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The Relation Between Operator and Path Integral Covariant Quantizations of the Green-Schwarz Superstring

E. Nissimov*, S. Pacheva*, and S. Solomon**

Department of Physics, Weizmann Institute of Science
Rehovot 76100, Israel

ABSTRACT

By further study of the geometry of the harmonic superspace constraints we make explicit the relation between the operator and path integral approaches to the manifestly covariant harmonic superstring.

In particular we find the correct complete set of functionally independent gauge symmetries for the auxiliary variables and identify the ones corresponding to the harmonic superfield postulate in the operator formalism.

Then, we deduce in a systematic way the lagrangian path integral from the well defined covariant hamiltonian formulation of the GS superstring.

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* Permanent address:Institute of Nuclear Research and Nuclear Energy, Boul. Lenin 72, 1784 Sofia, Bulgaria.

** Incumbent of the Recu Career Development Chair

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1. Introduction

In a series of papers [1-4] we performed the super-Poincare covariant canonical (operator) quantization of the D=10 Brink-Schwarz (BS) super-particle [5] and the Green-Schwarz (GS) superstring [6,7]. Furthermore we succeeded to construct, using the BFV-BRST ghost formalism [8], the relevant off-shell unconstrained superfield action for the point-particle limit of the GS superstring, i.e. the D=10 Super-Yang-Mills (SYM) theory [9,10].

The main tool was the introduction of auxiliary bosonic Lorentz-vector and Lorentz-spinor variables $(u^a_\mu, v_\alpha^{\pm\frac{1}{2}})$ called "harmonics" since they form a homogenous space related to the "moving-light-cone" homogenous space $SO(1,9)/SO(8) \times SO(1,1)$ [11].

In such an approach it is natural to construct the quantum theory by restricting the wave functions to be harmonic superfields [12] of a form which explicitly displays certain local harmonic symmetries of the extended superspace. Therefore it was very convenient (and in fact essential for getting the BRST (super)-field theory action [9,10]) to work within the operator quantization formalism.

However, much of our intuition about the string theory is comming from the path integral formulation which allows the use of the powerful 2-dimensional conformal field theory techniques.

In the present letter we reexpress the information codified in the structure of the "Hilbert space" of superfield wave functions (the space of harmonic superfields [1,10]) in the form of harmonic gauge symmetries which are handy for the the deduction of a covariant path-integral representation of the GS superstring.

The deduction of the path integral from a well structured hamiltonian formalism is essential in two ways. First, at the conceptual level , systems with variable structure "constants" in the constraint's algebra present certain ambiguities in the measure of the path integral [13] which can be resolved systematically only by using the hamiltonian techniques.

Second, the well structured algebraic set of constraints allows us to have complete control on the functional independence (BFV-irreducibility [8]) of the gauge invariances and to make sure that they really are eliminating all the auxiliary variables (see sect. 2).

In section 2 we reexpress in a convenient way our harmonic superspace geometry and identify the relation between certain harmonic gauge invariances and the form of the harmonic superfields composing the "Hilbert space" of superfield wave functions.

In section 3, starting from the well defined hamiltonian path integral we deduce the corresponding manifestly covariant lagrangian path integral for the GS superstring by integrating over the canonical string momenta.

2. Harmonic superspace geometry

The auxiliary variables $(u^a_\mu, v^{\pm\frac{1}{2}}_\alpha)$ needed for the covariant quantization of the GS superstring were introduced in [1,2] as follows. The indices μ and α transform as vector and Majorana-Weyl spinor under the global Lorentz SO(1,9), respectively, while the indices $a, \pm\frac{1}{2}$ transform respectively under the internal local SO(8) and SO(1,1). Due to the triality properties of SO(8) the indices $a = 1, ..., 8$ can be chosen to transform under any of the fundamental (s), (c), (v) representations of SO(8).

$(u^a_\mu, v^{\pm\frac{1}{2}}_\alpha)$ were taken in [1,2] to satisfy the following kinematical constraints:

$$\begin{aligned} &u^a_\mu u^{b\mu} - C^{ab} = 0, \\ &u^a_\mu (v^{\pm\frac{1}{2}} \sigma^\mu v^{\pm\frac{1}{2}}) = 0 \\ &(v^{+\frac{1}{2}} \sigma_\mu v^{+\frac{1}{2}})(v^{-\frac{1}{2}} \sigma^\mu v^{-\frac{1}{2}}) + 1 = 0 \end{aligned} \tag{2.1}$$

Here C^{ab} denotes the invariant metric tensor in the relevant $SO(8)$ representation space.

