IV МЕЖДУНАРОДНОЕ СОВЕЩАНИЕ ПО АНАЛИТИЧЕСКИМ ВЫЧИСЛЕНИЯМ НА ЭВМ В ФИЗИЧЕСКИХ ИССЛЕДОВАНИЯХ

(СБОРНИК АННОТАЦИЙ)

IV INTERNATIONAL CONFERENCE ON COMPUTER ALGEBRA IN PHYSICAL RESEARCH

Дубна 1990

ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

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Contents

REDUCE as a Numerical Tool

John Fitch
University of Bath
Great Britain

The use of computers for algebraic computations are well known, and are established as algebraic tools for physical sciences. In this talk concentration will be on the assistance that algebraic calculation can make for numerical calculations. This assistance includes analysis of problems, preparation of programs, and code generation for numerical programs and supercomputer architectures.

Particular mention will be made of the current cooperative project between AG Ltd. and the University of Bath.

Size Efficient Parallel Algebraic Circuits for Partial Derivatives

Erich Kaltofen[1] *and Michael F. Singer*[2]
[1]Department of Computer Science,
Rensselaer Polytechnic Institute,
Troy, NY 12180-3590; kaltofen@cs.rpi.edu
USA
[2]Department of Mathematics,
North Carolina State University,
Raleigh, NC 27695-8205; SINGER@NCUMATH.BITNET
USA

Given an algebraic circuit or straight-line program of depth d that computes multivariate rational function in l arithmetic operations (additions, multiplications, and divisions), we construct circuits that compute

(1) the first k partial derivatives in a single variable with depth $O(\log(k)(d+\log(k)))$ using $O(l \log(k) \log(\log k)d)$ arithmetic operations;

(2) all first partial derivatives in depth $O(d)$ using no more than $4l$ arithmetic operations;

Our first result is based on Taylor series expansion and essentially parallelizes the Leibniz formula. Our second result parallelizes a construction by Baur and Strassen. A crucial ingredient to the parallel solution is the fact that bounded fan-in computation graphs can be transformed to those where the fan-out is bounded as well while increasing the depth by no more than a constant factor.

EXCALC - A Package for Calculations in Modern Differential Geometry

Eberhard Schrüfer
Gesellschaft für Mathematik und Datenverarbeitung mbH,
Institut F1, Postfach 1240, D-5205 St. Augustin 1
F.R.G.

The capabilities and the design philosophy of the differential geometry package EXCALC are described. Examples from physical Field Theories and from the Cartan-Kähler theory of partial differential equations will illuminate the potentials of this system. The current status and projected future developments of the implementation will be discussed.

ADVANCES IN INTEGRATING SYMBOLIC, NUMERIC AND GRAPHICS COMPUTING

Paul S. Wang

Department of Mathematical Sciences, Kent State University, Kent, Ohio 44242

USA

Modern computer workstations are equipped with high resolution graphics displays, sufficient memory, ample processing power, and networking capabilities. The workstations are usually connected on a local area network (LAN) that also links powerful mainframe computers, and parallel processors. This hardware and operating environment provides a powerful platform to build integrated scientific computing systems that can significantly increase the productivity of contemporary scientists and engineers. When symbolic, numeric, graphics and document processing facilities and techniques are combined in an integrated environment, they reinforce one another so that the whole is bigger than the sum of the parts.

We envision an integrated system consisting of *compute servers* that are controlled by an overall *system-independent user interface* (SUI). SUI runs on a user's graphics workstation and each compute server can run on any processor on the LAN. Through SUI, the compute servers interact with one another and with the user. The user interface is window/mouse oriented with graphics and networking capabilities. The compute engines will include existing systems such as Vaxima, Reduce, Maple, Matlab, NAG, LINPACK, DITROFF, and LATEX. An interface architecture and protocol must be designed to accommodate existing as well as future compute engines.

Research in this direction has been going on at Kent and elsewhere. Presented are some recent developments at Kent: symbolic derivation of numerical code for finite element analysis; automatic numeric code generation based on derived formulas; graphical display of mathematical functions; automatic inclusion of mathematical expressions produced symbolically in documents; generation of code for parallel processors and CRAY super computers; architecture, design, protocol and implementation of SUI.

We also describe several other software packages in these directions.

AN EXAMPLE OF AN USER-WRITTEN EXTENSION PACKAGE TO THE REDUCE COMPUTER ALGEBRA SYSTEM FOR CALCULATIONS IN PHYSICS

M. Warns

Institute of Physics
University of Bonn
FRG

I will present a software package which extends the capability of the REDUCE Computer Algebra System in handling expressions containing non-scalar and non-commutative quantities, e.g. quantum mechanical vector operators or gamma matrices. Special emphasis is put on the way to link such a package into REDUCE system. As an example I will present some applications in the field of Feynman box graph calculations and for the calculations of commutators between quantum mechanical operators.

ANALYTIC CALCULATION OF HIGHER ORDER CORRECTIONS IN QUANTUM FIELD THEORY: METHODS AND RESULTS

K.G.CHETYRKIN, A.L.KATAEV

Institute for Nuclear Research of the Academy of Sciences of the USSR, Moscow

The report reviews, first, the current methods of analytic calculations of higher order corrections to renormalization group functions (β-functions and anomalous dimensions) and to the coefficient functions of the operator product expansion and, second, a variety of physical results obtained with the help of these methods. We consider the Gegenbauer polynomial technique in ρ and $\times$ spaces, the method of integration by parts in dimensional regularisation; various methods of infrared rearrangement of Feynman integrals, including the most universal one based on the R^* - operation; the uniqueness method and, finally, the method of projectors. A special attention is devoted to constructing effective computational algorithms on the base of these methods

directed at their implementation by means of the computer system for symbolic calculations. The physical results under discussion include multiloop calculations of renormalization group functions in various theories, in particular $g\phi^4$, QED, QCD and supersymmetric models; calculations of higher order corrections to $(g-2)_\mu$; to QCD sum rules for light and heavy quarks; finding of nonleading corrections to the characteristics of the deep inelastic scattering, to the decay width of the Higgs boson, and a number of others.

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For the related reviews see:

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Current Trends in Source-Code Optimization

J.A. van Hulzen
Twente University, Department of Computer Science
P.O. Box 217, 7500 AE Enschede, The Netherlands

An important application of computer algebra systems is the generation of code for numerical purposes via automatic or semi-automatic program generation. GENTRAN [1], a flexible general-purpose package, was especially designed to assist in such a task, when using MACSYMA or REDUCE.

Attendant to automatic program generation is the problem of automatic source-code optimization. This is a crucial aspect because code generated from symbolic computations often tends to be made up of lengthy expressions, to be grouped together in blocks of straightline code in a program for numerical purposes. The main objective of source-code optimization was up to now to minimize the number of (elementary) arithmetic operations in such blocks. SCOPE, a Source-Code Optimization PackageE for REDUCE [4], now available through the REDUCE network library, was especially designed for this purpose.

SCOPE is completely written in RLISP. It requires some GENTRAN facilities to interface with REDUCE. Its input is a set of syntactically correct REDUCE assignment-statements. Its output is the optimized version of the input, given in the syntax of one of GENTRAN's target languages. It is possible to combine it with a list of declarations, defining the type of the input names and the system generated subexpression names. This output is in fact produced as a side-effect of a SCOPE-application.

At present we are working on a new version of SCOPE, taking into account the design-considerations for a future REDUCE 4 version. An implication of this desire is to realize a functional behaviour of the future version, for instance by making both GENTRAN and SCOPE totally independent modules. Minimization of the arithmetic complexity of source-code is mainly attractive for the construction of programs, to be executed on von Neumann type machines. Therefore, other strategies leading to efficiently executable programs on other types of machines [2] will be incorporated. This demands a more flexible and object-

oriented approach and the inclusion of some new features, such as data dependence analysis. It will also influence the structure and possibilities of application packages based on the use of both GENTRAN and SCOPE. Worth mentioning is automatic generation of Jacobians and Hessians [3].
We discuss present and future features of SCOPE.

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METHODS AND ALGORITHMS OF PERTURBATIVE CALCULATIONS IN QUANTUM FIELD THEORY AND THEIR COMPUTER IMPLEMENTATION

(I) Theory and phenomenological applications

S.A.Larin, L.R.Surguladze and F.V.Tkachov

Institute for Nuclear Research of the USSR Academy of Sciences,
Moscow, 117312 , USSR

A review is presented of the current state of the art in calculational methods for a wide class of problems of applied quantum field theory: computation of renormalization group functions, coefficient functions of operator product expansions, and two-point vacuum correlators of local operators. Discussed are the infra-red rearrangement methods (A.A.Vladimirov's method, the R^*-operation and further developments) and the results of the theory of euclidean asymptotic expansions (explicit formulae for coefficient functions, the As-operation) which allow one to drastically reduce the classes of multiloop diagrams to be computed in these problems (massless propagator-type diagrams, massive vacuum diagrams). Algorithms for such calculations and feasibility of their computer implementation are discussed. It is stressed that the development of theoretical methods is strongly influenced by capabilities of the available computer systems of symbolic manipulations. A series of calculations is described, performed using the above methods in ' two-, three-, four- and five-loop approximations; their phenomenological consequences are briefly discussed and their importance is stressed (e.g. for estimating theoretical uncertainties

of perturbative calculations and determining applicability regions of perturbative calculations in quantum chromodynamics). Unsolved problems of this class and the corresponding difficulties are discussed, in particular, the absence of adequate programming tools (esp. for manipulation of graphs) for a fuller automation of the multiloop calculations.

METHODS AND ALGORITHMS OF PERTURBATIVE CALCULATIONS IN QUANTUM FIELD THEORY *AND THEIR COMPUTER IMPLEMENTATION* (II) Program packages

S.A.Larin, L.R.Surguladze and F.V.Tkachov

Institute for Nuclear Research of the USSR Academy of Sciences,
Moscow, 117312 , USSR

Program tools are described for calculation of multiloop massless dimensionally regularized propagator-type Feynman integrals (p-integrals) which emerge in problems of quantum field theory: the *SCHOONSCHIP* package *MINCER* [1] and the *REDUCE* package LOOPS (versions for mainframe and personal computers) [2]. The packages allow one to analytically calculate p-integrals at the two- (LOOPS) and three-loop (MINCER) levels, which enables one to perform unique calculations of renormalization group functions, coefficient functions of operator product expansions etc. LOOPS allows arbitrary tensor structures in the numerators of integrands and provides a basic set of tools to work with dimensionally regularized expressions within *REDUCE*. *MINCER* is specialized for large-volume calculations and can be used for calculations in various physical space-time dimensions (the latter possibility has been used for studying operator expansions in exactly solvable two-dimensional models). The requirements to symbolic manipulation systems imposed by such problems are discussed, and various versions of *REDUCE* and *SCHOONSCHIP* are compared from this point of view. A fundamental flaw in both systems is the lack of local context protection of symbolic names often used as local labels and substitution rules, which makes difficult writing large application packages. On the whole a well-designed powerful "primitive" algebraic processor with a carefully chosen set of functions and a laconic "system"-level language would be invaluable for the problems of the described type.

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COMPUTER-AIDED CLASSIFICATION OF GEOMETRIES IN GENERAL RELATIVITY

M. A. H. MacCallum

School of Mathematical Sciences, Queen Mary and Westfield College, Mile End Road, LONDON E1 4NS, England

Many solutions of Einstein's field equations are known. Due to the arbitrariness of coordinates, two or more apparently different such geometries may represent the same physical situation. The problem is to characterize a geometry invariantly, and thus provide a way to decide equivalence of geometries.

Differential geometric considerations show one has to compute components of the Riemannian curvature and its derivatives as functions on the frame bundle of the spacetime. An effective method is to put the results into canonical form, by change of basis, at each step of differentiation.

Practical programs to embody this method require sub-algorithms for such problems as testing rotation invariance, algebraic classification of quartics, specification of a minimal set of derivatives and testing functional independence. These have been devised and implemented as the package CLASSI in the specialized algebra system SHEEP. Some results will be shown, and possible developments discussed.

OPERATOR ORDERING AND G-QUANTIZATION SCHEMES IN COMPUTER ALGEBRA SYMBOL REPRESENTATIONS OF ENVELOPING ALGEBRAS

Wolfgang Lassner

Karl-Marx-Universität, Sektion Informatik

Leipzig 7010, GDR

Phase space methods in quantum theory make use of symbol representations of the Weyl algebra. The method can be generalized to enveloping algebras U(G) of Lie algebras G different from the Heisenberg Lie algebra. We restrict us to a pure algebraic view point.

The method starts from a one-to-one correspondence between the linear space of the enveloping algebra U(G) and that of the symmetric algebra S(G) over G. The algebra S(G) can be equipped with a so-called twisted product so that it becomes isomorphic to U(G).

