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QCD CORRECTIONS TO THE WEAK INTERACTIONS OF MESONS IN THE STATIC CAVITY

Lloyd C. L. Hollenberg and Bruce H. J. McKellar
Research Centre for High Energy Physics
School of Physics
University of Melbourne
Parkville, Victoria, 3052, Australia

Abstract

The effect of QCD loop corrections to the weak interactions of hadrons is studied in a non-perturbative framework in which the matrix elements of the bare effective weak Hamiltonian between hadronic states are computed. The QCD renormalisation is carried out in the static cavity, alleviating the scale matching problem of the standard short distance analysis. Specifically, we study the $\Delta I=1/2$ rule by computing the QCD corrected K-π matrix elements. The modified static cavity model used is characterised by four parameters; the effective quark-gluon coupling constant, the confinement pressure, the zero-point energy and a parameter governing the centre-of-mass corrections. These parameters are fitted to the π and ρ masses and the charge radii of π and K. Results are given for the ultra-relativistic ($m_u=m_d=0$) sector and for a region ($m_u=m_d=140$ MeV) where the $\Delta I=1/2$ rule is uniquely reproduced.

1. Introduction

Many developments in effective field theory[1-5] over the past few years have led to powerful and elegant techniques by which QCD corrections to the effective weak Hamiltonian can be computed perturbatively. It is expected that QCD corrections to the dynamics of hadrons will be of importance in the quantitative description of long outstanding phenomenon such as the $\Delta I = \frac{1}{2}$ rule observed in the non-leptonic decays of kaons and hyperons.

Although the techniques of effective field theory have resulted in reliable perturbative calculations of the leading order QCD corrections to the bare effective weak Hamiltoian, the lack of ability in computing hadronic matrix elements at a similar level of sophistication have frustrated these efforts to understand the phenomenology of non leptonic weak interactions. Most notably the lack of detail in current models of non-perturbative QCD leads to a mismatch between the effective weak Hamiltonian operator coefficients and the non-perturbative matrix elements of the effective operators. Whilst the Wilson coefficients possess a well defined scale dependence, μ, which in principle is cancelled by that of the effective operators, the evaluation of the matrix elements of the effective operators in current models gives no explicit scale dependence. For the case of the effective three quark (u,d,s) $\Delta S = 1$ interaction, relevant to kaon decay, the Wilson coefficients are valid only at a scale bounded above and below by the charm quark mass and non-perturbative modifications of the gluon propagator (the lower limit is commonly taken to be about 1 Gev, however, detailed studies suggest[4] that this value is too low) whilst the scale at which the corresponding matrix elements are computed in bag or oscillator models is most likely $O(m_K)$. This is the scale matching problem which provides the motivation for the work carried out in this paper.

Although in principle large scale simulations of QCD on the lattice can overcome this problem, in practice realistic results including dynamical quarks may not be possible for some time. Thus, phenomenological approaches such as that adopted in this paper are still of interest. Alternative methods usually involve a long distance analysis. The most notable of these, for the meson case, is the use of the effective chiral

Lagrangian[6]. However, we will in this work concentrate on the technical difficulties of the standard short distance analysis.

The approach we take involves a framework in which both QCD corrections to the bare effective weak Hamiltonian and the hadronic matrix elements are computed thus rendering the scale dependence irrelevant. To this end we employ the static cavity model of QCD bound states for which meson wavefunctions to first order in the quark-gluon coupling constant have been constructed[7]. Matrix elements of the bare effective weak Hamiltonian using these wavefunctions thus include most one loop QCD corrections, subject to the level of approximation involved in the original static cavity model. Although the attention here is exclusively on meson dynamics, the techniques developed are readily generalised to the baryon sector.

The paper is organised as follows. In section 2 the model employed is reviewed. Section 3 deals with the computation of the diagrams relevant for the K-π matrix elements. Results and discussion are given in section 4.

2. Large-Basis O(g) Wavefunction

The framework employed in this paper is based on QCD in the static cavity approximation, in which meson wavefunctions containing O(g) bremsstrahlung ($|q\bar{q}G\rangle$) and vacuum fluctuation ($|q\bar{q}q\bar{q}G\rangle$) states in addition to the usual valence configuration have been constructed and fitted in the ultra-relativistic ($m_u=m_d=0$) sector[7]. In the notation of Ref.7 the "full" O(g) meson wavefunction (containing all states in the static cavity basis of j=1/2 quark and l=1 gluon states with the exception of three gluon vacuum fluctuation states which only contribute to the overall normalisation in the applications considered here), denoted $|\psi\rangle^J_F$, is firstly decomposed into parts giving rise to connected (C) and disconnected (D) energy shifts to the unperturbed valence ground state as

$$|\psi\rangle^J_F = N^J_F \left\{ |\psi\rangle^J_C + |\psi\rangle^J_D \right\} \qquad (2.1)$$

where N_F^J is the Fock state normalisation and J the spin of the meson state. In terms of the quark and gluon basis states in the static cavity, the connected part, $|\psi\rangle_C^J$, is

$$|\psi\rangle_C^J = |1S1\bar{S}\rangle^J + \sum_{J_{q\bar{q}}}\Bigg\{\sum_{n_g}^{N_B}\Big[a_{TE1}^{J,J_{q\bar{q}}}(1,n_g) + a_{TE2}^{J,J_{q\bar{q}}}(1,n_g)\Big]|1S1\bar{S}G_{n_g}^{TE}\rangle_{J_{q\bar{q}}}^J$$

$$+ \sum_{n>1}^{N_B}\sum_{n_g}^{N_B}\Big[a_{TE1}^{J,J_{q\bar{q}}}(n,n_g)\,|nS1\bar{S}G_{n_g}^{TE}\rangle_{J_{q\bar{q}}}^J + a_{TE2}^{J,J_{q\bar{q}}}(n,n_g)\,|1Sn\bar{S}G_{n_g}^{TE}\rangle_{J_{q\bar{q}}}^J\Big]\Bigg\}$$

$$+ \sum_{n,n_g}^{N_B}\Bigg\{\sum_{J_{q\bar{q}}}\Big[a_{TM1}^{J,J_{q\bar{q}}}(n,n_g)\,|nP1\bar{S}G_{n_g}^{TM}\rangle_{J_{q\bar{q}}}^J + a_{TM2}^{J,J_{q\bar{q}}}(n,n_g)\,|1Sn\bar{P}G_{n_g}^{TM}\rangle_{J_{q\bar{q}}}^J\Big]$$

$$+ \sum_{\{i\}}\Big[a_{VF1}^J(n,n_g)_{\{i\}}\,|1Sn\bar{P}G_{n_g}^{TE}:1S1\bar{S}\rangle_{\{i\},q_a}^{VF1} + a_{VF2}^J(n,n_g)_{\{i\}}\,|nP1\bar{S}G_{n_g}^{TE}:1S1\bar{S}\rangle_{\{i\},q_b}^{VF2}$$

$$+ a_{VF3}^J(n,n_g)_{\{i\}}\,|1Sn\bar{S}G_{n_g}^{TM}:1S1\bar{S}\rangle_{\{i\},q_a}^{VF3} + a_{VF4}^J(n,n_g)_{\{i\}}\,|nS1\bar{S}G_{n_g}^{TM}:1S1\bar{S}\rangle_{\{i\},q_b}^{VF4}\Big]\Bigg\}$$

(2.2)

where the labels TE1 and TE2 refer to TE bremsstrahlung off q and $\bar{q}$ quarks respectively (and similarly for TM bremsstrahlung), $J_{q\bar{q}}$ is the spin of the $q\bar{q}$ pair in the $|q\bar{q}G\rangle$ state (since we are studying exclusively pseudoscalar states, i.e. $J=0$ and $J_{q\bar{q}}=1$, we will supress these labels when their meaning is clear), the flavour subscript on the vacuum fluctuation (VF) Fock states, $|q\bar{q}q\bar{q}G\rangle$, denotes the flavour of the "quark-sea", the labels VF1→VF4 correspond to the four VF orbital configurations and the label set, $\{i\}$, represents the spin and colour labels of the quarks and gluon in the VF state (left uncoupled at this stage for the VF states). The basis size, N_B, serves as the truncation parameter in the regularisation of quantities calculated from the wavefunction, (2.1).

The disconnected piece of the wavefunction, $|\psi\rangle_D^J$, is given by

$$|\psi\rangle_D^J = \sum_f \sum_{n,\bar{n},n_g}^{N_B} \sum_{\{i\}} \Big\{ a^J_{D1}(n,\bar{n},n_g)^f_{\{i\}} \Delta_n(q_a,f)\, |n S \bar{n} \bar{P} G^{TE}_{n_g} : 1S1\bar{S}\rangle^{D1}_{\{i\},f}$$

$$+ a^J_{D2}(n,\bar{n},n_g)^f_{\{i\}} \Delta_{\bar{n}}(q_b,f)\, |n P \bar{n} \bar{S} G^{TE}_{n_g} : 1S1\bar{S}\rangle^{D2}_{\{i\},f}$$

$$+ a^J_{D3}(n,\bar{n},n_g)^f_{\{i\}} \Delta_n(q_a,f)\, \Delta_{\bar{n}}(q_b,f)\, |n S \bar{n} \bar{S} G^{TM}_{n_g} : 1S1\bar{S}\rangle^{D3}_{\{i\},f}$$

$$+ a^J_{D4}(n,\bar{n},n_g)^f_{\{i\}}\, |n P \bar{n} \bar{P} G^{TM}_{n_g} : 1S1\bar{S}\rangle^{D4}_{\{i\},f} \Big\} . \qquad (2.3)$$

The flavour label, f, is summed over f=u,d,c,s,b. The VF configurations in the disconnected states have been classified by the labels D1→D4 . To avoid double counting of the VF states in the connected part of the wavefunction the factor, $\Delta_n(q_i,q_j)$, has been included which is given by

$$\Delta_n(q_i,q_j) = 1 - \delta_{n1}\delta_{q_i q_j} . \qquad (2.4)$$

One of the most serious problems associated with bag models in general is that the static cavity approximation breaks translational invariance and it is not possible to construct plane-wave states from static cavity wavefunctions using a Lorentz boost. One plausible method commonly used to overcome the problems of centre-of-mass corrections and the construction of momentum eigenstates, associated with the static cavity approximation, is the wave-packet, or Peierls-Yoccoz, ansatz[8-10]. In this approach the static cavity wavefunction, $|\psi\rangle$, centred at the origin X=0, is expanded as a superposition of plane-wave states, $|\psi(p)\rangle$, weighted with some distribution amplitude. Taking $|\psi(X{=}0)\rangle \equiv |\psi\rangle$ we have

$$|\psi\rangle = \int d^3p\, \frac{\phi(p)}{2E(p)} |\psi(p)\rangle\, e^{ip\cdot X} \Big|_{X=0} , \qquad (2.5)$$

where $E(p) = \sqrt{p^2 + M^2}$ and $\phi(p)$ is the momentum distribution amplitude which is parameterised as

$$\phi(p) = \left[\frac{2}{\pi\Lambda^2}\right]^{3/4} \sqrt{\frac{2E(p)}{(2\pi)^3}}\; e^{-p^2/\Lambda^2} . \qquad (2.6)$$

The above for $\phi(p)$ is based on the Gaussian approximation for the overlap, $\langle\psi(X=0)|\psi(X)\rangle$. Once the form of $\phi(p)$ has been specified the required plane wave matrix elements can be extracted quite simply from the associated static cavity matrix element using the above prescription.

The four parameters of the model govern the quark gluon coupling, the confinement pressure, the zero-point energy and the CM prescription. For the wavefunction (2.1) these parameters have been fitted (for $m_u=m_d=0$) to the light meson masses and charge radii for $N_B=1$ to 10 in Ref.7 and we use these values as input to the ultra-relativistic calculations presented here.

3. Matrix Elements of the Bare Effective Operator

Since we are working in time independent perturbation theory the inclusion of O(g) states to the meson wavefunction leads to a proliferation of contributions to the $\Delta S=1$ matrix elements. The $\Delta S=1$ and 2 bare effective operators are (the colour structure is diagonal)

$$\mathcal{O}_{\Delta S=1} = :\bar{s}\gamma^{\mu}(1-\gamma_5)u\,\bar{u}\gamma_{\mu}(1-\gamma_5)d: - :\bar{s}\gamma^{\mu}(1-\gamma_5)c\,\bar{c}\gamma_{\mu}(1-\gamma_5)d: \tag{3.1}$$

and

$$\mathcal{O}_{\Delta S=2} = :\bar{s}\gamma^{\mu}(1-\gamma_5)d\,\bar{s}\gamma_{\mu}(1-\gamma_5)d: \tag{3.2}$$

To begin a systematic approach to the computation of the large number of diagrams involved in the $\Delta S=1$ and $\Delta S=2$ matrix elements, the latter of which will studied in a separate paper, we first define an operator, $\mathcal{O}_k$, where the label, k, corresponds to the three distinct operators above as

$$O_k = \begin{cases} :\bar{s}\gamma^{\mu}(1-\gamma_5)u\ \bar{u}\gamma_{\mu}(1-\gamma_5)d:, & k=1 \\ :\bar{s}\gamma^{\mu}(1-\gamma_5)c\ \bar{c}\gamma_{\mu}(1-\gamma_5)d:, & k=2 \\ :\bar{s}\gamma^{\mu}(1-\gamma_5)d\ \bar{s}\gamma_{\mu}(1-\gamma_5)d:, & k=3 \end{cases} \quad . \tag{3.3}$$

This operator is written down in terms of its sixteen component operators as

$$O_k(\underset{\sim}{x},0) = \sum_{i=1}^{16} \mathcal{C}_k^{(i)}(\underset{\sim}{x}) . \tag{3.4}$$

Since we will refer to these component operators throughout this section it is a necessary exercise to write them down in full in order to establish the notation to be used. Explicitly ($\Gamma^{\mu}=\gamma^{\mu}(1-\gamma_5)$), we have

$$\mathcal{C}_k^{(1)}(\underset{\sim}{x}) = \sum_{\text{labels}} \bar{u}_1(\underset{\sim}{x})\Gamma^{\mu}u_2(\underset{\sim}{x})\bar{u}_3(\underset{\sim}{x})\Gamma_{\mu}u_4(\underset{\sim}{x})\delta_{c_1c_2}\delta_{c_3c_4}:b^{\dagger}(q_1,1)b(q_2,2)b^{\dagger}(q_3,3)b(q_4,4):$$

$$\mathcal{C}_k^{(2)}(\underset{\sim}{x}) = \sum_{\text{labels}} \bar{u}_1(\underset{\sim}{x})\Gamma^{\mu}u_2(\underset{\sim}{x})\bar{u}_3(\underset{\sim}{x})\Gamma_{\mu}v_4(\underset{\sim}{x})\delta_{c_1c_2}\delta_{c_3c_4}:b^{\dagger}(q_1,1)b(q_2,2)b^{\dagger}(q_3,3)d^{\dagger}(q_4,4):$$

$$\mathcal{C}_k^{(3)}(\underset{\sim}{x}) = \sum_{\text{labels}} \bar{u}_1(\underset{\sim}{x})\Gamma^{\mu}u_2(\underset{\sim}{x})\bar{v}_3(\underset{\sim}{x})\Gamma_{\mu}u_4(\underset{\sim}{x})\delta_{c_1c_2}\delta_{c_3c_4}:b^{\dagger}(q_1,1)b(q_2,2)d(q_3,3)b(q_4,4):$$

$$\mathcal{C}_k^{(4)}(\underset{\sim}{x}) = \sum_{\text{labels}} \bar{u}_1(\underset{\sim}{x})\Gamma^{\mu}u_2(\underset{\sim}{x})\bar{v}_3(\underset{\sim}{x})\Gamma_{\mu}v_4(\underset{\sim}{x})\delta_{c_1c_2}\delta_{c_3c_4}:b^{\dagger}(q_1,1)b(q_2,2)d(q_3,3)d^{\dagger}(q_4,4):$$

$$\mathcal{C}_k^{(5)}(\underset{\sim}{x}) = \sum_{\text{labels}} \bar{u}_1(\underset{\sim}{x})\Gamma^{\mu}v_2(\underset{\sim}{x})\bar{u}_3(\underset{\sim}{x})\Gamma_{\mu}u_4(\underset{\sim}{x})\delta_{c_1c_2}\delta_{c_3c_4}:b^{\dagger}(q_1,1)d^{\dagger}(q_2,2)b^{\dagger}(q_3,3)b(q_4,4):$$

$$\mathcal{C}_k^{(6)}(\underset{\sim}{x}) = \sum_{\text{labels}} \bar{u}_1(\underset{\sim}{x})\Gamma^{\mu}v_2(\underset{\sim}{x})\bar{u}_3(\underset{\sim}{x})\Gamma_{\mu}v_4(\underset{\sim}{x})\delta_{c_1c_2}\delta_{c_3c_4}:b^{\dagger}(q_1,1)d^{\dagger}(q_2,2)b^{\dagger}(q_3,3)d^{\dagger}(q_4,4):$$

$$\mathcal{C}_k^{(7)}(\underset{\sim}{x}) = \sum_{\text{labels}} \bar{u}_1(\underset{\sim}{x})\Gamma^{\mu}v_2(\underset{\sim}{x})\bar{v}_3(\underset{\sim}{x})\Gamma_{\mu}u_4(\underset{\sim}{x})\delta_{c_1c_2}\delta_{c_3c_4}:b^{\dagger}(q_1,1)d^{\dagger}(q_2,2)d(q_3,3)b(q_4,4):$$

$$\mathcal{C}_k^{(8)}(\underset{\sim}{x}) = \sum_{\text{labels}} \bar{u}_1(\underset{\sim}{x})\Gamma^{\mu}v_2(\underset{\sim}{x})\bar{v}_3(\underset{\sim}{x})\Gamma_{\mu}v_4(\underset{\sim}{x})\delta_{c_1c_2}\delta_{c_3c_4}:b^{\dagger}(q_1,1)d^{\dagger}(q_2,2)d(q_3,3)d^{\dagger}(q_4,4):$$

$$C_k^{(9)}(\underset{\sim}{x}) = \sum_{\text{labels}} \bar{v}_1(\underset{\sim}{x})\,\Gamma^{\mu} u_2(\underset{\sim}{x})\,\bar{u}_3(\underset{\sim}{x})\,\Gamma_{\mu} u_4(\underset{\sim}{x})\,\delta_{c_1c_2}\delta_{c_3c_4} : d(q_1,1)b(q_2,2)b^{\dagger}(q_3,3)b(q_4,4) :$$

$$C_k^{(10)}(\underset{\sim}{x}) = \sum_{\text{labels}} \bar{v}_1(\underset{\sim}{x})\,\Gamma^{\mu} u_2(\underset{\sim}{x})\,\bar{u}_3(\underset{\sim}{x})\,\Gamma_{\mu} v_4(\underset{\sim}{x})\,\delta_{c_1c_2}\delta_{c_3c_4} : d(q_1,1)b(q_2,2)b^{\dagger}(q_3,3)d^{\dagger}(q_4,4) :$$

$$C_k^{(11)}(\underset{\sim}{x}) = \sum_{\text{labels}} \bar{v}_1(\underset{\sim}{x})\,\Gamma^{\mu} u_2(\underset{\sim}{x})\,\bar{v}_3(\underset{\sim}{x})\,\Gamma_{\mu} u_4(\underset{\sim}{x})\,\delta_{c_1c_2}\delta_{c_3c_4} : d(q_1,1)b(q_2,2)d(q_3,3)b(q_4,4) :$$

$$C_k^{(12)}(\underset{\sim}{x}) = \sum_{\text{labels}} \bar{v}_1(\underset{\sim}{x})\,\Gamma^{\mu} u_2(\underset{\sim}{x})\,\bar{v}_3(\underset{\sim}{x})\,\Gamma_{\mu} v_4(\underset{\sim}{x})\,\delta_{c_1c_2}\delta_{c_3c_4} : d(q_1,1)b(q_2,2)d(q_3,3)d^{\dagger}(q_4,4) :$$

$$C_k^{(13)}(\underset{\sim}{x}) = \sum_{\text{labels}} \bar{v}_1(\underset{\sim}{x})\,\Gamma^{\mu} v_2(\underset{\sim}{x})\,\bar{u}_3(\underset{\sim}{x})\,\Gamma_{\mu} u_4(\underset{\sim}{x})\,\delta_{c_1c_2}\delta_{c_3c_4} : d(q_1,1)d^{\dagger}(q_2,2)b^{\dagger}(q_3,3)b(q_4,4) :$$

$$C_k^{(14)}(\underset{\sim}{x}) = \sum_{\text{labels}} \bar{v}_1(\underset{\sim}{x})\,\Gamma^{\mu} v_2(\underset{\sim}{x})\,\bar{u}_3(\underset{\sim}{x})\,\Gamma_{\mu} v_4(\underset{\sim}{x})\,\delta_{c_1c_2}\delta_{c_3c_4} : d(q_1,1)d^{\dagger}(q_2,2)b^{\dagger}(q_3,3)d^{\dagger}(q_4,4) :$$

$$C_k^{(15)}(\underset{\sim}{x}) = \sum_{\text{labels}} \bar{v}_1(\underset{\sim}{x})\,\Gamma^{\mu} v_2(\underset{\sim}{x})\,\bar{v}_3(\underset{\sim}{x})\,\Gamma_{\mu} u_4(\underset{\sim}{x})\,\delta_{c_1c_2}\delta_{c_3c_4} : d(q_1,1)d^{\dagger}(q_2,2)d(q_3,3)b(q_4,4) :$$

$$C_k^{(16)}(\underset{\sim}{x}) = \sum_{\text{labels}} \bar{v}_1(\underset{\sim}{x})\,\Gamma^{\mu} v_2(\underset{\sim}{x})\,\bar{v}_3(\underset{\sim}{x})\,\Gamma_{\mu} v_4(\underset{\sim}{x})\,\delta_{c_1c_2}\delta_{c_3c_4} : d(q_1,1)d^{\dagger}(q_2,2)d(q_3,3)d^{\dagger}(q_4,4) :$$

(3.5)

where the label sets are

$$\{q_i,i\} \equiv \{q_i,n_iL_i,s_i,c_i\} \; , \tag{3.6}$$

(the quark flavours, q_i, can be read directly from (3.3) for each value of k) and the notation has, for the present, been simplified by rewriting the spinors, $u^{(s_i)}_{n_iL_i}(\underset{\sim}{x})$, as $u_i(\underset{\sim}{x})$ (and similarly for the antiquark spinors). The notation is further developed by writing the integral over d^3x of the component operators, (3.5), as

$$\int d^3x\, C_k^{(i)}(\underset{\sim}{x}) = \sum_{\text{labels}} T_k^{(i)}(\vec{nL},\vec{s})\,\delta_{c_1c_2}\delta_{c_3c_4}\,[C_k^{(i)}]_{op} \tag{3.7}$$

where $[C_k^{(i)}]_{op}$ contains the Fock space operators and the integral over the spinors is given by $T_k^{(i)}(\vec{nL},\vec{s})$ with the vectors $\vec{nL}$ and $\vec{s}$ representing the various labels of the spinors, i.e.

$$\vec{nL} = (n_1L_1,n_2L_2,n_3L_3,n_4L_4), \quad \vec{s} = (s_1,s_2,s_3,s_4) \ . \tag{3.8}$$

Taking the i=1 case as an example, we have

$$T_k^{(1)}(\vec{nL},\vec{s}) = \int d^3x \, \bar{u}_1(\underset{\sim}{x}) \Gamma^\mu u_2(\underset{\sim}{x}) \bar{u}_3(\underset{\sim}{x}) \Gamma_\mu u_4(\underset{\sim}{x}) \ , \tag{3.9}$$

and

$$[C_k^{(1)}]_{op} = :b^\dagger(q_1,1)b(q_2,2)b^\dagger(q_3,3)b(q_4,4): \ . \tag{3.10}$$

When the labels of $T_k^{(i)}(\vec{nL},\vec{s})$ need to be written out explicitly we will adopt the following notation:

$$T_k^{(i)}(\vec{nL},\vec{s}) = T\left[\begin{matrix} n_1L_1 & n_2L_2 \\ n_3L_3 & n_4L_4 \end{matrix}\middle| \begin{matrix} s_1 & s_2 \\ s_3 & s_4 \end{matrix}\right]_k^{(i)} \ . \tag{3.11}$$

Nearly all of the component operators, $C_k^{(i)}$, will contribute to the calculation of the $\Delta S=1$ (and $\Delta S=2$) matrix elements of states contained in the full O(g) wavefunctions leading to a large number of distinct diagrams. We will begin the analysis by considering first the K^0-π^0 and then the K^+-π^+ matrix elements. We calculate the matrix elements in two stages. First the expressions in terms of the spinor integrals, $T_k^{(i)}(\vec{nL},\vec{s})$, with definite arguments, are derived for all the matrix elements required. The angular integrations over the static cavity spinors are then performed, giving the matrix elements in terms of simple overlap integrals.

Proceeding in this manner highlights the many similarities in the dynamics of the $\Delta S=1$ (and $\Delta S=2$) matrix elements and aids in the eventual numeric computations.

(1) K^0-π^0 matrix element.