Due to the remarkable D=10 Fierz identities (see e.g. [7]) the following composite Lorentz vectors:

$$u^\pm_\mu \equiv (v^{\pm\frac{1}{2}} \sigma_\mu v^{\pm\frac{1}{2}}) \tag{2.2}$$

are identically light-like.

Now we are going to present a simplified set of auxiliary variables with more transparent geometrical meaning of the associated gauge invariances rendering these auxiliary variables pure-gauge.

First, we observe that due to the D=10 Fierz identities:

$$(v^{+\frac{1}{2}} \sigma_\mu v^{-\frac{1}{2}})(v^{+\frac{1}{2}} \sigma^\mu v^{-\frac{1}{2}}) = -2(v^{+\frac{1}{2}} \sigma_\mu v^{+\frac{1}{2}})(v^{-\frac{1}{2}} \sigma^\mu v^{-\frac{1}{2}}) \tag{2.3}$$

Eqs. (2.3) and (2.1) tell us that we can take the following composite space-like vector:

$$u^8_\mu \equiv \sqrt{2}(v^{+\frac{1}{2}}\sigma_\mu v^{-\frac{1}{2}}) \tag{2.4}$$

as one of the eight u^a_μ entering $(2.1)^*$.

The identification (2.4) reduces simultaneously the internal $SO(8)$ to an internal $SO(7)$ which will act on the indices $p = 1, ..., 7$ of the remaining seven space-like vectors u^p_μ. This amounts to splitting the $SO(8)$ indices $a = (p, 8)$ into $SO(7)$-vector and $SO(7)$ singlet ones.

Thus, the simplified set $(u^p_\mu, v^{\pm\frac{1}{2}}_\alpha)$ of auxiliary variables now obeys the following kinematical constraints:

$$\begin{aligned} &u^p_\mu u^{q\mu} - \delta^{pq} = 0 \\ &u^p_\mu \sqrt{2}(v^{+\frac{1}{2}}\sigma^\mu v^{-\frac{1}{2}}) = 0 \\ &u^p_\mu (v^{\pm\frac{1}{2}}\sigma^\mu v^{\pm\frac{1}{2}}) = 0 \\ &(v^{+\frac{1}{2}}\sigma_\mu v^{+\frac{1}{2}})(v^{-\frac{1}{2}}\sigma^\mu v^{-\frac{1}{2}}) + 1 = 0 \end{aligned} \tag{2.5}$$

Next, we introduce a set of hamiltonian first class constraints describing the pure gauge dynamics of $(u^p_\mu, v^{\pm\frac{1}{2}}_\alpha)$ (2.5) (summation over Lorentz-indices μ, α is supressed):

$$D^{pq} \equiv -u^p \pi^q_u + u^q \pi^p_u - \frac{1}{2} v^{+\frac{1}{2}} \sigma^{pq} \pi_v^{-\frac{1}{2}} - \frac{1}{2} v^{-\frac{1}{2}} \sigma^{pq} \pi_v^{+\frac{1}{2}} \tag{2.6}$$

$$D^{8p} \equiv -u^8 \pi^p_u - \frac{1}{2} v^{+\frac{1}{2}} \sigma^8 \sigma^p \pi_v^{-\frac{1}{2}} - \frac{1}{2} v^{-\frac{1}{2}} \sigma^8 \sigma^p \pi_v^{+\frac{1}{2}} \tag{2.7}$$

$$D^{-+} \equiv -\frac{1}{2}(v^{+\frac{1}{2}} \pi_v^{-\frac{1}{2}} - v^{-\frac{1}{2}} \pi_v^{+\frac{1}{2}}) \tag{2.8}$$

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* Actually, since $\{u^a_\mu\}$ form a space-like frame on the surface (2.5), we can always rotate this frame by local $SO(8)$ rotations such that one of the frame vectors, e.g. u^8_μ, will coincide with the space-like unit vector $\sqrt{2}(v^{+\frac{1}{2}}\sigma_\mu v^{-\frac{1}{2}})$.

