a consequence of the fact that the "missing mass" from the basement acts as a "hole" in the otherwise uniform matter distribution of the Earth. It turns out that for typical buildings $y'' > y' > y$, and that as a consequence the sign of the expression in { } is determined by the third term.

We see from (5.3) that $\Delta\kappa$ is predicted to be directly proportional to $\Delta(B/\mu)$ with a proportionality constant γ given by { } in (5.3). The EPF data have been discussed extensively in Refs. 1,2,5, and 7 and our best fit to these data gives

$$\begin{aligned} \Delta\kappa &= \gamma\Delta(B/\mu) + \delta \\ \gamma &= (+5.60 \pm 0.71) \times 10^{-6} \\ \delta &= (+0.64 \pm 0.62) \times 10^{-9} \\ \chi^2 &= 2.2 \text{ (6 degrees of freedom)}. \end{aligned} \quad (5.5)$$

We see from the fit in (5.5), which is shown in Fig. 2 of Ref. 5, that a plot of $\Delta\kappa$ versus $\Delta(B/\mu)$ does indeed lead to a straight line, which passes through the origin as it should. Moreover, the sign and magnitude of the slope γ are in reasonably good agreement with what one expects from (5.3), given our crude knowledge of the various quantities in { }. We note that if the fifth force has a range of order 200 m, then the first term in (5.3) can be dropped. This is because the Earth is not spherical, as we assumed in deriving (5.3), which is a consequence of the fact that the Earth elastically deforms so that its mean surface (geoid) lies along an equipotential of the combined gravitational and rotational fields. For this reason $\vec{g}_\oplus$ is parallel to $(\vec{g}_N + \vec{a}_c)$, which determines the direction along which the fibre points. Hence $\vec{g}_\oplus$ has no component perpendicular to this direction, which is the only component that the EPF torsion balance can detect. The remaining terms in (5.3) can be numerically evaluated by knowing (at least approximately) the matter distribution at the time of the experiment. We have done so and find $\gamma \approx +1.7 \times 10^{-6}$. For the case of a force with range comparable to $R_\oplus$, $\vec{g}_\oplus$ and $\vec{g}_N$ are parallel, but since the fibre axis is determined by $\vec{g}_0$, a net torque due to $\vec{g}_\oplus$ results. We refer the reader to Refs. 5, 7, and 16 for further discussion.

VI. DECAYS INTO HYPERPHOTONS

As noted in Sec. II, the EPF results and the geophysical data point to a coupling of the fifth field to some linear combination of N, Z, and L but say nothing about couplings to other quantum numbers. As we discuss in Sec. VII below, evidence for a coupling to strangeness or hypercharge comes from indications of an anomalous energy-dependence of the $K^0 - \overline{K}^0$ parameters.[4] Although this evidence is relatively weak at present,[20] the possibility of a coupling of the fifth field to the $K^0 - \overline{K}^0$ system certainly cannot be ruled out. If such a coupling exists then decays of kaons into a real hyperphoton (γ_Y) should also occur, and the best way of looking for these is through the decays $K^\pm \to \pi^\pm\gamma_Y$, as has been noted recently by a number of authors.[12,21]

The observation that the rate of K-decay into γ_Y could be sufficiently large to be detectable (despite the fact that f is extremely small), was made earlier in another context by Weinberg,[22] and has been revived recently in Ref. 1. If γ_Y is a massive *vector* particle, then the amplitude T for emission of γ_Y can be written in the form

$$T = \bar{f}\epsilon_\mu(q)\mathcal{M}_\mu\,, \quad (6.1)$$

where $\epsilon_\mu(q)$ is the polarization vector for a γ_Y with momentum q, and $\bar{f} = (4\pi)^{1/2}f$ is the $KK\gamma_Y$ coupling (with f in Gaussian units and with c_S in (2.10) set equal to unity for simplicity). From Eq. (6.1)

$$|T|^2 = \bar{f}^2\left(\delta_{\mu\nu} + q_\mu q_\nu/m_Y^2\right)\mathcal{M}_\mu\mathcal{M}_\nu^*\,. \quad (6.2)$$