As a starting point we will first consider the non-VF diagonal contributions, i.e. the valence and bremsstrahlung diagrams contained in the diagonal matrix element of the connected part of the wavefunction. In terms of the amplitudes of the wavefunction this matrix element is (to distinguish π and K we place a bar over the pion Fock state amplitudes)

$$
\begin{aligned}
{}_F\langle\pi^0|O_{\Delta S=1}(\underset{\sim}{x},0)|K^0\rangle_F\Big|_{\text{connected}} = N_F^K N_F^\pi \Big\{ & {}_{\pi^0}\langle 1S\,1S|O_{\Delta S=1}(\underset{\sim}{x},0)|1S\,1S\rangle_{K^0} \\
+ \sum_{n,n',n_g}^{N_B} \Big[& \bar{a}^*_{TE1}(n',n_g)\, a_{TE1}(n,n_g)\, {}_{\pi^0}\langle n'S\,1S\,G^{TE}_{n_g}|O_{\Delta S=1}(\underset{\sim}{x},0)|n\,S\,1S\,G^{TE}_{n_g}\rangle_{K^0} \\
& + \bar{a}^*_{TE2}(n',n_g)\, a_{TE1}(n,n_g)\, {}_{\pi^0}\langle 1S\,n'S\,G^{TE}_{n_g}|O_{\Delta S=1}(\underset{\sim}{x},0)|n\,S\,1S\,G^{TE}_{n_g}\rangle_{K^0} \\
& + \bar{a}^*_{TE1}(n',n_g)\, a_{TE2}(n,n_g)\, {}_{\pi^0}\langle n'S\,1S\,G^{TE}_{n_g}|O_{\Delta S=1}(\underset{\sim}{x},0)|1S\,n\,S\,G^{TE}_{n_g}\rangle_{K^0} \\
& + \bar{a}^*_{TE2}(n',n_g)\, a_{TE2}(n,n_g)\, {}_{\pi^0}\langle 1S\,n'S\,G^{TE}_{n_g}|O_{\Delta S=1}(\underset{\sim}{x},0)|1S\,n\,S\,G^{TE}_{n_g}\rangle_{K^0} \\
& + \bar{a}^*_{TM1}(n',n_g)\, a_{TM1}(n,n_g)\, {}_{\pi^0}\langle n'P\,1S\,G^{TM}_{n_g}|O_{\Delta S=1}(\underset{\sim}{x},0)|n\,P\,1S\,G^{TM}_{n_g}\rangle_{K^0} \\
& + \bar{a}^*_{TM2}(n',n_g)\, a_{TM1}(n,n_g)\, {}_{\pi^0}\langle 1S\,n'P\,G^{TM}_{n_g}|O_{\Delta S=1}(\underset{\sim}{x},0)|n\,P\,1S\,G^{TM}_{n_g}\rangle_{K^0} \\
& + \bar{a}^*_{TM1}(n',n_g)\, a_{TM2}(n,n_g)\, {}_{\pi^0}\langle n'P\,1S\,G^{TM}_{n_g}|O_{\Delta S=1}(\underset{\sim}{x},0)|1S\,n\,P\,G^{TM}_{n_g}\rangle_{K^0} \\
& + \bar{a}^*_{TM2}(n',n_g)\, a_{TM2}(n,n_g)\, {}_{\pi^0}\langle 1S\,n'P\,G^{TM}_{n_g}|O_{\Delta S=1}(\underset{\sim}{x},0)|1S\,n\,P\,G^{TM}_{n_g}\rangle_{K^0} \Big] \\
& + \text{VF terms} \Big\} . \qquad (3.12)
\end{aligned}
$$

where N_F^K and N_F^π are the kaon and pion normalizations respectively.

The reason we have written the matrix element out in full is to facilitate the identification of the various diagrams involved. The first term in (3.12) is just the valence graph shown in Figure 1. The four TE-bremsstrahlung terms correspond to the diagrams of Figures 2a)-d) respectively. Similarly, the four TM-bremsstrahlung terms correspond to the diagrams of Figures 3a)-d).

The $\Delta S = 1$ operator component responsible for the valence and bremsstrahlung K^0-π^0 matrix elements is $C_1^{(13)}$. Below we present the initial aspects of the calculation of these matrix elements.

a) Valence K^0-π^0 matrix element.

The neutral kaon and pion valence states are given by,

$$K^\circ: \quad |1S\,1S\rangle = \frac{1}{\sqrt{3}} \sum_{s_a s_b} \sum_{c_a c_b} C^0_{q\bar{q}}(s_a,s_b)\, \delta_{c_a c_b}\, b^\dagger(d,1S,s_a,c_a)\, d^\dagger(s,1S,s_b,c_b)\, |0\rangle$$

and

$$\pi^\circ: \quad |1S\,1S\rangle_{\pi^0} = \frac{1}{\sqrt{2}} \left\{ \frac{1}{\sqrt{3}} \sum_{s'_a s'_b} \sum_{c'_a c'_b} C^0_{q\bar{q}}(s'_a,s'_b)\, \delta_{c'_a c'_b}\, b^\dagger(u,1S,s'_a,c'_a)\, d^\dagger(u,1S,s'_b,c'_b)\, |0\rangle \right\}. \tag{3.14}$$

where we have projected out the $u\bar{u}$ component of the neutral pion state, $b^\dagger(f,nL,s,c)$ and $d^\dagger(f,nL,s,c)$ are the quark and anti-quark creation operators for states with flavour, mode, orbital, spin and colour labels given by the arguments respectively and $C^J_{q\bar{q}}(s_a,s_b)$ are the Clebsch-Gordan coefficients for the $q\bar{q}$ pair coupled to spin J. From (3.14), the operator part of the $C_1^{(13)}$ valence matrix element is

$${}_{\pi^0}\langle 1S\,1S\,|[C_1^{(13)}]_{op}|\,1S\,1S\rangle_{K^0} = -\,\delta_{1b}\,\delta_{2b'}\,\delta_{3a'}\,\delta_{4a}\,. \tag{3.15}$$

where the label sets follow from (3.14) and (3.5). Inserting into (3.15) the full expression for $C_1^{(13)}$, summing over labels and including the integration over d^3x gives:

$${}_{\pi^0}\langle 1S\,1S\,|\int d^3x\, C_1^{(13)}(\underset{\sim}{x})|\,1S\,1S\rangle_{K^0} = -\frac{1}{2\sqrt{2}}\, S_1^{(13)}(k{=}1{:}1S,1S,1S,1S)\,. \tag{3.16}$$

where the spin summation $S_1^{(i)}$ is given by

$$S_1^{(i)}(k:\vec{nL}) = T\begin{bmatrix} n_1L_1 & n_2L_2 & \downarrow & \downarrow \\ n_3L_3 & n_4L_4 & \uparrow & \uparrow \end{bmatrix}^{(i)}_k - T\begin{bmatrix} n_1L_1 & n_2L_2 & \uparrow & \downarrow \\ n_3L_3 & n_4L_4 & \uparrow & \downarrow \end{bmatrix}^{(i)}_k$$

$$- T\begin{bmatrix} n_1L_1 & n_2L_2 & \downarrow & \uparrow \\ n_3L_3 & n_4L_4 & \downarrow & \uparrow \end{bmatrix}^{(i)}_k + T\begin{bmatrix} n_1L_1 & n_2L_2 & \uparrow & \uparrow \\ n_3L_3 & n_4L_4 & \downarrow & \downarrow \end{bmatrix}^{(i)}_k . \tag{3.17}$$

For clarity we will label the graphs as $G^{(i)}_{m}$, where (i) is the operator component(s) and the subscript, m, labels the diagram type from the corresponding figure. Hence, the valence contribution is

$$G_1^{(13)} = -\frac{1}{2\sqrt{2}} N_F^K N_F^\pi S_1^{(13)}(k{=}1{:}1S,1S,1S,1S). \qquad (3.18)$$

b) Diagonal bremsstrahlung $\Delta S = 1$ matrix elements.

The bremsstrahlung states (T_i=TE, TM) are written as

$$K^\circ: \quad |n_a L_a n_b L_b G^{T_i}_{n_g}>$$

and

$$\pi^\circ: \quad |n'_a L'_a n'_b L'_b G^{T_i}_{n_g}>, \qquad (3.19)$$

where explicitly we have

$$|n_a L_a n_b L_b G^{T_i}_{n_g}> = \frac{1}{\sqrt{8}} \sum_a \sum_{c_a c_b} \sum_{\substack{s_a s_b \\ j_3}} C^{0,1}_{q\bar{q}G}(s_a,s_b;j_3) \frac{1}{\sqrt{2}} \lambda^a_{c_a c_b}$$
$$\times\, c^{a\dagger}(T_i,n_g,j_3)\, b^\dagger(q_a,nS,s_a,c_a)\, d^\dagger(q_b,1S,s_b,c_b)\, |o>. \qquad (3.20)$$

where the gluon creation operator is denoted by $c^{a\dagger}(T_i,n_g,j_3)$ and the Clebsch-Gordan coefficient is $C^{J,J_{q\bar{q}}}_{q\bar{q}G}$. The algebra involved in the bremsstrahlung matrix element is analogous to the valence example above and need not be written down here. So, from (3.20) the bremsstrahlung matrix element of the operator $C_1^{(13)}$ is

$$_{\pi^\circ}<n'_a L'_a n'_b L'_b G^{T_i}_{n_g}| \int d^3x\, C_1^{(13)}(\underset{\sim}{x}) |n_a L_a n_b L_b G^{T_i}_{n_g}>_{K^\circ}$$

$$= -\frac{1}{6\sqrt{2}} \left[2\, S_2^{(13)}(k{=}1{:}n_b L_b, n'_b L'_b, n'_a L'_a, n_a L_a) + S_3^{(13)}(k{=}1{:}n_b L_b, n'_b L'_b, n'_a L'_a, n_a L_a) \right]. \qquad (3.21)$$

The spin summations, $S^{(i)}_{2,3}$, are given by

$$S_2^{(i)}(k:\vec{nL}) = T\begin{bmatrix} n_1L_1\ n_2L_2 & \uparrow\ \uparrow \\ n_3L_3\ n_4L_4 & \uparrow\ \uparrow \end{bmatrix}_k^{(i)} + T\begin{bmatrix} n_1L_1\ n_2L_2 & \downarrow\ \downarrow \\ n_3L_3\ n_4L_4 & \downarrow\ \downarrow \end{bmatrix}_k^{(i)}, \quad (3.22)$$

and

$$S_3^{(i)}(k:\vec{nL}) = T\begin{bmatrix} n_1L_1\ n_2L_2 & \downarrow\ \downarrow \\ n_3L_3\ n_4L_4 & \uparrow\ \uparrow \end{bmatrix}_k^{(i)} + T\begin{bmatrix} n_1L_1\ n_2L_2 & \uparrow\ \downarrow \\ n_3L_3\ n_4L_4 & \uparrow\ \downarrow \end{bmatrix}_k^{(i)}$$

$$+ T\begin{bmatrix} n_1L_1\ n_2L_2 & \downarrow\ \uparrow \\ n_3L_3\ n_4L_4 & \downarrow\ \uparrow \end{bmatrix}_k^{(i)} + T\begin{bmatrix} n_1L_1\ n_2L_2 & \uparrow\ \uparrow \\ n_3L_3\ n_4L_4 & \downarrow\ \downarrow \end{bmatrix}_k^{(i)}. \quad (3.23)$$

Using (3.21), the various bremsstrahlung contributions of Figures 2 and 3 can be obtained.

In order that the rest of the spin summations relevant to weak matrix elements need not be written out in full, we will adopt a convenient notation by introducing spin coefficients, $\varepsilon_j(\vec{s})$, defined as

$$S_j^{(i)}(k:\vec{nL}) = \sum_{\vec{s}} \varepsilon_j(\vec{s})\, T_k^{(i)}(\vec{nL},\vec{s}). \quad (3.27)$$

For reference the spin coefficients, $\varepsilon_j(\vec{s})$, required for the study of weak matrix elements in this framework are given in Table 1.

This completes the initial analysis of the K^0-π^0 matrix element with respect to the non-VF part of the wavefunction. We proceed now to the diagonal vacuum fluctuations.

c) Diagonal VF K^0-π^0 matrix elements.

(i) $u\bar{u}$ component of pion state.

The $u\bar{u}$ component contributions to the K^0-π^0 matrix element from the diagonal VF states originate from the component operators $C_1^{(13)}, C_1^{(1)}, C_1^{(7)}, C_1^{(10)}$ and $C_1^{(16)}$. The last four mix sea-quark flavour. In order to be able to identify the various processes graphically it is necessary to present the algebra involved in a measure of detail. To illustrate, we present the calculation of the VF matrix elements of $C_1^{(13)}$, beginning with the disconnected states which are simple since there is no Pauli interference in $|\psi\rangle_D^J$.

We write the fermionic part of the disconnected states, $D_j = D1 \rightarrow D4$, for the neutral kaon and pion, $|K^o\rangle^f_{D_j}$ and $|\pi^o\rangle^f_{D_j}$, in label set notation, as

$$K^o: \quad |K^o\rangle^f_{D_j} = b^\dagger(d,1')b^\dagger(f,2')d^\dagger(s,3')d^\dagger(f,4')|o\rangle,$$

and

$$\pi^o: \quad |\pi^o\rangle^f_{D_j} = \frac{1}{\sqrt{2}} b^\dagger(u,1'')b^\dagger(f,2'')d^\dagger(u,3'')d^\dagger(f,4'')|o\rangle. \tag{3.28}$$

The operator part of the matrix element is then given by

$${}^f_{D_j}\langle\pi^o|[C_1^{(13)}]_{op}|K^o\rangle^f_{D_j} = -\left[\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 + \Delta_5 + \Delta_6\right], \tag{3.29}$$

where

$$\Delta_1 = \delta_{13'}\delta_{23''}\delta_{31''}\delta_{41'}\delta_{2'2''}\delta_{4'4''}\,, \qquad \Delta_2 = -\delta_{df}\delta_{14'}\delta_{23''}\delta_{31''}\delta_{41'}\delta_{2'2''}\delta_{3'4''}\,,$$

$$\Delta_3 = -\delta_{uf}\delta_{13'}\delta_{24''}\delta_{31''}\delta_{41'}\delta_{2'2''}\delta_{4'3''}\,, \quad \Delta_4 = -\delta_{df}\delta_{13'}\delta_{23''}\delta_{31''}\delta_{42'}\delta_{1'2''}\delta_{4'4''}\,,$$

$$\Delta_5 = -\delta_{uf}\delta_{13'}\delta_{23''}\delta_{32''}\delta_{41'}\delta_{2'1''}\delta_{4'4''}\,, \quad \Delta_6 = \delta_{uf}\delta_{13'}\delta_{24''}\delta_{32''}\delta_{41'}\delta_{2'1''}\delta_{4'3''}\,. \tag{3.30}$$

The complete matrix element, after inclusion of the disconnected amplitudes[7] is

$${}^f_{D_j}\langle\pi^o| \int d^3x\, C_1^{(13)}(\underset{\sim}{x})|K^o\rangle^f_{D_j}$$

$$= -\frac{1}{\sqrt{2}}\left[\frac{g}{2\sqrt{3}}\right]^2 \sum_{\text{labels}} \left[\frac{R}{\omega^f_{n_{2'}L_{2'}} + \omega^f_{n_{4'}L_{4'}} + k^{T_i}_{n_g}}\right]\left[\frac{R}{\omega^f_{n_{2''}L_{2''}} + \omega^f_{n_{4''}L_{4''}} + k^{T_i}_{n_g}}\right] T\left[\begin{array}{cc|cc} n_1L_1 & n_2L_2 & s_1 & s_2 \\ n_3L_3 & n_4L_4 & s_3 & s_4 \end{array}\right]^{(13)}_1$$

$$\times \int d^3x'\, \underset{\sim}{W}^{qf}(\underset{\sim}{x}':n_{2'}L_{2'},n_{4'}L_{4'}:s_{2'},s_{4'}).\, \underset{\sim}{A}^{T_i^*}_{n_g j_3}(\underset{\sim}{x}')\, C_{q\bar{q}}(s_{1'},s_{3'})\, \lambda^a_{c_{2'}c_{4'}}\, \delta_{c_{1'}c_{3'}}$$

$$\times \int d^3x''\, \underset{\sim}{W}^{qf^*}(\underset{\sim}{x}'':n_{2''}L_{2''},n_{4''}L_{4''}:s_{2''},s_{4''}).\, \underset{\sim}{A}^{T_i}_{n_g j_3}(\underset{\sim}{x}'')\, C_{q\bar{q}}(s_{1''},s_{3''})\, \lambda^a_{c_{4''}c_{2''}}\, \delta_{c_{1''}c_{3''}}$$

$$\times \left[\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 + \Delta_5 + \Delta_6\right] \tag{3.31}$$

where $\underset{\sim}{A}^{T_i}_{n_g j_3}(\underset{\sim}{x})$ is the static cavity gluon field, ω^{f}_{nL} are the quark mode numbers, R is the confinement radius and the VF part of the quark current in the quark-gluon interaction is

$$\underset{\sim}{W}^q(\underset{\sim}{x}:n_1L_1,n_2L_2:s_1,s_2) = \overline{\mathcal{U}}^{(s_1)}_{n_1L_1}(\underset{\sim}{x})\,\underset{\sim}{\gamma}\,\mathcal{V}^{(s_2)}_{n_2L_2}(\underset{\sim}{x})\,. \tag{3.32}$$

It should be noted that the different radii of the kaon and pion states is to be read implicitly in the above expression; this can be done simply by recognizing the appropriate particle labels as defined in (3.31).

The identification of the terms in the above expression with the diagrams of Figure 4 can now be made by performing the summations over the operator indices, $\vec{nL}$ and $\vec{s}$, and the remaining labels contained in the Δ_i factors. By tracing the indices involved in the weak transition and the vacuum fluctuations we see that the first term, i.e. that associated with Δ_1, contracts over the indices involved in the vacuum fluctuation and hence corresponds to the disconnected diagrams, Figure 4a)-d). Similarly, we identify the remaining terms, Δ_2 to Δ_5, with respect to Figure 4, as corresponding to e) and f), g) and h), i) and j), k) and l) respectively. The last term, Δ_6, is zero, due to the colour structure, and corresponds to the $u\bar{u}$ component of the neutral pion originating directly from the vacuum fluctuation.

After summing over all indices and substituting the overlap integrals governing the vacuum fluctuations for particular gluon types these diagrams can be written down in analogous forms to the valence and bremsstrahlung cases already considered. However, before writing the expressions out, we make the important observation that the matrix element, (3.31), involving only disconnected states does not encompass the diagrams of Figure 4 entirely. Some modes in the internal quark and antiquark lines are excluded due to the counting factors, $\Delta_n(q_i,q_j)$, in the disconnected part of the wavefunction and come from either matrix elements of connected vacuum fluctuations or matrix elements which mix disconnected and connected parts of the wavefunction. The effect of these inclusions is to simply fill in the gaps of the mode summations occurring in (3.31).

After the contributions to these diagrams from connected vacuum fluctuations are also taken into account, the relevant expressions are given by

$$G^{(13)}_{4a)} = -\frac{3}{\sqrt{2}} N_F^K N_F^{\pi} \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{E^{(-)}_{q_f}(nS,\bar{n}P,n_g)}{\omega^f_{nS} + \omega^f_{\bar{n}P} + k^{TE}_{n_g}} \right\}_{\pi} \left\{ \frac{E^{(-)}_{q_f}(nS,\bar{n}P,n_g)}{\omega^f_{nS} + \omega^f_{\bar{n}P} + k^{TE}_{n_g}} \right\}_{K}$$

$$\times S_1^{(13)}(k{=}1{:}1S,1S,1S,1S) \qquad (3.33a)$$

$$G^{(13)}_{4b)} = -\frac{3}{\sqrt{2}} N_F^K N_F^{\pi} \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{E^{(-)}_{q_f}(nP,\bar{n}S,n_g)}{\omega^f_{nP} + \omega^f_{\bar{n}S} + k^{TE}_{n_g}} \right\}_{\pi} \left\{ \frac{E^{(-)}_{q_f}(nP,\bar{n}S,n_g)}{\omega^f_{nP} + \omega^f_{\bar{n}S} + k^{TE}_{n_g}} \right\}_{K}$$

$$\times S_1^{(13)}(k{=}1{:}1S,1S,1S,1S) \qquad (3.33b)$$

$$G^{(13)}_{4c)} = -\frac{3}{\sqrt{2}} N_F^K N_F^{\pi} \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{M^{(+)}_{q_f}(nS,\bar{n}S,n_g)}{\omega^f_{nS} + \omega^f_{\bar{n}S} + k^{TM}_{n_g}} \right\}_{\pi} \left\{ \frac{M^{(+)}_{q_f}(nS,\bar{n}S,n_g)}{\omega^f_{nS} + \omega^f_{\bar{n}S} + k^{TM}_{n_g}} \right\}_{K}$$

$$\times S_1^{(13)}(k{=}1{:}1S,1S,1S,1S) \qquad (3.33c)$$

$$G^{(13)}_{4d)} = -\frac{3}{\sqrt{2}} N_F^K N_F^{\pi} \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{M^{(+)}_{q_f}(nP,\bar{n}P,n_g)}{\omega^f_{nP} + \omega^f_{\bar{n}P} + k^{TM}_{n_g}} \right\}_{\pi} \left\{ \frac{M^{(+)}_{q_f}(nP,\bar{n}P,n_g)}{\omega^f_{nP} + \omega^f_{\bar{n}P} + k^{TM}_{n_g}} \right\}_{K}$$

$$\times S_1^{(13)}(k{=}1{:}1S,1S,1S,1S) \qquad (3.33d)$$

$$G^{(13)}_{4e)} = \frac{1}{\sqrt{2}} N_F^K N_F^{\pi} \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{E^{(-)}_{q_s}(nP,1S,n_g)}{\omega^s_{nP} + \omega^s_{1S} + k^{TE}_{n_g}} \right\}_{\pi} \left\{ \frac{E^{(-)}_{q_s}(nP,\bar{n}S,n_g)}{\omega^s_{nP} + \omega^s_{\bar{n}S} + k^{TE}_{n_g}} \right\}_{K}$$

$$\times \left[\frac{1}{2} S_1^{(13)}(k{=}1{:}\bar{n}S,1S,1S,1S) - \frac{\sqrt{2}}{3} S_4^{(13)}(k{=}1{:}\bar{n}S,1S,1S,1S) \right] \qquad (3.33e)$$

$$G^{(13)}_{4f)} = \frac{1}{\sqrt{2}} N_F^K N_F^{\pi} \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{M^{(+)}_{q_s}(nS,1S,n_g)}{\omega^s_{nS} + \omega^s_{1S} + k^{TM}_{n_g}} \right\}_{\pi} \left\{ \frac{M^{(+)}_{q_s}(nS,\bar{n}S,n_g)}{\omega^s_{nS} + \omega^s_{\bar{n}S} + k^{TM}_{n_g}} \right\}_{K}$$

$$\times \left[\frac{1}{2} S_1^{(13)}(k{=}1{:}\bar{n}S,1S,1S,1S) - \frac{\sqrt{2}}{3} S_4^{(13)}(k{=}1{:}\bar{n}S,1S,1S,1S) \right] \qquad (3.33f)$$

$$G^{(13)}_{4g)} = \frac{1}{\sqrt{2}} N^K_F N^\pi_F \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{E^{(-)}_{q_d}(1S,\bar{n}P,n_g)}{\omega^d_{1S} + \omega^d_{\bar{n}P} + k^{TE}_{n_g}} \right\}_\pi \left\{ \frac{E^{(-)}_{q_d}(nS,\bar{n}P,n_g)}{\omega^d_{nS} + \omega^d_{\bar{n}P} + k^{TE}_{n_g}} \right\}_K$$

$$\times \left[\frac{1}{2} S^{(13)}_1(k=1:1S,1S,1S,nS) + \frac{\sqrt{2}}{3} S^{(13)}_4(k=1:1S,1S,1S,nS) \right] \quad (3.33g)$$

$$G^{(13)}_{4h)} = \frac{1}{\sqrt{2}} N^K_F N^\pi_F \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{M^{(+)}_{q_d}(1S,\bar{n}S,n_g)}{\omega^d_{1S} + \omega^d_{\bar{n}S} + k^{TM}_{n_g}} \right\}_\pi \left\{ \frac{M^{(+)}_{q_d}(nS,\bar{n}S,n_g)}{\omega^d_{nS} + \omega^d_{\bar{n}S} + k^{TM}_{n_g}} \right\}_K$$

$$\times \left[\frac{1}{2} S^{(13)}_1(k=1:1S,1S,1S,nS) + \frac{\sqrt{2}}{3} S^{(13)}_4(k=1:1S,1S,1S,nS) \right] \quad (3.33h)$$

$$G^{(13)}_{4i)} = \frac{1}{\sqrt{2}} N^K_F N^\pi_F \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{E^{(-)}_{q_u}(nP,1S,n_g)}{\omega^u_{nP} + \omega^u_{1S} + k^{TE}_{n_g}} \right\}_K \left\{ \frac{E^{(-)}_{q_u}(nP,\bar{n}S,n_g)}{\omega^u_{nP} + \omega^u_{\bar{n}S} + k^{TE}_{n_g}} \right\}_\pi$$

$$\times \left[\frac{1}{2} S^{(13)}_1(k=1:1S,\bar{n}S,1S,1S) - \frac{\sqrt{2}}{3} S^{(13)}_5(k=1:1S,\bar{n}S,1S,1S) \right] \quad (3.33i)$$

$$G^{(13)}_{4j)} = \frac{1}{\sqrt{2}} N^K_F N^\pi_F \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{M^{(+)}_{q_u}(nS,1S,n_g)}{\omega^u_{nS} + \omega^u_{1S} + k^{TM}_{n_g}} \right\}_K \left\{ \frac{M^{(+)}_{q_u}(nS,\bar{n}S,n_g)}{\omega^u_{nS} + \omega^u_{\bar{n}S} + k^{TM}_{n_g}} \right\}_\pi$$

$$\times \left[\frac{1}{2} S^{(13)}_1(k=1:1S,\bar{n}S,1S,1S) - \frac{\sqrt{2}}{3} S^{(13)}_5(k=1:1S,\bar{n}S,1S,1S) \right] \quad (3.33j)$$

$$G^{(13)}_{4k)} = \frac{1}{\sqrt{2}} N^K_F N^\pi_F \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{E^{(-)}_{q_u}(1S,\bar{n}P,n_g)}{\omega^u_{1S} + \omega^u_{\bar{n}P} + k^{TE}_{n_g}} \right\}_K \left\{ \frac{E^{(-)}_{q_u}(nS,\bar{n}P,n_g)}{\omega^u_{nS} + \omega^u_{\bar{n}P} + k^{TE}_{n_g}} \right\}_\pi$$

$$\times \left[\frac{1}{2} S^{(13)}_1(k=1:1S,1S,nS,1S) + \frac{\sqrt{2}}{3} S^{(13)}_4(k=1:1S,1S,nS,1S) \right] \quad (3.33k)$$

$$G^{(13)}_{4l)} = \frac{1}{\sqrt{2}} N^K_F N^\pi_F \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{M^{(+)}_{q_u}(1S,\bar{n}S,n_g)}{\omega^u_{1S} + \omega^u_{\bar{n}S} + k^{TM}_{n_g}} \right\}_K \left\{ \frac{M^{(+)}_{q_u}(nS,\bar{n}S,n_g)}{\omega^u_{nS} + \omega^u_{\bar{n}S} + k^{TM}_{n_g}} \right\}_\pi$$

$$\times \left[\frac{1}{2} S^{(13)}_1(k=1:1S,1S,nS,1S) + \frac{\sqrt{2}}{3} S^{(13)}_4(k=1:1S,1S,nS,1S) \right] \quad (3.33l)$$

where the general TE and TM overlap integrals, resulting from the transverse part of the quark-gluon interaction, are defined in terms of quark and gluon normalizations, N_{nL} and $N^{T_i}_{n_g}$, quark shell momenta, p_{nL} and gluon mode numbers, $k^{T_i}_{n_g}$, as[11]

$$E^{(\pm)}_q(nL,n'L,n_g) = N^{TE}_{n_g} N_{nL} N_{n'L} \int_0^1 \xi^2 d\xi \left\{ F_{nL}\, j_1(p_{nL}R\xi)\, j_0(p_{n'L}R\xi) \pm F_{n'L}\, j_1(p_{n'L}R\xi)\, j_0(p_{nL}R\xi) \right\} j_1(k^{TE}_{n_g}\xi)\,, \tag{3.34}$$

and

$$M^{(\pm)}_q(nL,n'L',n_g) = N^{TM}_{n_g} N_{nL} N_{n'L'} \int_0^1 \xi^2 d\xi \left\{ j_0(p_{nL}R\xi)\, j_0(p_{n'L'}R\xi)\, j_0(k^{TM}_{n_g}\xi) \pm \frac{1}{3} F_{nL} F_{n'L'}\, j_1(p_{nL}R\xi)\, j_1(p_{n'L'}R\xi) \left[j_0(k^{TM}_{n_g}\xi) - 2 j_2(k^{TM}_{n_g}\xi) \right] \right\}. \tag{3.35}$$