$$D^{\pm p} \equiv -u^{\pm}\pi_u^p - \frac{1}{2}v^{\mp\frac{1}{2}}\sigma^{\pm}\sigma^p\pi_v^{\pm\frac{1}{2}} \tag{2.9}$$

$$D^{\pm 8} \equiv -\frac{1}{2}v^{\mp\frac{1}{2}}\sigma^{\pm}\sigma^8\pi_v^{\pm\frac{1}{2}} \tag{2.10}$$

$$\tilde{D}^{8p} \equiv -\frac{1}{2}v^{+\frac{1}{2}}\sigma^8\sigma^p\pi_v^{-\frac{1}{2}} + \frac{1}{2}v^{-\frac{1}{2}}\sigma^8\sigma^p\pi_v^{+\frac{1}{2}} \tag{2.11}$$

Here $(\pi_v^{\mp\frac{1}{2}})^{\alpha}$, $(\pi_u^p)_{\mu}$ denote canonical momenta conjugated to $v_{\alpha}^{\pm\frac{1}{2}}, u_{\mu}^p$:

$$\{(\pi_v^{\mp\frac{1}{2}})^{\alpha}(\xi), v_{\beta}^{\pm\frac{1}{2}}(\eta)\}_{PB} = -\delta_{\beta}^{\alpha}\delta(\xi-\eta),$$

$$\{(\pi_u^p)_{\mu}(\xi), u_{\nu}^q(\eta)\}_{PB} = -\delta^{pq}\eta_{\mu\nu}\delta(\xi-\eta), \tag{2.12}$$

(ξ, η denote the string world-sheet parameter at fixed world-sheet time τ; in most cases they will be supressed for brevity), $u_{\mu}^8, u_{\mu}^{\pm}$ are the same as in (2.2), (2.4),

$$\begin{gathered} \sigma^{\pm} \equiv \sigma^{\mu}u_{\mu}^{\pm}, \quad \sigma^8 \equiv \sigma^{\mu}u_{\mu}^8, \quad \sigma^p \equiv \sigma^{\mu}u_{\mu}^p, \\ \sigma^{p_1\ldots p_N} \equiv \sigma^{[p_1}\sigma^{p_2}\ldots\sigma^{p_N]} \end{gathered} \tag{2.13}$$

Comparing (2.6)-(2.10) with the set of constraints $D^{ab}, D^{-+}, D^{\pm a}$ used previously in [1-4,10] we immediately see that:
(i) D^{ab} spanning $SO(8)$ algebra coincide with the subset (D^{pq}, D^{8p}) (2.6), (2.7), where D^{pq} span the $SO(7)$ subalgebra and D^{8p} correspond to the coset $SO(8)/SO(7)$;
(ii) $D^{\pm a}$ spanning the coset $SO(1,9)/SO(8)\times SO(1,1)$ coincide with the subset $(D^{\pm p}, D^{\pm 8})$ (2.9),(2.10);
(iii) D^{-+} (the generator of the local $SO(1,1)$) is the same in both versions.

The meaning and the properties of the constraints $\tilde{D}^{8p}$ (2.11) are as follows. First, we observe that the number of independent components of $(v_\alpha^{\pm\frac{1}{2}}, u_\mu^p)$, accounting for the kinematical constraints (2.5), is 52 which exactly coincides with the number of hamiltonian constraints (2.6)-(2.11). Next, we find:

$$\{\tilde{D}^{8p}, (D^{qr}, D^{8p}, D^{-+}, D^{\pm q}, D^{\pm 8})\}_{PB} = 0 \tag{2.14}$$

$$\{\tilde{D}^{8p}(\xi), \tilde{D}^{8q}(\eta)\}_{PB} = \gamma^{pqr}(\xi)\tilde{D}^8_r(\xi)\delta(\xi - \eta) \tag{2.15}$$

$$\gamma^{pqr} \equiv \sqrt{2}(v^{-\frac{1}{2}}\sigma^{pqr}v^{+\frac{1}{2}})$$

$$\{\tilde{D}^{8p}, (u_\mu^\pm, u_\mu^8, u_\mu^q)\}_{PB} = 0 \tag{2.16}$$

In deriving (2.15) we used the identities:

$$\sqrt{2}(v^{-\frac{1}{2}}\sigma^{pqr}v^{+\frac{1}{2}})\frac{1}{2}(v^{\pm\frac{1}{2}}\sigma^8\sigma_r\pi_v^{\mp\frac{1}{2}}) = \pm\frac{1}{2}(v^{\pm\frac{1}{2}}\sigma^{pq}\pi_v^{\mp\frac{1}{2}}) \tag{2.17}$$

which follow from (2.5) and the D=10 Fierz identities.

