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Large Contribution To The Neutron Electric Dipole Moment From A Dimension Six Four Quark Operator

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In this paper we study the contribution of a dimension six four quark operator to the neutron electric dipole moment. We find that this contribution dominates over other contributions by at least one order of magnitude in Left-Right symmetric models and two orders of magnitude in di-quark scalar models.

In this paper we study possible large contribution to the neutron electric dipole moment (EDM) due to a dimension six four quark operator in several models. Using the vacuum saturation and factorization method, we find that such a contribution is the dominant one in Left-Right symmetric and di-quark scalar models. Among several dimension six four quark operators we find the following one dominates over the others,

$$C_q Q_q = C_q \bar{q} q \bar{q}' \gamma_5 q' , \tag{1}$$

where q is one of the six quarks and $q' = u$, or d. The coefficient C_q is model dependent. We will calculate the contribution of the above operator to neutron EDM by first calculating the CP violating coupling $f_{\pi NN}$ of πNN vertex induced by Q_q and then evaluating the one loop diagram shown in Fig. 1 to obtain the neutron EDM.

Let us first evaluate $< N\pi|\bar{q}q\bar{q}'\gamma_5 q'|N >$. In the vacuum saturation and factorization approximation, we have

$$< \pi N|Q_{u(d)}|N > = < \pi|\bar{u}(\bar{d})\gamma_5 u(d)|0 >< N|\bar{q}q|N > . \tag{2}$$

The CP violating coupling $f_{\pi^+ \bar{p} n}$ of the charged pion to nucleons due to four quark operators has been calculated in Ref.[1] and we will not repeat it here. We evaluate the CP violating coupling $f_{\pi n n}$ of the neutral pion to nucleons. We use

$$2m_u < \pi^0|\bar{u}\gamma_5 u|0 > = -2m_d < \pi^0|\bar{d}\gamma_5 d|0 > = -iF_\pi m_\pi^2 . \tag{3}$$

For the matrix element $< N|\bar{q}q| >$, we use the pion-nucleon Sigma-term, $\sigma_{\pi N} = \bar{m} < N|\bar{u}u + \bar{d}d|N > = 45 MeV$ extracted from experiments and calculations of the nucleon mass shift due to SU(3) breaking quark masses[2]. For heavy quarks, we use

$$< N|m_h \bar{h}h|N > = - < N|\frac{\alpha_s}{12\pi} G_{\mu\nu}G^{\mu\nu}|N > + O(\frac{\mu^2}{m_h^2}) . \tag{4}$$

The second term in the above equation will be neglected. Using $m_u = 4.2MeV$, $m_d = 7.5MeV$ and $m_s = 150MeV$, we obtain

$$m_u < n|\bar{u}u|n> = 14MeV \;, m_d < n|\bar{d}d|n> = 32MeV \;, \tag{5}$$
$$m_s < n|\bar{s}s|n> = 247MeV \;, < n| - \frac{\alpha_s}{12\pi} G_{\mu\nu}G^{\mu\nu}|n> = 48MeV \;.$$

There are also other four quark operators. However, if the coefficients of all operators are of the same order of magnitude, the operator Q_q dominates the contribution to $f_{\pi NN}$ because this operator has enhancement factors like $1/m_{u,d}$. Also this operator can be easily generated at the tree level in many models.

Using the above informtion, for a given model $f_{\pi nn}$ can be calculated. Evaluating the one loop diagram in Fig. 1, we have[3]

$$d_n = \frac{e g_{\pi nn} f_{\pi nn}}{8\pi^2} \frac{k_n}{2m_n} F_n(\frac{m_\pi^2}{m_n^2}) \;, \tag{6}$$

where $k_n = -1.91$ is the anomalous magnetic dipole moment of the neutron and $g_{\pi NN} = 13.26$ is the π-nucleon strong interaction coupling. The function $F_n(s)$ is from the loop integral and is given by

$$F_n(s) = \frac{3}{2} - s - \frac{3s - s^2}{2} ln s + \frac{s(5s - s^2) - 4s}{2\sqrt{s - s^2/4}} tan^{-1} \frac{\sqrt{s - s^2/4}}{s/2} \;. \tag{7}$$

In many models, the operator Q_q with a relatively large coefficient C_q can be generated and therefore induce a large value for $f_{\pi nn}$. C_q is extremely small in the minimal standard model. In order to have a sizable effect, we need to go beyond the standard model. In the following we will calculate the neutron EDM in several models using this mechanism.

In Left-Right symmetric models, there are new CP violating interactions in the charged current due to the existence of the right-handed gauge boson W_R of $SU(2)_R$.