In the above expressions the factor, F_{nL}, is given by

$$F_{nL} = \left[\frac{\omega_{nL} + \kappa m_q R}{\omega_{nL} - \kappa m_q R} \right]^{1/2}, \qquad (\kappa = \pm 1 \text{ for } L = P,\ S). \tag{3.36}$$

For the analysis of the $u\bar{u}$ component of the diagonal VF contributions we are left with the sea flavour mixing operators $C^{(1)}_1$, $C^{(7)}_1$, $C^{(10)}_1$ and $C^{(16)}_1$ each producing two diagrams as shown in Figure 5. The expressions for these diagrams are given by

$$G^{(1)}_{5a)} = -\frac{1}{6\sqrt{2}} N^K_P N^\pi_F \frac{8\alpha}{3} \sum_{n,n',n_g}^{N_B} \left\{ \frac{E^{(-)}_{q_u}(nP,1S,n_g)}{\omega^u_{nP} + \omega^u_{1S} + k^{TE}_{n_g}} \right\}_K \left\{ \frac{E^{(-)}_{q_s}(n'P,1S,n_g)}{\omega^s_{n'P} + \omega^s_{1S} + k^{TE}_{n_g}} \right\}_\pi \times \left[S^{(1)}_6(k{=}1{:}n'P,nP,1S,1S) + 2S^{(1)}_7(k{=}1{:}n'P,nP,1S,1S) \right] \tag{3.37a}$$

$$G^{(1)}_{5b)} = -\frac{1}{6\sqrt{2}} N^K_P N^\pi_F \frac{8\alpha}{3} \sum_{n,n',n_g}^{N_B} \left\{ \frac{M^{(+)}_{q_u}(nS,1S,n_g)}{\omega^u_{nS} + \omega^u_{1S} + k^{TM}_{n_g}} \right\}_K \left\{ \frac{M^{(+)}_{q_s}(n'S,1S,n_g)}{\omega^s_{n'S} + \omega^s_{1S} + k^{TM}_{n_g}} \right\}_\pi \times \left[S^{(1)}_6(k{=}1{:}n'S,nS,1S,1S) + 2S^{(1)}_7(k{=}1{:}n'S,nS,1S,1S) \right] \tag{3.37b}$$

$$G^{(7)}_{5c)} = \frac{1}{6\sqrt{2}} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{E^{(-)}_{q_u}(1S,\bar{n}P,n_g)}{\omega^u_{1S} + \omega^u_{\bar{n}P} + k^{TE}_{n_g}} \right\}_K \left\{ \frac{E^{(-)}_{q_s}(nP,1S,n_g)}{\omega^s_{nP} + \omega^s_{1S} + k^{TE}_{n_g}} \right\}_\pi$$

$$\times \left[S^{(7)}_6(k=1:1S,1S,\bar{n}P,nP) + 2S^{(7)}_8(k=1:1S,1S,\bar{n}P,nP) \right] \quad (3.37c)$$

$$G^{(7)}_{5d)} = \frac{1}{6\sqrt{2}} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{M^{(+)}_{q_u}(1S,\bar{n}S,n_g)}{\omega^u_{1S} + \omega^u_{\bar{n}S} + k^{TM}_{n_g}} \right\}_K \left\{ \frac{M^{(+)}_{q_s}(nS,1S,n_g)}{\omega^s_{nS} + \omega^s_{1S} + k^{TM}_{n_g}} \right\}_\pi$$

$$\times \left[S^{(7)}_6(k=1:1S,1S,\bar{n}S,nS) + 2S^{(7)}_8(k=1:1S,1S,\bar{n}S,nS) \right] \quad (3.37d)$$

$$G^{(10)}_{5e)} = \frac{1}{6\sqrt{2}} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{E^{(-)}_{q_u}(nP,1S,n_g)}{\omega^u_{nP} + \omega^u_{1S} + k^{TE}_{n_g}} \right\}_K \left\{ \frac{E^{(-)}_{q_d}(1S,\bar{n}P,n_g)}{\omega^d_{1S} + \omega^d_{\bar{n}P} + k^{TE}_{n_g}} \right\}_\pi$$

$$\times \left[2S^{(10)}_8(k=1:1S,nP,1S,\bar{n}P) + S^{(10)}_6(k=1:1S,nP,1S,\bar{n}P) \right] \quad (3.37e)$$

$$G^{(10)}_{5f)} = \frac{1}{6\sqrt{2}} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{M^{(+)}_{q_u}(nS,1S,n_g)}{\omega^u_{nS} + \omega^u_{1S} + k^{TM}_{n_g}} \right\}_K \left\{ \frac{M^{(+)}_{q_d}(1S,\bar{n}S,n_g)}{\omega^d_{1S} + \omega^d_{\bar{n}S} + k^{TM}_{n_g}} \right\}_\pi$$

$$\times \left[2S^{(10)}_8(k=1:1S,nS,1S,\bar{n}S) + S^{(10)}_6(k=1:1S,nS,1S,\bar{n}S) \right] \quad (3.37f)$$

$$G^{(16)}_{5g)} = -\frac{1}{6\sqrt{2}} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{\bar{n},\bar{n}',n_g}^{N_B} \left\{ \frac{E^{(-)}_{q_u}(1S,\bar{n}P,n_g)}{\omega^u_{1S} + \omega^u_{\bar{n}P} + k^{TE}_{n_g}} \right\}_K \left\{ \frac{E^{(-)}_{q_d}(1S,\bar{n}'P,n_g)}{\omega^d_{1S} + \omega^d_{\bar{n}'P} + k^{TE}_{n_g}} \right\}_\pi$$

$$\times \left[S^{(16)}_6(k=1:1S,1S,\bar{n}P,\bar{n}'P) + 2S^{(16)}_7(k=1:1S,1S,\bar{n}P,\bar{n}'P) \right] \quad (3.37g)$$

$$G^{(16)}_{5h)} = -\frac{1}{6\sqrt{2}} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{\bar{n},\bar{n}',n_g}^{N_B} \left\{ \frac{M^{(+)}_{q_u}(1S,\bar{n}S,n_g)}{\omega^u_{1S} + \omega^u_{\bar{n}S} + k^{TM}_{n_g}} \right\}_K \left\{ \frac{M^{(+)}_{q_d}(1S,\bar{n}'S,n_g)}{\omega^d_{1S} + \omega^d_{\bar{n}'S} + k^{TM}_{n_g}} \right\}_\pi$$

$$\times \left[S^{(16)}_6(k=1:1S,1S,\bar{n}S,\bar{n}'S) + 2S^{(16)}_7(k=1:1S,1S,\bar{n}S,\bar{n}'S) \right] \quad (3.37h)$$

We now move on to the $d\bar{d}$ component case.

(ii) $d\bar{d}$ component of pion state.

The $d\bar{d}$ component diagonal VF matrix elements are essentially a class of Penguin diagrams containing both $u\bar{u}$ and $c\bar{c}$ loop quarks. The relevant operators are $C_k^{(4)}, C_k^{(10)}$ and $C_k^{(13)}$ (with $k=1,2$) and correspond to the diagrams of Figure 6a)–d), e)–h) and i)–l) respectively. For loop quark flavour $f=u,c$ (corresponding to $k=1,2$) we obtain

$$G_{6a)}^{(4)} = \frac{1}{6\sqrt{2}} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{\substack{n,\bar{n}\\ n',n_g}}^{N_B} \left\{ \frac{E_{q_f}^{(-)}(nS,\bar{n}P,n_g)}{\omega_{nS}^f + \omega_{\bar{n}P}^f + k_{n_g}^{TE}} \right\}_K \left\{ \frac{E_{q_s}^{(-)}(n'P,1S,n_g)}{\omega_{n'P}^s + \omega_{1S}^s + k_{n_g}^{TE}} \right\}_\pi$$

$$\times\left[2S_2^{(4)}(k:n'P,nS,\bar{n}P,1S) + S_3^{(4)}(k:n'P,nS,\bar{n}P,1S)\right] \quad (3.38a)$$

$$G_{6b)}^{(4)} = \frac{1}{6\sqrt{2}} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{\substack{n,\bar{n}\\ n',n_g}}^{N_B} \left\{ \frac{E_{q_f}^{(-)}(nP,\bar{n}S,n_g)}{\omega_{nP}^f + \omega_{\bar{n}S}^f + k_{n_g}^{TE}} \right\}_K \left\{ \frac{E_{q_s}^{(-)}(n'P,1S,n_g)}{\omega_{n'P}^s + \omega_{1S}^s + k_{n_g}^{TE}} \right\}_\pi$$

$$\times\left[2S_2^{(4)}(k:n'P,nP,\bar{n}S,1S) + S_3^{(4)}(k:n'P,nP,\bar{n}S,1S)\right] \quad (3.38b)$$

$$G_{6c)}^{(4)} = \frac{1}{6\sqrt{2}} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{\substack{n,\bar{n}\\ n',n_g}}^{N_B} \left\{ \frac{M_{q_f}^{(+)}(nS,\bar{n}S,n_g)}{\omega_{nS}^f + \omega_{\bar{n}S}^f + k_{n_g}^{TM}} \right\}_K \left\{ \frac{M_{q_s}^{(+)}(n'S,1S,n_g)}{\omega_{n'S}^s + \omega_{1S}^s + k_{n_g}^{TM}} \right\}_\pi$$

$$\times\left[2S_2^{(4)}(k:n'S,nS,\bar{n}S,1S) + S_3^{(4)}(k:n'S,nS,\bar{n}S,1S)\right] \quad (3.38c)$$

$$G_{6d)}^{(4)} = \frac{1}{6\sqrt{2}} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{\substack{n,\bar{n}\\ n',n_g}}^{N_B} \left\{ \frac{M_{q_f}^{(+)}(nP,\bar{n}P,n_g)}{\omega_{nP}^f + \omega_{\bar{n}P}^f + k_{n_g}^{TM}} \right\}_K \left\{ \frac{M_{q_s}^{(+)}(n'S,1S,n_g)}{\omega_{n'S}^s + \omega_{1S}^s + k_{n_g}^{TM}} \right\}_\pi$$

$$\times\left[2S_2^{(4)}(k:n'S,nP,\bar{n}P,1S) + S_3^{(4)}(k:n'S,nP,\bar{n}P,1S)\right] \quad (3.38d)$$

$$G_{6e)}^{(10)} = \frac{1}{\sqrt{2}} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{\substack{n,\bar{n}\\ n',n_g}}^{N_B} \left\{ \frac{E_{q_f}^{(-)}(nS,\bar{n}P,n_g)}{\omega_{nS}^f + \omega_{\bar{n}P}^f + k_{n_g}^{TE}} \right\}_K \left\{ \frac{E_{q_f}^{(-)}(n'S,\bar{n}P,n_g)}{\omega_{n'S}^f + \omega_{\bar{n}P}^f + k_{n_g}^{TE}} \right\}_\pi$$

$$\times\left[\frac{1}{2} S_9^{(10)}(k:1S,nS,n'S,1S) - \frac{\sqrt{2}}{3} S_{10}^{(10)}(k:1S,nS,n'S,1S)\right] \quad (3.38e)$$

$$G^{(10)}_{6f)} = \frac{1}{\sqrt{2}} N^K_f N^\pi_F \frac{8\alpha}{3} \sum_{\substack{n,\bar{n} \\ n',n_g}}^{N_B} \left\{ \frac{E^{(-)}_{q_f}(nP,\bar{n}S,n_g)}{\omega^f_{nP} + \omega^f_{\bar{n}S} + k^{TE}_{n_g}} \right\}_K \left\{ \frac{E^{(-)}_{q_f}(n'P,\bar{n}S,n_g)}{\omega^f_{n'P} + \omega^f_{\bar{n}S} + k^{TE}_{n_g}} \right\}_\pi$$

$$\times \left[\frac{1}{2} S^{(10)}_9(k{:}1S,nP,n'P,1S) - \frac{\sqrt{2}}{3} S^{(10)}_{10}(k{:}1S,nP,n'P,1S) \right] \qquad (3.38f)$$

$$G^{(10)}_{6g)} = \frac{1}{\sqrt{2}} N^K_F N^\pi_F \frac{8\alpha}{3} \sum_{\substack{n,\bar{n} \\ n',n_g}}^{N_B} \left\{ \frac{M^{(+)}_{q_f}(nS,\bar{n}S,n_g)}{\omega^f_{nS} + \omega^f_{\bar{n}S} + k^{TM}_{n_g}} \right\}_K \left\{ \frac{M^{(+)}_{q_f}(n'S,\bar{n}S,n_g)}{\omega^f_{n'S} + \omega^f_{\bar{n}S} + k^{TM}_{n_g}} \right\}_\pi$$

$$\times \left[\frac{1}{2} S^{(10)}_9(k\cdot 1S,nS,n'S,1S) - \frac{\sqrt{2}}{3} S^{(10)}_{10}(k{:}1S,nS,n'S,1S) \right] \qquad (3.38g)$$

$$G^{(10)}_{6h)} = \frac{1}{\sqrt{2}} N^K_F N^\pi_F \frac{8\alpha}{3} \sum_{\substack{n,\bar{n} \\ n',n_g}}^{N_P} \left\{ \frac{M^{(+)}_{q_f}(nP,\bar{n}P,n_g)}{\omega^f_{nP} + \omega^f_{\bar{n}P} + k^{TM}_{n_g}} \right\}_K \left\{ \frac{M^{(+)}_{q_f}(n'P,\bar{n}P,n_g)}{\omega^f_{n'P} + \omega^f_{\bar{n}S} + k^{TM}_{n_g}} \right\}_\pi$$

$$\times \left[\frac{1}{2} S^{(10)}_9(k{:}1S,nP,n'P,1S) - \frac{\sqrt{2}}{3} S^{(10)}_{10}(k{:}1S,nP,n'P,1S) \right] \qquad (3.38h)$$

$$G^{(13)}_{6i)} = \frac{1}{6\sqrt{2}} N^K_F N^\pi_F \frac{8\alpha}{3} \sum_{\substack{n,\bar{n} \\ n',n_g}}^{N_B} \left\{ \frac{E^{(-)}_{q_d}(n'P,1S,n_g)}{\omega^d_{n'P} + \omega^d_{1S} + k^{TE}_{n_g}} \right\}_K \left\{ \frac{E^{(-)}_{q_f}(nS,\bar{n}P,n_g)}{\omega^f_{nS} + \omega^f_{\bar{n}P} + k^{TE}_{n_g}} \right\}_\pi$$

$$\times \left[2S^{(13)}_2(k{:}1S,\bar{n}P,nS,n'P) + S^{(13)}_4(k{:}1S,\bar{n}P,nS,n'P) \right] \qquad (3.38i)$$

$$G^{(13)}_{6j)} = \frac{1}{6\sqrt{2}} N^K_F N^\pi_F \frac{8\alpha}{3} \sum_{\substack{n,\bar{n} \\ n',n_g}}^{N_B} \left\{ \frac{E^{(-)}_{q_d}(n'P,1S,n_g)}{\omega^d_{n'P} + \omega^d_{1S} + k^{TE}_{n_g}} \right\}_K \left\{ \frac{E^{(-)}_{q_f}(nP,\bar{n}S,n_g)}{\omega^f_{nP} + \omega^f_{\bar{n}S} + k^{TE}_{n_g}} \right\}_\pi$$

$$\times \left[2S^{(13)}_2(k{:}1S,\bar{n}S,nP,n'P) + S^{(13)}_3(k{:}1S,\bar{n}S,nP,n'P) \right] \qquad (3.38j)$$

$$G^{(13)}_{6k)} = \frac{1}{6\sqrt{2}} N^K_F N^\pi_F \frac{8\alpha}{3} \sum_{\substack{n,\bar{n} \\ n',n_g}}^{N_B} \left\{ \frac{M^{(+)}_{q_d}(n'S,1S,n_g)}{\omega^d_{n'S} + \omega^d_{1S} + k^{TM}_{n_g}} \right\}_K \left\{ \frac{M^{(+)}_{q_f}(nS,\bar{n}S,n_g)}{\omega^f_{nS} + \omega^f_{\bar{n}S} + k^{TM}_{n_g}} \right\}_\pi$$

$$\times \left[2S^{(13)}_2(k{:}1S,\bar{n}S,nS,n'S) + S^{(13)}_3(k{:}1S,\bar{n}S,nS,n'S) \right] \qquad (3.38k)$$

$$G^{(13)}_{6l)} = \frac{1}{6\sqrt{2}} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{\substack{n,\bar{n}\\ n',n_g}}^{N_B} \left\{ \frac{M^{(+)}_{q_d}(n'S,1S,n_g)}{\omega^d_{n'S} + \omega^d_{1S} + k^{TM}_{n_g}} \right\}_K \left\{ \frac{M^{(+)}_{q_f}(nP,\bar{n}P,n_g)}{\omega^f_{nP} + \omega^f_{\bar{n}P} + k^{TM}_{n_g}} \right\}_\pi$$

$$\times \left[2S^{(13)}_2(k:1S,\bar{n}P,nP,n'S) + S^{(13)}_3(k:1S,\bar{n}P,nP,n'S) \right] \qquad (3.38l)$$

When the diagrams of Figure 6 are related to the K^o-π^o matrix element of the $\Delta S = 1$ operator, (3.1), one must include the negative sign (in the operator(3.1)) for the charm quark case.

The above expressions completes the analysis of the diagonal VF contributions to the the K^o-π^o matrix element. We now proceed to the off diagonal contributions.

d) Off diagonal VF-bremsstrahlung K^o-π^o matrix elements.

(i) $u\bar{u}$ component of pion state.

The two operators which give non zero contributions to the off diagonal VF-bremsstrahlung for the $u\bar{u}$ component are $C^{(5)}_1$ and $C^{(14)}_1$. In Figure 7 we give the resulting diagrams. The expressions for these diagrams are given by

$$G^{(5)}_{7a)} = -\frac{1}{2\sqrt{2}} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{n,n',n_g}^{N_B} \left\{ \frac{E^{(+)}_{q_d}(1S,nS,n_g)}{\omega^d_{1S} - \omega^d_{nS} - k^{TE}_{n_g}} \right\}_K \left\{ \frac{E^{(-)}_{q_s}(n'P,1S,n_g)}{\omega^s_{n'P} + \omega^s_{1S} + k^{TE}_{n_g}} \right\}_\pi$$

$$\times S^{(5)}_{11}(k=1:n'P,1S,1S,nS) \qquad (3.39a)$$

$$G^{(5)}_{7b)} = -\frac{1}{2\sqrt{2}} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{n,n',n_g}^{N_B} \left\{ \frac{M^{(-)}_{q_d}(1S,nP,n_g)}{\omega^d_{1S} - \omega^d_{nP} - k^{TM}_{n_g}} \right\}_K \left\{ \frac{M^{(+)}_{q_s}(n'S,1S,n_g)}{\omega^s_{n'S} + \omega^s_{1S} + k^{TM}_{n_g}} \right\}_\pi$$

$$\times S^{(5)}_{11}(k=1:n'S,1S,1S,nP) \qquad (3.39b)$$

$$G^{(5)}_{7c)} = -\frac{1}{2\sqrt{2}} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{E^{(+)}_{q_s}(1S,\bar{n}S,n_g)}{\omega^s_{1S} - \omega^s_{\bar{n}S} - k^{TE}_{n_g}} \right\}_K \left\{ \frac{E^{(-)}_{q_s}(nP,\bar{n}S,n_g)}{\omega^s_{nP} + \omega^s_{\bar{n}S} + k^{TE}_{n_g}} \right\}_\pi$$

$$\times S^{(5)}_{11}(k=1:nP,1S,1S,1S) \qquad (3.39c)$$

$$G^{(5)}_{7d)} = -\frac{1}{2\sqrt{2}} N^K_F N^\pi_F \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{M^{(-)}_{q_s}(1S,\bar{n}P,n_g)}{\omega^s_{1S} - \omega^s_{\bar{n}P} - k^{TM}_{n_g}} \right\}_K \left\{ \frac{M^{(+)}_{q_s}(nP,\bar{n}P,n_g)}{\omega^s_{nP} + \omega^s_{\bar{n}P} + k^{TM}_{n_g}} \right\}_\pi$$

$$\times S^{(5)}_{11}(k=1:nP,1S,1S,1S) \qquad (3.39d)$$

$$G^{(14)}_{7e)} = \frac{1}{2\sqrt{2}} N^K_F N^\pi_F \frac{8\alpha}{3} \sum_{\bar{n},\bar{n}',n_g}^{N_B} \left\{ \frac{E^{(+)}_{q_s}(1S,\bar{n}S,n_g)}{\omega^s_{1S} - \omega^s_{\bar{n}S} - k^{TE}_{n_g}} \right\}_K \left\{ \frac{E^{(-)}_{q_d}(1S,\bar{n}'P,n_g)}{\omega^d_{1S} + \omega^d_{\bar{n}'P} + k^{TE}_{n_g}} \right\}_\pi$$

$$\times S^{(14)}_{11}(k=1:\bar{n}S,1S,1S,\bar{n}'P) \qquad (3.39e)$$

$$G^{(14)}_{7f)} = \frac{1}{2\sqrt{2}} N^K_F N^\pi_F \frac{8\alpha}{3} \sum_{\bar{n},\bar{n}',n_g}^{N_B} \left\{ \frac{M^{(-)}_{q_s}(1S,\bar{n}P,n_g)}{\omega^s_{1S} - \omega^s_{\bar{n}P} - k^{TM}_{n_g}} \right\}_K \left\{ \frac{M^{(+)}_{q_d}(1S,\bar{n}'S,n_g)}{\omega^d_{1S} + \omega^d_{\bar{n}'S} + k^{TM}_{n_g}} \right\}_\pi$$

$$\times S^{(14)}_{11}(k=1:\bar{n}P,1S,1S,\bar{n}'S) \qquad (3.39f)$$

$$G^{(14)}_{7g)} = \frac{1}{2\sqrt{2}} N^K_F N^\pi_F \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{E^{(+)}_{q_d}(1S,nS,n_g)}{\omega^d_{1S} - \omega^d_{nS} - k^{TE}_{n_g}} \right\}_K \left\{ \frac{E^{(-)}_{q_d}(nS,\bar{n}P,n_g)}{\omega^d_{nS} + \omega^d_{\bar{n}P} + k^{TE}_{n_g}} \right\}_\pi$$

$$\times S^{(14)}_{11}(k=1:1S,1S,1S,\bar{n}P) \qquad (3.39g)$$

$$G^{(14)}_{7h)} = \frac{1}{2\sqrt{2}} N^K_F N^\pi_F \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{M^{(-)}_{q_d}(1S,nP,n_g)}{\omega^d_{1S} - \omega^d_{nP} - k^{TM}_{n_g}} \right\}_K \left\{ \frac{M^{(+)}_{q_d}(nP,\bar{n}P,n_g)}{\omega^d_{nP} + \omega^d_{\bar{n}P} + k^{TM}_{n_g}} \right\}_\pi$$

$$\times S^{(14)}_{11}(k=1:1S,1S,1S,\bar{n}P) \qquad (3.39h)$$

(ii) $d\bar{d}$ component of pion state.