From (2.14)-(2.16) we notice that the subset of constraints $\tilde{D}^{8p}$ (2.11) form a closed subalgebra commuting with the $SO(1,9)$ algebra of the constraints (2.6)-(2.10) (introduced in our previous papers [1-4,10]) and, moreover, $\tilde{D}^{8p}$ leaves **invariant** the frame $(u_\mu^\pm, u_\mu^8, u_\mu^p)$ (cf. (2.16)).

Now the geometric meaning of (2.6)-(2.11) becomes extremely transparent . First, with the help of (2.6)-(2.10) (subset of constraints spanning the $SO(1,9)$ algebra) we can always rotate the frame $(u_\mu^\pm, u_\mu^8, u_\mu^p)$ to any fixed orientation in space time. Second, with the help of (2.11) which do not move anymore the frame (cf.(2.16)), we can fix completely the remaining freedom in the Lorentz spinors $v_\alpha^{\pm\frac{1}{2}}$ entering (2.2) , (2.4).

This is the geometric manifestation of the pure-gauge property of our auxiliary variables $(u^p_\mu, v^{\pm\frac{1}{2}}_\alpha)$.

In order to make contact with the covariant canonical (operator) quantization of the GS superstring and its point-particle limit - the BS superparticle - performed in [1-4,10], let us point out that the space of harmonic superfield wave functions for the BS superparticle used in [1,3,10]:

$$\Phi(p,\theta,u,v) = \sum (\frac{u^+_{\lambda_1}}{p^+})...(\frac{u^-_{\nu_1}}{p^-})...u^+_{\kappa_1}...u^-_{\mu_1}...\Phi^{\{\lambda\}\{\nu\}\{\kappa\}\{\mu\}}(p,\theta) \qquad (2.18)$$

$$(p^\pm \equiv u^\pm_\mu p^\mu \equiv v^{\pm\frac{1}{2}} \not{p} v^{\pm\frac{1}{2}})$$

is nothing but the space of general solutions of the following Dirac constraint equations entering the covariant first-quantization formalism:

$$\begin{aligned} D^{pq}\phi &= 0, \\ D^{8p}\phi &= 0, \\ D^{-+}\phi &= 0, \\ \tilde{D}^{8p}\phi &= 0 \end{aligned} \qquad (2.19)$$

where $D^{pq}, ..., \tilde{D}^{8p}$ denote the quantized versions of (2.6)-(2.8), (2.11).

Thus, now it is not needed to postulate that the wave functions belong to the space (2.18) of harmonic superfields where all the internal $SO(8) \times SO(1,1)$ indices are saturated among the u's and v's and the coefficients in the expansion are ordinary superfields which do not carry internal indices. We just take completely arbitrary wave functions $\phi(p,\theta,u,v)$ with an arbitrary dependence on (u,v) and arrive systematically at the form (2.18) after solving the Dirac constraint eqs. (2.19).

To conclude this section let us comment on the recent papers by Kallosh and Rahmanov (KR) [14] where a modification of our covariant quantization procedure for the GS superstring was proposed using essentially the same auxiliary variables (u, v) and almost the same set of hamiltonian constraints for them as those introduced previously in [1,2].

There is, however, an essential difference. The set of constraints in [14] does not include D^{-a} (2.9), (2.10), but rather introduces constraints of the form (a,b =1,...,8):

$$K^{ab} \equiv \frac{1}{2} v^{+\frac{1}{2}} \sigma^{ab} \pi_v^{-\frac{1}{2}} \tag{2.20}$$

Now, recalling the identity (2.17), we find that (2.20) are actually **functionally dependent** (p,q,r=1,...,7):

$$K^{pq} - \gamma^{pq}{}_r K^{8r} = 0 \tag{2.21}$$

(actually, the r.h.s. of (2.21) is proportional to the kinematical constraints (2.5)). Therefore, in spite of the fact that the constraints in the KR version [14] are equal in number with the auxiliary variables (u, v), they are insufficient , in view of the functional dependence (2.21), to render all (u, v) pure-gauge.