The twist product technique allows one to derive formulae which are useful for a fast multiplication in non-commutative algebra. Furthermore, the twist product techniques can be extended to subrings of the quotient division ring D(G) of the enveloping algebra U(G).

Operator orderings (resp. quantization schemes) have been described in mathematical and physical literature by various teohniques. Symbol representations are powerful tools in this context. A generalization to enveloping algebras U(G) resp. to differential operators over Lie groups will be of theoretical and practical interest.The ordering problem consists first of all in a

description of different operator orderings. Different ordering rules correspond to different bases in U(G). The elements of U(G) are represented by different symbols in dependence of the ordering rule chosen. The twisted product depends on the ordering rule in a non-trivial way. We generalize the notion of an ordering defining function ϕ. It is possible to calculate ϕ for various ordering rules. This helps to determine a transformation between different types of symbols and is useful in order to use the same fast multiplication algorithms for different orderings.

Symbol representations of enveloping algebras are useful for the so-called G-quantizations, i.e. for systems with symmetries related to G. The method has been used in Lie optics in order to calculate aberrations.

Computer algebra calculations in non-commutative algebra need new software engineering for the implementation of efficient algorithms into commercial integrated systems for scientific computation.

AUTOMATIC CALCULATION OF SCATTERING AMPLITUDES

Toshiaki Kaneko

Faculty of General Education, Meiji Gakuin University, Totsuka, Yokohama 244, Japan

As the available energy for high energy experiments is increasing, a large number of perturbative calculations are required. Many parts of this kind of work must be performed on computers. The conventional applications of computers are limited only to numerical calculations and some parts of symbolic manipulations so far. The purpose of the present work is to utilize the ability of computers in wider range.

The requirements for the program package is composed of the following components: 1. Feynman graph generator: This program reads the type of external particles and the order of perturbation for a scattering reaction. It generates all the relevant Feynman graphs. Output on a graphic device is also indispensable for checking each step of calculation. 2. Source program generator: Source programs for the numerical calculation of amplitudes should be generated automatically in accordance with the obtained Feynman graphs. This source program is used as an integrand of numerical integration. 3. Library of Kinematics: To perform the numerical integrations of the square of amplitudes to get the cross sections, one has to prepare kinematics. They should be standardized and collected into a library. 4. Numerical integration package: The value of scattering cross section is obtained by multi-dimensional integration over the phase space. The source program should be generated in suitable forms for the integration package. 5. Event generator: The final form of output of this package should be an event generator.

We have developed several parts of this package. The Feynman graph generator is already available for QED in arbitrary order of perturbation[1]. We added another program which is applicable up to one-loop corrections in the Weinberg-Salam model and QCD. Generated Feynman graphs are automatically converted to FORTRAN source code. In this stage, we consider only the tree graphs. The method of calculation used in the FORTRAN source code is described in ref.2, in which spinors are treated numerically and Feynman amplitudes are calculated directly. A feature of this method is that we can get rid of the use of symbolic manipulating programs. We use

BASES/SPRING[3] as a multi-dimensional integration package. This program package contains general event generator based on the resulting data of the integration.

Now we have enough tools to calculate scattering cross sections within the lowest order of perturbation of the standard model. We have tested our program in the polarized cross section of $e^+e^- \to \mu^+\mu^-\gamma$. The result of the integration of created FORTRAN source program is compared with the following three programs: 1. A program generated by REDUCE which calculates square of amplitudes by the conventional method. 2. A program written by hand in the same method as automatically generated FORTRAN source program. 3. A program generated by REDUCE which calculates Feynman amplitude directly. The results are in good agreement among these four programs within the statistical error of Monte Carlo integration.

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EXTENDING POSSIBILITIES OF SOME COMPUTER ALGEBRA ALGORITHMS FOR SOLVING LINEAR DIFFERENTIAL AND DIFFERENCE EQUATIONS

S. A. Abramov

Acad. of Sciences of USSR, Computing Center, Moscow
USSR

Many of computer algebra systems contain programs for solving differential or difference equations of some kind. The linear equations with variable coefficients are quite interesting in this context, and will be dealt with in this lecture.

Let the coefficients of equations be in some function field **P** closed with respect to an operation **d/dx** (or Δ in the case of difference equations). Let algorithm $\mathcal{A}_L$ for finding all solutions of the equations in some linear space **L** be known. We extend $\mathcal{A}_L$ for **L** not closed with respect to the operation of integration (or summation). Let, for example, **L** be the rational or algebraic function field, the quasirational function space, etc. How can we construct all the solutions with their higher order derivatives (or differences) in **L** with the help of $\mathcal{A}_L$? This lecture demonstrates two algorithms of this construction. The role of this result for equations with polynomial coefficients is discussed.

APPLICATION OF COMPUTER ALGEBRA IN INVESTIGATION OF DIFFERENCE SCHEMES STABILITY

S.I.Serdjucova
Joint Institute for Nuclear Research, Dubna

The state of affairs and perspectives of application of computer algebra in the investigation of the difference schemes stability are discussed:

ON THE DESIGN OF AN ARTIFICIAL INTELLIGENCE ENVIRONMENT FOR COMPUTER ALGEBRA SYSTEMS

J. Calmet and **I.A. Tjandra**
Institut für Algorithmen und Kognitive Systeme, Universität Karlsruhe, FRG

Knowledge Representation is the field of Artificial Intelligence (AI) that studies how to represent and manipulate knowledge. We adopt the point of view that algebraic algorithms are a special sort of knowledge which is implemented in Computer Algebra Systems (CAS). This leads to reformulate the concept of a CAS in the framework of AI.

The first task was to design a general hybrid knowledge representation system capable of accommodating mathematical knowledge. This task has been completed and a system called MANTRA is available for this purpose.

The second task is to define the concept of Mathematical Knowledge. This study is based on the definition of computing domains (which are called either categories or types) and on the inference procedure which permits the system to ensure that a mathematical operation on a given domain is valid. This approach leads to a feasible solution to the problem of type inference in CA which is known to be undecidable (i.e. not solvable by a universal algorithm).

Such an environment is well suited to extend the capabilities of CAS. For instance, it is trivial to integrate graphics since the kernel of MANTRA is based upon a graphical representation of knowledge. The design of

explanation or tutoring capabilities is straightforward since they are simply built upon existing tools of the environment.

A short term goal is to embed an existing CAS into the environment. A longer term goal is to design a computer algebra system better suited to this environment and whose main features have been described by the first author in *Intelligent Computer Algebra System : Myth, Fancy or Reality ?* In "Trends in Computer Algebra", R. Janßen Ed., Springer-Verlag LNCS **296**, pp. 2-11, 1988.

A PROJECT OF TOOL FOR IMPLEMENTATION OF A COMPUTER ALGEBRA SYSTEM

A.V.Astrelin, E.V.Pankrat'ev, A.F.Slepuhin
Moscow State University, Moscow, USSR

The system in progress is a tool for object-oriented languages creation. It consists of a kernel and a shell. The kernel includes basic operations available for different objects: creating, deleting, copying, taking of an element and call of them (theoretically not only subroutine or function but any object can be called). All these operations can be extended when new types are created; the types may be dynamically created and they form a quasi ordered set. New types creating operations are contained in the kernel too. The type hierarchies are needed for polymorphic function (i.e. functions with identically names but different parameter types) creating.

The shell consists of libraries to be linked to the kernel. As a rule they contain new types and operations over them. Even such functions as interpreter of an object (that allows to write programs in a special language, not only in C language) and a compiler creating such objects from text information can be among the library functions. All new created libraries must satisfy some conditions.

The proposal usage of system is development of easy adapted computer algebra system on its base.

ON THE SYSTEM OF COMPUTER ALGEBRA OPERATING AS A PART OF POWERFUL COMPUTER SYSTEM

M.V.Mikhayljuk, A.N.Sotnikov, A.A.Trushina
Institute of cybernetic problems
of the USSR Academy of Sciences, USSR

The analysis of modern approaches to the development of computer algebra systems operating as a part of powerful computer system shows that the main user's requirements and consequently the creator's efforts are concentrated on one side on the expansion of the problems range to be solved using the computer algebra system and on the other side on the ensuring of "friendly" interface for working with them.

The success of the second trend demands the intellectualization of means for working with system enabling in a rather simple way to introduce in view new objects (for example in mathematical physics, celectial mechanics and so on) and to perform operations on them. To a great degree this approach is realized through interfaces of a "menu" type.

The works carried out at the Institute of cybernetic problems of the USSR Academy of Sciences in the field of analytical manipulations are aimed at creating the basic components of the computer algebra system operating as a part of powerful computer. They embrace a set of instrumental means for creating the menu type system, basic operations on algebraic objects also in Boolean algebra and series theory.

The menu system enables the user to create a menu tree in an interactive mode, provides all possibilities for moving along the tree, connecting applied programmes to it and their run.

Basic operatoins include a software package for arithmetic operations with unlimited accuracy numbers and with Poisson series.

Working in the Boolean algebra allows to realize formulas manipulations specified in various bases and to find minimal formulas for certain classes.

As many problems of computer algebr. demand a rather big memory capacity and much time for their solution, so the creation of components of computer algebra system in powerful computer environment will surely expand their possibilities.

ROOTS AND FRUITS OF DELIA SYSTEM

A.V.Bocharov
Program Systems Institute , Pereslavl , USSR
+ Optimum Integrated and Program Systems Inc.
Indianapolis , USA

DELiA emerged as a commercial successor of a prototype personal SCoLAr computer-algebraic program [1]. Its original root has been the geometric theory of differential equations , including the theory of classical [2] and *generalized [3] symmetry*.

But since its birth DELiA went through a long way towards solving certain classes of differential systems : so presently it includes an elaborate simplifier/solver for finite type overdetermined systems. The thing to be included into DELiA in the nearest future is the symbolic/numeric interface including numeric solving facilities for differential equations.

Still another thing to be included is a set of integrability tests for ordinary and evolution differential equations and systems .

The present state of DELiA and its nearest prospection is discussed in the report.

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ON THE AUTOMATIC CONSTRUCTION OF REPRESENTATIONS OF NON-COMMUTATIVE ALGEBRAS

Gilles Labonté
Department of Mathematics and Computer Science
Royal Military College of Canada

The problem of constructing representations of non-commutative algebras is often met in mathematical physics, especially in domains as Quantum Mechanics, and applications of Lie

algebras. We have developed a mathematical algorithm[1,2] to solve this problem when the algebra is specified by a finite presentation, that is, when it is defined as being generated by a set of elements $\{X_1, X_2, \ldots X_k\}$ which are related according to a set of polynomial equations as

$$p_i(X_1, X_2, \ldots X_k) = 0 \quad \text{for } i = 1 \text{ to } n . \qquad (1)$$

Thus, the problem is that of producing matrices $M_1, M_2, \ldots M_k$ which satisfy Eqs (1). Note that the vector space carrying the representation, as well as its dimension, is unknown at the start. All that is perhaps known about the vector space, is that there exists in it a (or more) vector with some desired properties, expressed in equations as:

$$q_i(X_1, X_2, \ldots X_k)\ v = 0 \quad \text{for } i = 1 \text{ to } m. \qquad (2)$$

Such conditions serve to specify which one of the various representations of the given algebra one wants constructed. If no such constraint is given, the algorithm will produce the regular representation.

An overview of the algorithm will be presented. It will be seen to be closely related to the Dehn's and Todd and Coxeter's algorithms, which are well known in Group Theory.

Some examples of physical problems for which it is useful will be given.

Footnote: Demonstrations and clonings of the program (in muLISP) implementing this algorithm will be available.

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CONSTRUCTION OF ECONOMICAL COMPUTATION FORMULAE IN COMPUTER ALGEBRA SYSTEMS

E.V.Zima
Facul. BMK MGU, Moscow
USSR

The situation when the numerical solution of a problem on computer is preceeded by symbolic transformations is typical enough. Some of computer algebra systems allow one to generate the programs on a suitable programming language with the help of formulae which are obtained as a result of symbolic transformations. However the

formulas which we get in computer algebra systems are usually unwieldy and direct computations on them aren't economical, especially in the case when computations on these formulas must be done in cycle.

The problem of automatic construction of economical iterative computation formulas is being considered. A method for comparision of functions, calculated in the cycle with systems of recurrence relations, which connect the next value of a function with the results of computations on previous steps of the cycle was proposed in /1,2/ . A generalization of recurrence relations systems class is being considered. A special algebraic methods for construction of such relations are extended that provides for increasing of computation economy. The methods for generation of parallel computational programs with the help of recurrence relations systems are being proposed. The possibility of using these methods in the case of symbolic computations is demonstrated. Besides the implementation of these methods in the Mumath and Reduce-3.2 systems is explained. Many examples and programs are given.