The off diagonal matrix elements of the $d\bar{d}$ component give rise to Penguin class diagrams, as was the case for the diagonal VF $d\bar{d}$ component contributions. The relevant operators are $C^{(12)}_k$ and $C^{(14)}_k$ (with $k=1,2$) corresponding to the diagrams of Figures 8 and 9 respectively. These diagrams are given by the expressions

$$G^{(12)}_{8a)} = -\frac{1}{6\sqrt{2}} N_F^K N^\pi \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{E^{(+)}_{q_d}(1S,1S,n_g)}{k^{TE}_{n_g}} \right\}_\pi \left\{ \frac{E^{(-)}_{q_f}(nS,\bar{n}P,n_g)}{\omega^f_{nS} + \omega^f_{\bar{n}P} + k^{TE}_{n_g}} \right\}_K$$

$$\times \left[2S^{(12)}_{12}(k{:}1S,nS,\bar{n}P,1S) + S^{(12)}_{13}(k{:}1S,nS,\bar{n}P,1S) \right] \qquad (3.40a)$$

$$G^{(12)}_{8b)} = -\frac{1}{6\sqrt{2}} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{E^{(+)}_{q_d}(1S,1S,n_g)}{k^{TE}_{n_g}} \right\}_\pi \left\{ \frac{E^{(-)}_{q_f}(nP,\bar{n}S,n_g)}{\omega^f_{nP} + \omega^f_{\bar{n}S} + k^{TE}_{n_g}} \right\}_K$$

$$\times \left[2S^{(12)}_{12}(k{:}1S,nP,\bar{n}S,1S) + S^{(12)}_{13}(k{:}1S,nP,\bar{n}S,1S) \right] \qquad (3.40b)$$

$$G^{(12)}_{8c)} = \frac{1}{6\sqrt{2}} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{\substack{n,\bar{n} \\ \bar{n}',n_g}}^{N_B} \left\{ \frac{E^{(+)}_{q_d}(1S,\bar{n}'S,n_g)}{\omega^d_{1S} - \omega^d_{\bar{n}'S} - k^{TE}_{n_g}} \right\}_\pi \left\{ \frac{E^{(-)}_{q_f}(nS,\bar{n}P,n_g)}{\omega^f_{nS} + \omega^f_{\bar{n}P} + k^{TE}_{n_g}} \right\}_K$$

$$\times \left[2S^{(12)}_{12}(k{:}1S,nS,\bar{n}P,\bar{n}'S) + S^{(12)}_{13}(k{:}1S,nS,\bar{n}P,\bar{n}'S) \right] \qquad (3.40c)$$

$$G^{(12)}_{8d)} = \frac{1}{6\sqrt{2}} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{\substack{n,\bar{n} \\ \bar{n}',n_g}}^{N_B} \left\{ \frac{E^{(+)}_{q_d}(1S,\bar{n}'S,n_g)}{\omega^d_{1S} - \omega^d_{\bar{n}'S} - k^{TE}_{n_g}} \right\}_\pi \left\{ \frac{E^{(-)}_{q_f}(nP,\bar{n}S,n_g)}{\omega^f_{nP} + \omega^f_{\bar{n}S} + k^{TE}_{n_g}} \right\}_K$$

$$\times \left[2S^{(12)}_{12}(k{:}1S,nP,\bar{n}S,\bar{n}'S) + S^{(12)}_{13}(k{:}1S,nP,\bar{n}S,\bar{n}'S) \right] \qquad (3.40d)$$

$$G^{(12)}_{8e)} = \frac{1}{6\sqrt{2}} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{\substack{n,\bar{n} \\ \bar{n}',n_g}}^{N_B} \left\{ \frac{M^{(-)}_{q_d}(1S,\bar{n}'P,n_g)}{\omega^d_{1S} - \omega^d_{\bar{n}'P} - k^{TM}_{n_g}} \right\}_\pi \left\{ \frac{M^{(+)}_{q_f}(nS,\bar{n}S,n_g)}{\omega^f_{nS} + \omega^f_{\bar{n}S} + k^{TM}_{n_g}} \right\}_K$$

$$\times \left[2S^{(12)}_{12}(k{:}1S,nS,\bar{n}S,\bar{n}'P) + S^{(12)}_{13}(k{:}1S,nS,\bar{n}S,\bar{n}'P) \right] \qquad (3.40e)$$

$$G^{(12)}_{8f)} = \frac{1}{6\sqrt{2}} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{\substack{n,\bar{n} \\ \bar{n}',n_g}}^{N_B} \left\{ \frac{M^{(-)}_{q_d}(1S,\bar{n}'P,n_g)}{\omega^d_{1S} - \omega^d_{\bar{n}'P} - k^{TM}_{n_g}} \right\}_\pi \left\{ \frac{M^{(+)}_{q_f}(nP,\bar{n}P,n_g)}{\omega^f_{nP} + \omega^f_{\bar{n}P} + k^{TM}_{n_g}} \right\}_K$$

$$\times \left[2S^{(12)}_{12}(k{:}1S,nP,\bar{n}P,\bar{n}'P) + S^{(12)}_{13}(k{:}1S,nP,\bar{n}P,\bar{n}'P) \right] \qquad (3.40f)$$

$$G^{(14)}_{9a)} = -\frac{1}{6\sqrt{2}} N^K_F N^\pi_F \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{E^{(+)}_{q_d}(1S,1S,n_g)}{k^{TE}_{n_g}} \right\}_K \left\{ \frac{E^{(-)}_{q_f}(nS,\bar{n}P,n_g)}{\omega^f_{nS} + \omega^f_{\bar{n}P} + k^{TE}_{n_g}} \right\}_\pi$$

$$\times \left[S^{(14)}_{13}(k:1S,\bar{n}P,nS,1S) + 2S^{(14)}_{14}(k:1S,\bar{n}P,nS,1S) \right] \quad (3.41a)$$

$$G^{(14)}_{9b)} = -\frac{1}{6\sqrt{2}} N^K_F N^\pi_F \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{E^{(+)}_{q_d}(1S,1S,n_g)}{k^{TE}_{n_g}} \right\}_K \left\{ \frac{E^{(-)}_{q_f}(nP,\bar{n}S,n_g)}{\omega^f_{nP} + \omega^f_{\bar{n}S} + k^{TE}_{n_g}} \right\}_\pi$$

$$\times \left[S^{(14)}_{13}(k:1S,\bar{n}S,nP,1S) + 2S^{(14)}_{14}(k:1S,\bar{n}S,nP,1S) \right] \quad (3.41b)$$

$$G^{(14)}_{9c)} = \frac{1}{6\sqrt{2}} N^K_F N^\pi_F \frac{8\alpha}{3} \sum_{\substack{n,\bar{n} \\ \bar{n}',n_g}}^{N_B} \left\{ \frac{E^{(+)}_{q_s}(1S,\bar{n}'S,n_g)}{\omega^s_{1S} - \omega^s_{\bar{n}'S} - k^{TE}_{n_g}} \right\}_K \left\{ \frac{E^{(-)}_{q_f}(nS,\bar{n}P,n_g)}{\omega^f_{nS} + \omega^f_{\bar{n}P} + k^{TE}_{n_g}} \right\}_\pi$$

$$\times \left[S^{(14)}_{13}(k:\bar{n}'S,\bar{n}P,nS,1S) + 2S^{(14)}_{14}(k:\bar{n}'S,\bar{n}P,nS,1S) \right] \quad (3.41c)$$

$$G^{(14)}_{9d)} = \frac{1}{6\sqrt{2}} N^K_F N^\pi_F \frac{8\alpha}{3} \sum_{\substack{n,\bar{n} \\ \bar{n}',n_g}}^{N_B} \left\{ \frac{E^{(+)}_{q_s}(1S,\bar{n}'S,n_g)}{\omega^s_{1S} - \omega^s_{\bar{n}'S} - k^{TE}_{n_g}} \right\}_K \left\{ \frac{E^{(-)}_{q_f}(nP,\bar{n}S,n_g)}{\omega^f_{nP} + \omega^f_{\bar{n}S} + k^{TE}_{n_g}} \right\}_\pi$$

$$\times \left[S^{(14)}_{13}(k:\bar{n}'S,\bar{n}S,nP,1S) + 2S^{(14)}_{14}(k:\bar{n}'S,\bar{n}S,nP,1S) \right] \quad (3.41d)$$

$$G^{(14)}_{9e)} = \frac{1}{6\sqrt{2}} N^K_F N^\pi_F \frac{8\alpha}{3} \sum_{\substack{n,\bar{n} \\ \bar{n}',n_g}}^{N_B} \left\{ \frac{M^{(-)}_{q_s}(1S,\bar{n}'P,n_g)}{\omega^s_{1S} - \omega^s_{\bar{n}'P} - k^{TM}_{n_g}} \right\}_K \left\{ \frac{M^{(+)}_{q_f}(nS,\bar{n}S,n_g)}{\omega^f_{nS} + \omega^f_{\bar{n}S} + k^{TM}_{n_g}} \right\}_\pi$$

$$\times \left[S^{(14)}_{13}(k:\bar{n}'P,\bar{n}S,nS,1S) + 2S^{(14)}_{14}(k:\bar{n}'P,\bar{n}S,nS,1S) \right] \quad (3.41e)$$

$$G^{(14)}_{9f)} = \frac{1}{6\sqrt{2}} N^K_F N^\pi_F \frac{8\alpha}{3} \sum_{\substack{n,\bar{n} \\ \bar{n}',n_g}}^{N_B} \left\{ \frac{M^{(-)}_{q_s}(1S,\bar{n}'P,n_g)}{\omega^s_{1S} - \omega^s_{\bar{n}'P} - k^{TM}_{n_g}} \right\}_K \left\{ \frac{M^{(+)}_{q_f}(nP,\bar{n}P,n_g)}{\omega^f_{nP} + \omega^f_{\bar{n}P} + k^{TM}_{n_g}} \right\}_\pi$$

$$\times \left[S^{(14)}_{13}(k:\bar{n}'P,\bar{n}P,nP,1S) + 2S^{(14)}_{14}(k:\bar{n}'P,\bar{n}P,nP,1S) \right] \quad (3.41f)$$

This completes the analytic analysis of the K°-π° matrix element.

(2) K^+-π^+ matrix elements.

a) Valence K^+-π^+ matrix element.

The operator giving rise to the valence component of the K^+-π^+ transition, Figure 10, is $C_1^{(10)}$. Using the same notation as used in (3.14) for the K°-π° case, the matrix element of this operator and the valence states is given by

$$ {}_{\pi^+}\langle 1S\,1S|\int d^3x\, C_1^{(10)}(\underset{\sim}{x})|1S\,1S\rangle_{K^+} = \frac{3}{2} S_1^{(10)}(k{=}1{:}1S,1S,1S,1S)\ . \qquad (3.42) $$

Hence the valence contribution is

$$ G_{10}^{(10)} = \frac{3}{2} N_F^K N_F^\pi\, S_1^{(10)}(k{=}1{:}1S,1S,1S,1S)\ . \qquad (3.43) $$

An important difference between the K°-π° and K^+-π^+ matrix elements is the fact that the latter does not receive contributions from diagonal bremsstrahlung matrix elements. This is due to the singlet colour structure of the bare effective operator and the fact that in the bremsstrahlung states the $q\bar{q}$ pair are in a colour octet.

b) Diagonal VF matrix elements.

Diagrams diagonal in sea quark flavour come from $C_k^{(10)}$ whilst the operators $C_k^{(4)}$ and $C_k^{(13)}$ $(k=1,2)$ generate non diagonal sea quark graphs. The operator $C_k^{(10)}$, gives rise to the diagrams shown in Figure 11.

The expressions for the spectator diagrams, Figures 11m) to p) can be obtained directly from the K°-π° case already considered (without the amplitude factor of $\frac{1}{\sqrt{2}}$) from the expressions corresponding to the Figures 6e) to h).

For the non diagonal sea quark flavour operators, $C_k^{(4)}$ and $C_k^{(13)}$ $(k=1,2)$, we obtain the diagrams of Figure 12 and Figure 13 respectively. Note that there are only Penguin class diagrams because the graphs analogous to those of Figure 5 are zero by colour.

The expressions corresponding to the diagrams of Figures 11, 12 and 13 are given by

$$G^{(10)}_{11a)} = 9\, N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{E_{q_f}^{(-)}(nS,\bar{n}P,n_g)}{\omega^f_{nS} + \omega^f_{\bar{n}P} + k^{TE}_{n_g}} \right\}_\pi \left\{ \frac{E_{q_f}^{(-)}(nS,\bar{n}P,n_g)}{\omega^f_{nS} + \omega^f_{\bar{n}P} + k^{TE}_{n_g}} \right\}_K$$

$$\times S^{(10)}_{15}(k{=}1{:}1S,1S,1S,1S) \qquad (3.44a)$$

$$G^{(10)}_{11b)} = 9\, N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{E_{q_f}^{(-)}(nP,\bar{n}S,n_g)}{\omega^f_{nP} + \omega^f_{\bar{n}S} + k^{TE}_{n_g}} \right\}_\pi \left\{ \frac{E_{q_f}^{(-)}(nP,\bar{n}S,n_g)}{\omega^f_{nP} + \omega^f_{\bar{n}S} + k^{TE}_{n_g}} \right\}_K$$

$$\times S^{(10)}_{15}(k{=}1{:}1S,1S,1S,1S) \qquad (3.44b)$$

$$G^{(10)}_{11c)} = 9\, N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{M_{q_f}^{(+)}(nS,\bar{n}S,n_g)}{\omega^f_{nS} + \omega^f_{\bar{n}S} + k^{TM}_{n_g}} \right\}_\pi \left\{ \frac{M_{q_f}^{(+)}(nS,\bar{n}S,n_g)}{\omega^f_{nS} + \omega^f_{\bar{n}S} + k^{TM}_{n_g}} \right\}_K$$

$$\times S^{(10)}_{15}(k{=}1{:}1S,1S,1S,1S) \qquad (3.44c)$$

$$G^{(10)}_{11d)} = 9\, N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{M_{q_f}^{(+)}(nP,\bar{n}P,n_g)}{\omega^f_{nP} + \omega^f_{\bar{n}P} + k^{TM}_{n_g}} \right\}_\pi \left\{ \frac{M_{f}^{(+)}(nP,\bar{n}P,n_g)}{\omega^f_{nP} + \omega^f_{\bar{n}P} + k^{TM}_{n_g}} \right\}_K$$

$$\times S^{(10)}_{15}(k{=}1{:}1S,1S,1S,1S) \qquad (3.44d)$$

$$G^{(10)}_{11e)} = -\, N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{E_{q_u}^{(-)}(1S,\bar{n}P,n_g)}{\omega^u_{1S} + \omega^u_{\bar{n}P} + k^{TE}_{n_g}} \right\}_\pi \left\{ \frac{E_{q_u}^{(-)}(nS,\bar{n}P,n_g)}{\omega^u_{nS} + \omega^u_{\bar{n}P} + k^{TE}_{n_g}} \right\}_K$$

$$\times \left[\frac{3}{2} S^{(10)}_{15}(k{=}1{:}1S,nS,1S,1S) - \sqrt{2}\, S^{(10)}_{16}(k{=}1{:}1S,nS,1S,1S) \right] \qquad (3.44e)$$

$$G^{(10)}_{11f)} = -\, N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{M_{q_u}^{(+)}(1S,\bar{n}S,n_g)}{\omega^u_{1S} + \omega^u_{\bar{n}S} + k^{TM}_{n_g}} \right\}_\pi \left\{ \frac{M_{q_u}^{(+)}(nS,\bar{n}S,n_g)}{\omega^u_{nS} + \omega^u_{\bar{n}S} + k^{TM}_{n_g}} \right\}_K$$

$$\times \left[\frac{3}{2} S^{(10)}_{15}(k{=}1{:}1S,nS,1S,1S) - \sqrt{2}\, S^{(10)}_{16}(k{=}1{:}1S,nS,1S,1S) \right] \qquad (3.44f)$$

$$G^{(10)}_{11g)} = -\, N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{E_{q_u}^{(-)}(1S,\bar{n}P,n_g)}{\omega^u_{1S} + \omega^u_{\bar{n}P} + k^{TE}_{n_g}} \right\}_K \left\{ \frac{E_{q_u}^{(-)}(nS,\bar{n}P,n_g)}{\omega^u_{nS} + \omega^u_{\bar{n}P} + k^{TE}_{n_g}} \right\}_\pi$$

$$\times \left[\frac{3}{2} S^{(10)}_{15}(k{=}1{:}1S,1S,nS,1S) - \sqrt{2}\, S^{(10)}_{17}(k{=}1{:}1S,1S,nS,1S) \right] \qquad (3.44g)$$

$$G^{(10)}_{11h)} = -N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{M_{q_u}^{(+)}(1S,\bar{n}S,n_g)}{\omega^u_{1S} + \omega^u_{\bar{n}S} + k^{TM}_{n_g}} \right\}_K \left\{ \frac{M_{q_u}^{(+)}(nS,\bar{n}S,n_g)}{\omega^u_{nS} + \omega^u_{\bar{n}S} + k^{TM}_{n_g}} \right\}_\pi$$
$$\times \left[\frac{3}{2} S_{15}^{(10)}(k=1:1S,1S,nS,1S) - \sqrt{2}\, S_{17}^{(10)}(k=1:1S,1S,nS,1S) \right] \quad (3.44h)$$

$$G^{(10)}_{11i)} = -N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{E_{q_s}^{(-)}(nP,1S,n_g)}{\omega^s_{nP} + \omega^s_{1S} + k^{TE}_{n_g}} \right\}_\pi \left\{ \frac{E_{q_s}^{(-)}(nP,\bar{n}S,n_g)}{\omega^s_{nP} + \omega^s_{\bar{n}S} + k^{TE}_{n_g}} \right\}_K$$
$$\times \left[\frac{3}{2} S_{15}^{(10)}(k=1:\bar{n}S,1S,1S,1S) + \sqrt{2}\, S_{16}^{(10)}(k=1:\bar{n}S,1S,nS,1S) \right] \quad (3.44i)$$

$$G^{(10)}_{11j)} = -N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{M_{q_s}^{(+)}(nS,1S,n_g)}{\omega^s_{nS} + \omega^s_{1S} + k^{TM}_{n_g}} \right\}_\pi \left\{ \frac{M_{q_s}^{(+)}(nS,\bar{n}S,n_g)}{\omega^s_{nS} + \omega^s_{\bar{n}S} + k^{TM}_{n_g}} \right\}_K$$
$$\times \left[\frac{3}{2} S_{15}^{(10)}(k=1:\bar{n}S,1S,1S,1S) + \sqrt{2}\, S_{16}^{(10)}(k=1:\bar{n}S,1S,nS,1S) \right] \quad (3.44j)$$

$$G^{(10)}_{11k)} = -N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{E_{q_d}^{(-)}(nP,1S,n_g)}{\omega^d_{nP} + \omega^d_{1S} + k^{TE}_{n_g}} \right\}_K \left\{ \frac{E_{q_d}^{(-)}(nP,\bar{n}S,n_g)}{\omega^d_{nP} + \omega^d_{\bar{n}S} + k^{TE}_{n_g}} \right\}_\pi$$
$$\times \left[\frac{3}{2} S_{15}^{(10)}(k=1:1S,1S,1S,\bar{n}S) + \sqrt{2}\, S_{17}^{(10)}(k=1:1S,1S,1S,\bar{n}S) \right] \quad (3.44k)$$

$$G^{(10)}_{11l)} = -N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{M_{q_d}^{(+)}(nS,1S,n_g)}{\omega^d_{nS} + \omega^d_{1S} + k^{TM}_{n_g}} \right\}_K \left\{ \frac{M_{q_d}^{(+)}(nS,\bar{n}S,n_g)}{\omega^d_{nS} + \omega^d_{\bar{n}S} + k^{TM}_{n_g}} \right\}_\pi$$
$$\times \left[\frac{3}{2} S_{15}^{(10)}(k=1:1S,1S,1S,\bar{n}S) + \sqrt{2}\, S_{17}^{(10)}(k=1:1S,1S,1S,\bar{n}S) \right] \quad (3.44l)$$

$$G^{(10)}_{11m)} = N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{\substack{n,\bar{n} \\ n',n_g}}^{N_B} \left\{ \frac{E_{q_f}^{(-)}(nS,\bar{n}P,n_g)}{\omega^f_{nS} + \omega^f_{\bar{n}P} + k^{TE}_{n_g}} \right\}_K \left\{ \frac{E_{q_f}^{(-)}(n'S,\bar{n}P,n_g)}{\omega^f_{n'S} + \omega^f_{\bar{n}P} + k^{TE}_{n_g}} \right\}_\pi$$
$$\times \left[\frac{1}{2} S_9^{(10)}(k:1S,nS,n'S,1S) - \frac{\sqrt{2}}{3} S_{10}^{(10)}(k:1S,nS,n'S,1S) \right] \quad (3.44m)$$

$$G^{(10)}_{11n)} = N^K_F N^\pi_F \frac{8\alpha}{3} \sum_{\substack{n,\bar{n}\\ n',n_g}}^{N_B} \left\{ \frac{E^{(-)}_{q_f}(nP,\bar{n}S,n_g)}{\omega^f_{nP} + \omega^f_{\bar{n}S} + k^{TE}_{n_g}} \right\}_K \left\{ \frac{E^{(-)}_{q_f}(n'P,\bar{n}S,n_g)}{\omega^f_{n'P} + \omega^f_{\bar{n}S} + k^{TE}_{n_g}} \right\}_\pi$$

$$\times \left[\frac{1}{2} S^{(10)}_9(k:1S,nP,n'P,1S) - \frac{\sqrt{2}}{3} S^{(10)}_{10}(k:1S,nP,n'P,1S) \right] \quad (3.44n)$$

$$G^{(10)}_{11o)} = N^K_F N^\pi_F \frac{8\alpha}{3} \sum_{\substack{n,\bar{n}\\ n',n_g}}^{N_B} \left\{ \frac{M^{(+)}_{q_f}(nS,\bar{n}S,n_g)}{\omega^f_{nS} + \omega^f_{\bar{n}S} + k^{TM}_{n_g}} \right\}_K \left\{ \frac{M^{(+)}_{q_f}(n'S,\bar{n}S,n_g)}{\omega^f_{n'S} + \omega^f_{\bar{n}S} + k^{TM}_{n_g}} \right\}_\pi$$

$$\times \left[\frac{1}{2} S^{(10)}_9(k:1S,nS,n'S,1S) - \frac{\sqrt{2}}{3} S^{(10)}_{10}(k:1S,nS,n'S,1S) \right] \quad (3.44o)$$

$$G^{(10)}_{11p)} = N^K_F N^\pi_F \frac{8\alpha}{3} \sum_{\substack{n,\bar{n}\\ n',n_g}}^{N_B} \left\{ \frac{M^{(+)}_{q_f}(nP,\bar{n}P,n_g)}{\omega^f_{nP} + \omega^f_{\bar{n}P} + k^{TM}_{n_g}} \right\}_K \left\{ \frac{M^{(+)}_{q_f}(n'P,\bar{n}P,n_g)}{\omega^f_{n'P} + \omega^f_{\bar{n}S} + k^{TM}_{n_g}} \right\}_\pi$$

$$\times \left[\frac{1}{2} S^{(10)}_9(k:1S,nP,n'P,1S) - \frac{\sqrt{2}}{3} S^{(10)}_{10}(k:1S,nP,n'P,1S) \right] \quad (3.44p)$$

$$G^{(4)}_{12a)} = \frac{1}{6} N^K_F N^\pi_F \frac{8\alpha}{3} \sum_{\substack{n,\bar{n}\\ n',n_g}}^{N_B} \left\{ \frac{E^{(-)}_{q_f}(nS,\bar{n}P,n_g)}{\omega^f_{nS} + \omega^f_{\bar{n}P} + k^{TE}_{n_g}} \right\}_K \left\{ \frac{E^{(-)}_{q_s}(n'P,1S,n_g)}{\omega^s_{n'P} + \omega^s_{1S} + k^{TE}_{n_g}} \right\}_\pi$$

$$\times \left[2S^{(4)}_2(k:n'P,nS,\bar{n}P,1S) + S^{(4)}_3(k:n'P,nS,\bar{n}P,1S) \right] \quad (3.45a)$$

$$G^{(4)}_{12b)} = \frac{1}{6} N^K_F N^\pi_F \frac{8\alpha}{3} \sum_{\substack{n,\bar{n}\\ n',n_g}}^{N_B} \left\{ \frac{E^{(-)}_{q_f}(nP,\bar{n}S,n_g)}{\omega^f_{nP} + \omega^f_{\bar{n}S} + k^{TE}_{n_g}} \right\}_K \left\{ \frac{E^{(-)}_{q_s}(n'P,1S,n_g)}{\omega^s_{n'P} + \omega^s_{1S} + k^{TE}_{n_g}} \right\}_\pi$$

$$\times \left[2S^{(4)}_2(k:n'P,nP,\bar{n}S,1S) + S^{(4)}_3(k:n'P,nP,\bar{n}S,1S) \right] \quad (3.45b)$$

$$G^{(4)}_{12c)} = \frac{1}{6} N^K_F N^\pi_F \frac{8\alpha}{3} \sum_{\substack{n,\bar{n}\\ n',n_g}}^{N_B} \left\{ \frac{M^{(+)}_{q_f}(nS,\bar{n}S,n_g)}{\omega^f_{nS} + \omega^f_{\bar{n}S} + k^{TM}_{n_g}} \right\}_K \left\{ \frac{M^{(+)}_{q_s}(n'S,1S,n_g)}{\omega^s_{n'S} + \omega^s_{1S} + k^{TM}_{n_g}} \right\}_\pi$$

$$\times \left[2S^{(4)}_2(k:n'S,nS,\bar{n}S,1S) + S^{(4)}_3(k:n'S,nS,\bar{n}S,1S) \right] \quad (3.45c)$$

$$G^{(4)}_{12d)} = \frac{1}{6} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{\substack{n,\bar{n}\\ n',n_g}}^{N_B} \left\{ \frac{M_{q_\gamma}^{(+)}(nP,\bar{n}P,n_g)}{\omega^f_{nP} + \omega^f_{\bar{n}P} + k^{TM}_{n_g}} \right\}_K \left\{ \frac{M_{q_s}^{(+)}(n'S,1S,n_g)}{\omega^f_{n'S} + \omega^f_{1S} + k^{TM}_{n_g}} \right\}_\pi$$

$$\times \left[2S_2^{(4)}(k:n'S,nP,\bar{n}P,1S) + S_3^{(4)}(k:n'S,nP,\bar{n}P,1S) \right] \quad (3.45d)$$

$$G^{(4)}_{13a)} = \frac{1}{6} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{\substack{n,\bar{n}\\ n',n_g}}^{N_B} \left\{ \frac{E_{q_d}^{(-)}(n'P,1S,n_g)}{\omega^d_{n'P} + \omega^d_{1S} + k^{TE}_{n_g}} \right\}_K \left\{ \frac{E_{q_\gamma}^{(-)}(nS,\bar{n}P,n_g)}{\omega^f_{nS} + \omega^f_{\bar{n}P} + k^{TE}_{n_g}} \right\}_\pi$$

$$\times \left[2S_2^{(13)}(k:1S,\bar{n}P,nS,n'P) + S_3^{(13)}(k:1S,\bar{n}P,nS,n'P) \right] \quad (3.46a)$$

$$G^{(4)}_{13b)} = \frac{1}{6} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{\substack{n,\bar{n}\\ n',n_g}}^{N_{\bar{B}}} \left\{ \frac{E_{q_d}^{(-)}(n'P,1S,n_g)}{\omega^d_{n'P} + \omega^d_{1S} + k^{TE}_{n_g}} \right\}_K \left\{ \frac{E_{q_\gamma}^{(-)}(nP,\bar{n}S,n_g)}{\omega^f_{nP} + \omega^f_{\bar{n}S} + k^{TE}_{n_g}} \right\}_\pi$$

$$\times \left[2S_2^{(13)}(k:1S,\bar{n}S,nP,n'P) + S_3^{(13)}(k:1S,\bar{n}S,nP,n'P) \right] \quad (3.46b)$$

$$G^{(4)}_{13c)} = \frac{1}{6} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{\substack{n,\bar{n}\\ n',n_g}}^{N_B} \left\{ \frac{M_{q_d}^{(+)}(n'S,1S,n_g)}{\omega^d_{n'S} + \omega^d_{1S} + k^{TM}_{n_g}} \right\}_K \left\{ \frac{M_{q_\gamma}^{(+)}(nS,\bar{n}S,n_g)}{\omega^f_{nS} + \omega^f_{\bar{n}S} + k^{TM}_{n_g}} \right\}_\pi$$

$$\times \left[2S_2^{(13)}(k:1S,\bar{n}S,nS,n'S) + S_3^{(13)}(k:1S,\bar{n}S,nS,n'S) \right] \quad (3.46c)$$

$$G^{(4)}_{13d)} = \frac{1}{6} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{\substack{n,\bar{n}\\ n',n_g}}^{N_B} \left\{ \frac{M_{q_d}^{(+)}(n'S,1S,n_g)}{\omega^d_{n'S} + \omega^d_{1S} + k^{TM}_{n_g}} \right\}_K \left\{ \frac{M_{q_\gamma}^{(+)}(nP,\bar{n}P,n_g)}{\omega^f_{nP} + \omega^f_{\bar{n}P} + k^{TM}_{n_g}} \right\}_\pi$$

$$\times \left[2S_2^{(13)}(k:1S,\bar{n}P,nP,n'S) + S_3^{(13)}(k:1S,\bar{n}P,nP,n'S) \right] \quad (3.46d)$$

c) Off diagonal VF-bremsstrahlung matrix elements.