The break-down of the pure-gauge nature of (u, v) in the KR version is manifested on the first-quantized level through the fact that in the superparticle limit one obtains, using the constraints of [14], an infinite number of unphysical supermultiplets instead of the D=10 SYM multiplet [15].

3. From Hamiltonian to Lagrangian Path integral formulation

In this section we briefly sketch the systematic derivation of the functional integral representation of the covariantly quantized heterotic GS superstring. The correct treatment, especially in the case of field dependent structure "constants" of the algebra of gauge symmetries, starts from the hamiltonian (phase space form) of the functional integral [16,8].

For the case at hand we have:

$$\begin{aligned} Z = \int & DX^{\mu} D\theta_{\alpha} Du Dv DP^{\mu} Dp_{\theta}^{\alpha} D\pi_u D\pi_v \\ & D\Lambda_L D\Lambda_R D\Lambda^{-\frac{1}{2}a} DM^{-\frac{1}{2}a} D\Lambda_{ab} D\Lambda^{+-} D\Lambda_a^{\pm} D\tilde{\Lambda}_p^8 \\ & exp\{i\tilde{S}\} \delta(\chi^{(rep)}) \Delta_{FP}^{(rep)} \delta(\chi^{(harm)}) \Delta_{FP}^{(harm)} \\ & det^{-12}[\Pi^+] \delta(v^{+\frac{1}{2}} \sigma^a \theta) \delta(\Psi^{AB}) \delta(\Omega^{AB}) \end{aligned} \tag{3.1}$$

In eq. (3.1) the following notations are used. $\tilde{S}$ denotes the hamiltonian form of the heterotic GS action [2-4] (in what follows we are supressing the internal string degrees of freedom corresponding to the left-moving sector):

$$\begin{aligned} \tilde{S} = \int d\tau d\xi [& P_{\mu} \partial_{\tau} X^{\mu} + p_{\theta}^{\alpha} \partial_{\tau} \theta_{\alpha} - \Lambda_L T_L - \Lambda_R T_R - \Lambda_a^{-\frac{1}{2}} D^{+\frac{1}{2}a} - M_a^{-\frac{1}{2}} G^{+\frac{1}{2}a} \\ & + \pi_v^{\pm\frac{1}{2}} \partial_{\tau} v^{\mp\frac{1}{2}} + \pi_u^p \partial_{\tau} u_p - \Lambda_{ab} D^{ab} - \Lambda^{+-} D^{-+} - \Lambda_a^{\mp} D^{\pm a} - \tilde{\Lambda}_p^8 \tilde{D}^{8p}] \end{aligned} \tag{3.2}$$

Each $SO(8)$ index a appearing in (3.1) (3.2) and below is short hand for $a = (p, 8)$ (pair of $SO(7)$ vector and $SO(7)$ singlet indices). $D^{ab}, D^{-+}, D^{\pm a}, \tilde{D}^{8p}$ are the same as in (2.6)-(2.11). $T_{L,R}$ are the left (right-) reparametrization (Virasoro) constraints (primes indicate differentiation w.r.t. string parameter ξ):

$$T_L \equiv (P_\mu - X'_\mu)^2 - 4(\pi^p_u u'_p + \pi_v^{\mp\frac{1}{2}}(v^{\pm\frac{1}{2}})') \tag{3.3}$$

$$T_R \equiv \Pi^2 - 4i\theta'_\alpha D^\alpha \tag{3.4}$$

where

$$\Pi^\mu \equiv P^\mu + X'^\mu + 2i\theta\sigma^\mu\theta'$$

Note the second term in T_L (3.3) which says that the auxiliary variables (u, v) transform under reparametrizations as left-moving world-sheet scalars. Similarly, the harmonic constraints (2.6)-(2.11) transform as conformal spin one left moving world-sheet fields:

$$\{T_L(\xi), D^{pq}(\eta)\}_{PB} = -4D^{pq}(\xi)\delta'(\xi - \eta),$$

$$\{T_R(\xi), D^{pq}(\eta)\}_{PB} = 0, \quad etc.$$

$D^{+\frac{1}{2}a}$ and $G^{+\frac{1}{2}a}$ denote the covariantly disentangled [2-4] first-class part and second-class part of the fermionic string constraints D^α:

$$D^\alpha \equiv -ip^\alpha_\theta - (P^\mu + X'^\mu + i\theta\sigma^\mu\theta')(\sigma_\mu\theta)^\alpha \tag{3.5}$$

$$D^{+\frac{1}{2}a} \equiv v^{+\frac{1}{2}}\sigma^a \not{\Pi} D \tag{3.6}$$

$$G^{+\frac{1}{2}a} \equiv \frac{1}{2}v^{-\frac{1}{2}}\sigma^a\sigma^+ D \tag{3.7}$$

$\Lambda_L, \Lambda_R, ..., \tilde{\Lambda}^8_p$ denote Lagrange multipliers for the corresponding constraints. Let us stress that all constraints in $\tilde{S}$ (3.2) except $G^{+\frac{1}{2}a}$ (3.7) are first-class. The Faddeev-Popov (FP) measure in (3.1) consists of factors corresponding to:

(i) fixing $\chi^{(rep)} = 0$ of the reparametrization invariance;

(ii) fixing $\chi^{(harm)} = 0$ of the harmonic gauge invariances $D^{ab}, ..., \tilde{D}^{8p}$;

(iii) covariant fixing $v^{+\frac{1}{2}}\sigma^a\theta = 0$ of the fermionic κ-gauge invariance (generated by $D^{+\frac{1}{2}a}$ (3.6)) with FP factor $det^{-8}[\Pi^+]\delta(v^{+\frac{1}{2}}\sigma^a\theta)$ where $\Pi^+ \equiv u^+_\mu \Pi^\mu = v^{+\frac{1}{2}} \not{\Pi} v^{+\frac{1}{2}}$;

(iv) factor $det^{-\frac{1}{2}}[\{G^{+\frac{1}{2}a}, G^{+\frac{1}{2}a}\}_{PB}] = det^{-4}[\Pi^+]$ corresponding to the second class constraints (3.7);

(v) fixing $\Omega^{AB} = 0$ of the first-class kinematical constraints (2.5) collectively denoted by Ψ^{AB}. The explicit form of Ω^{AB} reads [1,2]:

$$\begin{aligned}\Omega^{AB} \equiv \{&\frac{1}{2}\pi_u^{(p}u^{q)}, (\pi_u^p)_\mu(v^{\pm\frac{1}{2}}\sigma^\mu v^{\pm\frac{1}{2}}),\\ &(\pi_u^p)_\mu\sqrt{2}(v^{+\frac{1}{2}}\sigma^\mu v^{-\frac{1}{2}}), \frac{1}{4}(v^{+\frac{1}{2}}\pi_v^{-\frac{1}{2}} + v^{-\frac{1}{2}}\pi_v^{+\frac{1}{2}})\}\end{aligned} \tag{3.8}$$

The FP determinant corresponding to Ω^{AB} is constant on the surface $\Psi^{AB} = 0$ (2.5).

Now, it is straightforward to perform explicitly the integrations over p_θ^α, $\Lambda^{-\frac{1}{2}a}$, $M^{-\frac{1}{2}a}$, P^μ. Indeed, integrating over p_θ^α one gets the δ-function:

$$\delta(\dot{\theta}_\alpha + 4\Lambda_R\theta'_\alpha + i(v^{+\frac{1}{2}}\sigma^a \not{\Pi})_\alpha\Lambda_a^{-\frac{1}{2}} + \frac{i}{2}(v^{-\frac{1}{2}}\sigma^a\sigma^+)_\alpha M_a^{-\frac{1}{2}})$$

Integrating then over $\Lambda_a^{-\frac{1}{2}}$, $M_a^{-\frac{1}{2}}$ yields a gaussian integral over P^μ which, in turn, is easy performed. The resulting functional integral can be easily rewritten in a manifestly reparametrization invariant form by introducing the world sheet metric as:

$$\begin{aligned}\sqrt{-g}g^{00} &= -[2(\Lambda_L + \Lambda_R)]^{-1},\\ \sqrt{-g}g^{01} &= (\Lambda_R - \Lambda_L)(\Lambda_L + \Lambda_R)^{-1},\\ \sqrt{-g}g^{11} &= 8\Lambda_L\Lambda_R(\Lambda_L + \Lambda_R)^{-1}\end{aligned} \tag{3.9}$$