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INFORMATION SYSTEM

ALGEBRAIC COMPUTING SYSTEMS

FOR PERSONAL COMPUTERS

S.A.Pritomanov

Chemical and technological institute, Dnepropetrovsk

USSR

We elaborated the information system Algebraic Computing Systems (ACS) for the aim of collection and systematization of all information about computer algebra and algebraic computing systems. The system was written on FRED and realised in FRAMEWORK-II packet for perconal computers IBM PC/XT,AT and compatible with RAM 512 K.

The ACS system contains:

-the information about algebraic computing systems and its technical features;

-comparative features of algebraic computing systems;
-special features of algebraic computing systems for personal computers;
-description of REDUCE-3.2 system for personal computers IBM PC/XT,AT and compatible;

The **ACS** system allow to create, correct and sort the databases:
-active users database in Soviet Union and foreign;
-computer algebra reviews database;
-books and articles database (soviet and foreign);
-**symposiums** and conferences database

User may easy correct and modify the information system with own requirements in different fields of physics and mathematics. At present time the systems orientate on the information about using the computer algebra in general relativity and gravitation.

HECAS-2: NEW VERSION OF THE COMPUTER ALGEBRA SYSTEM FOR HIGH ENERGY PHYSICS

S.N.Grudtsin, V.N.Larin

Institute for high energy physics, Protvino

USSR

The new version of computer algebra system HECAS is described. The general structure of system is represented, the possibilities of earlier versions are listed. Some more detailed is discussion of the new possibilities: calculation of the derivatives in vector, covariant differentiation, traces calculation from γ-matrix products in N-dimensional space (dimensional regularization), working with tensor components and user noncommutative variables, new service, etc.

Special attention was paid to important component of practically any CAS - the substitutions. New types and new management mechanism for substitution (at user instructions level) were introduced into HECAS-2, so system flexibility was essentially increased.

The construction principle of two-level CAS and perspectives of further development of system (version HECAS-3) are shortly discussed in the final part of that communication.

INTEGRATED SYSTEM INTERCOMP AND COMPUTER LANGUAGE FOR PHYSICISTS

S.N.Grudtsin, V.N.Larin
Institute for high energy physics, Protvino
USSR

It is described the general approach to physical software integration, development history and modern stage of INTERCOMP system, containing a large set of language and program means for description and computer analysis of physical models. The system has a high level interpreted language and includes a powerful analytic calculation subsystem, numeric algorithms library, relational DBMS, graphic package, editor and text processor. It allows to describe all stages of physical problem solving (such as analytical stage, selection of experimental data required, numeric analysis, text and graphic representation of results) as united compact program, written in one language, convenient to study and use. Relational DBMS and resident compiler allows one to hold, select and execute the INTERCOMP programs. This opens the possibilities to create the "Physical Models Bank" as system of data and algorithms with complex mutual relations, which can be independently maintained and dynamically connected.

Some more detailed is the description of analytical calculation subsystem HECAS, which is most important and most "nonstandard" part of the system. It has high efficiency and large language possibilities, it is widely used by physicists, and may be a thing of independent interest.

ALGEBRAIC PROGRAM DIRAC ON IBM PC

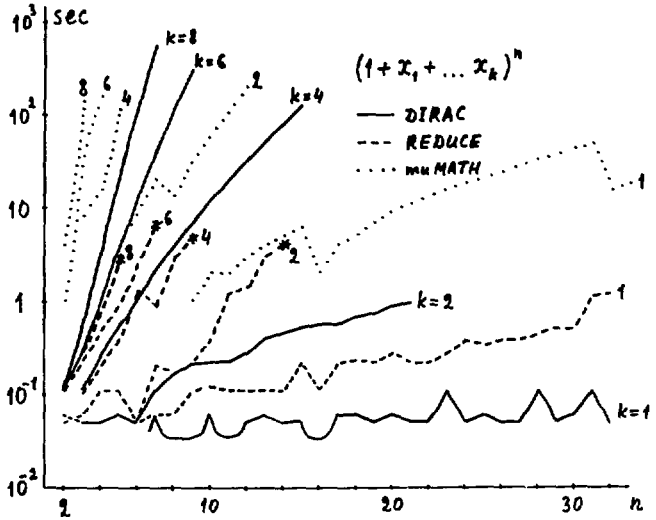
A.G.Grozin
Institute of Nuclear Physics, Novosibirsk, USSR
H.Perlt
K.Marx University, Leipzig, GDR

The program DIRAC for algebraic calculations with polynomials and tensors [1,2] was recently adapted for IBM PC compatible personal computers [3].

It is compared in time and storage efficiency with REDUCE [4] and muMATH [5] on these computers. Results of the polynomial algebra test are shown at the figure. muMATH appeared to be the less efficient system (it is seen to prefer powers of the form 2^n). DIRAC is more time efficient than REDUCE on polynomials up to 4 variables. REDUCE uses storage much less efficiently: stars on the figure denote that the lines can't be continued due to the lack of storage.

As a tensor algebra test, expressions

$$\frac{\partial}{\partial x_{i_1}} \cdots \frac{\partial}{\partial x_{i_n}} \frac{I}{X^2}$$



were calculated (the bar means averaging over x directions, n is even). DIRAC worked for n=4 - 0.4 sec, n=6 - 5.4 sec, n=8 - 390.8 sec. REDUCE n=4 - 12.1 sec, it can't calculate the 6-th derivative due to the lack of storage. muMATH with SYMBOL-HIT package [6] can't calculate the 3-d derivative.

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COMPUTER-INTERACTIVE SYSTEM FOR CALCULATION OF PARTICLE COLLISION CHARACTERISTICS AT HIGH ENERGIES

E.Boos, M.Dubinin, V.Edneral, V.Ilyin, A.Kryukov, A.Pukhov, A.Rodionov, V.Savrin, D.Slavnov, A.Taranov
Research Institute for Nuclear Physics
Moscow State University, 119899, Moscow, USSR

At present time new generation of accelerators (1-10 TeV) are under construction. Theoretical foundation of its physical program is an important part of the whole accelerator project. This foundation contains the detailed computation of basic characteristics for all possible collision processes and decays necessary for the reliable identification of the interesting processes and separation them from the background.

Developed software exists for some components of high energy physics calculations. The symbolic computation systems REDUCE, MACSYMA, MAPLE, a number of programs for numerical calculation and Monte-Carlo simulation can be mentioned. Unfortunately these programs are not connected with each other. Only highly qualificated user is able to use them in the full scope. Therefore the creation of a system which joins symbolic and numeric computation, plots etc. (from Lagrangians to cross-section) under interactive user shell is necessary for HEP investigations.

We emphasize some features essential for this task.

1. There is a large number of theoretical models for the particle interactions at the TeV energies (Standard Model, nonstandard electroweak models, compositeness, SUSY, GUT, etc.). Therefore the system should contain some standard models and give user a possibility to change these models or to create a new one.

2. A large number of Feynman diagrams appears for every physically interesting process. For instance, in the Standard Model the number of vertices is 72 and the number of diagrams for the 2-->3 processes can be close to 1000. Furthermore there are several background processes for every one under consideration. Therefore the task consists of the large number of simple similar calculations and the system must be very fast if we want to work in interactive mode.

3. The lowest order of perturbation theory is only necessary in the task. One can also use the limited number of exclusive processes (subprocesses) types: 2->2, 2->3 (probably 2->4), 1->2, 1->3, 1->4. A limited number of final characteristics (cross-sections, asymmetrics and decay rates in formulas and plots) is needed of interest.

4. The user interface should be simple with the implications of the well-known high energy physics notations. The knowledge of computational techniques and programming should not be necessary for the user.

We present CompHEP the interactive system for above-stated task. In the framework of the CompHEP project the following possibilities are realized now:

1) choice of the model lagrangian;
2) selection of the process (and subprocess);
3) generation and graphical representation of Feynman diagrams;
4) construction of analytical expressions corresponding to the diagrams;
5) symbolic calculation of the squared matrix element.

SYMBOLIC COMPUTATIONS SYSTEM ON PERSONAL COMPUTERS FOR BOOLEAN ALGEBRA

S.V.Kolyada
Glushkov Institute of Cybernetics, Kiev, USSR

Boolean algebra as scientific discipline has a few features. It is a pure mathematical theory and, on the other hand, an applied mathematical theory too. Boolean algebra is applied, for instance, to improve intelligence of software, to automate theorem proving as it can be used to model situation analysis and decision making.

Computer algebra system for boolean algebra (APAL-PC) allows to write and process logical formulae in usual manner. The system APAL-PC consists of the following components:
- standard functional bases (0,1,&,∨,¬) and (0,1,+,*) for writing formulae;
- standard systems of relations for each standard functional basis, each of the systems is applied for usual boolean algebra formula processing;
- a set of operators for formula processing. These operators can be used to evaluate, order and expand formulae, to move brackets in formulae, to transfer formulae from one functional basis to another equivalently, to prove equivalency and graphical identity of boolean formulae, to apply relations and systems of relations, as well as to copy and substitute into formulae;
- control tools - operators to save and restore results for given number of processing steps and the set of canonical forms.

The system APAL-PC is designed for IBM PC personal computers on the basis of the programming language C and universal formula processing tools which are developed at Glushkov Institute of Cybernetics by A.A.Letichevsky and his colleagues. The experience of development of a similar system APAL-ES (APAL-ES implemented in OS/360 environment) is taken in consideration in designing of the system APAL-PC.

SYMBOLIC MANIPULATION IN THE UNICALC SYSTEM

V. E. Dmitriev, A. L. Semenov, I. E. Shvetsov
Computing Center of the Siberian Division
USSR Academy of Sciences, Novosibirsk

Computational models [1] based on the concept of subdefinite objects allow a new kind of programming systems to be created by means of constructing functional networks and their subsequent data flow interpretation. The UNICALC system [2] intended for solving joint systems of equations and inequalities on IBM PC -compatible personal computers is implemented on the basis of these models.

A user of the system is provided by a multi-windowed editor for data input and editing and by a menu to choose necessary commands. To initiate a process of calculations, a system of equations and inequalities (which may have three types of subdefinite variables: integer, real, Boolean and integer and real constants) should be entered and an accuracy of calculations should be defined. The results are either exact solutions, or intervals which the solution belongs to.

In the framework of the UNICALC system symbolic manipulations are currently carried out at the stage preceding calculation. Since all the equations and equalities are translated into the net representation before being calculated, a size of the net sufficiently depends on both a number of the equations and inequalities and a number of their variables. Thus, efficiency of calculations may be raised by means of similar term reduction, extraction of common factors, selection of common subexpressions and by decreasing a number of the equations and inequalities by analytical solution of some equations. These actions are performed by a symbolic manipulation processor considered in the paper. This processor allows all the equations and inequalities to be presented in a special form, being a variant of Cantorovich schemes, and then a calculator transforms these schemes into a net representation. Further the system intends to integrate numerical and symbolic capabilities in a scientific computing system.

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EXPERIMENTAL SOFTWARE TOOL FOR DEVELOPING COMPUTER ALGEBRA SYSTEMS FOR "PRAVETZ-16" MICROCOMPUTERS

M.M.Nisheva-Pavlova

Department of Mathematics and Computer Science, Sofia University

Bulgaria

In the last years computer algebra systems (CAS) have been widely used in science, engineering and education. In many countries CAS have been applied in teaching mathematics in the secondary schools and universities.

The paper examines some problems of the application of CAS in research, design and education. Some difficulties concerning the application of existing CAS in these fields are discussed and a project of an intelligent software tool for developing computer algebra systems for "Pravetz-16" compatible microcomputers is presented. The design and the implementation of the experimental version of this instrument computer algebra system are described.

The functional facilities of the system include some means for performing the following main mathematical operations:

- infinite precision integer and rational arithmetic;
- polynomial arithmetic;
- formal differentiation;
- definition of substitution rules in algebraic expressions;
- automatic and user-drived algebraic simplification;
- computation of algebraic expressions.

Some possibilities for rational and elementary transcendental function manipulation and some graphic means are provided too.

The user's work with the software tool is supported by some proper means for filling the system knowledge base with information about new mathematical object types and new symbolic manipulation rules and algorithms.

MAS
Modula–2 Algebra System

Heinz Kredel
University of Passau,
Innstraße 33, D–8390 Passau, FRG

December 13, 1989

The MAS system is discussed with emphasis on two points: the *non–commutative polynomial ring* package and the *symbolic–numeric* interface.