There are just two operators responsible for the non zero off diagonal matrix elements. These are $C_k^{(12)}$ and $C_k^{(14)}$ $(k=1,2)$ and correspond to the diagrams in Figures 14 and 15 respectively. The expressions for these diagrams are

$$G^{(12)}_{14a)} = \frac{3}{2} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{E^{(+)}_{q_u}(1S,nS,n_g)}{\omega^u_{1S} - \omega^u_{nS} - k^{TE}_{n_g}} \right\}_\pi \left\{ \frac{E^{(-)}_{q_u}(nS,\bar{n}P,n_g)}{\omega^u_{nS} + \omega^u_{\bar{n}P} + k^{TE}_{n_g}} \right\}_K$$

$$\times S^{(12)}_{18}(k{=}1{:}1S,1S,\bar{n}P,1S) \qquad (3.47a)$$

$$G^{(12)}_{14b)} = \frac{3}{2} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{M^{(-)}_{q_u}(1S,nP,n_g)}{\omega^u_{1S} - \omega^u_{nP} - k^{TM}_{n_g}} \right\}_\pi \left\{ \frac{M^{(+)}_{q_u}(nP,\bar{n}P,n_g)}{\omega^u_{nP} + \omega^u_{\bar{n}P} + k^{TM}_{n_g}} \right\}_K$$

$$\times S^{(12)}_{18}(k{=}1{:}1S,1S,\bar{n}P,1S) \qquad (3.47b)$$

$$G^{(12)}_{14c)} = \frac{3}{2} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{\bar{n},\bar{n}',n_g}^{N_B} \left\{ \frac{E^{(+)}_{q_d}(1S,\bar{n}'S,n_g)}{\omega^d_{1S} - \omega^d_{\bar{n}'S} - k^{TE}_{n_g}} \right\}_\pi \left\{ \frac{E^{(-)}_{q_u}(1S,\bar{n}P,n_g)}{\omega^u_{1S} + \omega^u_{\bar{n}P} + k^{TE}_{n_g}} \right\}_K$$

$$\times S^{(12)}_{18}(k{=}1{:}1S,1S,\bar{n}P,\bar{n}'S) \qquad (3.47c)$$

$$G^{(12)}_{14d)} = \frac{3}{2} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{\bar{n},\bar{n}',n_g}^{N_B} \left\{ \frac{M^{(-)}_{q_d}(1S,\bar{n}'P,n_g)}{\omega^d_{1S} - \omega^d_{\bar{n}'P} - k^{TM}_{n_g}} \right\}_\pi \left\{ \frac{M^{(+)}_{q_u}(1S,\bar{n}S,n_g)}{\omega^u_{1S} + \omega^u_{\bar{n}S} + k^{TM}_{n_g}} \right\}_K$$

$$\times S^{(12)}_{18}(k{=}1{:}1S,1S,\bar{n}S,\bar{n}'P) \qquad (3.47d)$$

$$G^{(12)}_{14e)} = -\frac{1}{6} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{E^{(+)}_{q_u}(1S,1S,n_g)}{k^{TE}_{n_g}} \right\}_\pi \left\{ \frac{E^{(-)}_{q_f}(nS,\bar{n}P,n_g)}{\omega^f_{nS} + \omega^f_{\bar{n}P} + k^{TE}_{n_g}} \right\}_K$$

$$\times \left[2S^{(12)}_{12}(k{:}1S,nS,\bar{n}P,1S) + S^{(12)}_{13}(k{:}1S,nS,\bar{n}P,1S) \right] \qquad (3.47e)$$

$$G^{(12)}_{14f)} = -\frac{1}{6} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{E^{(+)}_{q_u}(1S,1S,n_g)}{k^{TE}_{n_g}} \right\}_\pi \left\{ \frac{E^{(-)}_{q_f}(nP,\bar{n}S,n_g)}{\omega^f_{nP} + \omega^f_{\bar{n}S} + k^{TE}_{n_g}} \right\}_K$$

$$\times \left[2S^{(12)}_{12}(k{:}1S,nP,\bar{n}S,1S) + S^{(12)}_{13}(k{:}1S,nP,\bar{n}S,1S) \right] \qquad (3.47f)$$

$$G^{(12)}_{14g)} = \frac{1}{6} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{\substack{n,\bar{n}\\ \bar{n}',n_g}}^{N_B} \left\{ \frac{E^{(+)}_{q_d}(1S,\bar{n}'S,n_g)}{\omega^d_{1S} - \omega^d_{\bar{n}'S} - k^{TE}_{n_g}} \right\}_\pi \left\{ \frac{E^{(-)}_{q_f}(nS,\bar{n}P,n_g)}{\omega^f_{nS} + \omega^f_{\bar{n}P} + k^{TE}_{n_g}} \right\}_K$$

$$\times \left[2S^{(12)}_{12}(k{:}1S,nS,\bar{n}P,\bar{n}'S) + S^{(12)}_{13}(k{:}1S,nS,\bar{n}P,\bar{n}'S) \right] \quad (3.47g)$$

$$G^{(12)}_{14h)} = \frac{1}{6} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{\substack{n,\bar{n}\\ \bar{n}',n_g}}^{N_B} \left\{ \frac{E^{(+)}_{q_d}(1S,\bar{n}'S,n_g)}{\omega^d_{1S} - \omega^d_{\bar{n}'S} - k^{TE}_{n_g}} \right\}_\pi \left\{ \frac{E^{(-)}_{q_f}(nP,\bar{n}S,n_g)}{\omega^f_{nP} + \omega^f_{\bar{n}S} + k^{TE}_{n_g}} \right\}_K$$

$$\times \left[2S^{(12)}_{12}(k{:}1S,nP,\bar{n}S,\bar{n}'S) + S^{(12)}_{13}(k{:}1S,nP,\bar{n}S,\bar{n}'S) \right] \quad (3.47h)$$

$$G^{(12)}_{14i)} = \frac{1}{6} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{\substack{n,\bar{n}\\ \bar{n}',n_g}}^{N_B} \left\{ \frac{M^{(-)}_{q_d}(1S,\bar{n}'P,n_g)}{\omega^d_{1S} - \omega^d_{\bar{n}'P} - k^{TM}_{n_g}} \right\}_\pi \left\{ \frac{M^{(+)}_{q_f}(nS,\bar{n}S,n_g)}{\omega^f_{nS} + \omega^f_{\bar{n}S} + k^{TM}_{n_g}} \right\}_K$$

$$\times \left[2S^{(12)}_{12}(k{:}1S,nS,\bar{n}S,\bar{n}'P) + S^{(12)}_{13}(k{:}1S,nS,\bar{n}S,\bar{n}'P) \right] \quad (3.47i)$$

$$G^{(12)}_{14j)} = \frac{1}{6} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{\substack{n,\bar{n}\\ \bar{n}',n_g}}^{N_B} \left\{ \frac{M^{(-)}_{q_d}(1S,\bar{n}'P,n_g)}{\omega^d_{1S} - \omega^d_{\bar{n}'P} - k^{TM}_{n_g}} \right\}_\pi \left\{ \frac{M^{(+)}_{q_f}(nP,\bar{n}P,n_g)}{\omega^f_{nP} + \omega^f_{\bar{n}P} + k^{TM}_{n_g}} \right\}_K$$

$$\times \left[2S^{(12)}_{12}(k{:}1S,nP,\bar{n}P,\bar{n}'P) + S^{(12)}_{13}(k{:}1S,nP,\bar{n}P,\bar{n}'P) \right] \quad (3.47j)$$

$$G^{(14)}_{15a)} = -\frac{3}{2} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{E^{(+)}_{q_u}(1S,nS,n_g)}{\omega^u_{1S} - \omega^u_{nS} - k^{TE}_{n_g}} \right\}_K \left\{ \frac{E^{(-)}_{q_u}(nS,\bar{n}P,n_g)}{\omega^u_{nS} + \omega^u_{\bar{n}P} + k^{TE}_{n_g}} \right\}_\pi$$

$$\times S^{(14)}_{19}(k{=}1{:}1S,\bar{n}P,1S,1S) \quad (3.48a)$$

$$G^{(14)}_{15b)} = -\frac{3}{2} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{M^{(-)}_{q_u}(1S,nP,n_g)}{\omega^u_{1S} - \omega^u_{nP} - k^{TM}_{n_g}} \right\}_K \left\{ \frac{M^{(+)}_{q_u}(nP,\bar{n}P,n_g)}{\omega^u_{nP} + \omega^u_{\bar{n}P} + k^{TM}_{n_g}} \right\}_\pi$$

$$\times S^{(14)}_{19}(k{=}1{:}1S,\bar{n}P,1S,1S) \quad (3.48b)$$

$$G^{(14)}_{15c)} = -\frac{3}{2} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{\bar{n},\bar{n}',n_g}^{N_B} \left\{ \frac{E^{(+)}_{q_g}(1S,\bar{n}'S,n_g)}{\omega^s_{1S} - \omega^s_{\bar{n}'S} - k^{TE}_{n_g}} \right\}_K \left\{ \frac{E^{(-)}_{q_g}(1S,\bar{n}P,n_g)}{\omega^u_{1S} + \omega^u_{\bar{n}P} + k^{TE}_{n_g}} \right\}_\pi$$

$$\times S^{(14)}_{19}(k{=}1{:}\bar{n}'S,\bar{n}P,1S,1S) \qquad (3.48c)$$

$$G^{(14)}_{15d)} = -\frac{3}{2} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{\bar{n},\bar{n}',n_g}^{N_B} \left\{ \frac{M^{(-)}_{q_g}(1S,\bar{n}'P,n_g)}{\omega^s_{1S} - \omega^s_{\bar{n}'P} - k^{TM}_{n_g}} \right\}_K \left\{ \frac{M^{(+)}_{q_g}(1S,\bar{n}S,n_g)}{\omega^u_{1S} + \omega^u_{\bar{n}S} + k^{TM}_{n_g}} \right\}_\pi$$

$$\times S^{(14)}_{19}(k{=}1{:}\bar{n}'P,\bar{n}S,1S,1S) \qquad (3.48d)$$

$$G^{(14)}_{15e)} = -\frac{1}{6} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{E^{(+)}_{q_g}(1S,1S,n_g)}{k^{TE}_{n_g}} \right\}_K \left\{ \frac{E^{(-)}_{q_g}(nS,\bar{n}P,n_g)}{\omega^f_{nS} + \omega^f_{\bar{n}P} + k^{TE}_{n_g}} \right\}_\pi$$

$$\times \left[S^{(14)}_{13}(k{:}1S,\bar{n}P,nS,1S) + 2S^{(14)}_{14}(k{:}1S,\bar{n}P,nS,1S) \right] \qquad (3.48e)$$

$$G^{(14)}_{15f)} = \frac{1}{6} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{n,\bar{n},n_g}^{N_B} \left\{ \frac{E^{(+)}_{q_g}(1S,1S,n_g)}{k^{TE}_{n_g}} \right\}_K \left\{ \frac{E^{(-)}_{q_g}(nP,\bar{n}S,n_g)}{\omega^f_{nP} + \omega^f_{\bar{n}S} + k^{TE}_{n_g}} \right\}_\pi$$

$$\times \left[S^{(14)}_{13}(k{:}1S,\bar{n}S,nP,1S) + 2S^{(14)}_{14}(k{:}1S,\bar{n}S,nP,1S) \right] \qquad (3.48f)$$

$$G^{(14)}_{15g)} = \frac{1}{6} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{\substack{n,\bar{n}\\ \bar{n}',n_g}}^{N_B} \left\{ \frac{E^{(+)}_{q_g}(1S,\bar{n}'S,n_g)}{\omega^s_{1S} - \omega^s_{\bar{n}'S} - k^{TE}_{n_g}} \right\}_K \left\{ \frac{E^{(-)}_{q_g}(nS,\bar{n}P,n_g)}{\omega^f_{nS} + \omega^f_{\bar{n}P} + k^{TE}_{n_g}} \right\}_\pi$$

$$\times \left[S^{(14)}_{13}(k{:}\bar{n}'S,\bar{n}P,nS,1S) + 2S^{(14)}_{14}(k{:}\bar{n}'S,\bar{n}P,nS,1S) \right] \qquad (3.48g)$$

$$G^{(14)}_{15h)} = \frac{1}{6} N_F^K N_F^\pi \frac{8\alpha}{3} \sum_{\substack{n,\bar{n}\\ \bar{n}',n_g}}^{N_B} \left\{ \frac{E^{(+)}_{q_g}(1S,\bar{n}'S,n_g)}{\omega^s_{1S} - \omega^s_{\bar{n}'S} - k^{TE}_{n_g}} \right\}_K \left\{ \frac{E^{(-)}_{q_g}(nP,\bar{n}S,n_g)}{\omega^f_{nP} + \omega^f_{\bar{n}S} + k^{TE}_{n_g}} \right\}_\pi$$

$$\times \left[S^{(14)}_{13}(k{:}\bar{n}'S,\bar{n}S,nP,1S) + 2S^{(14)}_{14}(k{:}\bar{n}'S,\bar{n}S,nP,1S) \right] \qquad (3.48h)$$

$$G^{(14)}_{15i)} = \frac{1}{6} N^K_F N^{\kappa}_F \frac{8\alpha}{3} \sum_{\substack{n,\bar{n}\\ \bar{n}',n_g}}^{N_B} \left\{ \frac{M^{(-)}_{q_g}(1S,\bar{n}'P,n_g)}{\omega^{s}_{1S} - \omega^{s}_{\bar{n}'P} - k^{TM}_{n_g}} \right\}_K \left\{ \frac{M^{(+)}_{q_g}(nS,\bar{n}S,n_g)}{\omega^{f}_{nS} + \omega^{f}_{\bar{n}S} + k^{TM}_{n_g}} \right\}_\kappa$$

$$\times \left[S^{(14)}_{13}(k:\bar{n}'P,\bar{n}S,nS,1S) + 2S^{(14)}_{14}(k:\bar{n}'P,\bar{n}S,nS,1S) \right] \quad (3.48i)$$

$$G^{(14)}_{15j)} = \frac{1}{6} N^K_F N^{\kappa}_F \frac{8\alpha}{3} \sum_{\substack{n,\bar{n}\\ \bar{n}',n_g}}^{N_B} \left\{ \frac{M^{(-)}_{q_g}(1S,\bar{n}'P,n_g)}{\omega^{s}_{1S} - \omega^{s}_{\bar{n}'P} - k^{TM}_{n_g}} \right\}_K \left\{ \frac{M^{(+)}_{q_g}(nP,\bar{n}P,n_g)}{\omega^{f}_{nP} + \omega^{f}_{\bar{n}P} + k^{TM}_{n_g}} \right\}_\kappa$$

$$\times \left[S^{(14)}_{13}(k:\bar{n}'P,\bar{n}P,nP,1S) + 2S^{(14)}_{14}(k:\bar{n}'P,\bar{n}P,nP,1S) \right] \quad (3.48j)$$

We have now completed the first stage of the analysis in so far as the matrix elements of the operators O_k for the full O(g) wavefunctions, as classified by the diagrams $G^{(i)}_m$, have been derived in terms of the spin summations listed in Table 1. The terms in the various spin summations must now be further reduced to one dimensional overlap integrals by performing the angular integrations. To do this it is a matter of substituting the spinors with the appropriate spin and parity assignments and integrating. Although the procedure is straightforward enough the large number of seemingly disparate spin summations with various combinations of parity assignments leads to a prohibitively large amount of work. However, the problem can be overcome, to some extent, by employing a symbolic algebra code.

To present the results of these manipulations we first introduce the radial functions, $f_{nL}(r)$ and $g_{nL}(r)$, which appear in the quark spinors as

$$u^{(\alpha)}_{nS}(\underset{\sim}{x}) = \frac{1}{\sqrt{4\pi}} \begin{pmatrix} i f_{nS}(r)\chi_\alpha \\ g_{nS}(r)\,\underset{\sim}{\sigma}\cdot\hat{\underset{\sim}{r}}\,\chi_\alpha \end{pmatrix}, \qquad u^{(\alpha)}_{nP}(\underset{\sim}{x}) = \frac{1}{\sqrt{4\pi}} \begin{pmatrix} -i g_{nP}(r)\,\underset{\sim}{\sigma}\cdot\hat{\underset{\sim}{r}}\,\chi_\alpha \\ f_{nP}(r)\chi_\alpha \end{pmatrix}, \quad (3.49)$$

(χ_α is a two component Pauli spinor) and are given by

$$f_{nL}(r) = R^{-\frac{3}{2}} N_{nL}\, j_0(p_{nL} r) ,$$

$$g_{nL}(r) = -\,R^{-\frac{3}{2}}\, N_{nL}\left[\frac{\omega_{nL} + \kappa m R}{\omega_{nL} - \kappa m R}\right]^{\frac{1}{2}} j_1(p_{nL} r)\ . \tag{3.50}$$

In addition, we define integrals over the eight possible combinations of four spinor functions, containing even numbers of each, as $h_j^k(\vec{nL})$, where $j=1,\ldots,8$ and the label k gives the flavour types corresponding to the convention used in the definition of $\mathcal{O}_k$. Explicitly, we have

$$\begin{aligned}
h_1^k(\vec{nL}) &= \frac{1}{\pi}\int_0^R r^2\,dr\ g_{n_1L_1}(r)\,g_{n_2L_2}(r)\,g_{n_3L_3}(r)\,g_{n_4L_4}(r) \\
h_2^k(\vec{nL}) &= \frac{1}{\pi}\int_0^R r^2\,dr\ g_{n_1L_1}(r)\,g_{n_2L_2}(r)\,f_{n_3L_3}(r)\,f_{n_4L_4}(r) \\
h_3^k(\vec{nL}) &= \frac{1}{\pi}\int_0^R r^2\,dr\ g_{n_1L_1}(r)\,f_{n_2L_2}(r)\,f_{n_3L_3}(r)\,g_{n_4L_4}(r) \\
h_4^k(\vec{nL}) &= \frac{1}{\pi}\int_0^R r^2\,dr\ g_{n_1L_1}(r)\,f_{n_2L_2}(r)\,g_{n_3L_3}(r)\,f_{n_4L_4}(r) \\
h_5^k(\vec{nL}) &= \frac{1}{\pi}\int_0^R r^2\,dr\ f_{n_1L_1}(r)\,g_{n_2L_2}(r)\,f_{n_3L_3}(r)\,g_{n_4L_4}(r) \\
h_6^k(\vec{nL}) &= \frac{1}{\pi}\int_0^R r^2\,dr\ f_{n_1L_1}(r)\,g_{n_2L_2}(r)\,g_{n_3L_3}(r)\,f_{n_4L_4}(r) \\
h_7^k(\vec{nL}) &= \frac{1}{\pi}\int_0^R r^2\,dr\ f_{n_1L_1}(r)\,f_{n_2L_2}(r)\,g_{n_3L_3}(r)\,g_{n_4L_4}(r) \\
h_8^k(\vec{nL}) &= \frac{1}{\pi}\int_0^R r^2\,dr\ f_{n_1L_1}(r)\,f_{n_2L_2}(r)\,f_{n_3L_3}(r)\,f_{n_4L_4}(r)
\end{aligned} \tag{3.51}$$

In terms of these basic integrals over spinor functions we define two sets of integrals $A_k^{(m)}$, $m=1,\ldots,5$, and $B_k^{(n)}$, $n=1,\ldots,6$, as

$$A_k^{(m)} = \sum_{j=1}^{8} \alpha_j^{(m)} h_j^k(\vec{nL}) \tag{3.52}$$

and

$$B_k^{(n)} = \sum_{j=1}^{8} \beta_j^{(n)} h_j^k(\vec{nL}) \tag{3.53}$$

where the coefficients $\alpha_j^{(m)}$ and $\beta_j^{(n)}$ are given in Table 2.

All the spin summations required can ultimately be written in terms of the integrals $A_k^{(m)}$ and $B_k^{(n)}$, after angular integration. In Table 3 the spin summations for the diagrams listed in this section are given in terms of the overlap integrals defined above.

The results of the numerical evaluation of the weak matrix elements will be presented in the next section.

4. Results and Discussion

Before presenting the results of the computation of the matrix elements of the preceding section, we will briefly summarize the quantities to be calculated. The analysis of $K \to \pi\pi$ decays is complicated considerably by the two body final state. It is usual to avoid this through current algebra and low energy theorems which relate the $K \to \pi\pi$ matrix elements to the more tractable single particle $K-\pi$ matrix elements. This is the case even for lattice simulations where the physical size of the lattice is not large enough to accommodate the two body final state. The low energy theorem giving the $K \to \pi\pi$ matrix elements of the weak Hamiltonian in terms of the $K-\pi$ matrix elements is[12]

$$\langle \pi^+\pi^0 | \mathcal{H}_W | K^+ \rangle = \frac{i}{f_\pi} h \left\{ \frac{1}{\sqrt{2}} \langle \pi^+(p) | \mathcal{H}_W | K^+(p) \rangle + \langle \pi^0(p) | \mathcal{H}_W | K^0(p) \rangle \right\}$$

$$\langle \pi^+\pi^- | \mathcal{H}_W | K^0 \rangle = \frac{i}{f_\pi} h \; \langle \pi^+(p) | \mathcal{H}_W | K^+(p) \rangle$$

$$\langle \pi^0\pi^0 | \mathcal{H}_W | K^0 \rangle = -\frac{i\sqrt{2}}{f_\pi} h \; \langle \pi^0(p) | \mathcal{H}_W | K^0(p) \rangle \tag{4.1}$$

where p is the common four-momentum in the $K-\pi$ transition and

$$h = \frac{m_K^2 - m_\pi^2}{p^2}. \tag{4.2}$$

If the $K-\pi$ matrix elements are written as

$$\langle \pi^+(p)|\mathcal{H}_W|K^+(p)\rangle = \mathcal{A}^+ p^2 ,$$

$$\langle \pi^o(p)|\mathcal{H}_W|K^o(p)\rangle = \mathcal{A}^o p^2 , \tag{4.3}$$

then the experimental data for $K \to \pi\pi$ decays are reproduced from the low energy theorem, (1.18), for the following "experimental" values of the single particle matrix elements(12):

$$|\mathcal{A}^+|_{Expt.} = 1.555 \times 10^{-7} \text{ and } |\mathcal{A}^o|_{Expt.} = 1.052 \times 10^{-7}. \tag{4.4}$$

The question of the common four momentum, p, to be used when comparing the calculated $K-\pi$ matrix elements to the "experimental" values above is unclear in this framework and so we will be more concerned with the ratio of the $K-\pi$ matrix elements.

As we have ignored mixing from the top quark in our effective $\Delta S = 1$ operator, (3.1), the bare effective $\Delta S = 1$ Hamiltonian is

$$\mathcal{H}_{eff}^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} \lambda_c O_{\Delta S=1} . \tag{4.5}$$

For the numerical calculations we will be interested only with the real part of the interaction in order to compare with the extrapolated experimental values of the $K-\pi$ matrix elements, (4.4). Furthermore, we can assume that the KM factor, λ_c, is determined by the first two generations, i.e. $\lambda_c \approx s_1 c_1$. Writing the $K-\pi$ matrix elements in the notation (4.3) and employing the wavepacket prescription we have ($\beta = o$ or $+$)

$$\mathcal{A}^{\beta} p^2 = -\frac{G_F}{\sqrt{2}} \lambda_c \frac{\int d^3x \, {}_F\langle \pi^{\beta} | O_{\Delta S=1}(\underset{\sim}{x}, 0) | K^{\beta} \rangle_F}{(2\pi)^3 \int d^3p \, \frac{\phi_K(p)}{2E_K(p)} \frac{\phi_\pi(p)}{2E_\pi(p)}} . \tag{4.6}$$

A similar and not unrelated problem to the common four momentum in the $K-\pi$ matrix elements is the question of confinement radius to be used in the weak interaction overlap integrals. Fortunately, the difference in pion and kaon confinement radii in the model is very small. For example, in the ultra-relativistic sector at $N_B = 1$ the difference is 0.4% of R_π, and decreases to 0.2% at $N_B = 10$ (in the original MIT model[13] the difference is quite pronounced, about 2.4%). Hence we will assume a common confinement radius, even for the wavefunction amplitudes, and compute the matrix elements at three appropriate values, namely R_π, R_K and $\frac{1}{2}(R_\pi + R_K)$. In this manner we are able to estimate the sensitivity of the weak matrix elements to the confinement radius. It is interesting to note in passing that this approximate "degeneracy" in confinement radii for π and K states reflects the fact that the various observables, such as mass and charge radii, appear to be strongly dependent on the underlying dynamics.