One obtains :

$$Z = \int DX^{\mu} D\theta_{\alpha} Du Dv D(\pi_u)_m D(\pi_v)_m Dg_{mn} D\Lambda_n^{MN} D\mathcal{M}^{AB}$$
$$exp\{iS\}\delta(\chi^{(rep)})\Delta_{FP}^{(rep)}\delta(\chi^{(harm)})\Delta_{FP}^{(harm)}\delta(v^{+\frac{1}{2}}\sigma^a\theta)\delta(\Omega^{AB})\Delta_{local} \tag{3.10}$$

$$S \equiv S_{GS}^{heterotic} + S_{harmonic} \tag{3.11}$$

$$S_{harmonic} \equiv \int d\tau d\xi \sqrt{-g}[(\pi_v^{\mp\frac{1}{2}})_n P_-^{nm}\partial_m v^{\pm\frac{1}{2}} + (\pi_u^q)_n P_-^{nm}\partial_m u_q$$
$$- \Lambda_n^{MN} P_+^{nm} D_m^{MN} - \mathcal{M}^{AB}\Psi^{AB}]$$

Δ_{local} is a local determinant factor of the form :

$$\Delta_{local} \equiv \int D\chi^{-\frac{1}{2}a}$$
$$exp\{i \int d\tau d\xi \sqrt{-g}[\bar{\chi}^{-\frac{1}{2}a}\rho_n \frac{1}{2}(1-\rho_5)\chi_a^{-\frac{1}{2}}]P_-^{nm}\partial_m X^{\mu} u_{\mu}^{+}\} \tag{3.12}$$

where $\chi^{-\frac{1}{2}a}$ are world-sheet spinor bosonic ghosts and ρ_n, ρ_5 denote the world-sheet Dirac matrices.

In eqs. (3.11) and (3.12) the following notations are used. $P_{\pm}^{mn}$ are the D=2 (anti-) self-duality projectors:

$$P_{\pm}^{mn} \equiv \frac{1}{2}(g^{mn} \pm \frac{\epsilon^{mn}}{\sqrt{-g}})$$

The canonical momenta of (u, v) enter (3.11) as:

$$\pi_v^{\pm\frac{1}{2}} = \sqrt{-g} P_+^{0m} (\pi_v^{\pm\frac{1}{2}})_m \quad , \quad \pi_u^q = \sqrt{-g} P_+^{0m} (\pi_u^q)_m \tag{3.13}$$

Similarly, D_m^{MN} have exactly the same form as the hamiltonian constraints D^{MN} (2.6)-(2.11) with all canonical momenta $\pi_v^{\pm\frac{1}{2}}$, π_u^q substituted with $(\pi_v^{\pm\frac{1}{2}})_m$, $(\pi_u^q)_m$. The first term in the r.h.s. of (3.11) is precisely the usual heterotic GS action [7]:

$$S_{GS}^{heterotic} = \int d\tau d\xi \sqrt{-g}[-\frac{1}{2}g^{mn}\partial_m X^\mu \partial_n X_\mu - 2i(P_-^{nm}\partial_m X^\mu)(\theta\sigma_\mu\partial_n\theta) + \frac{1}{2}g^{mn}(\theta\sigma_\mu\partial_n\theta)(\theta\sigma^\mu\partial_m\theta)] \tag{3.14}$$

Further simplifications in (3.10) are achieved by changing variables [2]:

$$\theta_\alpha \rightarrow \theta^{\pm\frac{1}{2}a} = (v^{\pm\frac{1}{2}}\sigma^a\theta) \tag{3.15}$$

with subsequent rescaling (in the conformal gauge for g_{mn}) :

$$\theta^{-\frac{1}{2}a} \rightarrow \psi^a = -2(\partial_{\bar{z}}X^\mu u_\mu^+)^{\frac{1}{2}}\theta^{-\frac{1}{2}a} \tag{3.16}$$

such that the corresponding Jacobian will exactly cancell Δ_{local} (3.12).