As noted by Weinberg,[22] if γ_Y couples to the (nonconserved) hypercharge current then $q_\mu\mathcal{M}_\mu \neq 0$ in which case the term proportional to m_Y^{-2} survives. For sufficiently large $\lambda = m_Y^{-1}$, as in the present case, this term gives the dominant contribution to $|T|^2$ so that

$$|T|^2 \cong \bar{f}^2\lambda^2(q_\mu\mathcal{M}_\mu)(q_\nu\mathcal{M}_\nu)^*. \quad (6.3)$$

From (6.3) the decay rate Γ for $K^\pm \to \pi^\pm\gamma_Y$ is given by[12]

$$\Gamma(K^\pm \to \pi^\pm\gamma_Y) = \frac{|\vec{p}|}{2m_K^2}f^2\lambda^2|a(K^\pm - \pi^\pm)|^2, \quad (6.4)$$

where $|\vec{p}|$ is the pion 3-momentum in the rest frame of the decaying kaon. In arriving at (6.4) we have assumed that the decay proceeds in two steps: The γ_Y is first emitted by the $K^\pm$, which then (virtually) converts to $\pi^\pm$ by the weak interaction. The amplitude for this transition, denoted by $a(K^\pm - \pi^\pm)$ in Eq. (6.4), has been calculated in a number of different models by the authors of Refs. 12 and 21. Since this is an unphysical amplitude, various extrapolations of the $K^\pm$ and $\pi^\pm$ momenta arise, and differing ways of treating these lead to the appearance or disappearance of factors of order m_π^2/m_K^2 in $a(K^\pm - \pi^\pm)$. In $\Gamma(K^\pm \to \pi^\pm\gamma_Y)$, and hence in the branching ratio $\Gamma(K^\pm \to \pi^\pm\gamma_Y)/\Gamma(K^\pm \to all)$, these extrapolations lead to uncertainties of order $(m_{\pi^0}/m_{K^0})^4 = 5.4 \times 10^{-3}$. This in large measure explains the differences obtained by the various authors. Clearly more work is required to produce a reliable calculation of the $K^\pm \to \pi^\pm\gamma_Y$ branching ratio, from which one can extract a bound on $f^2\lambda^2$ by comparison to experiment. To facilitate this comparison, for a given model of $a(K^\pm - \pi^\pm)$, we write the branching ratio in the form

$$\boxed{\begin{aligned} \text{B.R.}\,(K^\pm \to \pi^\pm\gamma_Y) &= 5.6 \times 10^{13}\text{eV}^2 \\ &\times \left|\frac{a(K^\pm - \pi^\pm)}{1.0 \times 10^9\text{eV}^2}\right|^2 (f^2/e^2)\,\lambda^2, \end{aligned}} \quad (6.5)$$

where e is the electric charge (in Gaussian units), and $a(K^\pm - \pi^\pm)$ is normalized as in Ref. 12.

To illustrate the implications of the experimental data we consider the results of Ref. 12 which obtains

$$a(K^{\pm} - \pi^{\pm}) = 2.9 \times 10^{9}\mathrm{eV}^{2}, \qquad (6.6)$$

$$\mathrm{B.R.}(K^{\pm} \to \pi^{\pm}\gamma_{Y}) = 4.7 \times 10^{14}\mathrm{eV}^{2}(f^{2}/e^{2})\lambda^{2}. \qquad (6.7)$$

Eq. (6.7) can be compared to the experimental bounds implied by searches for $K^{\pm} \to \pi^{\pm}\tilde{a}$ ($\tilde{a}$ =axion), which would have the same signal as the γ_Y decay mode we are interested in: In both case the neutral particle would go undetected, and so the signal for these modes would be a $\pi^{\pm}$ with $|\vec{p}| = 227$ MeV and nothing else observed. The experimental limit on this decay mode is,[23]

$$\mathrm{B.R.}(K^{\pm} \to \pi^{\pm}\gamma_{Y}) < 4.6 \times 10^{-8} \qquad (6.8)$$

which implies

$$\left|\frac{\alpha}{1+\alpha}\right| \left(\frac{\lambda}{1\,\mathrm{m}}\right)^{2} \leq 4.7\,. \qquad (6.9)$$

For the HST value $\alpha = -0.0075(38)$ this gives $\lambda \lesssim 25$ m, which is very much smaller than their "best value" $\lambda \approx 200$ m. Moreover, the estimates of $a(K^{\pm} - \pi^{\pm})$ given in Refs. 21 are substantially larger than (6.6), and would thus imply even smaller values of λ.