MAS is an experimental computer algebra system combining Modula–2 program development, a LISP interpreter with a Modula–2 like language. MAS further supports generic functions and access to the comprehensive ALDES/SAC–2 algebraic algorithm libraries by G. E. Collins and R. Loos. Current implementations run on an Atari 1040ST / GEM–TOS, and IBM–PC / MS DOS (or compatible). It is completely written in the programming language Modula–2.

We will discuss the implementation and usage of a package for arithmetic in **non–commutative polynomial rings** of solvabe type. Such polynomial rings have been defined by A. Kandri–Rodi and V. Weispfenning and include e.g. enveloping algebras of Lie algebras. The package allows the computation of left and two–sided Gröbner Bases with respect to various term orders, which can be specified also by linear forms.

We describe a **symbolic–numeric** interface between MAS and parts of a numerical program library developed by G. Engeln–Muellges and F. Reutter. The interface allows not only so call numerical programs from MAS, but moreover it is also possible to call symbolic MAS routines from the numerical programms.

USER-FRIENDLY FEATURES OF ORTOCARTAN

Andrzej Kraśiński

N. Copernicus Astronomical Center, Polish Academy of Sciences

Bartycka 18, 00 716 Warszawa, Poland

The talk will discuss the algorithms of those parts of the ORTOCARTAN system [1] whose counterparts in other systems tend to cause troubles for the users. The purpose of the talk will be to present to other programmers the already tested solutions to those problems. It will concentrate on two of those solutions: differentiation and substitutions. ORTOCARTAN has the chain rule for differentiation built-in. Consequently, if a function depends on a variable x not directly, but through a chain of secondary quantities, e.g. $f(g(u(x)))$, then $\frac{df}{dx} = f_g\, g_u\, u_x$ will be calculated automatically. This is often a problem in other systems (references omitted in order to avoid negative advertising). In ORTOCARTAN the difficulty is solved by storing the arguments of each function in its property list and by a simple use of recursion.

Each substitution is automatically followed by an algebraic simplification. This principle should in fact become an axiom of algebraic programming because the only purpose of substitutions is to cause simplifications. ORTOCARTAN's substitution package was optimized so that it avoids unnecessary copying of the list structures and simplifies the result beginning with the level where the substitution actually occurred. It is apparently either the single or one of very few working systems that can do pattern-matching. The basic ideas of the substitution algorithm will be presented.

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STENSOR, a System for Tensor- and Noncommutative Algebra

Lars Hörnfeldt
Physics, Univ.of Stockholm, Vanadisv.9, S-113 46 Stockholm, SWEDEN

STENSOR is a computer algebra system devoted to tensors with symbolic indices and noncommuting objects. During 14 intense years the following unique features have evolved:

•Full Simplification of Symbolic Tensor Expressions

-ie with symbolic indices on tensors. Any index symmetries are allowed, and consequences understood, eg that: $A_{ij}S^{ij} = 0$, if (A)S are (anti)symmetric respectively.

•Noncommutative- and Operator Algebra

Handling octonians, Clifford/Grassman algebra, Poisson brackets, variations, difforms...

•Kaluza-Klein, Supergravity, Spinors

Splitting into any number of subspaces, multiple indextypes/conventions for these subspaces, Fiertz-transformations, gamma algebra and trace for any dimension/metric....

•Sum-Substitutor fully Exploit Trig- and other Sumrelations

Eg: $-cos^6x sin^2x + cos^6x - cos^2x sin^2x + sin^2x \Rightarrow sin^6x + cos^6x$. Other systems tend towards pure sin (or cos) results, while **STENSOR** returns the minimal mixture! From the cyclic Riemann symmetry (sumrelation) **STENSOR** derived the seemingly new identities: $R_{ijkl}R_{klmn}R_{mnij} = 4R_{ijkl}R_{ikmn}R_{jmln} = 2R_{ijkl}R_{ikmn}R_{jlmn}$ from the (accidental) LHS input. The two reformulations must be among the most 'creative' responses in computer algebra!

•Generate Tensor Algorithms for Component Computation

From symbolic formulae,algorithms are generated that compute tensor components often faster than handcoded algorithms. Especially this is done in symbiosis with the relativity system **SHEEP**.

•Disk-Bucketing for Multi - Million Term Calculations

Expression parts (buckets) can be automatically shuffled to/from disk at need. So was the general Ricci-tensor in 5 dimensions computed; R_{mn} had 263,598 terms, exceeding eg Reduce/Macsyma capability by orders of magnitude. Time was a reasonable 100h, with a negligible part spent in disk i/o, so much larger calculations than this are possible.

The last item above is provided by one other system too: Schoonship. The other features are elsewhere offered only partly or not at all.

User interface: •Formula input close to textbook conventions. •Do-What-I-Mean toplevel with menus. •History and modification mechanism a la unix cshell. •On line help that looks up appropriate documentation. •Extensively commented lectures/demos that you step through interactively. •A TEX-mode cause **STENSOR** create a session file you can send directly to TEX:-the handiest way to produce papers. •User guide, which was produced by **STENSOR** in its TEX-mode. •Reference manual,120 p. •**STENSOR/SHEEP** can run alone or inside **REDUCE**, fully exploiting its features like GCD and factoring.

Machines: VAX Unix or VMS, ORION (RLH ltd), SUN, Atari ST, Amiga (untested). Also on the 8800 Atari ST 1040 + hard disk. **STENSOR** can with disk-bucketing still handle millions of terms, more than conventional systems on a mainframe! Versions for Acorn machines are planned: the BBC + NS32016 coprocessor, and Acorn Archimedes. May now also run on PSL-lisp, for eg APOLLO, CRAY and IBM3090 machines.

Literature: Beside guide + manual see Springer-Verlag *Lecture Notes in Computer Science* vol.72,p279-290(EUROSAM 1979,ed Ng) "A System for Automatic Generation of Tensor..", vol.144,p188-195(EUROCAM 1982,ed Calmet) "A Sum-substitutor used as Trigonomet.." and eg International Conferences on General Relativity and Gravitation: GR12,11,9,8.

ALGEBRAIC PROGRAMMING SYSTEM APS-1

J. V. Kapitonova, A. A. Letichevsky, S. V. Konozenko
Glushkov Institute of Cybernetics, Kiev
USSR

APS system is intended for creating of environment and mechanisms of applied problems solving, which involve functional, algebraic and logical programming as tools. The main idea of APS is to the use of algebraic objects to represent data and knowledge about subject domains and to use rewriting system technique for data manipulation.

The APS-1 version of the system is implemented in c language on IBM PC compatible computer. Source language of the system is algebraic programming language APLAN which makes it possible to combine algebraic and procedural programming: rewriting rules and efficient strategies for their application.

Experiments in the system were carried out in such subject domains: algebra (polynomials, analytics, algebra of logic, free groups), computer logic (solving of problems on relational models, solving of boolean equations), discrete systems (Petry Nets, systems, defined by means of productions), set theoretical models of subject domains.

System is developed on followed directions:

- increasing of algebraic programming language possibilities of expression and adaptation for various subject domains;
- creating of mathematical models of subject domains and solving problems on such models[1];
- implementation of tools for optimization of algebraic programs by means of mixed computations (transformation of interpreters with rewriting systems into C programs).

Comparison with well known systems (MACSYMA, REDUCE e.t.c.) show that APS is tool, in which along with more universal techniques of data structures creating and computations the high efficiency of problem solving in specialized environments can be achieved.

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ONE PARTICULAR CODE FOR POLYNOMIAL MANIPULATIONS

J. Obdržálek
Faculty of Mathematics and Physics – Charles University
Ke Karlovu 3, 121 16 Prague, Czechoslovakia

For the general symbolical manipulations, the list form of coding is usually used; namely, the expression $a^2b - 6abd$ is coded in a form like

a% ^% 2% *% b% –% 6% *% a% *% b% *% d%, resp.
a% 2% ^* b% *% 6% a% *% b% *% d% *% –%, using RPN,

where the symbol % represents a pointer to the next atom. This is the case of REDUCE, mu–Math etc.

Handling polynomials only, more compact manner may be used. An example is in POLYMAN [1], using for the above mentioned expression code

1.000	2	1	0	0
–6.000	1	1	0	1
0.000	0	0	0	0 ,

i.e. each term coded by one real coefficient and set of integers giving the exponents by corresponding variables. Terms are ordered lexicographically, a zero term denoting the end of the polynomial. This manner is speed increasing and space saving, both factors being crucial for small computers. It is supposed that the number of variables is limited and kept fixed in frame of one calculation. Sophisticated methods can be used to overcome "compatibility problems" when chaining more calculations together.

Particular physical problem presented by Cordts, Deus and Frei [2] led to an evaluation of the determinants of matrices M, M^2, M^3. Elements of 24 * 24 matrix M are taken from + 50 different symbols A_1..H_6. Symbols A..H have to be further expressed as simple polynomials in 10 variables a,b... This problem has been solved by a program written in PASCAL, handling homogeneous polynomials of limited degree (here 3) in many (50) variables coded as

1.000	1	1	2
–6.000	1	2	4
0.000	0	0	0 ,

i.e. each term coded by one real coefficient and three integers denoting the ordinal number of the variables making one term of the homogeneous polynomial (of the degree of 3, in this case). Terms are ordered lexicographically, zero term denoting the end of the polynomial.

This form of coding enabled even an 8-bit microcomputer BBC Acorn to perform rather large symbolical calculations connected to the topics of [2].

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FORMIS - AN INTERACTIVE SYSTEM OF ANALYTICAL CALCULATIONS

A.Yu. Zharkov, B.L. Faifel
Saratov State University, 410600 Saratov, U.S.S.R.

In the present work FORMIS (Formula Manipulation Interactive System) which is an interactive superstructure of the well-known PL/1 - FORMAC system is described. In the proposed FORMIS system EC-7920 devices are used; it can operate both in the PRIMUS environment and in the OS IBM environment (in the batch regime).

The input language of the FORMIS system consists of two subsets: the symbolic subset including the FORMAC system statements and functions for symbolic calculations, as well as the command environment which is a BASIC-type language. FORMIS works in the interpretation regime which enables to eliminate macrogeneration, compilation and linkage edition steps and facilitates communication with the system.

The main statements of the FORMIS command environment are SET, PRINT, IF, GOTO, FOR, NEXT, CALL, PROCEDURE, LOAD, DELETE. SET is an assignment statement for the command environment variables. PRINT realizes output of command variables. Each statement can be labelled; any combination of letters and digits not exceeding 8 symbols may serve as a label. GOTO realizes an unconditional branch to a label while IF is used for a conditional branch. FOR and NEXT are used for loop organization; CALL and PROCEDURE organize a call for procedures; LOAD and DELETE are used for dynamic loading and deleting procedures. Command variables have the integer type not exceeding 2^{31} in absolute value.

When entering a procedure by CALL the command variables environment is being generated anew while the old one is stored which enables to organize recursive procedures.

Possible recursion depth is limited only by the dimensions of an accessible region of the main storage.

Command variables may be used in symbolic expressions, in this case they must be double-quoted. For the comparison of two symbolic expressions, IDENT and EQUIV functions are used in the described FORMIS system which return the integer values - 1 and 0 - in the case of identity and difference of their arguments, respectively.

COMPUTER ALGEBRA SYSTEM FOR PERSONAL COMPUTER ON FORTH

E.N. Kruchkova
Altay politechnic institute, Barnaul, USSR

It is a consideration of Forth's possibilities as a programming tool for personal computers system of computer algebra. The use of Forth allows to realize a computer algebra with maximum simple in the following ways:

1) some mode variable's declaration in program provides the creation of corresponding vocabulary entry, in consequense of it the system would be suitable for the context in which programming data appears;

2) a value of each analytic variable is kept as a Reverse Polish Notation, it corresponds to Forth's principles and allows easily to realize all data operations;

3) any declared variable used in program will cause appearance of this value address on the stack, since this address is kept in code field of corresponding vocabulary entry both during the declaration (reference to own name field) and in executing (reference to the calculated value);

4) each operator type (assignation, differentiation, etc.) corresponds its vocabulary; switching from one operator to another one is performed by switching over to corresponding vocabulary;

5) there is every operation symbol (+ , - ets.) in all vocabularys and code operation field has data processing in accordance with their mode (integer, symbolic, etc.) and operator type;

6) any operation appearance provides the execution kept in code field of this current vocabulary.

System's realization on Forth enables the user to change, add or delete any system part adapting it to a required applied field.

SOME ASPECTS OF SYMBOLIC MANIPULATIONS CONCEPT IN THE SYSTEM FOR SYMBOL-ANALYTIC TRANSFORMATIONS SANTRA-2

I.B.Shchenkov

Keldysh Institute for Applied Mathematics,

The USSR Academy of Sciences, Moscow

USSR

The specific feature of the Santra-2 system is that symbolic manipulations means inhering in the implementation languages of such advanced systems for analytic computations as Macsyma and Reduce are included at the source language level. These means take an intermediate place between Refal language statements and mathematical recurrence relations and have facilities of them both. They provide universality of analytic computations due to primary nature of symbolic manipulations. The source language with such features provides adequacy of mathematical objects representation in various forms.