In Figure 16 we plot the values of $\mathcal{A}^{\circ} p^2$ (Figure 16a)) and $\mathcal{A}^{+} p^2$ (Figure 16b)) as a function of basis size in the ultra-relativistic sector using the parameters of Ref 7. Finally, the ratio of the $K-\pi$ matrix elements, which is free from the p^2 ambiguity, is plotted in Figure 17.

All values shown are for the "mean" confinement radius, $\frac{1}{2}(R_\pi + R_K)$, which is used as the reference so that the "error" due to the difference of R_π and R_K is estimated from the values obtained at the extreme confinement radii, R_π and R_K. The resulting changes in the $K-\pi$ matrix elements are shown as error bars.

The general impressions of the results given in Figures 16 and 17 are that although the magnitudes of the $K-\pi$ matrix elements are reasonable, the signs are incorrect. In so far as the $K-\pi$ matrix elements are concerned, the "experimental" values, (4.4), only give the magnitudes and not the signs. However, for the single particle matrix elements to reproduce the exact $\Delta I = \frac{1}{2}$ rule they must satisfy the condition

$$\left.\frac{\mathcal{A}^{0}}{\mathcal{A}^{+}}\right|_{\text{Exact } \Delta I=1/2} = -\frac{1}{\sqrt{2}} . \tag{4.7}$$

Figure 17 demonstrates the fact the $K-\pi$ matrix elements calculated here have the wrong sign and poor relative magnitude in comparison with the "experimental" values.

Thus the full wavefunction, with $m_{u,d}=0$, appears to do badly in describing the weak interaction dynamics. However, we have still to exploit the one remaining degree of freedom, namely the up, down quark mass. We have refitted the parameters for various values of $m_{u,d}$ up to 200 MeV. Since the non-zero quark mass calculations require a great deal of computer time we have investigated the quark mass dependence for $N_B=1$ to 6 only (this required a cumulative total of O(100) hours on a CYBER 990).

The results for the $K-\pi$ matrix elements and their ratio are shown in detail for $N_B=1$ in Figure 18. The K^0-π^0 matrix element changes sign at about $m_{u,d}=113$ MeV whist the K^+-π^+ matrix element changes sign at about $m_{u,d}=146$ MeV (note that the sensitivity of the $K-\pi$ ratio to the confinement radius is more pronounced in the region of the asymptote). Hence, there exists a (unique) region from 113 to 146 Mev where the $K-\pi$ ratio has the correct sign. In fact at $m_{u,d}=138$ MeV the ratio reproduces the "experimental" $\Delta I=\frac{1}{2}$ rule. Since the valence $K-\pi$ ratio is always positive (the valence weak overlap integrals are the same) this observation of the $\Delta I=\frac{1}{2}$ rule is due to the presence of the QCD corrections. The common four momentum required to give the correct magnitudes, (4.4), for the $K-\pi$ matrix elements for $N_B=1$ is, $|p|=96$ MeV.

The corresponding change in the meson mass spectrum, for these larger values of $m_{u,d}$, will be small as indicated by the behaviour of K^* and ϕ masses which were found to change by only about 1 MeV over the entire range of $m_{u,d}$.

Due to limitations on computer time we have performed the calculations for $N_B=1$, 2, 3 and 6 only. At each basis size we find unique solutions for the value of the light quark mass required to reproduce the experimental $\Delta I=\frac{1}{2}$ rule. The computation for $N_B=6$ is given in Figure 19 which shows that the matrix elements have changed sign with respect to the small basis size values ($N_B<4$). This change of sign is probably not significant as it reflects a typical instability, at small basis sizes, of quantities computed from the wavefunction.

The aim of this work was to study QCD corrections to the bare effective weak Hamiltonian in a framework where the non-perturbative calculation of the hadronic matrix elements is made. Specifically, we have computed the resulting QCD corrections to weak interactions of mesons from O(g) wavefunctions in the static cavity fitted to the light meson sector.

Whilst we have demonstrated that the non-perturbative calculation of QCD corrections to the bare effective weak Hamiltonian is possible within the context of the static cavity model, the limitations of this approach to the scale matching problem, due to the nature of the static cavity model itself must be kept in mind. Nevertheless, we believe that the major problem of the static cavity model, namely the CM corrections have been treated here in a reasonable manner, although we acknowledge that, at present, a complete and rigorous treatment of the CM corrections can only be performed in the soliton formalism[14,15].

It appears to be significant that the O(g) wavefunctions at $m_{u,d}=0$ produce quite good results for the meson mass spectrum with respect to the potential models[16] and certainly in comparison to the valence MIT model[13], whereas the weak interaction dynamics does not seem to be well described by this wavefunction. The fact that the weak matrix elements do not come out satisfactorily may be an indicator that various higher order contributions, left out in this analysis, will be important.

Yet, if we are prepared to discard the notion that the light quark dynamics must be ultra-relativistic, i.e. that the up, down quark mass is small (in comparison with the static cavity energy scale 1/R), then we find that for $m_{u,d}=138$ to 146 MeV (at $N_B=1$ to 6 so far) the full O(g) wavefunction gives an excellent account of the K-π system as well as charge radii and the ground state meson spectrum. The significance of such a large up, down quark mass in this relativistic framework is unclear. A possible conjecture is that after confinement has been taken into account by the static cavity mechanism, the bare quark masses may be dressed non-perturbatively such that the quark masses appearing in the resulting QCD are effective masses. Alternatively, the large value of the up, down mass may also reflect the absence of the higher order contributions.

The most obvious of these contributions are those where the QCD process occurs before or after the weak interaction. Such corrections to the matrix elements of the bare effective weak Hamiltonian will arise from $O(g^2)$ states in the wavefunction. For example, the excited valence configuration $|q\bar{q}\rangle$, produced from transverse and Coulomb gluon exchange, and the sea-state, $|q\bar{q}q\bar{q}\rangle$, will play an important role here. Furthermore, the wavefunction should ultimately be extended to include higher angular momentum states, i.e $j>\frac{1}{2}$, for the quarks (and hence $l>1$ for the gluons). Since the meson mass spectrum is already well described by the model considered here it may be argued that, in so far as the mass calculations are concerned, the higher j modes and $O(g^2)$ states may change the overall scale whilst leaving the quark mass dependence unaltered in such a way as to allow the model parameters to readjust.

The numerical calculations presented in this paper are quite lengthy, but perhaps not prohibitively so with respect to the above modifications of higher j modes and $O(g^2)$ states. However, whilst the inclusion of higher j modes will most likely only increase the amount of computational work, the inclusion of all $O(g^2)$ states in the wavefunction will lead to a vast, and perhaps unmanageable number of diagrams. An alternative way to proceed may be to perform the calculations using the Multiple Reflection Expansion techniques[17] in a covariant gauge where the number of diagrams will be manageable. The fact that this has not yet been attempted may reflect the numerical difficulties in using these methods for fitting purposes in order to obtain reliable parameters, and the problems associated with CM corrections.

Within the framework of the O(g) wavefunction for mesons we have been able to demonstrate two important aspects of the weak interaction of hadrons. Firstly, using the wavefunction construction and fitting procedures outlined in this thesis, the calculation of all $O(g^2)$ QCD corrections to the matrix elements of the bare effective weak Hamiltonian, including higher order contributions, is in principle possible for the static cavity. Secondly, the $O(g^2)$ QCD corrections to the matrix elements of the bare effective weak Hamiltonian computed here are certainly significant with respect to the valence contributions of the naive quark model and appear to be crucial to the understanding of the $\Delta I=\frac{1}{2}$ rule.

References

1. M. K. Gaillard and B. W. Lee, Phys. Rev. Lett 33 (1974) 108.
2. A. I. Vainshtein, V. Zakharov and M. A. Shifman, JETP Lett. 22 (1975) 55.
3. F. J. Gilman and M. B. Wise, Phys. Rev. D20 (1979) 2392.
4. R. D. C. Miller and B. H. J. McKellar, Phys. Rep 106 (1984) 169.
5. B. Guberina and R. D. Peccei, Nucl. Phys B163 (1980) 289.
6. C. Bernard, T. Draper, A. Soni, H. Politzer and M. Wise, Phys. Rev. D32 (1985) 2343.
7. L. C. L. Hollenberg and B. H. J. McKellar, University of Melbourne Preprint UM-P-89/50.
8. C. W. Wong, Phys. Rev. D24 (1981), 1416.
9. L. C. L. Hollenberg and B. H. J. McKellar, to appear in Phys. Rev. D.
10. J. F. Donoghue and K. Johnson, Phys. Rev. D21 (1980) 1975.
11. L. C. L. Hollenberg and B. H. J. McKellar, Int. J. Mod. Phys A 4 (1989) 1949.
12. H. Y. Cheng, Preprint IUHET-132 (1987).
13. T. DeGrand, R.L. Jaffe, K. Johnson and J. Kiskis, Phys. Rev. D12 (1975) 2060.
14. E. G. Lubeck, M. C. Birse, E. M. Henley and L. Wilets, Phys. Rev. D33 (1986) 234.
15. E. G. Lubeck, E. M. Henley and L. Wilets, Phys. Rev. D35 (1986) 2809.
16. S. Godfrey and N. Isgur, Phys. Rev. D32 (1985) 189.
17. T. H. Hansson and R. L. Jaffe, Phys. Rev D29 (1983) 882.

Table 1 Coefficients, $\varepsilon_j(\vec{s})$, for the spin summations, $S_j^{(i)}(k:n\vec{L})$.

j	↑↑↑↑	↑↑↑↓	↑↑↓↑	↑↑↓↓	↑↓↑↑	↑↓↑↓	↑↓↓↑	↑↓↓↓	↓↑↑↑	↓↑↑↓	↓↑↓↑	↓↑↓↓	↓↓↑↑	↓↓↑↓	↓↓↓↑	↓↓↓↓
1	0	0	0	1	0	-1	0	0	0	0	-1	0	1	0	0	0
2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
3	0	0	0	1	0	1	0	0	0	0	1	0	1	0	0	0
4	0	0	-1	0	1	0	0	0	0	0	0	1	0	-1	0	0
5	0	-1	0	0	0	0	0	1	1	0	0	0	0	-1	0	0
6	1	0	0	0	0	0	-1	0	0	-1	0	0	0	0	0	1
7	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0
8	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0
9	0	0	0	0	0	0	0	-1	-1	0	0	1	0	1	0	0
10	1	0	0	0	0	0	1	0	0	1	0	0	0	0	0	1
11	0	0	-1	0	1	0	0	0	0	0	0	-1	0	1	0	0
12	0	0	1	0	1	0	0	0	0	0	0	1	0	1	0	0
13	0	0	0	0	0	0	0	0	1	0	0	0	0	0	-1	0
14	0	0	-1	0	-1	0	0	0	0	0	0	1	0	1	0	0
15	0	0	0	0	0	-1	1	0	0	1	-1	0	0	0	0	0
16	1	0	0	0	0	0	-1	0	0	-1	0	0	0	0	0	1
17	0	-1	1	0	0	0	0	0	0	0	0	0	0	1	-1	0
18	0	0	0	0	1	0	0	1	-1	0	0	-1	0	0	0	0
19	0	1	-1	0	0	0	0	0	0	0	0	0	0	1	-1	0
20	0	1	0	0	0	0	0	1	-1	0	0	0	0	0	-1	0

Table 5.2 Integral coefficients $\alpha_j^{(m)}$ and $\beta_j^{(n)}$.

	$\alpha_j^{(m)}$					$\beta_j^{(n)}$					
j	1	2	3	4	5	1	2	3	4	5	6
1	1	1	1	1	1	1	1	1	1	1	1
2	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	1	-1	-1	1	-1	1
3	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	1	-1	1	-1	-1	1
4	-1	1	-1	1	-1	-1	-1	1	1	-1	-1
5	-1	1	1	-1	-1	1	1	-1	-1	-1	-1
6	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	-1	1	-1	1	-1	1
7	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	-1	1	1	-1	-1	1
8	1	1	-1	-1	1	-1	-1	-1	-1	1	1

Table 5.3 (i)-(iii) The overlap integrals resulting from angular integration of the spin summations relevant to the weak matrix elements. For ease of reference mode numbers have been suppressed so that only the orbital assignments, shown for the spin summations, have been given. e.g. the entry, $S_6^{(1)}$: SSPP, denotes the spin summation, $S_6^{(1)}(k{:}n_1S,n_2S,n_3P,n_4P)$.

Table 5.3 (i)

Spin Summation	Integral	Spin Summation	Integral
$S_6^{(1)}$: SSSS	$A_k^{(2)}$	$S_3^{(4)}$: SSSS	$A_k^{(5)}$
$S_6^{(1)}$: SSPP	$A_k^{(2)}$	$S_3^{(4)}$: PSPS	$-A_k^{(5)}$
$S_6^{(1)}$: PPSS	$A_k^{(2)}$	$S_3^{(4)}$: PPSS	$A_k^{(5)}$
$S_7^{(1)}$: SSSS	$A_k^{(2)}$	$S_3^{(4)}$: SPPS	$A_k^{(5)}$
$S_7^{(1)}$: SSPP	$A_k^{(2)}$	$S_{18}^{(5)}$: SSPS	$B_k^{(1)}$
$S_7^{(1)}$: PPSS	$A_k^{(2)}$	$S_{18}^{(5)}$: SSSP	$-B_k^{(1)}$
$S_{20}^{(2)}$: SSPS	$-B_k^{(1)}$	$S_{11}^{(5)}$: PSSS	$-B_k^{(2)}$
$S_{20}^{(2)}$: SPSS	$B_k^{(1)}$	$S_{11}^{(5)}$: SSSP	$B_k^{(2)}$
$S_{19}^{(2)}$: PSSS	$B_k^{(2)}$	$S_{15}^{(7)}$: SSSS	$B_k^{(5)}$
$S_{19}^{(2)}$: SPSS	$-B_k^{(2)}$	$S_8^{(7)}$: SSSS	$-A_k^{(1)}$
$S_{19}^{(3)}$: SPSS	$-B_k^{(3)}$	$S_8^{(7)}$: SPSP	$A_k^{(1)}$
$S_{19}^{(3)}$: PSSS	$B_k^{(3)}$	$S_8^{(7)}$: PSPS	$A_k^{(1)}$
$S_{11}^{(3)}$: SSSP	$B_k^{(4)}$	$S_8^{(7)}$: SSPP	$-A_k^{(1)}$
$S_{11}^{(3)}$: PSSS	$-B_k^{(4)}$	$S_6^{(7)}$: SSSS	$-A_k^{(1)}$
$S_2^{(4)}$: SSSS	$A_k^{(5)}$	$S_6^{(7)}$: SPSP	$A_k^{(1)}$
$S_2^{(4)}$: PSPS	$-A_k^{(5)}$	$S_6^{(7)}$: PSPS	$A_k^{(1)}$
$S_2^{(4)}$: PPSS	$A_k^{(5)}$	$S_6^{(7)}$: SSPP	$-A_k^{(1)}$
$S_2^{(4)}$: SPPS	$A_k^{(5)}$	$S_8^{(8)}$: SPSS	$B_k^{(3)}$

Table 5.3 (ii)

Spin Summation	Integral	Spin Summation	Integral
$S_{20}^{(8)}$: SSPS	$-B_k^{(3)}$	$S_{18}^{(12)}$: SSPS	$B_k^{(1)}$
$S_{18}^{(8)}$: SSSP	$-B_k^{(4)}$	$S_{18}^{(12)}$: SSSP	$-B_k^{(1)}$
$S_{18}^{(8)}$: SSPS	$B_k^{(4)}$	$S_{11}^{(12)}$: SSSP	$B_k^{(2)}$
$S_{20}^{(9)}$: SPSS	$B_k^{(3)}$	$S_{11}^{(12)}$: PSSS	$-B_k^{(2)}$
$S_{20}^{(9)}$: SSPS	$-B_k^{(3)}$	$S_{12}^{(12)}$: SSPS	$-A_k^{(3)}$
$S_{18}^{(9)}$: SSSP	$-B_k^{(4)}$	$S_{12}^{(12)}$: SPSS	$A_k^{(3)}$
$S_{18}^{(9)}$: SSPS	$B_k^{(4)}$	$S_{12}^{(12)}$: SSSP	$A_k^{(3)}$
$S_{15}^{(10)}$: SSSS	$B_k^{(5)}$	$S_{12}^{(12)}$: SPPP	$A_k^{(3)}$
$S_{8}^{(10)}$: SSSS	$-A_k^{(1)}$	$S_{13}^{(12)}$: SSPS	$-A_k^{(3)}$
$S_{8}^{(10)}$: SPSP	$A_k^{(1)}$	$S_{13}^{(12)}$: SPSS	$A_k^{(3)}$
$S_{6}^{(10)}$: SSSS	$-A_k^{(1)}$	$S_{13}^{(12)}$: SSSP	$A_k^{(3)}$
$S_{6}^{(10)}$: SPSP	$A_k^{(1)}$	$S_{13}^{(12)}$: SPPP	$A_k^{(3)}$
$S_{16}^{(10)}$: SSSS	0	$S_{1}^{(13)}$: SSSS	$-B_k^{(5)}$
$S_{17}^{(10)}$: SSSS	0	$S_{2}^{(13)}$: SSSS	$A_k^{(5)}$
$S_{9}^{(10)}$: SSSS	$-B_k^{(6)}$	$S_{2}^{(13)}$: SPSP	$-A_k^{(5)}$
$S_{9}^{(10)}$: SPPS	$-B_k^{(6)}$	$S_{2}^{(13)}$: SSPP	$A_k^{(5)}$
$S_{10}^{(10)}$: SSSS	0	$S_{2}^{(13)}$: SPPS	$A_k^{(5)}$
$S_{10}^{(10)}$: SPPS	0	$S_{2}^{(13)}$: PSPS	$-A_k^{(5)}$

Table 5.3 (iii)

Spin Summation	Integral	Spin Summation	Integral
$S_2^{(13)}$: PPSS	$A_k^{(5)}$	$S_{13}^{(14)}$: PSSS	$A_k^{(4)}$
$S_3^{(13)}$: SSSS	$A_k^{(5)}$	$S_{13}^{(14)}$: PPPS	$A_k^{(4)}$
$S_3^{(13)}$: PSPS	$-A_k^{(5)}$	$S_{14}^{(14)}$: SPSS	$-A_k^{(4)}$
$S_3^{(13)}$: PPSS	$A_k^{(5)}$	$S_{14}^{(14)}$: SSPS	$A_k^{(4)}$
$S_3^{(13)}$: SPSP	$-A_k^{(5)}$	$S_{14}^{(14)}$: PSSS	$A_k^{(4)}$
$S_3^{(13)}$: SSPP	$A_k^{(5)}$	$S_{14}^{(14)}$: PPPS	$A_k^{(4)}$
$S_3^{(13)}$: SPPS	$A_k^{(5)}$	$S_{20}^{(15)}$: SPSS	$B_k^{(1)}$
$S_5^{(13)}$: SSSS	0	$S_{20}^{(15)}$: SSPS	$-B_k^{(1)}$
$S_4^{(13)}$: SSSS	0	$S_{19}^{(15)}$: PSSS	$B_k^{(2)}$
$S_{19}^{(14)}$: SPSS	$-B_k^{(3)}$	$S_9^{(15)}$: SPSS	$-B_k^{(2)}$
$S_{19}^{(14)}$: PSSS	$B_k^{(3)}$	$S_6^{(16)}$: SSSS	$A_k^{(2)}$
$S_{11}^{(14)}$: SSSP	$B_k^{(4)}$	$S_6^{(16)}$: SSPP	$A_k^{(2)}$
$S_{11}^{(14)}$: PSSS	$-B_k^{(4)}$	$S_7^{(16)}$: SSSS	$A_k^{(2)}$
$S_{13}^{(14)}$: SPSS	$-A_k^{(4)}$	$S_7^{(16)}$: SSPP	$A_k^{(2)}$
$S_{13}^{(14)}$: SSPS	$A_k^{(4)}$		

Figure Captions

Fig. 1 Valence diagram for the $K^o - \pi^o$ transition.

Fig. 2 Diagonal TE-bremsstrahlung diagrams for $K^o - \pi^o$.

Fig. 3 Diagonal TM-bremsstrahlung diagrams for $K^o - \pi^o$.

Fig. 4 Diagonal VF diagrams for the $u\bar{u}$ component contributions from the operator $C_1^{(13)}$ to the $K^o - \pi^o$ matrix element.

Fig. 5 Diagonal VF diagrams for the $u\bar{u}$ component contributions from the operators $C_1^{(1)}, C_1^{(7)}, C_1^{(10)}$ and $C_1^{(16)}$ to the $K^o - \pi^o$ matrix element.

Fig. 6 Diagonal VF diagrams for the $d\bar{d}$ component contributions to the $K^o - \pi^o$ matrix element. Diagrams a) - d) , e) - h) and i) - l) correspond to the operators $C_k^{(4)}, C_k^{(10)}$ and $C_k^{(13)}$ respectively.

Fig. 7 Off diagonal VF-bremsstrahlung diagrams for the $u\bar{u}$ component contributions from the operators $C_1^{(5)}$ and $C_1^{(14)}$ to the $K^o - \pi^o$ matrix element.

Fig. 8 Off diagonal VF-bremsstrahlung diagrams for the $d\bar{d}$ component contributions from the operator $C_k^{(12)}$ to the $K^o - \pi^o$ matrix element.

Fig. 9 Off diagonal VF-bremsstrahlung diagrams for the $d\bar{d}$ component contributions from the operator $C_k^{(14)}$ to the $K^o - \pi^o$ matrix element.

Fig. 10 Valence diagram for the $K^+ - \pi^+$ transition.

Fig. 11 Diagonal VF diagrams from the the operator $C_k^{(10)}$ for the $K^+ - \pi^+$ matrix element.

Fig. 12 Diagonal VF diagrams from the operator $C_k^{(4)}$ for the $K^+ - \pi^+$matrix element.

Fig. 13 Diagonal VF diagrams from the operator $C_k^{(13)}$ for the $K^+ - \pi^+$matrix element.

Fig. 14 Off diagonal VF-bremsstrahlung diagrams from the operator $C_k^{(12)}$ for the $K^+ - \pi^+$ matrix element.

Fig. 15 Off diagonal VF-bremsstrahlung diagrams from the operator $C_k^{(14)}$ for the $K^+ - \pi^+$ matrix element.

Fig. 16 Results for $K - \pi$ matrix elements ($m_{u,d}=0$) for $N_B = 1$ to 10, showing the various contributions. a) $K^0 - \pi^0$ matrix element, $\mathcal{A}^0 p^2$. b) $K^+ - \pi^+$ matrix element, $\mathcal{A}^+ p^2$.

Fig. 17 $K - \pi$ ratio, $\mathcal{A}^0/\mathcal{A}^+$ ($m_{u,d}=0$) for $N_B = 1$ to 10. The dashed line indicates the value of the $K - \pi$ ratio for the exact $\Delta I = \frac{1}{2}$ rule.

Fig. 18 Light quark mass dependence of the $N_B = 1$ results. a) $\mathcal{A}^0 p^2$ and $\mathcal{A}^+ p^2$. b) $K - \pi$ ratio.

Fig. 19 Light quark mass dependence of the $N_B = 6$ results. a) $\mathcal{A}^0 p^2$ and $\mathcal{A}^+ p^2$. b) $K - \pi$ ratio.