These results have led various authors to suggest that the limit in (6.8) is already incompatible with the HST results. This is premature for the following reasons: To start with, the most recent analysis of HST[3] acknowledges a much greater uncertainty in λ than is implied by the value originally quoted in Ref. 1, namely $\lambda = (200 \pm 50)$ m. In addition there is the more fundamental question of whether the $KK\gamma_Y$ and $pp\gamma_Y$ couplings necessarily have the same strength. As noted in Sec. II, this was assumed for simplicity to be the case in Ref. 1, where the coupling strength was taken to be proportional to the hypercharge $Y = B + S$. Although this is roughly compatible with the existing data, as shown in Fig. 1 of Ref. 5, the data are also compatible with a coupling to a generalized hypercharge, as defined in Ref. 12. Moreover, there is no fundamental reason to exclude couplings of the fifth force to other quantum numbers, as we discuss in Sec. II. By analogy, such couplings to higher flavors could be studied[24] in decays such as $D \to K\gamma_Y$. In whatever scheme would thus emerge, it is not necessary for the coupling strengths of nucleons and kaons to γ_Y to be precisely the same. For this reason it is best to simply reinstate the constant c_S from (2.10), which is then determined by B.R.$(K \to \pi\gamma_Y)$. Since this constant also appears in the expression for the energy-dependence of the $K^0 - \overline{K}^0$ parameters, we can in principle combine both determinations to check the consistency of the picture we are using.

We conclude this section by asking how γ_Y might be distinguished from $\tilde{a}$, should a decay into a light neutral eventually be seen. From the derivation of Eq. (6.4) we note that the decay of any particle into a scalar ($J = 0$) analog of γ_Y would not be enhanced by the Weinberg mechanism, and hence such a decay mode could never be detected. However, since decays into the $J = 1$ γ_Y proceed only through the longitudinal ($J_z = 0$) component, they behave as if the γ_Y were in fact a scalar particle like $\tilde{a}$. Thus if $\vec{\zeta}$ denotes a polarized $J = 1$ particle (*e.g.*, ψ/J), then the decays $\vec{\zeta} \to \gamma\tilde{a}$ and $\vec{\zeta} \to \gamma\gamma_Y$ would lead to the same angular distribution for the outgoing γ. This is a consequence of the fact that the angular distribution of a product in a decay depends on its *helicity*, rather than on its total spin.[24] It follows that if a light noninteracting neutral is observed as a decay product, one can only distinguish between γ_Y and $\tilde{a}$ on the basis of dynamical considerations, such as absolute rates, branching ratios, etc.

VII. THE $K^0 - \overline{K}^0$ SYSTEM

Since the anomalous energy-dependence of the fundamental $K^0 - \overline{K}^0$ parameters Δm, τ_S, and $\eta_{\pm}$ has been extensively discussed in Ref. 4, we focus the present discussion on those features of the problem which are specifically relevant to the fifth force. In the absence of external fields, the time evolution of the $K^0 - \overline{K}^0$ wavefunction $\Psi(t)$ in the proper frame of the kaons is given by

$$-\frac{\partial\Psi(t)}{\partial t} = iH_0\Psi(t), \qquad (7.1)$$

$$iH_0 \equiv \Gamma + iM = h_0 I + \vec{h}\cdot\vec{\sigma}, \qquad (7.2)$$

where the h's are complex quantities, and $\vec{\sigma}$ denotes the usual 2×2 Pauli matrices. To describe the effects of an external field one introduces the matrix[25] F defined by

$$iH_0 \to iH = \Gamma + iM + iF, \qquad (7.3)$$

$$iF = u_0 I + \vec{u}\cdot\vec{\sigma}. \qquad (7.4)$$

The u's are complex position-dependent numbers which are functions of $\gamma = E_K/m_K = (1-\beta^2)^{-1/2}$, where m_K, E_K (β) are the mass and the laboratory energy (velocity) of the kaons. When H is diagonalized, the γ-dependence of the $u_a (a = 0, x, y, z)$ endows the various kaon parameters with an anomalous energy-dependence in the kaon proper frame. Working backwards from an experimentally observed γ-dependence, one can infer the existence of the u_a, and hence the presence of the external field whose coupling to the $K^0 - \overline{K}^0$ system the u_a describe.