The symbolic manipulations means give the system such properties as

- possibility for creation of problem-oriented packages with complex data structure;
- data abstraction support due to some way of generalised pattern matching;
- possibility to create additional superstructure source languages.

Superstructure language example is source language of Dislan system for difference scheems construction on irregular nets. Every

constructed scheem is represented by Fortran program for coefficients of this scheem computation. The system Santra-2 is also in use for nonlinear differential equations group properties investigation.

Implementation language of the system is Refal. It allowes to apply supercompiler methods developed by V.Turchin both to programs written in the source language and to the system description itself.

The system is implemented on IBM 370 computers and on IBM PC XT & AT.

VIBRAN IMPLEMENTATION FOR PERSONAL COMPUTER

R.Kulvietiene, G.Kulvietis

HCLI, Vilnius, USSR

For Vibran implementation was used the Microsoft Fortran Optimizing Compiler for the MS-DOS operating system, which generates fast, efficient native code. The compiler includes the following possibilities: a choice of miltiple and mixed memory models; math support, including floating-point emulation, coprocessor support; large program support.

Vibran uses one floppy disk, which consists of Vibran interpreter, object code library and fortran-code generation program. The vibran version is fully compatible with earlier versions for ES and CM computers.

PREPROCESSOR "ALGEBRA-88" IN THE REPRESENTATION THEORY

YU.P.Razmyslov, V.V.Borisenko

Moscow University, Department of Mathematics, USSR

One of the main problems in the theory of representations is investigation of linear operators, presented by formulas in some fixed elementary operators. To solve this problem using computer we divided it in three parts:

(i) to create a library of subroutines realising actions of elementary operators,

(ii) to write programs realising operators, presented by formulas in elementary operators,

(iii) to find eigenvalues and factors of characteristic polynomials of such "compound" operators, to solve systems of linear algebraical equations with these operators, etc.

The authors have created the FORTRAN-preprocessor executing the second step for algebraical formulas in vectors and operators (formulas written in the form used by mathematicians).

We suppose that each elementary linear operator is presented by a subroutine calculating the image of any given n-vector. The preprocessor transforms strings of the shape <vector> = <expression> , where <expression> is an arbitrary formula in vectors, operators, polynomials in vectors and operators, signs "+", "-" and "*", into sequences of CALL-statementes.

The preprocessor enables one to define different parameters of representations, to work with arrays of vectors, parametrized operators, polynomials, etc.

The text of the preprocessor consists of about 2000 lines in the C programming language. The program can be easily modified if one wants to use C instead of FORTRAN.

The authors use the preprocessor together with a package of programs for representation theory, including programs for computation of eingenvalues and factors of characteristic polynomials in the spirit of the Lancosh method and the conjugate gradient methrod. All algorithms are based on computation of images of vectors under operator action and do not require the full matrices to be stored in the computer memory. A size of necessary memory linearly depends on a dimension of representation. It makes possible to deal with large dimensions even on mini-computers (e.g. PDP-11/70).

ON A CONCEPT OF KNOWLEDGE REPRESENTATION ON METHODS OF INVESTIGATION OF DIFFERENTIAL EQUATIONS.

A. V. Bocharov & D. L. Shishkov

IPS AS USSR, Pereslavle-Zalessky, Mech.-math. MSU, Moscow, USSR

Programs are widely spreading nowadays based on consulting techniques connected with a certain knowledge base and with certain input data processing as well as with previous experience on analogous problems.

We pesent a progress report on an approach to development of a knowledge base on the methods of exact analytic and algorithmic investigation of differential equations systems on computers. A feasibility of building such a knowledge base with frames and rules of Personal Consultant Plus type is discussed.

Each frame is a structure, including the following principal objects : Goals, Parameters, Variables, Rules, Meta-Rules and Functions. We discass in more detail the semantics of these objects in the context of the differential equation theory. The Goals associate here with the Strategies for investigation of differential equetion, Parameters are associated with certain Features allowing to classify the equations as belonging to certain type, class or subclass and so on.

The theory of differential equations knows many assertions on solvability and resolution algorithms for differential equations which far from being exact theorems are still valid sources of likely conjectures (liable to verification in each particular case). This allows to attach to each frame together with the exact algorithms also a finite number of viable strategies, for solutions of equuations, and this attaching may serve as a base for the work of a deductive machine.

A GEOMETRICAL APPROACH IN TENSOR ALGEBRA COMPUTER MANIPULATIONS

V.Ilyin, A.Kryukov, A.Rodionov, A.Taranov
Research Institute for Nuclear Physics
Moscow State University, 119899 Moscow, USSR

There are well-known problems in tensor algebra manipulations when tensors have a symmetry group, and/or dummy indices are used, and/or linear identities (Bianchi ones in the case of Riemann tensor for example) are presented. The problems due to symmetry and dummy indices can be solved in the combinatorial approach. This problem may be formulated in terms of double cosets in symmetric group (refs /1,2/). However linear identities can not be treated similarly since the linear operations may be implemented only in the group algebra of a permutation group.

We suggest a geometrical approach to formulation and solution all of the problems. A linear space is constructed for this purposes. A point of this space corresponds to a sum of tensors with numeric coefficients. We name this sums as T-expressions. A linear identity can be treated now as a vector (designated as I below). It can be easily verified that two T-expressions are "equivalent" (one may be transformed into another using the identity) if the difference between the corresponding vectors is proportional to the vector I. Let us designate the orthogonal to I subspace as V_T. We conclude that the orthogonal projection to subspace V_T solves problems due to the linear identities.

Note that symmetry group relations and ones due to dummy indices can be presented by some vectors. In other words these relations may be viewed as linear identities. There are a number of vectors (identities) $I_1,\ldots,I_N$ and corresponding orthocomplement V in this case. Now we can formulate the main

conclusion: the orthogonal projection to V_r solves all tensor manipulation problems due to a group symmetry, dummy indices and linear identities.

Let us make some remarks about the algorithm, which is induced by this conclusion. The task is reduced to orthogonal projection in our approach. Such projection may be implemented via scalar product calculations only. Operations connected with scalar product may be performed with high efficiency and can be easily vectorized or parallelized. It can be added that in applications group symmetry coefficients and identity ones are rational numbers often. In this case the rational (indeed natural) arithmetic is used in our algorithm only.

Note that tensor manipulations are faced with other problems, like term reduction for example. This problem can be solved by transformation from the linear space to a corresponding projective space. The subspace V_r will have an image in this projective space and the orthogonal projection to V_r will have a corresponding analogue too. There are similar natural development of our approach for some other complicated features in tensor algebra manipulation.

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ON A DEFINITE INTEGRAL OF THE PRODUCT OF TWO POLYLOGARITHMS

K.S. Kölbig

CERN, European Laboratory for Particle Physics
1211 Geneva 23, Switzerland

As a result of progress in non-numerical computing, especially symbolic algebra, it is becoming increasingly possible to replace the method of numerical quadrature, widely used for the evaluation of integrals occurring in scientific and technical applications of computers, by representations in closed form (finite and infinite). For indefinite integrals of elementary functions, the Risch algorithm is an established part of most symbolic algebra system. However, for indefinite or definite integrals of special functions, only partial results are available (apart from reference to integral tables).

Even if one considers only definite integrals involving special functions, the absence of a generally applicable method is hardly surprising in view of the disparate techniques used for their evaluation and the widely differing forms of the results to be found in the tables. We shall illustrate this by means of the integral

$$I_{n,m}(\alpha,\sigma,\omega,r) = \int_0^\infty x^{\alpha-1}\, \mathrm{Li}_n(-\sigma x)\, \mathrm{Li}_m(-\omega x^r)\, dx$$

$(m,n \in \mathbb{N}, \alpha,\sigma,\omega \in \mathbb{C}, r \neq 0 \in \mathbb{R})$, where $\mathrm{Li}_k(z)$ is the polylogarithm. In the degenerate case $n = m = 1$ this integral reduces to

$$I_{1,1}(\alpha,\sigma,\omega,r) = \int_0^\infty x^{\alpha-1} \ln(1+\sigma x) \ln(1+\omega x^r)\, dx.$$

(The general result, obtained by residue calculus, and some special cases, have been published elsewhere.) The talk will present some other special cases and discuss the use of symbolic algebra to simplify sub-expressions, represent infinite series in terms of known functions, etc.

Symbolic Integration in Computer Algebra

Manuel Bronstein

IBM Research Division
T.J.Watson Research Center
Yorktown Heights, NY 10598
USA

One major goal of symbolic integrators is to determine under what circumstances the integrals of the elementary functions of calculus can themselves be expressed as elementary functions. While using tables and the *ad hoc tricks taught* in calculus courses can have some limited success, a decision procedure is necessary in all but the most trivial cases. The first complete algorithm for solving this problem was presented by Risch /1,2/ in 1969, but its complexity, specially when algebraic functions are present in the integrand, has prevented it from being fully implemented. Over the past 20 years, the Risch integration algorithm *has been completed, extended*, and improved *to* such a point that recent computer algebra systems can integrate elementary functions without using any of the heuristics traditionally taught in calculus courses and used by older systems. In *this talk*, we give an overview and description of the algorithms used in the Scratchpad symbolic integrator, and illustrate them with integrals drawn from the physical sciences.

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ANALYTICAL APPROXIMATE SOLUTION OF SINGULAR ORDINARY DIFFERENTIAL EQUATIONS

K.Hantzschmann, N.X.Thinh

Wilhelm-Pieck-Universität, Rostock, GDR

The main problem of the computer analysis consists of finding of formula approximate solutions, which should be adapted to the properties of the problem, simple and transparent. This includes in any case error boundaries which are evaluated completely by the computer.

For the approximate solution of ordinary differential equations the following method has proved a success: the approximation takes place in two steps. In the first step the given problem will be adapted by a suitable chosen neighbour problem, whose closed definable solutions form the base for the approximate ansatz.

We consider the initial value problem

$$L(y) = \sum_{i=0}^{k} x^i R_i(x) y^{(i)}(x) = 0 \qquad 0 < x \leq b$$

$$y^{(i)}(b) = \beta_i \qquad (i = 0(1)k-1),$$

where x=o is the regular singularity. In the step of adaption an Euler's differential equation will be determined, which should reflect the behaviour of the solution in the neighbourhood of the singular point qualitatively good. Various algorithms result in dependence of the chosen adaptive criterion. With the fundamental solutions of these Euler's differential equations approximate solutions will be determined in usual sense by various approximate criteria in the step of approximation. In this paper some of these algorithms will be presented. Good results could be achieved by different types of differential equations in the step of adaption too. These allow an extended rational approximation.

For error representation and error estimation we use the formula determinable Green's function of a neighbour Euler's differential operator k-th order. For the error function $f=y_N-y$ of the approximate solution y_N we can prove the following estimation:

$$|f(x)| \leq \frac{\|G(x,\cdot)\|_x}{1 - \|L_\Delta(G(\xi,t))\|_x} \|d(y_N)\|_x$$

with the defect $d(y_N)$, if $\|L_\Delta(G(\xi,t))\|_x = \left(\int_x^b \int_x^b L_\Delta^2(G(\xi,t))\,dt\,d\xi\right)^{1/2} < 1$

($L_\Delta = L_k - L$, $\|u\|_x$ L^2-norm on $[x,b]$).

The difficulties of numerical and analytical evaluating of error boundaries could be mastered by the aid of suitable relief measures. For the realization of the described algorithms and error estimations we have developed a programme system based on the CAS FORMAC.

USE OF META-LEVEL INFERENCE FOR SYMBOLIC SOLVING OF EQUATIONS

P.Zielczyński, E.Pierzchała

Atomic Energy Institute, 05-400 Otwock-Świerk, Poland

We have implemented in Atomic Energy Institute a system for trigonometric equation solving. This system is based on meta-level inference method proposed by the team of computer scientists from Edinbourgh /1/.

Usually a very large solving state space is required during symbolic solving of equations. In order to decrease this state space, use of two inference levels is proposed. In every step of equation transformation three operations are realized. First, with the aid of few conditions the "method" (meta-level problem-solving operator) is chosen. Next, a low-level operator is chosen from a relatively small set of operators of this method (and not from a large set of all possible operators). After applying chosen operator the procedure is repeated for the next processing step.

The system was implemented in two versions - in Lisp and in Prolog in order to compare usefullness of these languages to symbolic processing on IBM PC XT and AT /2/. The system is based on expert system techniques what made a knowledge base with method and operator description very easy to maintain.It allows one to add new methods and operators in order to increase the power of the system.