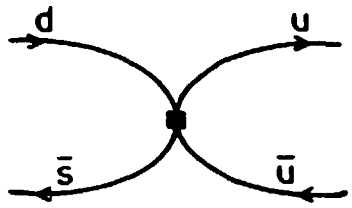

Figure 1

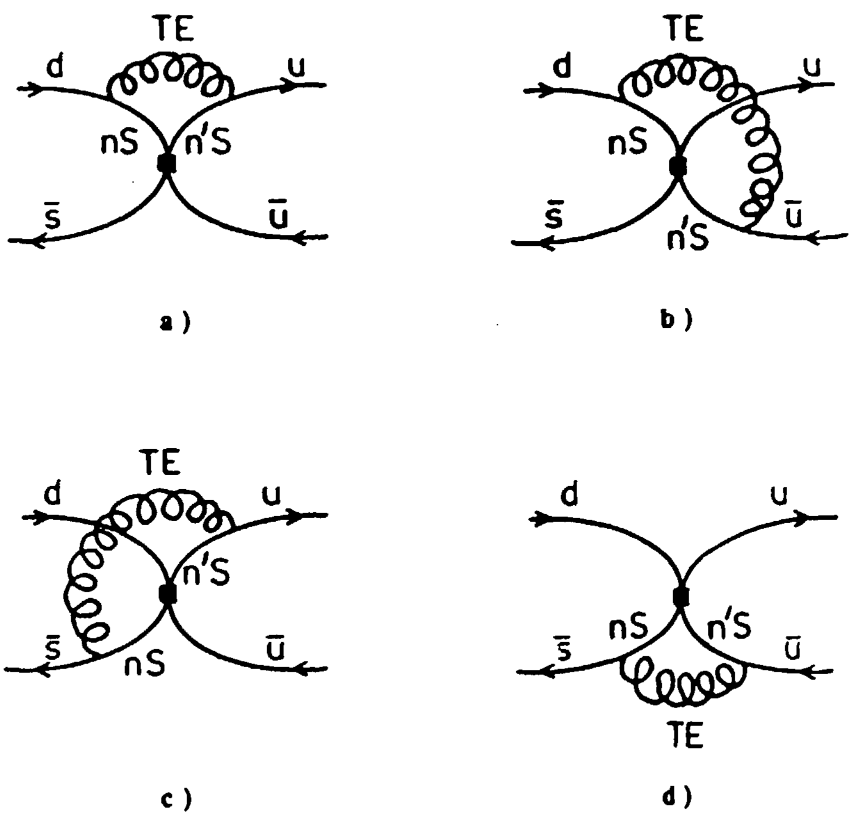

Figure 2

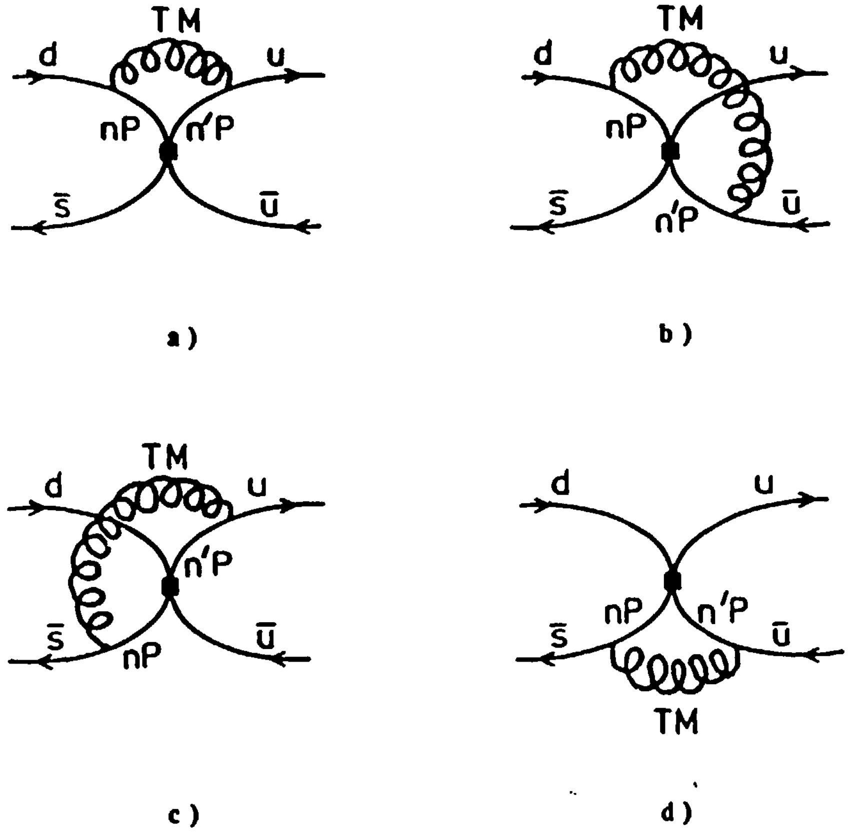

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Figure 3

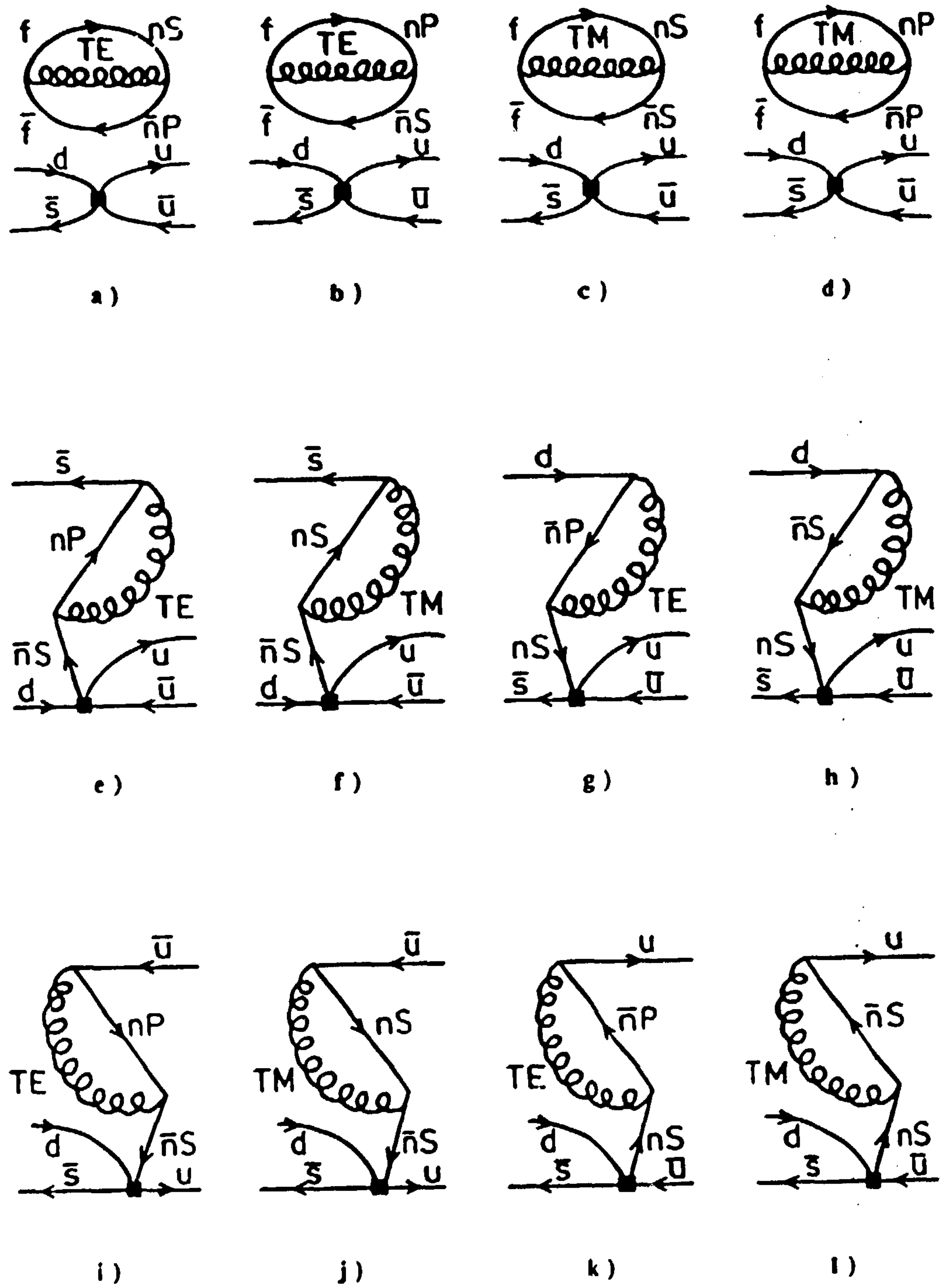
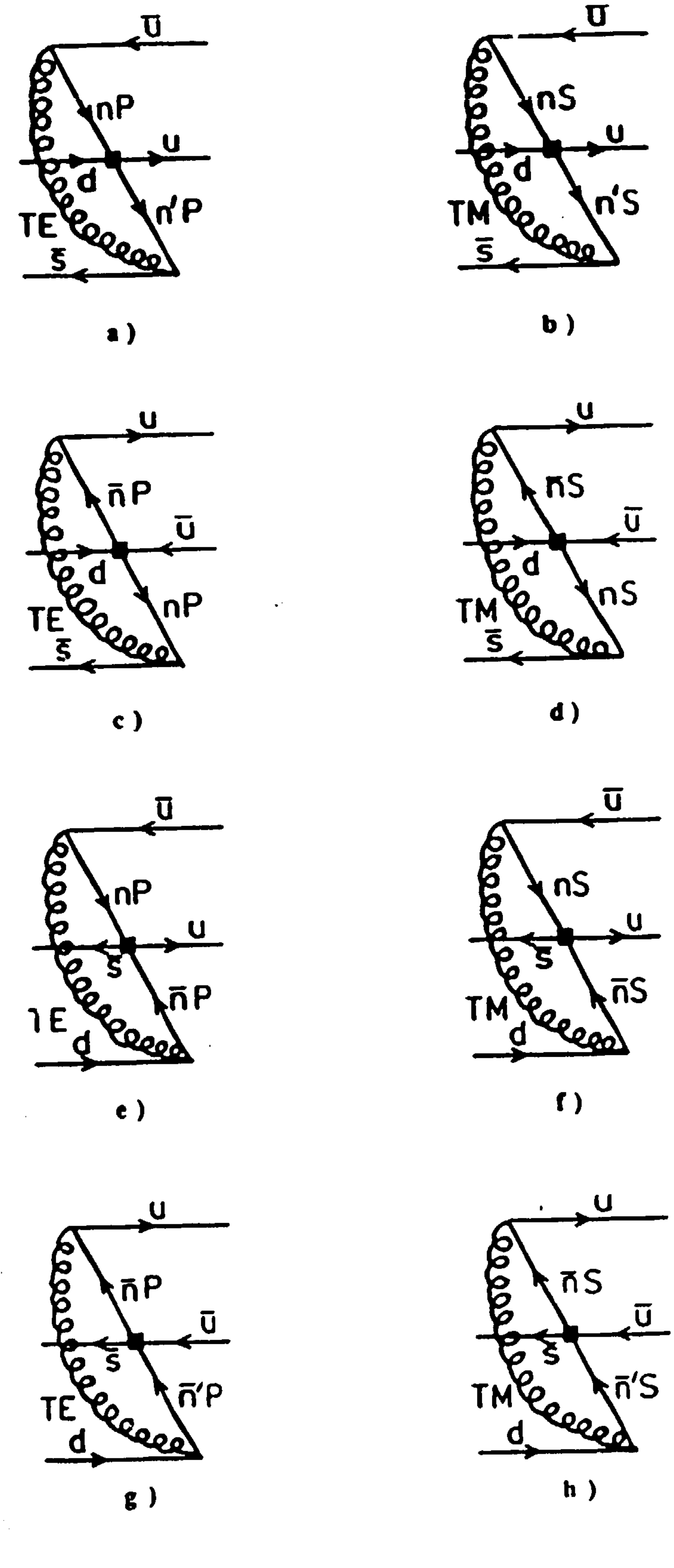


Figure 4



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u
s̄
n̄P
TE
d
e)
ū
nS
u
s̄
n̄S
TM
d
f)
u
n̄P
ū
s̄
n̄'P
TE
d
g)
u
n̄S
ū
s̄
n̄'S
TM
d
h)

Figure 5

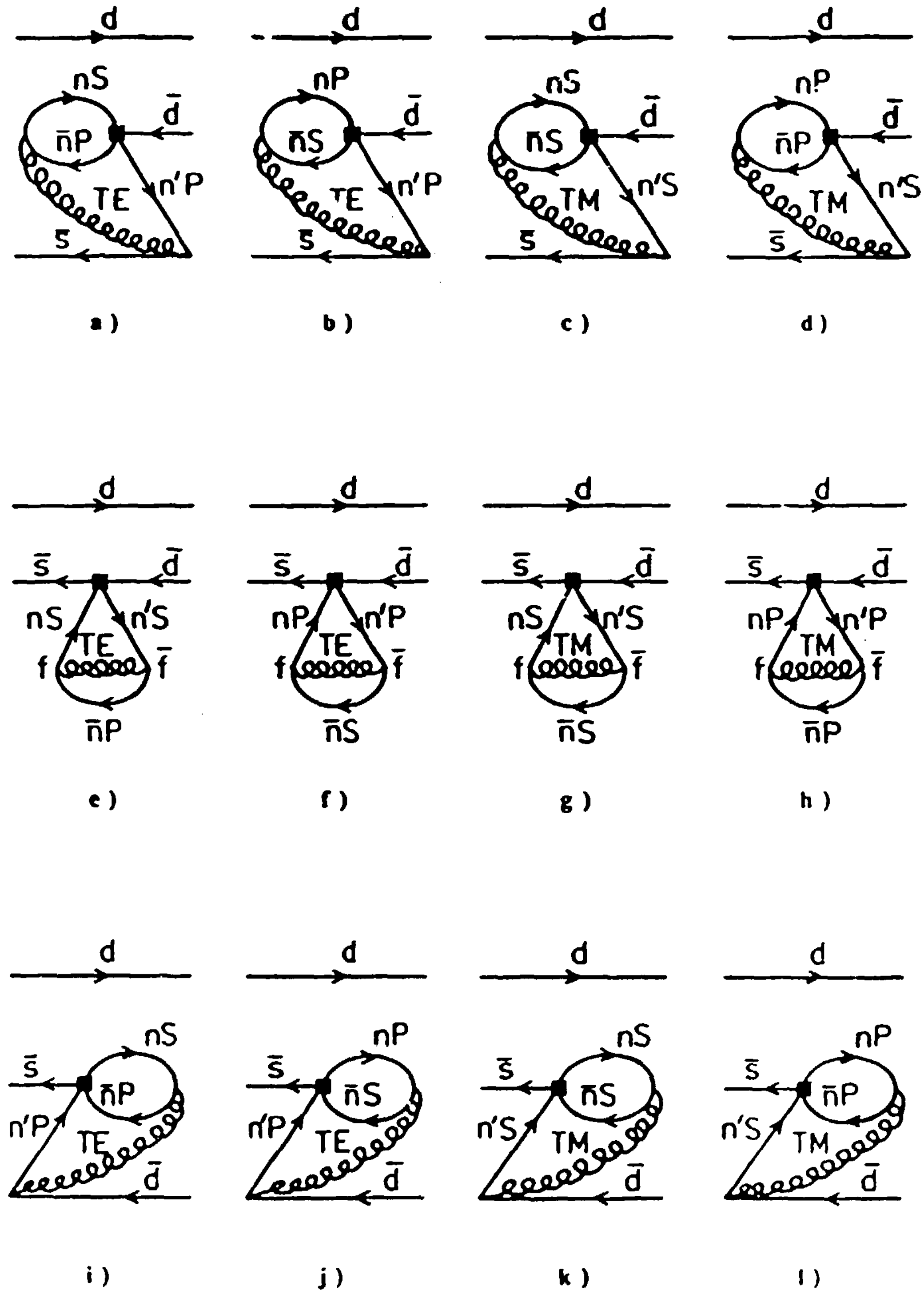


Figure 6

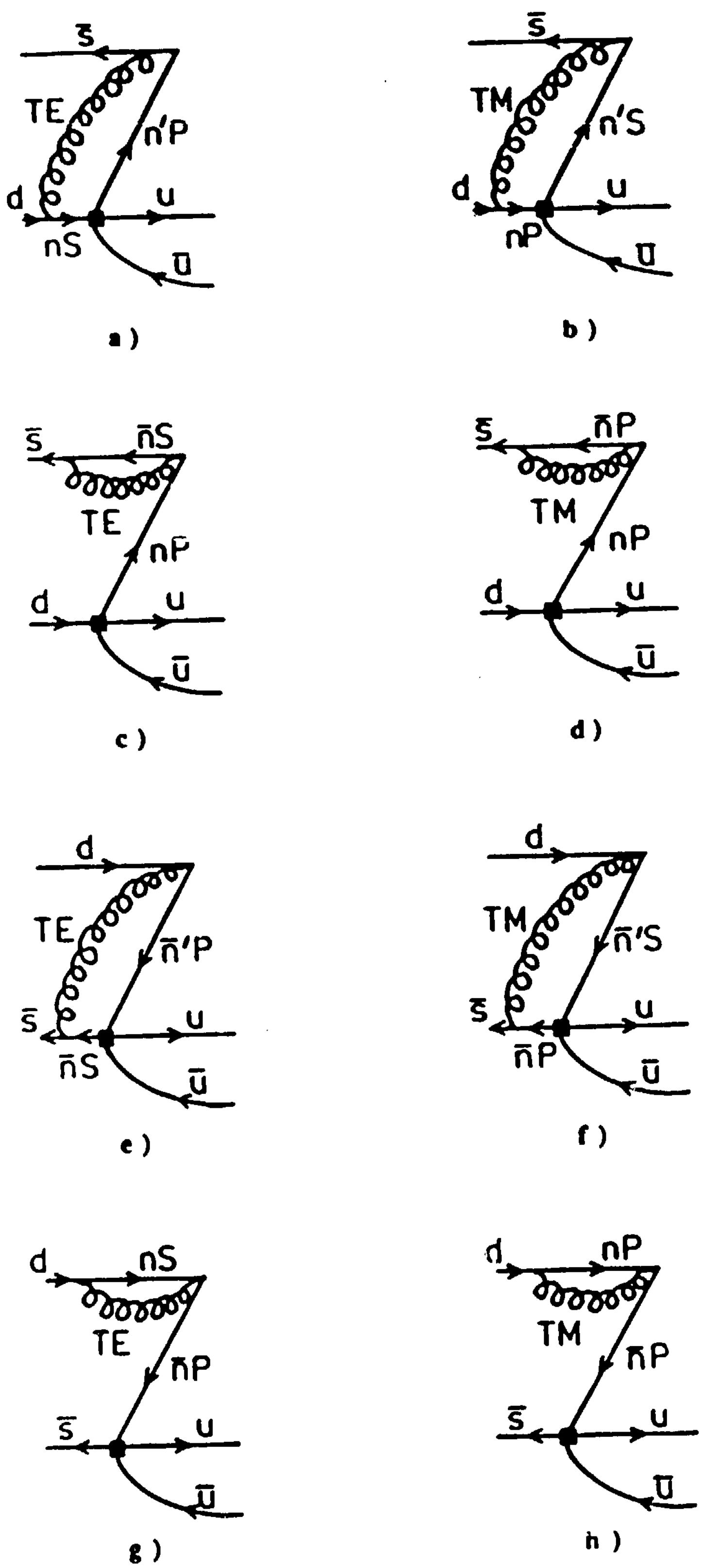

s̄
TE
n'P
d
u
nS
ū
a)
s̄
TM
n'S
d
u
nP
ū
b)
s̄
n̄S
TE
nP
d
u
ū
c)
s̄
n̄P
TM
nP
d
u
ū
d)
d
TE
n̄'P
s̄
u
n̄S
ū
e)
d
TM
n̄'S
s̄
u
n̄P
ū
f)
d
nS
TE
n̄P
s̄
u
ū
g)
d
nP
TM
n̄P
s̄
u
ū
h)

Figure 7

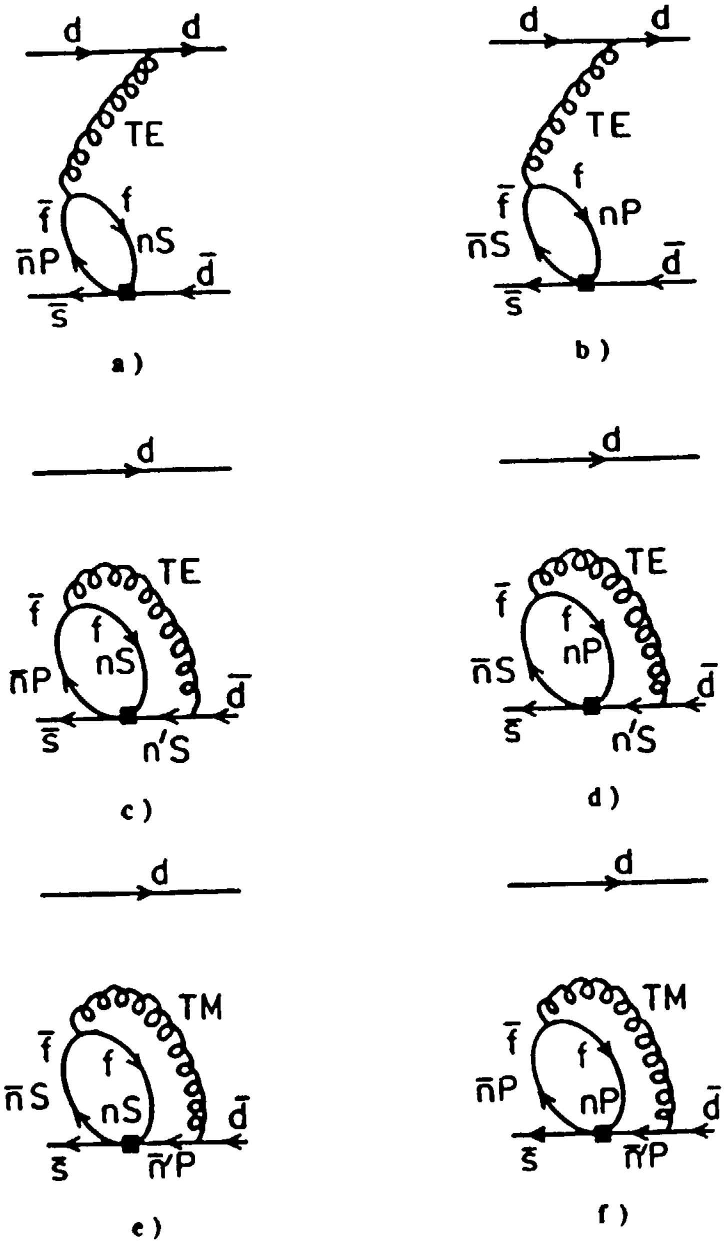


Figure 8

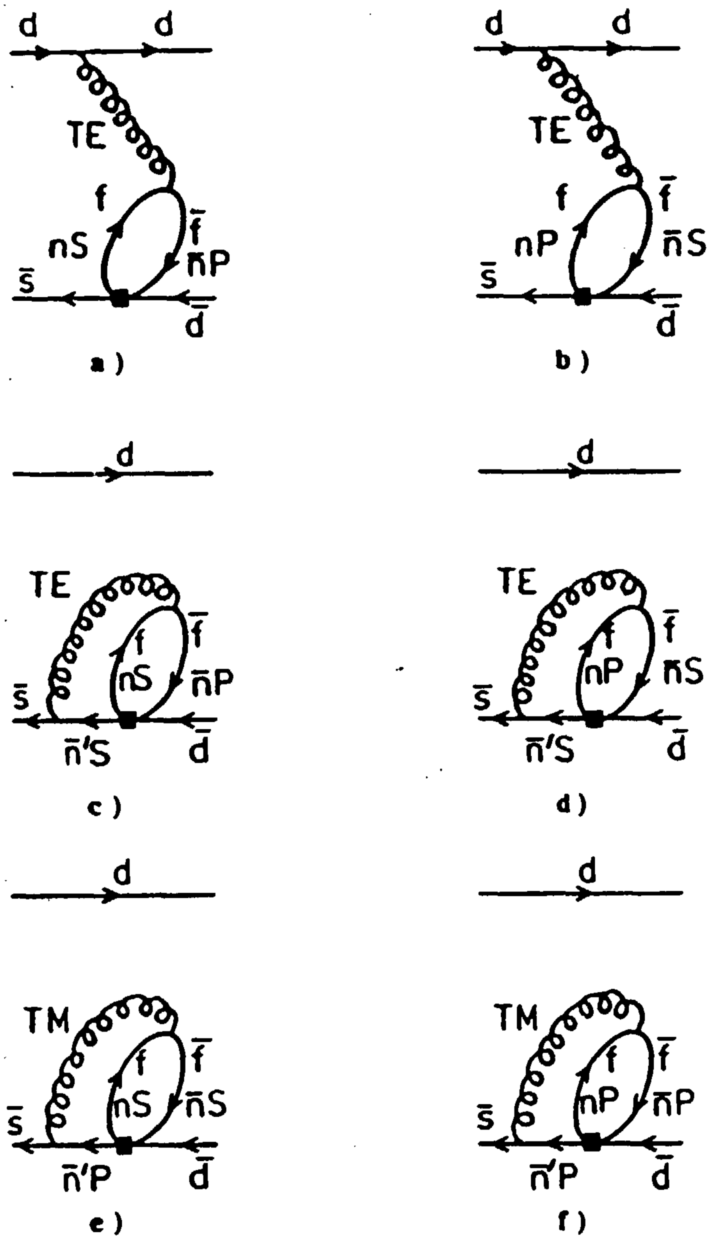


Figure 9

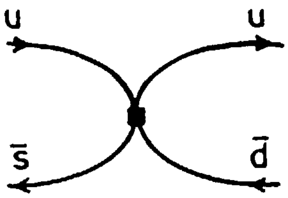
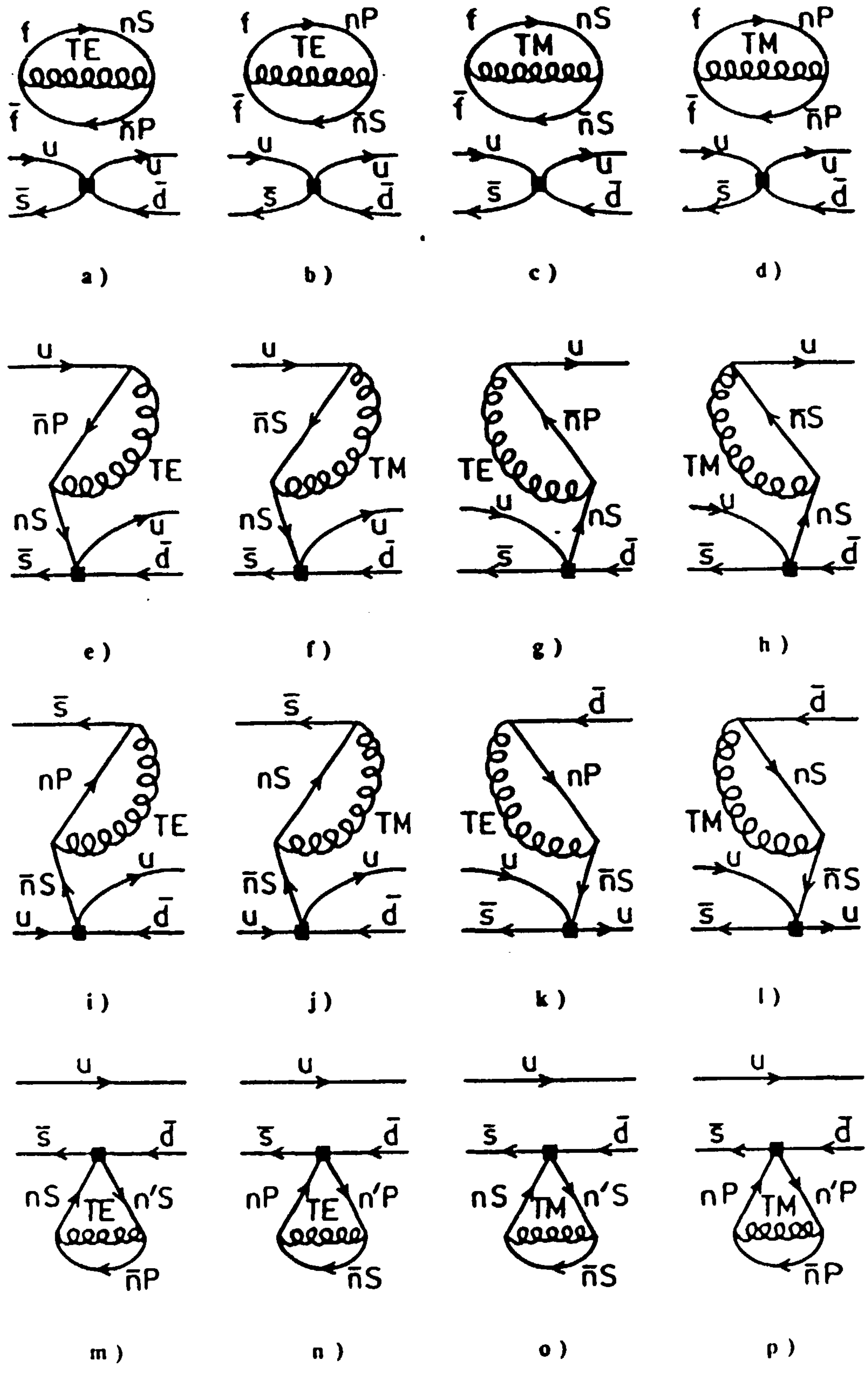