The interactions of mass eigenstates of the charged gauge bosons, $W_{1,2}$, with quarks have the form

$$L_{W_q} = \frac{1}{\sqrt{2}}[g_L \bar{U}_L \gamma_\mu V_L D_L cos\zeta + g_R \bar{U}_R \gamma_\mu V_R D_R sin\zeta] W_1^\mu$$
$$+ \frac{1}{\sqrt{2}}[-g_L \bar{U}_L \gamma_\mu V_L D_L sin\zeta + g_R \bar{U}_R \gamma_\mu V_R D_R cos\zeta] W_2^\mu \ . \quad (8)$$

where all fields are in their mass eigenstates, ζ is the mixing angle between W_L of $SU(2)_L$ and W_R of $SU(2)_R$. V_L is the KM matrix for the left-handed charged current and V_R is an analogous KM matrix involving the right-handed charged current. For simplicity we will set $g_L = g_R$.

The four quark effective Lagrangian which contains the CP violating interaction is induced at the tree level by exchange of W_1. We have

$$L_{LR} = 4\frac{G_F}{\sqrt{2}} cos\zeta sin\zeta [\bar{U}_L \gamma_\mu V_L D_L \bar{D}_R \gamma_\mu V_R^+ U_R \bar{U}_R \gamma_\mu V_R D_R \bar{D}_L \gamma_\mu V_L^+ U_L] \ . \quad (9)$$

After a Fierz rearrangement, we obtain the operator which gives the dominant contribution to $f_{\pi NN}$

$$L = -i\frac{4G_F}{3\sqrt{2}} cos\zeta sin\zeta [Im(V_{Lud}V_{Rud}^*)(\bar{u}\gamma_5 u\bar{d}d - \bar{u}u\bar{d}\gamma_5 d) + Im(V_{Lus}V_{Rus}^*)\bar{u}\gamma_5 u\bar{s}s] \ . \quad (10)$$

The CP violating πnn vertex is

$$f_{\pi nn} = <\pi n|L_{LR}|n> = -i\frac{4}{3}\frac{G_F}{\sqrt{2}} cos\zeta \sin\zeta$$
$$\times [Im(V_{Lud}V_{Rud}^*) < \pi^0|2m_u \bar{u}\gamma_5 u|0 >< n|\frac{\bar{u}u}{2m_u} + \frac{\bar{d}d}{2m_d}|n > \quad (11)$$
$$+ Im(V_{Lus}V_{Rud}^*) < \pi^0|\bar{u}\gamma_5 u|0 >< n|\bar{s}s|n >$$
$$\approx -90\frac{G_F}{\sqrt{2}} m_\pi^2 cos\zeta sin\zeta Im(V_{Lud}V_{Rud}^*) \ .$$

From this we obtain

$$|d_n| = 6.3 \times 10^{-20} |cos\zeta sin\zeta Im(V_{Lud} V^*_{Rud})| \; ecm \; . \qquad (12)$$

In the case where mixings of the first two generations with the third one are neglected, we have

$$|d_n| = 6.3 \times 10^{-20} |cos\zeta sin\zeta sin(\gamma - \delta_2)| \; ecm \; . \qquad (13)$$

The experimental upper bound[4] $|d_n(exp)| < 1.2 \times 10^{-25} ecm$ implies

$$|cos\zeta sin\zeta sin(\lambda - \delta_2)| < 2 \times 10^{-6} \; . \qquad (14)$$

This is to be compared with the total contribution $d_{n,total}$ from the valence quark EDM, the quark colour dipole moment and the hadron loop contributions[3, 5],

$$d_{n,total} = 2cos\zeta sin\zeta [4.5 sin(\gamma - \delta_2) + 74 sin(\gamma + \delta_1) - 1.1 sin(\gamma - \delta_1)$$
$$+ 11.2 sin(\gamma + \delta_2)] \times 10^{-23} ecm \; . \qquad (15)$$

We see that the new contribution is bigger by at least one order of magnitude. Of course if some delicate cancellation occurs, in particular if $sin(\gamma - \delta_2) = 0$, the new mechanism will give zero contribution to the neutron EDM, but the other mechanisms will still contribute.