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THE COMPUTER ALGEBRA PACKAGE CRACKSTAR AND EXAMPLES FOR ITS APPLICATION FOR THE EXACT SOLUTION AND ANALYTICAL INVESTIGATION OF DIFFERENTIAL EQUATIONS

Thomas Wolf, Andreas Brand

Friedrich Schiller Universitaet Jena
Rechenzentrum, Humboldt Str. 2,
GDR 6900 Jena, GDR

When carrying out ansaetze for investigating nonlinear differential equations (DEs) one usually has to solve overdetermined systems of di ferential equations. Whereas the formulation of those systems is performed with different programms, it is intended to use one program package for their solution. We report on improvements of the underlying algorithms for decoupling DEs and for the integration of exact partial DEs with arbitrary functions. A further point is the implementaion of CRACKSTAR in REDUCE.
Examples for possible applications are given, as
- symmetry investigations (point-, contact-, dynamical sym.),
- determination of integrating factors,
- arbitrary differential transformations,
- investigation of separation ansaetze,
- factorization of DEs,
- determination of Lagrangians for given DEs.

REPRESENTATIONS OF ALGEBRAIC CURVES

Franz Winkler

Institut für Mathematik and Research Inst. for Symbolic Computation
J. Kepler Universität, A-4040 Linz, Austria

Algebraic varieties, the main object of study in algebraic geometry, can be represented in essentially two different ways, namely as the set of zeros of finitely many polynomial equations, e.g. $V = \{(x,y) \mid 2x^4 - 3x^2y + y^2 - 2y^3 + y^4 = 0,\ x,y \in \mathbf{C}\}$, or as the set of values of rational functions

$V = \{(\phi(t),\chi(t)) \mid \phi(t) = -\frac{18t^4+21t^3-7t-2}{18t^4+48t^3+64t^2+40t+9}, \chi(t) = \frac{36t^4+84t^3+73t^2+28t+4}{18t^4+48t^3+64t^2+40t+9},\quad t \in \mathbf{C}\}$.

We call the first representation implicit and the second explicit or parametric. The representation of choice is of course determined by the operations one wants to perform with the variety. For determining whether a given point is a point of the variety, or for computing singular points of the variety, the implicit representation is more desirable than the parametric one. On the other hand, the parametric representation lends itself very easily to the determination of the curvature, to tracing of varieties, and in particular to visualizing them on a computer screen. The intersection of varieties can be determined rather easily if one of the varieties is given implicitly and the other one explicitly. For this reason it is essential to be able to switch between different representations.

Arnon and Sederberg investigated the problem of computing the implicit equation from a given parametric representation. This is a problem of elimination. The problem of computing a rational parametrization from the given implicit equations, especially for plane curves, is a classical problem in algebraic geometry. Intuitively speaking, a curve is parametrizable if it has enough singularities. The method suggested in Walker's book on algebraic curves and elaborated by Abhyankar and Bajaj proceeds by computing these singularities and sufficiently many simple points on the given curve of degree d. Through these points a pencil of curves of degree $d-2$ is passed, such that every element of the pencil

intersects the given curve in exactly one additional point. The additional intersection point yields the desired parametrization. We show that it is also possible to work with pencils of degree $d-1$ and d. In fact, these pencils are more attractive from a computational point of view. The determination of simple points on the curve introduces a lot of algebraic numbers. If they are not controlled, any further computations with the parametrization soon become too inefficient. We show that a pencil can be passed through a set of points on the given curve without having to compute these points explicitly.

Our main result /1/ can be stated as follows.

THEOREM: *Let C be an irreducible plane algebraic curve of degree d. Let $F(\alpha)$ be an algebraic extension of F such that all singular points of C have coordinates in $F(\alpha)$. Then a parametrization of C can be constructed in an extension of degree d over $F(\alpha)$.*

The parametrization computed by the algorithm has the lowest possible degrees in the numerators and denominators.

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IMPLEMENTATIONAL ASPECTS FOR NON-COMMUTATIVE DOMAINS

Joachim Apel, Uwe Klaus
Sektion Informatik
Universität Leipzig
DDR-7010

In this paper we discuss fundamental problems connected with the implementation of arithmetics of non-commutative domains.

In general the representation of such algebraic structures can be difficult since the existence of canonical simplifiers for the elements and algorithms for the basic operations are not ensured.

Subject of this paper is a special class of such domains for which there exist suitable data structures and algorithms for computing the basic operations +, -, * and $()^{-1}$.

Important members of the above class are enveloping algebras of Lie algebras, u-algebras of restricted Lie algebras, Clifford algebras, modules over these algebras and in the case of existence the quotient skew fields of them.

Finally, we describe our experiences with a small experimental computer algebra system especially designed for this field on IBM-PC.

RELATIONAL APPROACH TO MODELING OF ALGEBRA OF MULTIPLE SERIES MATRICES

E.A. Grebenikov
Institute of Cybernetics Problems, Moscow
N.D. Lozovsky, I.L. Tolmachev
Nation's Friendship University, Moscow
USSR

Algorithms of asymptotic theory for nonlinear multi-frequency differential equations extensively use operations on matrices of multiple trigonometric and power series.These equations appear in the field of nonlinear and celestial mechanics.

Universal computer algebra systems (REDUCE, MuMATH) running on personal computers have low speed and restricted size of mathematical objects. So implementation of our matrix algebra using the systems is not efficient.

This paper suggests to use relational database managment system for the implementation of matrix algebra on personal computers. We suggest to extend classical relational data model incorporating some ideas of object-oriented programming. We demonstrate that matrices of multiple series and base operations on them (multiplication, addition, construction of Jacobian matrix, extraction of resonant terms) can be naturally expressed in terms of the extended relational data model.

We use relational language Dbase and C for programming the prototype system based on this approach. Object-oriented virtual memory manager for the system is described.

ALGORITHMS FOR OPERATIONS WITH DIFFERENCE OPERATORS AND GRID FUNCTIONS IN SYMBOL FORM

V. G. Ganzha
Institute of Theoretical and Applied Mechanics
Siberian Department of the USSR Academy of Sciences, Novosibirsk
A. V. Solovjev, M. Ju. Shashkov
Keldysh Institute of Applied Mathematics
USSR Academy of Sciences, Moscow
USSR

The notion of grid function and a difference operator are used in constructing difference schemes. We perform arithmetic operation with grid functions, summation of operators, multiplication of an operator by a number and a function. The basic operations here are the superposition of difference operator and taking adjoint operator. In terms of symbol transformation, the grid functions are variable with indices, the latter varying in limits given by letters or numbers. The grid function may have various symbols or numerical value in various intervals of symbolic variations. It means that in analysing the grid functions in terms of symbol transformations a new notion arises such as region for unified assignment of symbol expressions (it is a region of the index variations where identical symbol representations are used for given function). A similar notion is introduced also for coefficients of difference operators. When we use the symbol transformations to execute operation with grid function and difference operators, our main task is to define the uniformity domain; for example, the uniformity domain of operator sums may prove to differ from those of both the first and second summands. Even a more complicated situation arises in the superposition of operators and adjoint operators. In this report the algorithms are given for constructing the uniformity domain for operation with grid function and difference operators. It is shown that the problem is reduced to determining joins and intersections of some sets obtained from original uniformity domain with involvement of scales of difference operators. The algorithm realization in the REDUCE language is described.

DECOMPOSITION THEOREMS FOR THE ZERO-SET OF AN ORDINARY OR DIFFERENTIAL POLYNOMIAL SET AND THEIR APPLICATIONS

Wu Wen-tsun

Institute of Systems Science, Academia Sinica, China

By a POLSET will be meant a finite set of ordinary polynomials or POLs in K[X1,...,Xn], K being a field of characteristic 0. A polset PS is called an ASCENDING SET (abbr. ASC-SET) if the following is true:

The variables Xi can be separated into two parts U = (U1,...,Ud) and Y = (Y1,...,Ye) with d+e=n such that the pols in PS can be arranged in an order F1,...,Fe with

Fi = Ii * Yi**Di + lower degree terms in Yi, Di>0,

verifying: (1) Fi is in K[U,Yj / j < i]. (2) The degree of Ii, called the INITIAL of Fi, in any Yj is less than Dj for each j<i.

For any polset PS the set of all zeros of PS in arbitrary extension field of K will be denoted by Zero(PS). Put for any pol G, Zero(PS/G) = Zero(PS) \ Zero(G). Then we have (cf.e.g. [1]):

THEOREM. There are algorithms such that for any polset PS we have

Zero(PS) = Zero(CS/J) + Zero(PS+J) = SUMj Zero(ASCj/Jj)

= SUMk Zero(IRRk) = SUMk Var[IRRk].

In these DECOMPOSITION FORMULAE CS, ASCj, IRRk are all asc-sets with IRRk irreducible, J and Jj are products of all initials of pols in the respective asc-sets, and Var[IRRk] is the irreducible algebraic variety determined from the irreducible asc-set IRRk via its generic zero.

The above theorem and the decomposition formulae have also been extended to the case of differential polynomials, cf. e.g. [2]. These formulae have diverse applications for equations-solving and mechanical theorem-proving and even problems beyond pure mathematics, e.g. automatic derivation of Newton's gravitational laws from Kepler's laws, etc.

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CONSTRUCTION OF A LIE ALGEBRA BY A SUBSET OF GENERATORS AND COMMUTATION RELATIONS

Akselrod I.R.,*) Gerdt V.P., Kovtun V.E.**), Robuk V.N.*)
Joint Institute for Nuclear Research, Dubna

The problem of constructing the factor algebra for a free Lie algebra on an ideal which is given by a subset of generators and commutation relations is investigated. The method proposed to solve this problem can be applied in particular for constructing the L-A pair for nonlinear evolution equations [1]. The algorithm is based on the concept of Hall basis for a free Lie algebra. The initial row of the Hall basis is determined in accordance with the commutation relations defined by the concrete problem to be solved. The higher rows of the Hall basis are computed stepwise. In this process one of two alternatives can occur, which are analyzed by dialog to correct the process of computation: (i) The computation of some row gives a commutative relation belonging to one of the previous rows. In this case the previous row is modified correspondingly and the computation continues from this row. (ii) All leading Lie monomials in some of the commutation relations obtained so far have scalar coefficients. In this case all possible variants of linear dependencies amongst these monomials are considered, and the computation continues for each possible variant separately. The process comes to an end when either the Lie algebra is finite, or the structure of all sequences of rows can be recognised by induction. As a result, all linear independent Lie monomials and commutation relations of a given order are obtained. The program is written in the computer algebra language REDUCE. For a number of cases (KdV,MkDV,etc.), the infinite Lie algebra is constructed.

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*) Phys.-Techn. Inst. of Low Temperature, Kharkov, USSR.
**) Kharkov State University, Kharkov, USSR.

THE DIFFERENCES OF ROOTS OF POLYNOMIALS AND SOLUTIONS OF ODE

V.I.Galiev, A.F.Polupanov, I.E.Shparlinski
IRE AS USSR, J/E "Interface", Moscow
USSR

The problem of the determination of the existence of nonzero integers among differences of roots of the polynomial $f(x) \in \mathbb{Z}[x]$ often arises when constructing formal solutions of some systems of ordinary differential equations. For example, the properties of solutions in a neighbourhood of a regularly singular point (see[1,2]) and the algorithm of their construction (see[3]) depend drastically on the answer to this question. Besides, for a numerical stability of this algorithm an exact lower bound for the deviation of the named differences from nonzero integers must be known. Note that in the case of radial Schroedinger equation with some matrix Hamiltonians the problem of the roots of corresponding polynomial (roots of the characteristic equation) has been solved exactly and it was shown that these roots are integers.

In this note we show that for any polynomial $f(x) \in \mathbb{Z}[x]$ this problem can be solved in time bounded by a polynomial on the degree $n = \deg f$ and the logarithm of its height

$$H(f) = \max\{|a_n|, \ldots, |a_0|\},$$

where $f(x) = a_n x^n + \ldots + a_0$, i.e. a polynomial in a size of its writing in bits.

Let us denote by $\|\gamma\|$ the distance from a complex γ to the nearest integer (usually this function is defined only for real γ, in our case may be $\|\gamma\| > 1/2$).

For the polynomial $f(x) \in \mathbb{Z}[x]$ we put

$$\delta(f) = \min_{1 \le i < j \le m} \|\lambda_j - \lambda_i\|,$$

where $\lambda_1, \ldots, \lambda_m$ are all pairwise different roots of $f(x)$. Then the following theorem is valid.