To illustrate the effects of the putative fifth force on the kaon parameters, we consider the vector coupling in (2.10) for the simplified case $c_L = 0$ and $c_S = 1$, so that $Q_1 = Yf$. Since K and $\bar{K}$ have opposite eigenvalues of Y, the additional contribution to H from the u_a arises from the term $u_z\sigma_z$, where the actual expression for u_z will depend on the functional form of the 4-vector potential $A_\mu(x)$ produced by the external hypercharge source. For a kaon moving with respect to the source, the potential A'_μ that is sees in its own (proper) frame is

$$A'_\mu(x) = \Lambda_{\mu\nu}A_\nu(\Lambda^{-1}x), \qquad (7.5)$$

where $\Lambda_{\mu\nu} = \Lambda_{\mu\nu}(\gamma)$ specifies the Lorentz boost. Note that the γ-dependence of the potential A'_μ seen by the moving kaons comes both from the vectorial nature of the field (which is the origin of $\Lambda_{\mu\nu}$) and from the Lorentz transformation of the coordinates ($x'_\rho = \Lambda_{\rho\sigma}x_\sigma$). Two special cases are of interest:

a) $A_0 =$ constant, $\vec{A} = 0$: This is the situation considered in Ref. 4, where A_0 was assumed to be of galactic origin. From (7.5) the potential seen in the kaon proper frame is

$$u_z = fA'_0 = \gamma f A_0, \qquad u_x = u_y = 0. \tag{7.6}$$

As shown in Ref. 4, the γ-dependence in (7.6) is not supported by the experimental data.

b) $A_0 = \sigma\delta(z)$, $\vec{A} = 0$: This is the other extreme, a potential of zero range with strength σ, which crudely simulates the effects of the short-range potential V_5 that we are studying. If the x and x' coordinate systems coincide at $t = t' = 0$ then, for a boost in the z-direction,

$$u_z = fA'_0 = \gamma f\sigma\delta(z) = \gamma f\sigma\delta(\gamma\beta t') \cong f\sigma\delta(t'), \tag{7.7}$$

where in the last step we have set $\beta \cong 1$, which is appropriate for high-energy kaons. We see from (7.7) that for a potential of zero range the two sources of γ-dependence in (7.5) exactly offset each other, so that the potential seen by a high-energy kaon in its proper frame is actually independent of γ.

The significance of (7.7) for the present discussion is this: The arguments in Ref. 4, which ruled out a vector coupling of the type we are now considering, were based on (7.6), which is appropriate to a field of very long range. We learn from (7.7) that as the range decreases, the γ-dependence becomes "softer", and hence the arguments of Ref. 4 no longer apply. For intermediate values of the range, the description of the $K^0 - \overline{K}^0$ system is much more involved than in the two extreme cases considered above, and to date a general analysis of the kaon parameters under these conditions is not yet available. For this reason it is not possible at present to establish one way or the other whether a fifth force with the properties we have been attributing to it could in fact explain the $K^0 - \overline{K}^0$ data of Ref. 4, should these turn out to be correct. Calculations that we have carried out point to a complicated situation, in which the predicted behavior of the fundamental parameters depends in a sensitive way on several different length scales in the problem. In addition to the ranges λ_k of the external fields, there is the scale set by $\hbar/\Delta mc = 5.56$ cm, which usually enters in the form $\gamma\hbar/\Delta mc$. For a typical kaon energy of 100 GeV, this corresponds to a length of ~ 11 m, which is comparable to the limits discussed previously for the ranges λ_k. Due to the presence of competing length scales, the parametrization of the energy-dependence used in Ref. 4 (see Eq. (7.8) below) is no longer appropriate, particularly at very high energies. Model calculations suggest that the $K^0 - \overline{K}^0$ parameters do not vary monotonically with γ, as was assumed in Ref. 4, but can be either an increasing or decreasing function of γ depending on the value of γ. Hence the average value of a parameter over some energy range may be close to the usual low-energy value, even when the parameter is energy-dependent, so that looking for a deviation of the average from the low-energy value is not the best means of searching for a γ-dependence.