Figure 10



f
nS
TE
f̄
n̄P
u
s̄
d̄
b)
nP
n̄S
c)
TM
d)
e)
n̄P
TE
nS
f)
n̄S
TM
g)
h)
i)
nP
n̄S
j)
k)
l)
m)
n'S
n)
n'P
o)
p)
a)

Figure 11

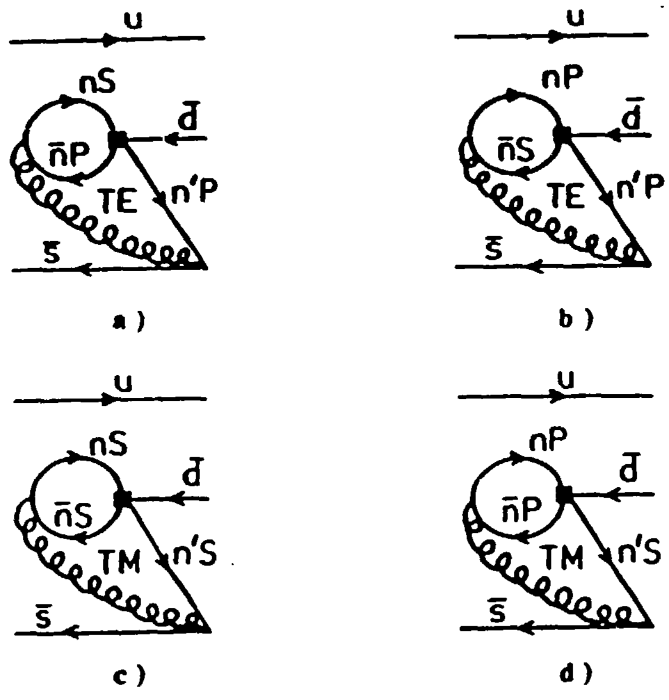


Figure 12

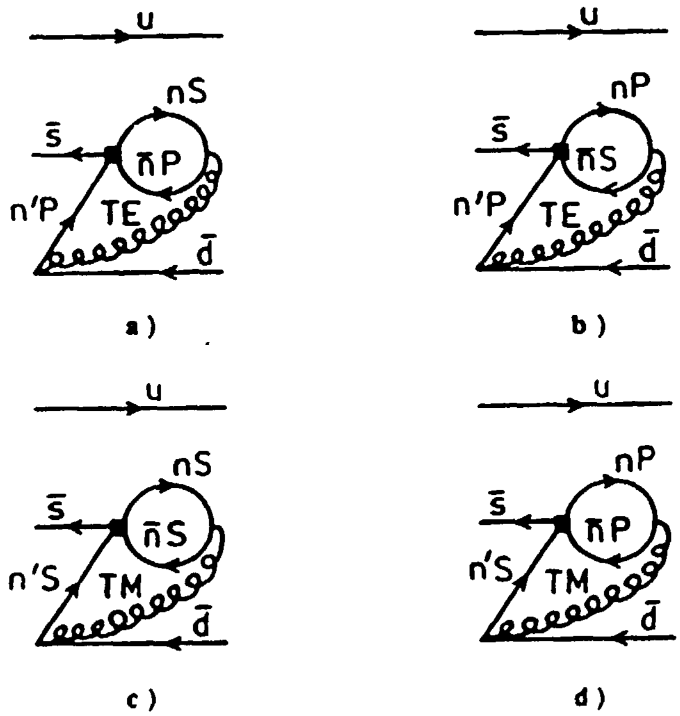
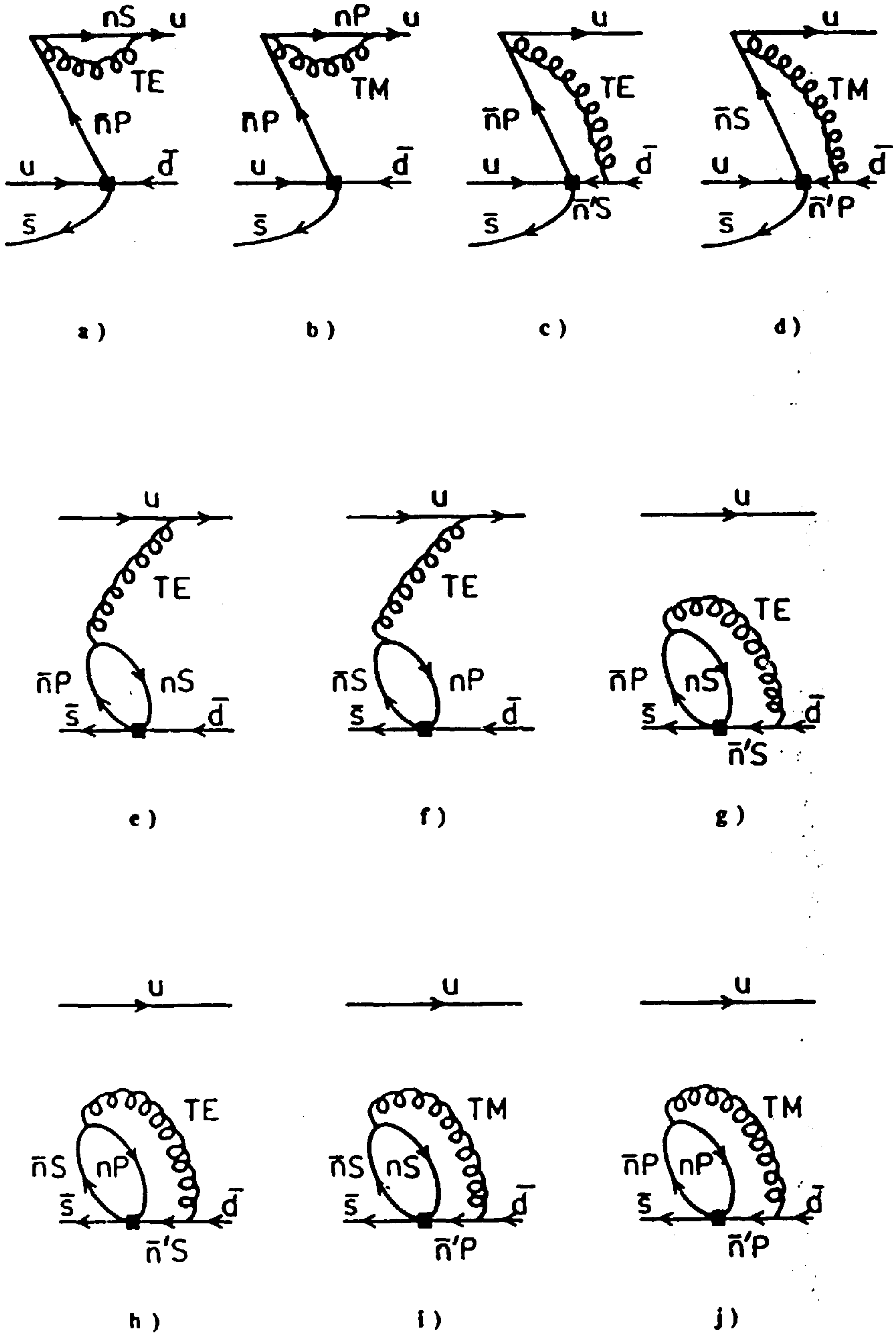


Figure 13



nS u
TE
n̄P
u d̄
s̄
nP u
TM
n̄P
u d̄
s̄
u
TE
n̄P
u d̄
n̄'S
s̄
u
TM
n̄S
u d̄
n̄'P
s̄
a)
b)
c)
d)
u
TE
n̄P nS
s̄ d̄
u
TE
n̄S nP
s̄ d̄
u
TE
n̄P nS
s̄ d̄
n̄'S
e)
f)
g)
u
TE
n̄S nP
s̄ d̄
n̄'S
u
TM
n̄S nS
s̄ d̄
n̄'P
u
TM
n̄P nP
s̄ d̄
n̄'P
h)
i)
j)

Figure 14

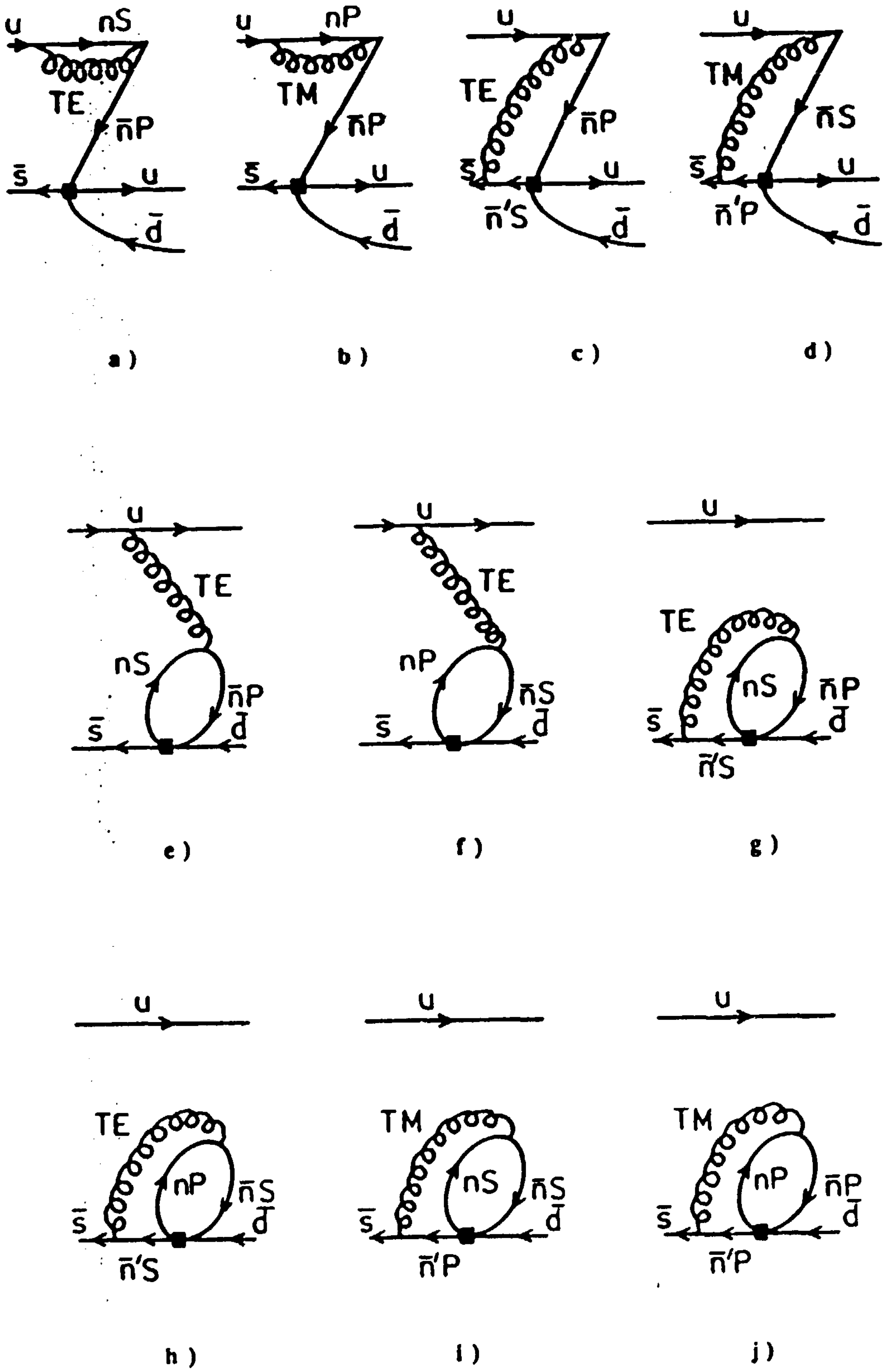


Figure 15

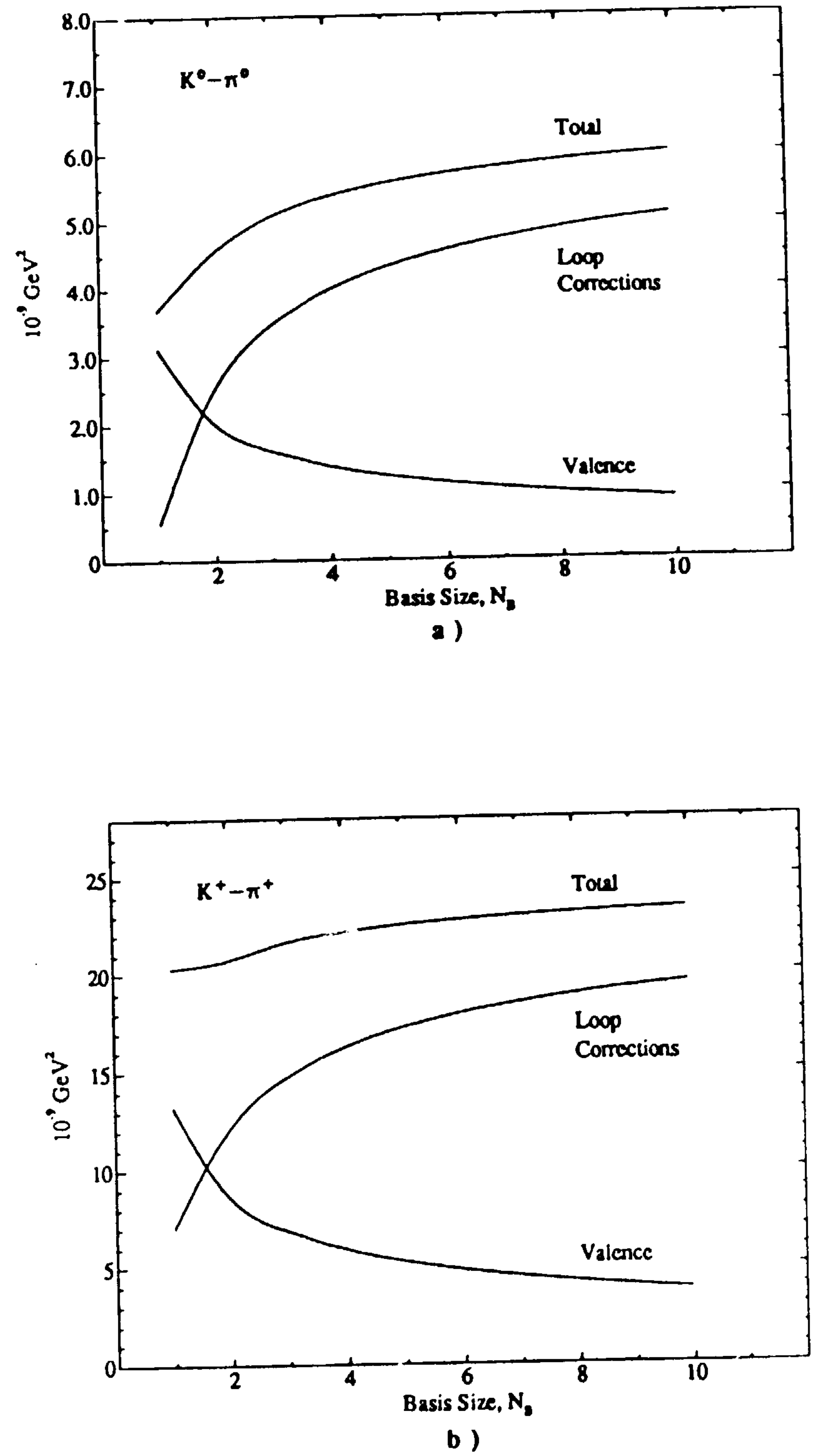

8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
$K^0-\pi^0$
Total
Loop
Corrections
Valence
10^{-9} GeV^2
2
4
6
8
10
Basis Size, N_B
a)
25
20
15
10
5
0
$K^+-\pi^+$
Total
Loop
Corrections
Valence
10^{-9} GeV^2
2
4
6
8
10
Basis Size, N_B
b)

Figure 16

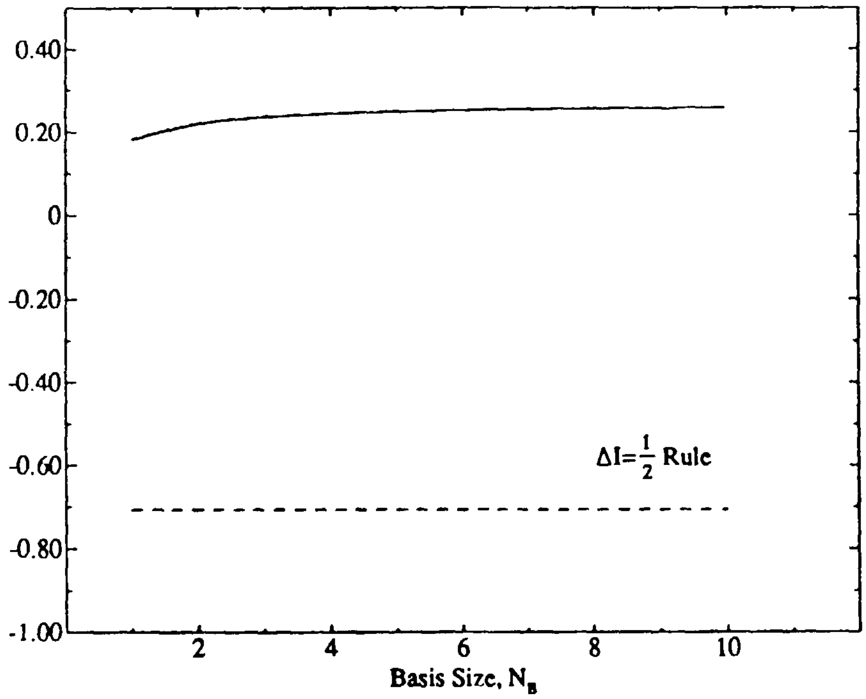
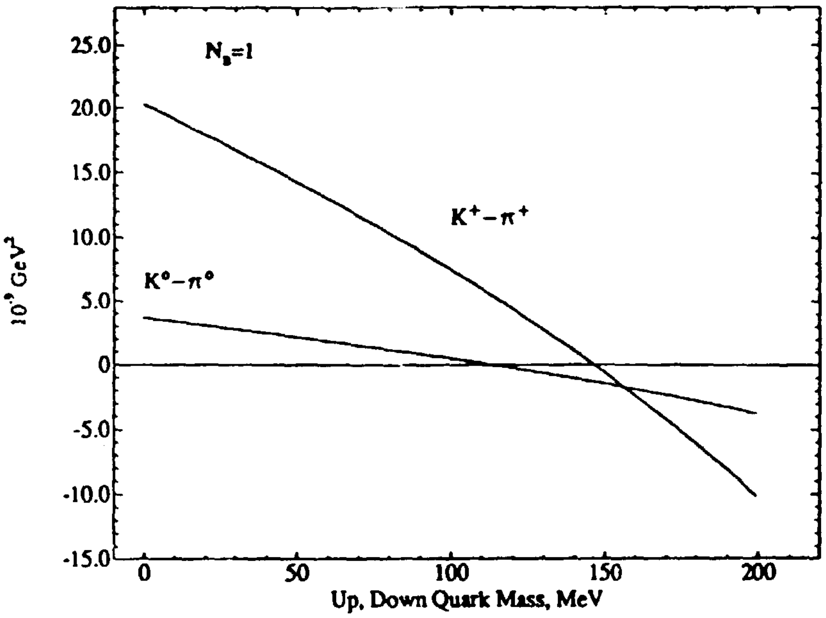
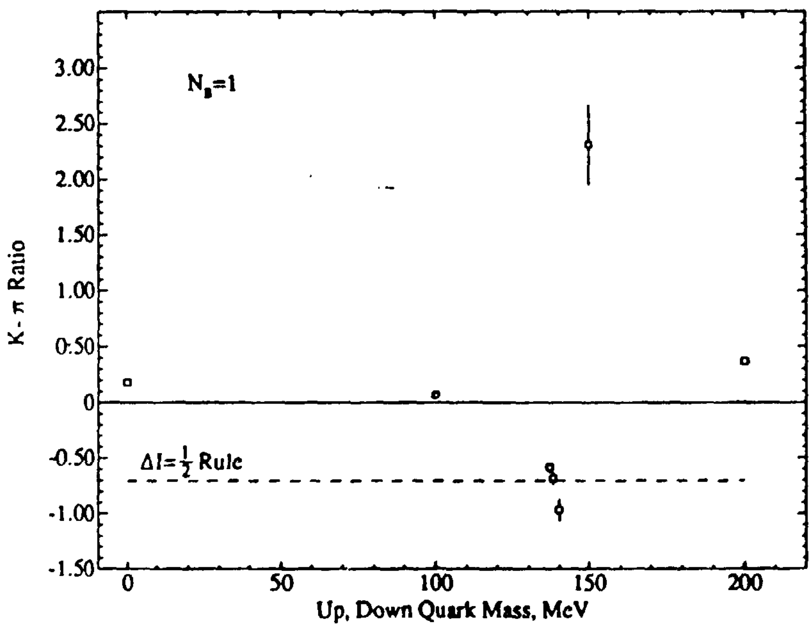


Figure 17

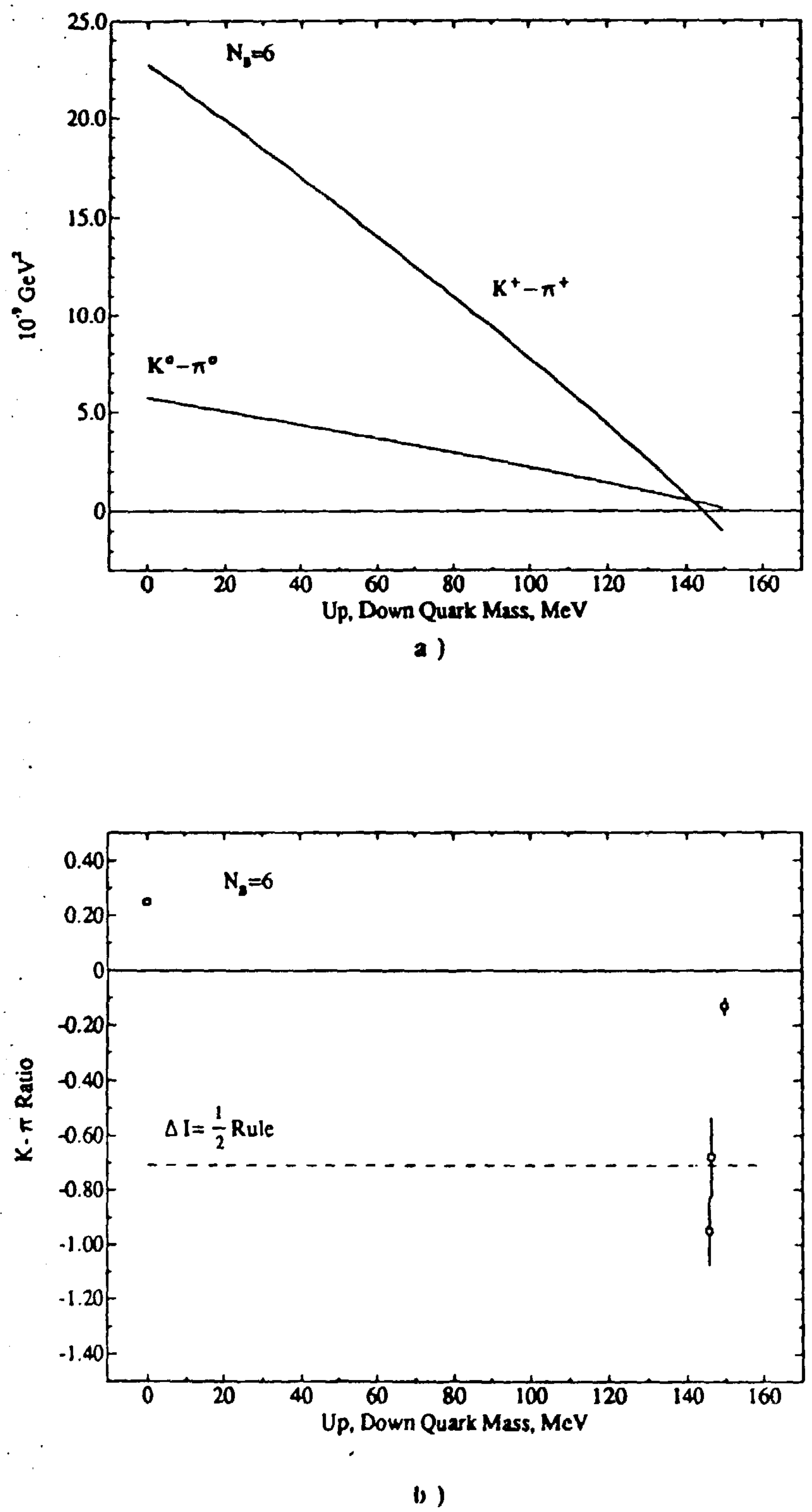

N_B=1
$K^+-\pi^+$
$K^0-\pi^0$
10^{-9} GeV^2
25.0
20.0
15.0
10.0
5.0
0
-5.0
-10.0
-15.0
0
50
100
150
200
Up, Down Quark Mass, MeV

a)


N_B=1
K - π Ratio
$\Delta I = \frac{1}{2}$ Rule
3.00
2.50
2.00
1.50
1.00
0.50
0
-0.50
-1.00
-1.50
0
50
100
150
200
Up, Down Quark Mass, MeV

b)

Figure 18



25.0
20.0
15.0
10.0
5.0
0
10^{-9} GeV^2
N_s=6
$K^+-\pi^+$
$K^o-\pi^o$
0 20 40 60 80 100 120 140 160
Up, Down Quark Mass, MeV
a)
0.40
0.20
0
-0.20
-0.40
-0.60
-0.80
-1.00
-1.20
-1.40
K-π Ratio
N_s=6
$\Delta I=\frac{1}{2}$ Rule
0 20 40 60 80 100 120 140 160
Up, Down Quark Mass, MeV
b)

Figure 19