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A Model for a Dia-Electric*

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André Martin is the master of the Schrödinger equation. Where others made do with cheap approximations he would not buy anything but a conclusive result proved by hard analysis. In recent years he applied his skill to meson spectroscopy where one deals with confining potentials. As a birthday present we will offer him a charged medium which gives such a potential purely in the framework of electrodynamics. We realize that this is just a toy and not the mechanism realized in nature. There it is supposed to emerge from QCD where also nonlinear and quantum effects are important. But our model may provide a formal basis to the heuristic discussions one finds in textbooks [1,2].

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The Higgs field is a model for a perfect diamagnetic fluid. Its current obeys the relativistic London equations of superconductivity. They express that the current can follow freely the electric field and a magnetic field induces eddi currents which tend to cancel it. The coupled Maxwell-London equations are most concisely written in the notation of differential geometry [3] where the various fields are considered as elements of E_p, the p-forms. As derivations appear the exterior derivative $d : E_p \to E_{p+1}$ and its adjoint $\delta : E_p \to E_{p-1}$. Then the electromagnetic field $F \in E_2$, the Higgs current $J \in E_1$ and an external current $j \in E_1$ obey

$$\delta F = J + j, \qquad dJ = \mu^2 F, \qquad dF = \delta J = \delta j = 0 \tag{1}$$

(μ^{-1} is the penetration length). It follows $(d\delta - \mu^2)F = dj$ or if we work in $\mathbf{R}^4$ then by Fourier-transform ($k^2 = \vec{k}^2 - k_0^2$)

$$\tilde{F}(k) = -\frac{\widetilde{dj}}{k^2 + \mu^2} = -\frac{\widetilde{dj}}{k^2}\varepsilon(k)^{-1} \tag{2}$$

which shows the exponential screening of all fields generated by j. In fact, the dielectric constant is

$$\varepsilon(k) = 1 + \frac{\mu^2}{k^2}$$

and goes to ∞ for $k \to 0$.

For the magnetic susceptibility κ one finds

$$\kappa = \frac{-1}{1 + k^2/\mu^2}, \qquad \varepsilon(1 + \kappa) = 1,$$

such that $\kappa \to -1$ for $k \to 0$. To get $\varepsilon < 1$ and $\kappa > 0$ we have to pervert the situation and change the sign of μ^2. Then the charges of the Higgs current run opposite to the usual way. Since this would produce a tachion we stabilize the situation by coupling J to another field $G \in E_2$ such that J and G alone behave normally. Correspondingly the field equations for the perverted Higgs model are

$$\delta F = J + j, \qquad dJ = \mu^2(G - F), \qquad \delta G = J, \qquad dF = dG = \delta J = \delta j = 0. \tag{3}$$

From these we conclude

$$\delta dJ = -\mu^2 j \Longrightarrow d\delta d\delta F = (-\mu^2 + d\delta)dj.$$

Turning to Fourier space we have

$$\tilde{F} = -\left(\frac{\mu^2}{k^4} + \frac{1}{k^2}\right)\widetilde{dj}. \tag{4}$$

This corresponds to $\varepsilon(k) = k^2/(k^2 + \mu^2)$ and for $k \to 0$ we have $\varepsilon \to 0$ or a perfect dia-electric. Similarly $\kappa = \mu^2/k^2$ and (for $k_0 = 0$) $\kappa > 0$, the stuff is paramagnetic. The static Green function becomes

$$\int d^3k e^{ikx}\left(\frac{\mu^2}{k^4} + \frac{1}{k^2}\right) = \frac{1}{4\pi}\left(\frac{1}{r} - \frac{\mu^2}{2}r\right) + \text{an infrared divergent constant.}$$

Thus we get just the potential popular among the quark confiners.

Remarks

1. When one constructs the Lagrangian and Hamiltonian for (3) one finds that J and G contribute negatively to the energy. On a simplified scalar version of this mechanism we have previously shown [4] that one can quantize nevertheless with a positive metric in Hilbert space. However, there are no space-time translation invariant states and the time evolution is not unitarily implemented. Indefinite metric does not really help [5]. Nevertheless a unitary S-matrix might exist.

2. There are other classical models for a dia-electric. J. Hôsek has constructed one with charged gravitons [6].

3. This electrodynamic type of theory would be catastrophic for strong interactions since it would lead to van der Waals type potentials $\sim 1/r^4$ between two quark-antiquark pairs. How these long range forces are avoided in QCD has yet to be demonstrated.

References

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