Theorem. For any polynomial $f(x) \in \mathbb{Z}[x]$, $\deg f = n$, one can find in time $(n \log[H(f) + 1])^{O(1)}$ such δ that $\delta \le \delta(f) \le 2\delta$.

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TWO CLASSICAL SUBRESULTANT PRS METHODS

Alkiviadis G. Akritas,
University of Kansas, Lawrence, Kansas 66045-2192 , USA

Given two (univariate) polynomials over the ring of integers, the computation of their greatest common divisor (gcd) along with their polynomial remainder sequence (prs), is an old topic that has been extensively studied. While performing these computations over the integers, the basic problem is that of restricting the coefficient growth *without* integer gcd computations.

We examine in detail the two classical methods that exist in the literature for controlling this coefficient growth *without* integer gcd computations[1]: the Sylvester-Habicht pseudodivisions subresultant prs method (consisting of two algorithms) which was initiated by Sylvester in 1853 and was completed by Habicht in 1948, and the matrix-triangularization subresultant prs method which was developed by the author in 1986 (see also Figure 1).

It should be noted that Sylvester's paper of 1853 had been completely ignored; it was used only once by Van Vleck in 1899.

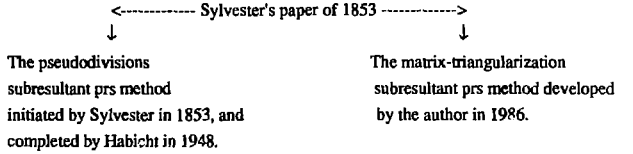

Figure 1. Overview of the historical development of the two classical subresultant prs methods. The method developed by Sylvester should be used only when the prs is complete, whereas the one by Habicht should be used when the prs is incomplete. The matrix-triangularization method can be used for both kinds of prs's, and, in certain cases, the coefficients obtained are smaller than those obtained with the Sylvester-Habicht method.

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APPLICATION OF GRÖBNER BASIS METHODS TO POLYNOMIAL EQUATION

Herbert Melenk

Konrad-Zuse-Zentrum für Informationstechnik Berlin

In recent years since Buchberger's intitial publication in 1965, Gröbner bases and related techniques were developed for dealing with equation systems of the type

$$0 = P_i(x_1, \ldots, x_k), i = 1, \ldots, n,$$

where the P_i are polynomials of some domain, in most technical relevant cases, $P_i \in \mathbb{Z}[x_1, \ldots, x_k]$. The theoretical completeness of the Gröbner basis approach contrasts with its limitation to "small" problems because of its explosion in computing time and memory requirements. Together with some examples, techniques are demonstrated which enlarge the application range by factorization, boundary conditions, etc., for real valued problem classes.

PARAMETRIC GRÖBNER BASES FOR NON-COMMUTATIVE POLYNOMIALS

V. Weispfenning
Universität Passau, FRG

It was shown in [KRW] that the Gröbner basis technique with all its immediate applications (see [B]) can be extended from commutative polynomials to a large class of non-commutative polynomial rings over fields. More precisely, the method works for one- and two-sided ideals in **solvable polynomial rings**, a class of rings that comprises e.g. enveloping algebras of finitely dimensional Lie algebras, Weyl algebras and iterated Ore-extensions of fields. So the method is a strong generalization of the results in [AL].

On the other hand, it is well-known that for commutative polynomials with parametric coefficients, Gröbner bases are extremely unstable under specialization of the parameters. This problem was overcome by the author by the construction of **comprehensive Gröbner bases**, i.e. bases for commutative polynomial ideals that are Gröbner bases in every specialization of the parametric coefficients (presented at the CoCoA-II conference, Genova 1989).

The talk will present the construction of comprehensive Gröbner bases for solvable polynomial rings, where both the coefficients of the polynomials and the commutator relations defining the ring may be parametric. Applications will be indicated.

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SEARCH OF THE RATIONAL SOLUTIONS OF THE LINEAR DIFFERENTIAL EQUATIONS IN REDUCE

K. Yu. Kvashenko

Moscow State University, USSR

Many computer algebra systems contain ordinary differential equations solvers. The linear differential equations with polynomial coefficients excite permanent interest. The paper deals with the REDUCE program for search of all rational solutions of such equations.

Let us consider the equation $\sum_{k=0}^{n} a_k(x)y^{(k)} = b(x)$, where $a_0(x)$, ... $a_n(x)$, $b(x) \in Q[x]$ (Q - rational numbers field). Search of the rational solutions of this equation includes two steps. First step is the calculation of solution's denominator $d(x)$. Then after replacement of the depended variable $y=u/d$, where $u(x)$ is a new unknown function, the common polynomial solution of the obtained equation is calculated.

The program of search of the rational solution doesn't involve integration and factorization of algebraic expressions. Only polynomial arithmetic functions and the function of polynomial resultant computation are used. This fact makes it possible to exploit the program with incomplete REDUCE versions, which are installed on the computers with small memory. It also determines high speed of work of the program.

Some aspects of the realization of the program are discussed in the paper. Some algorithms for raising efficiency of the solution calculation process are suggested.

MATRIX OPERATION CAS REDUCE EXTENSION

M.U. Drozdov, V.V. Malanin

The Perm State University, Perm

USSR

The experience of using computer algebra system (CAS) Reduce (version 3.0 - 3.2) to solve applied problems of mechanics shows the lack of matrix-type data facilities available for an ordinary user. To remove these drawbacks a

package of functions has been developed (similar to Solve et al.) which saves much effort in matrix-type data programming.

The package is written in Rlisp. Its size is 990 lines of the source text. Autoloading in terms of all the package functions available in Algebraic mode is implemented. There is an IBM PC compatible Reduce 3.2 package version written in Uo-Lisp.

The package used to study a number of dynamics problems for rigid and elastic body systems resulted in substantial simplifying and accelerating the process of simulation program writing for systems of large dimensions.

ENLARGEMENT OF SCA RUDUCE 3.0 TO TENSORS OPERATION

V.V. Malanin, V.M. Mikriukov
The Perm State University, Perm, USSR

Enlargement of SCA REDUCE 3.0 to tensors operation was performed, so that to conduct analytical evaluation with tensor final dimentionality. The enlargement was written on standart LISP in RLISP's format and was realized in Algebraic mode REDUCE 3.0. Tensor is the object REDUCE with specific qulity. Enlargement of SCA by tensor's operations permits to do functions:

- describe and/or conduct first initial of tensor object;
- define element's tensors one by one and/or by blocks: vector or matrix;
- execute all operations of tensor's analysis: letferine, summing up and subtract, external product, differentiate tensor by tensor or by scalar and scalar by tensor;
- describe element of tensor as tensor;
- execute one-dementionality tensors or its elements operations of vector's or scalar product;
- presentation of the results of tensors and/or matrix-vectors evaluation, which were written in natural mathematic forms by scalar forms;
- for operation, which needs large resources of operation storage, change non-numerical elements by operators with the same name;
- presentation of the results as of FORTRAN's program.

This enlargement of SCA REDUCE 3.0 is used for combining equations for systems of solid bodies. Also this enlargement is used by students in their laboratory, caurses and diploma works.

ON A COMPACTNESS OF TWO DIMENSIONAL OUTPUT

A.A.Markov
Republic engineering center of automation design,G.P.I.,Gorky
USSR

The important aspect of working out computer algebra systems (CAS) is the ensuring of adequate service, particularly, the ensuring of two-dimensional (many-level) formulas' output, that is a formulas' representation in natural mathematical notation. The problem of obtaining two dimensional formulas' representation has been solved long ago, but the problem of transfer from line to another of mathematical expressions is usually solved either by the change to a one-dimensional (FORTRAN-similar) outputs' form, or by noncompactness transfer, which increases the real size of the formulas.

Formulas in a many-level notation are characterized by length and height (the number of levels). In the report the representation of such formulas in the following way is described. With the system of re-designations, let's pass on to a system of formulas, which are length-bounded (Restriction 1), in this case the problem of transfer is irrelevant, and, maybe, they are height-bounded (Restriction 2). Representing the formulas by rooted trees, on their vertices the length-function and height-function are defined, let's solve the optimization problem of outputs' compactness, such as the minimization of the number of formulas in the system (Problem 1) and the minimization of the sum of formulas' heights (Problem 2).

The methods of rooted trees' decompositions, developed by the author, can be applied to solve these problems. The greedy algorithm G , solving the Problem 1 under Restriction 1 in a linear- bounded time, is proposed in the report; NP-hardness of the Problem 2 under Restriction 1 and the estimation of algorithm G error for the later are proved. The Problem 1 under Restrictions 1 and 2 remains in the class P (polynomial-bounded time algorithm is given), but this one is not to be solved in the class of finite automats over trees with finite delay; the simple approximate algorithm with its' error is given. The estimations of some other two dimensional outputs' characters are obtained.

SYMBOLIC COMPUTATION OF THE GENERAL TERM OF TAYLOR SERIES

Ivan V. Dikov

Institute of Mathematics, Bulgarian Academy of Sciences
Bulgaria

An intelligent computer algebra system for power series manipulations is under development in Artificial Intelligence Department of the Institute of Mathematics, Bulgarian Academy of Sciences under the leadership of prof V.Tomov.

In this system mathematical knowledge is being applied, represented by rules and procedures.

This paper concerns one capability of the mentioned system - the calculation of Taylor series of functions. An approach for obtaining such series is suggested by symbolic computation of the general term. The availability of a formula for the general term allows convenient and efficient application of Taylor series.

The coefficient of the general term of the Taylor series of the function f(x) in the point x_o has the form:

$$a_n = \frac{f^{(n)}(x_o)}{n!},$$

The suggested approach consists in the calculation of the n^{th} derivative of the function f(x) by successive application of rules for the n^{th} derivative of the basic elementary functions and procedures for the n^{th} derivative of expressions of such functions.

The implementation of the whole system mentioned above and in particular, the described method, is being accomplished using REDUCE system.

This approach is easily generalized for functions of several variables.

ALGEBRAS OF PSEUDODIFFERENTIAL OPERATORS: CONSTRUCTIVE ASPECT

S.M.Zverev, P.S.Shpak
Institute for Applied Problems of Mechanics and Mathematics,
Academy of Sciences of the Ukrainian SSR, Lviv, USSR

In terms of the n-dimensional Fourier transform $\hat{u}(\xi) = \int \exp(ix\cdot\xi)u(x)dx$, $x\cdot\xi \equiv x_1\xi_1 + \ldots + x_n\xi_n$, the pseudodifferential operators $p(x,D_x)u(x) = (2\pi)^{-n}\int \exp(ix\cdot\xi)$ with the symbols $p(x,\xi)$, $p_1(x,\xi)$, $p_2(x,\xi)$, $p^*(x,\xi)$, where $D_{x_j} \equiv -i\partial/\partial x_j \equiv -i\partial_j$, belong to an

algebra with the product $p(x,D_x)=p_1(x,D_x)\cdot p_2(x,D_x)$ and the adjoint $p^*(x,D_x)$ operators having the symbols defined by the asymptotic series /1/-/4/:

$$p(x,\xi)=\sum_{|\alpha|\le N}(1/\alpha!)\partial_\xi^\alpha p_1(x,\xi)(-i\partial_x)^\alpha p_2(x,\xi)+R_N(x,\xi),$$

$$p^*(x,\xi)=\sum_{|\alpha|\le N}(1/\alpha!)(-i\partial_x)^\alpha\partial_\xi^\alpha\overline{p(x,\xi)}+R_N^*(x,\xi).$$

We build the symbols of the operators in the PL/1-FORMAC computer algebra system environment and use them further to construct the parametrices of some boundary value and mixed boundary-contact problems of the mechanics, electromagnetics and mathematical physics for non-canonical domains and/or variable coefficients. Some asymptotics (high and low frequency domains, small corrugation) are also considered.