Let us now consider the situation in di-quark scalar models. Di-quark scalars are potential sources for large CP violation in flavour conserving processes. A list of possible di-quark scalars which couple to standard model quarks and some of their phenomenological implications can be found in Ref.[6, 7]. There are two di-quark scalars which can induce CP violation at the tree level. These are H_9 and H_{10} in the notation of Ref.[6]. They transform under the standard model group as $(3, 1, -2/3)$ and $(6, 1, -2/3)$ respectively. In the following we consider the contribution to d_n from H_9. The contribution of H_{10} can be similarly worked out. The couplings of H_9 to up and down quarks are

$$L = (\lambda_9 \bar{u}^c_{Ri} d_{Rj} + \lambda'_9 \bar{u}^c_{Li} d_{Lj})\epsilon^{ijk} H_{9k} + H.C. \,, \tag{16}$$

where i, j, k are colour indices, c indicates charge conjugation, and λ_9 and λ'_9 are complex numbers. Exchange of H_9 will generate a four quark interaction

$$\begin{aligned} L_{int} = -\frac{1}{m_H^2}(\lambda_9 \bar{u}^c_{Ri} d_{Rj} + \lambda'_9 \bar{u}^c_{Li} d_{Lj}) \\ \times (\lambda_9^* \bar{d}_{Rj'} u^c_{Ri'} + \lambda'_9 \bar{d}_{Lj'} u^c_{Li'})\epsilon^{ijk}\epsilon^{i'j'k} \,. \end{aligned} \tag{17}$$

The dominant terms which contribute to $f_{\pi nn}$ are

$$L_{int} = \frac{i}{m_H^2}\frac{2}{3} Im(\lambda_9 \lambda'^*_9)[\bar{u}u\bar{d}\gamma_5 d + \bar{d}d\bar{u}\gamma_5 \cdot] + \dots \,. \tag{18}$$

Here a Fierz rearrangement has been performed. We obtain

$$|f_{\pi nn}| = 3.5 \times 10^{-5} |Im(\lambda_9 \lambda'^*_9)| \frac{100^2 GeV^2}{m_H^2} \,. \tag{19}$$

and thus

$$|d_n| = 1.6 \times 10^{-19} |Im(\lambda_9 \lambda'^*_9)| \frac{100^2 GeV^2}{m_H^2} \,. \tag{20}$$

Requiring d_n to satisfy the experimental upper bound on the neutron EDM, we have

$$|Im(\lambda_9 \lambda'^*_9)| \frac{100^2 GeV^2}{m_H^2} < 10^{-6} \,. \tag{21}$$

This constraint is about two orders of magnitude stronger than the one obtained from the valence quark EDM constraint[6].

In multi-Higgs doublet models, exchange of the charged and the neutral Higgs particles at the tree level will also generate the operator Q_q. It turns out that the neutral Higgs contribution is much bigger than the one from the charged Higgs. The contribution to $f_{\pi nn}$ from neutral Higgs exchange is

$$\begin{aligned} f_{\pi nn} = \frac{G_F}{\sqrt{2}} \frac{m_\pi^2}{m_H^2} \times [0.06(GeV)^2(Im Z_{dd} - Im Z_{du} \\ + 0.02(GeV^2)(Im Z_{uu} - Im Z_{du})] \,, \end{aligned} \tag{22}$$

where $ImZ_{qq'} = 2\alpha_q\beta_{q'}$. α_q, β_q are the couplings of the neutral Higgs particles to quarks and are defined by

$$L_{H^0} = (2\sqrt{2}G_F)^{1/2}(\alpha_{d_i}\bar{D}M_D D + \beta_{d_i}\bar{D}iM_D\gamma_5 D + \alpha_{u_i}\bar{U}M_U U + \beta_{u_i}\bar{U}iM_U\gamma_5 U)H_i^0 . \tag{23}$$

The contribution to the neutron EDM is typically

$$|d_n| \approx 0.45 \times 10^{-26}|ImZ_{qq'}|\frac{100^2 GeV^2}{m_H^2} . \tag{24}$$

This contribution is comparable with contributions which have been discussed in the literature[8].

In conclusion we have found a new contribution to the neutron EDM which dominates over other contributions in Left-Right symmetric and di-quark scalar models. Some other implications of the operator Q_q have been discussed in Ref.[9, 10].

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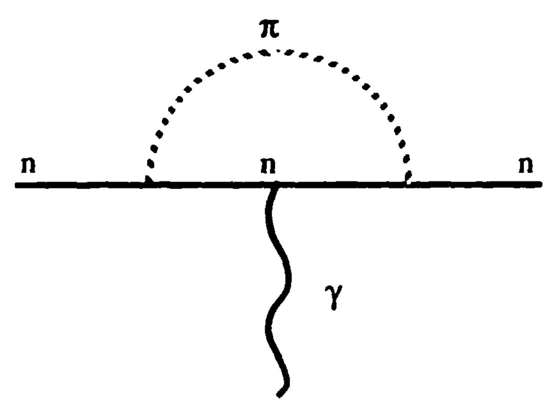

Fig.1 The one loop diagram contribution to the neutron EDM.