The final simplification is achieved by changing the gauge-fixing condition $\Omega^{AB} = 0$ (3.8) into a new one - $\mathcal{M}^{AB} = 0$ by just inserting the standard FP unity:

$$1 = \bar{\Delta}_{FP}\int D\omega^{AB}\delta(\mathcal{M}^{AB}(\omega)) \tag{3.17}$$

where the integration is over the abelian group generated by the kinematical constraints Ψ^{AB} (2.5). The explicit fermionic ghost representation of $\bar{\Delta}_{FP}$ reads:

$$\bar{\Delta}_{FP} = \int D\zeta_n D\bar{\zeta} exp\{\int d\tau d\xi \sqrt{-g}\zeta_n^{AB}P_-^{nm}\partial_m\bar{\zeta}_{AB}\} \tag{3.18}$$

Choosing the harmonic gauge-fixing conditions in the form $\chi^{(harm)} \equiv \Lambda^{MN} = 0$, the corresponding FP determinant has exactly the same form as (3.18) with ghosts $\eta_n^{MN}, \bar{\eta}_{MN}$.

Thus, we arrive at the following manifestly covariant functional integral :

$$Z = \int DX^{\mu} D\psi^{a} Du Dv D(\pi_u)_z D(\pi_v)_z D\zeta_z D\bar{\zeta} D\eta_z D\bar{\eta} exp\{i\hat{S}\} \Delta_{FP}^{(rep)} \tag{3.19}$$

$$\hat{S} \equiv 2 \int d\tau d\xi [-\partial_z X^{\mu} \partial_{\bar{z}} X_{\mu} - i\psi_a \nabla_z^{ab} \psi_b - (\pi_v)_z \partial_{\bar{z}} v + (\pi_u)_z \partial_{\bar{z}} u \\ + \zeta_z^{AB} \partial_{\bar{z}} \bar{\zeta}_{AB} + \eta_z^{MN} \partial_{\bar{z}} \bar{\eta}_{MN}] \tag{3.20}$$

with the following notations :

$$\nabla_z^{ab} = \delta^{ab} \partial_z + \Gamma_z^{ab} - (\partial_{\bar{z}} X^{\mu} u_{\mu}^{c})(\partial_{\bar{z}} X^{\nu} u_{\nu}^{+})^{-1} \Gamma_z^{+d} (\tilde{S}^{ab})_{cd} \tag{3.21}$$

$$\begin{aligned} \Gamma_z^{ab} &\equiv u_{\mu}^{a} \partial_z u^{b\mu} + v^{-\frac{1}{2}} \sigma^{ab} \sigma^{+} \partial_z v^{-\frac{1}{2}}, \\ \Gamma_z^{+a} &\equiv u_{\mu}^{+} \partial_z u^{a\mu}, \\ (S^{ab})_{cd} &\equiv \frac{1}{2} v^{-\frac{1}{2}} \sigma_c \sigma^{ab} \sigma^{+} \sigma_d v^{-\frac{1}{2}} \end{aligned} \tag{3.22}$$

(the latter are precisely the matrices of the $SO(8)$ generators in the harmonic (c)-spinor representation [4,10]).

Now it is straightforward to see the absence of the conformal anomalies. Indeed, integrating in (3.19) over $u, v, \pi_u, \pi_v, \zeta_z, \bar{\zeta}, \eta_z, \bar{\eta}$, one gets :

$$Z = \int DX^{\mu} D\psi^{a} \Delta_{FP}^{(rep)} exp\{i2 \int d\tau d\xi (-\partial_z X^{\mu} \partial_{\bar{z}} X_{\mu} - i\psi_a \nabla_z^{ab} \psi_b)\} \tag{3.23}$$

Eq.(3.23) is similar to the non-covariant expression in [17]. It was shown there (see also [18]) that careful computation of the integral over ψ^a (which is $\frac{1}{2}\bar{z}$-differential) yields complete cancellation of the conformal anomalies.

A very interesting problem is to compare the functional integral representation (3.19) or (3.10) of the covariantly quantized heterotic GS superstring with the functional integral representation found in [19] where a specific form of the moving frame $(u^{\pm}_{\mu}, u^{s}_{\mu}, u^{p}_{\mu})$ is used, namely, $u^{\pm}_{\mu}$ are expressed in terms of the tangent vectors $\partial_m X_{\mu}$ to the string world sheet, whereas $(u^{s}_{\mu}, u^{p}_{\mu})$ are chosen to span the normal frame w.r.t. the world-sheet.

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