Although a detailed calculation of the behavior of the $K^0 - \overline{K}^0$ system in the presence of the fifth force is not possible at the present time, we can still ask whether the effects expected in a simplified model are at least of the same magnitude as those seen in the data. The parameter most sensitive to an external C-odd field is $|\eta_\pm|$, and we can characterize its energy-dependence in terms of the slope parameter $b_\eta^{(2)}$ defined by[4]

$$|\eta_\pm|_u = |\eta_\pm|\left[1 + b_\eta^{(2)}\gamma^2\right]. \tag{7.8}$$

Here $|\eta_\pm| = 2.274(22) \times 10^{-3}$ is the usual low-energy value, and the data of Ref. 4 set the limit

$$b_\eta^{(2)} \leq +0.57 \times 10^{-6}, \quad (99.7\% C.L.). \tag{7.9}$$

If we assume that the theoretical analysis of Ref. 4 is at least roughly applicable down to ranges $\lambda \lesssim 200$ m, and that the only external source is the Earth itself, then $b_\eta^{(2)}$ is given by

$$\boxed{\begin{aligned} b_\eta^{(2)} &\approx \frac{1}{|\epsilon|^2(\Delta m)^2}\Big\{(f^2/e^2)e^{-x}\Phi(x) \\ &\qquad \times \left[8.1 \times 10^{35}\,\text{eV}\right]\Big\}^2, \\ &\cong (3.4 \times 10^{60})\left[(f^2/e^2)(\lambda/1\,\text{m})^2\right]^2, \end{aligned}} \tag{7.10}$$

where $|\epsilon| = 4.548 \times 10^{-3}$ is the CP violating parameter defined in Ref. 4, $x = R_\oplus/\lambda$, and $\Phi(x)$ is the function defined in (2.21). It must be emphasized that a more detailed model is needed to correctly describe the *signs* of the various slope parameters, which depend sensitively on the relative importance of the effects arising from the different length scales discussed above. However, if we assume that (7.10) gives a reasonable estimate of the *magnitude* of the effects expected in the $K^0 - \overline{K}^0$ system, then the combination of (7.9) and (7.10) leads to the constraint curve shown in Fig. 1 of Ref. 5. As we see from this figure, the geophysical and $K^0 - \overline{K}^0$ constraint curves intersect at a value of λ which is compatible with the "best" geophysical value ≈ 200 m. This observation motivated a more detailed search for other systems where the effects of such a force would show up, and this eventually led to the constraint curve shown in the same figure for the Eötvös experiment. This curve, which was drawn under the assumption that EPF could see an effect at the level of $\Delta\kappa \cong 1 \times 10^{-9}$, passes near the intersection of the other constraint curves. The suggestion of this figure, that the source of the anomalous effects in the $K^0 - \overline{K}^0$ system and in the geophysical data might also show up in the EPF experiment, was the specific motivation for the reanalysis of the Eötvös experiment described in Ref. 1.

The preceding considerations lead suggestively to the question of whether a fifth force with characteristics compatible with existing data may be the origin of CP-violation in the $K^0 - \overline{K}^0$ system. The idea that CP-violation is the manifestation of a vector cosmological field was put forward[26] shortly after the original observation of the $K_L \to 2\pi$ mode, and was quickly ruled out on the basis of three arguments: *i*) $|\eta_\pm|^2$ would be proportional to γ^2, contrary to what is observed experimentally. *ii*) $\arg(\eta_\pm) \equiv \phi_\pm \cong +45°$ was predicted instead to be $-45°$. *iii*) The decay of kaons into hyperphotons would be unphysically large (the "Weinberg catastrophe").[22] We have already seen, however, that the objections in *i*) and *iii*) may be overcome if the vector field has an appropriately short range, and it appears that this could also be the case for $\phi_\pm$ as well. Should it turn out that the objections in *i*)–*iii*) above can in fact be overcome, then it would be appropriate to reexamine the external field hypothesis as a possible origin of CP-violation.

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