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EQUAL-LEVELLED APPROXIMATION OF MATHEMATICAL FUNCTIONS BY SPLINES

B.A.Popov

G.V.Karpenko Physico-Mechanical Institute of the Ukr.SSR Academy of Sciences, Lviv, USSR

Let us have the mathematical function expanded into a series $f(x)=\sum_0^\infty a_i x^{ip+s}$ ($|x|<R; p,s$ are integral, $p\neq 0$). For its quick calculation in the interval $x\in[a,b]$ it is convenient to use an equal-levelled approximation by the Chebyshev apline $S_{k,\ell}(x)$ /1/ with links of the form

$$R_{k,\ell}(x)=x^s\sum_{i=0}^{k}a_i x^{ip}/(1+\sum_{i=1}^{\ell}b_i x^{ip}).$$

For some degrees of denominator ℓ and weights $w(x)$ deduced are the expressions for the approximation errors and equations for

nodes z_i $(i=\overline{0,r})$ of spline $S_{K,\ell}(x)$. So, at $a=0$, $w(x)=1$, $\ell=0$ the formula for the error is

$$\varepsilon_K = \frac{\alpha_{K+1}\, \delta^{p(K+1)+S}}{2^{2K+2}\, r^{K+1}}\, C^{K+1}\left[1+\sum_{j=1}^{\infty}\beta_j\, \delta^{pj}\right]^{K+1}\left[1+\mathcal{O}\left(\frac{\delta}{r}\right)\right],$$

where

$$\beta_j = \frac{p(K+1)+S}{(j+1)p(K+1)+S}\,\beta_j', \quad j=1,2,\ldots; \quad C=\frac{p(K+1)}{p(K+1)+S},$$

and the equation for the nodes looks like

$$z_i^p\left(1+\sum_{j=1}^{\infty}\gamma_j' z_i^{pj}\right) = \delta^p\left(1+\sum_{j=1}^{\infty}\gamma_j'\,\delta^{pj}\right)\left(\frac{i}{r}\right)^C, \quad i=\overline{0,r}.$$

The analytical expressions for β_j', γ_j', $j=1,2,\ldots$ are found on the computer with the help of the system for analytical computations. In particular:

$$\beta_1' = \frac{\alpha_{K+2}}{\alpha_{K+1}}\,\frac{K+2}{K+1}, \quad \beta_2' = \frac{K+2}{2!(K+1)}\left[\frac{\alpha_{K+5}}{\alpha_{K+1}}(K+3)+\left(\frac{\alpha_{K+2}}{\alpha_{K+1}}\right)^2\frac{K(K+2)}{K+1}\right],$$

$$\gamma_1' = \frac{p(K+1)}{2p(K+1)+S}\,\frac{\alpha_{K+2}(K+2)}{\alpha_{K+1}(K+1)}, \ldots$$

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GENERATION OF REDUCE-PROGRAMS BY THE PLANNER SYSTEM USING THE EXAMPLE OF EVALUATING THE DETERMINANT

V.A.Eltekov

Department of Physics, Moscow State University,
Moscow, 117234, USSR

Some techniques for evaluating the determinant are examined, namely, expansion by a formula ensuing from definition, expansion in line elements, and backward expansion in line elements. The REDUCE-program for evaluating the determinant is generated automatically by the PLANNER-system using a recursive pattern based on the formula for expansion in line elements with supression of repeated calculations of identical minors. The sequense of expressions producing the ascending chain of substitutions expressed in the REDUCE laguage is remembered in inverse order in each step of the recursion.

AUTOMATION OF NUMERICAL SOLUTION OF PDE SYSTEMS

R. Liska
Faculty of Nuclear Science and Physical Engineering
Technical University of Prague
Czechoslovakia

Computer algebra is used for automation of the process of numerical solving partial differential equations systems. The computer algebra system REDUCE and the numerical programming language FORTRAN are used in the presented methodology. The main aim of this methodology is to speed up the process of preparing numerical programs for solving systems of partial differential equations by the grid method. This process is quite often, especially for complicated systems, tedious and time consuming task.

In the process one can find several stages in which computer algebra can be used for performing routine analytical calculations, namely: transforming differential equations into different coordinate systems, discretization of differential equations, analysis of difference schemes and generation of numerical programs. The following REDUCE programs have been build, tested and documented:

EXPRES for transforming differential equations into any orthogonal coordinate system

IIMET for discretization of partial differential equations systems by integro-interpolation method

APPROX for determining the order of approximation of difference scheme

CHARPOL for calculation of amplification matrix and characteristic polynomial of difference scheme, which are needed in Fourier stability analysis

HURWP for polynomial roots locating necessary in verifying von Neumann stability condition

LINBAND for generating the block of FORTRAN code, which solves a system of linear algebraic equations with band matrix appearing quite often in difference schemes.

The possibilities of all these programs are demonstrated by examples.

The presented methodology is applied to two physical problems. The first one is the nonlinear Schrödinger equation, which describes several physical phenomenons, e.g. Langmuir waves in plasma. The second one is the Fokker-Planck equation in diffusive approximation, which is used for description of the kinetics of electrons in plasma, including interactions of electrons with laser beam through inverse bremsstrahlung. The numerical programs have been tested and compared with similar published calculations.

SYMBOLIC COMPUTATION OF ALTERNATIVE LAGRANGIANS

S. G. Shorokhov

Friendship of Nations University, Moscow

Recently there has been a renewed interest in the problem of determination of alternative variational formulations for different evolution systems, especially in connection with the problem of their integrability [1] and quantization [2]. Hence the number of examples with alternative lagrangian (hamiltonian) descriptions found is relatively few.

The methods of the inverse problem of the calculus of variations can be applied to the construction of alternative lagrangians. It is known that the inverse problem of the calculus of variations for a system of second order ODE's can be reduced to the integration of a system of linear homogeneous algebraic equations and first order PDE's with respect to the elements of a nonsingular matrix (matrix variational multiplier). The system is thoroughly investigated in [3] for the case of two ODE's.

The realization of the algorithm of [3], including some algorithms of linear algebra and theory of compatibility of differential systems (computation of the range of a matrix, n-dimensional vector product, Frobenius theorem, the method of Jacobi's brackets , etc) is obtained using computer algebra system REDUCE. The existence of alternative lagrangians for different classical systems, including rigid body with a fixed point in a potential force field, is investigated.

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COMPUTER ALGEBRA CALCULATIONS OF MULTIPARAMETRIC FAMILIES OF EXACT SOLUTIONS OF EINSTEIN – MAXWELL FIELD EQUATIONS

G.A.Alekseev

Steklov Mathematical Institute, Moscow, USSR

The general approach to the description of the space of local solutions of Einstein – Maxwell field equations for the class of electrovacuum space–times, depending on two coordinates only [1,2], provide us with the algorithm for

computer-aided calculations of wide families of exact solutions with any finite number of arbitrary parameters.

This general approach is based on the existence of one-to-one mapping of the whole space of local solutions and pairs of arbitrary functions $u(w)$ and $v(w)$, depending on a complex parameter w only, which are holomorphic in some disconnected region. This mapping is defined by the linear scalar (e.g. nonmatrix) singular integral equation. The kernel and right hand side of this equation contain the functions $u(w)$ and $v(w)$ and the unique solution of this equation (for given $u(w),v(w)$) uniquely determines all gravitational and electromagnetic field components in quadratures.

For arbitrary chosen rational functions $u(w),v(w)$ the integral equation may be transformed to the linear algebraic system and the corresponding solution may be found explicitly in elementary functions.

The algorithm for construction of such solutions, using the usual facilities of a computer algebra calculations in semiautomatical regime may be realized as a small collection of muMath programs, made to run on IBM PC XT/AT. However, appearing of too cumbersome calculations in the case of not very short rational expressions for $u(w)$ and $v(w)$ make it rather desirable to implement this algorithm using more powerful symbolic algebra systems and larger machines.

The new families of exact solutions, being obtained by this method, possessed of a rich physical content and include many known solutions (for example – solitons) as a particular cases, as well as their numerous generalizations and analytical continuations. Thus, for the construction of new interesting solutions it would be very useful to make computer implementations of many calculations, concerning geometrical and physical properties and the classification of these families of solutions.

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BASES OF SYMBOLIC TRANSFORMATIONS FOR MATHEMATICAL GEODESY PROBLEMS

I. Trenkov
Central Laboratory of High Geodesy,
Bulgarian Academy of Sciences

M. Spiridonova
Institute of Mathematics with Computer Center,
Bulgarian Academy of Sciences

M. Daskalova
Faculty of Geodesy,
High Institute of Architecture and Construction
Bulgaria

An approach for solving some problems in mathematical geodesy using the computer algebra system REDUCE and bases of symbolic transformations (BST) is presented.

The accepted structure of BST, as well as tools for their building, maintenance and use were developed in Artificial Intelligence Department at the Institute of Mathematics with Computer Center, Bulgarian Academy of Sciences.

The formulae for the main quantities of mathematical geodesy and their derivatives are presented by substitution rules of REDUCE system. The algorithms for geodesic calculations are defined by a set of procedures having heterogeneous structure. The substitution rules and procedures are stored as LISP expressions.

During the process of problem solving a chain of calls to BST, data base and REDUCE operations is constructed and performed.

Computation of the geographical coordinates of a point on the earthly ellipsoid when the azimuths of other points with known coordinates are observed is considered as an example of geodesy problem.

CONSTRUCTION OF THE ANALYTICAL PERIODIC SOLUTION FOR THE HILL'S PROBLEM IN THE REDUCE SYSTEM

V.M.Dekhtyarenko[1], V.N.Enachii[2], C.V.Mironov[3]

1 - RVC MNO MSSR, Kishinev,

2 - KGU "V.I.Lenin", Kishinev,

3 - IPC AN SSSR, Moscow

USSR

The Hill's method of the analytical construction of 2 -periodic solutions for the main problem of the Moon movement theory is modified. Some new and effective recurrence relations are found which ensure the computational construction of periodic solutions to an arbitrary preassigned precision of the order of μ^N, where μ is a small parameter and N is an a priori given natural number. On the basis of the recurrence formulae obtained an algorithm is realized in the REDUCE system to construct the expressions for the Hill's coefficients, implicit with respect to the small parameter μ, to an arbitrary preassigned precision. An iterative method is developed to construct automatically expressions, explicit with respect to the small parameter, for the coefficients of 2π-periodic solutions of the plane circular finite three-bodies problem. In order to construct desired solutions a computer algebra over special-type fractional-relational functions is developed and implemented in the REDUCE system. The "fractional-relational functions" method and that of "power series" are realized in the extended REDUCE sys-

tem. By means of the Hill system of procedures 2π-periodic solutions of the Hill's problem are constructed to an accuracy of μ^{29} by the "power series" method and to an accuracy of μ^{11} by the "fractional - rational functions" method .

The programme facilities which extend the possibilities of the REDUCE system of analytical calculations may be effectively used to solve the problems of celestial mechanics and mathematical physics.

AUTOMATION OF CONSTRUCTION OF BOSON REALIZATIONS

Č. Burdík
Nuclear Centre of the Faculty of Mathematics and Physics
Charles University, Czechoslovakia

Computer algebra is used for construction of boson representation of a Lie algebras. A realization (also canonical realization or boson representation) of a Lie algebra g denoted an expression of elements of g by means of polynomials in quantum canonical variables p_i, q_i, which preserves the commutation relation. Several types of the boson representations are used in physical applications.
In our paper /1/, the method of constructing realizations for an arbitrary semisimple Lie algebra g was presented. The construction starts from a triangle decomposition; it employs substantially induced representations of g with respect to a suitable representation of the subalgebra.
The main purpose of this article is the presentation of this method using the computer algebra system REDUCE. The following REDUCE program has been build, tested and documented:

INDUCE for construction boson realizations for semisimple Lie algebra.

The possibilities of this program are demonstrated by examples. The presented methodology is applied to two algebras. The first one is the symplectic algebra sp(6,R). The realizations of this algebra are physically interesting in connection with a microscopic model for the system of nucleons. The second one is the algebra sl(3,R). Canonical realizations have many applications in the representation theory.

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CALCULATION OF BEAM DYNAMICS IN MAGNETIC FIELD WITH PLANE SYMMETRY USING THE METHOD OF IMMERSION IN THE MOMENT PHASE SPACE

Ivanov E.L., Kostova Z.T., Perelstein E.A.
Joint Institute for Nuclear Research, Dubna

Nowadays there is a rapid growth of using computer algebra methods for obtaining approximate analytical solutions of complicated nonlinear differential equations. In many cases the results are obtained with the full computer accuracy in a much shorter time by using analytical formulae than by resorting to a numerical integration.

In the present paper we study the charged beam dynamics in magnetic field with plane symmetry using the method of immersion in the moment phase space. The system of nonlinear differential equations which describes the particle dynamics in the phase space $[x,y,x',y']$ approximately can be rewritten as a system of linear differential equations in the space of phase moments of n-th degree. We obtain the exact analytical solution of this linear system using computer algebra REDUCE 3 on ES-1061 (IBM 370) computer. Based on this solution the Fortran program is used to study the particle dynamics in the isochronic cyclotron. As an example the results of computations for the cyclotron LNR JINR CI-100 are given.

COMPLEX REDUCE-PROGRAMS FOR ANALYTICAL SOLVING SOME PROBLEMS OF BEAM TRANSPORT SYSTEMS

A.L.URINTSEV*, A.V.SAMOILOV

Institute of Nuclear Research, Moscow
*Rostov-on-Don State University
USSR

A complex of REDUCE-programs for analytical calculations of different properties of various beam-transport systems is described. The size of the complex is about 4800 lines. This complex permits one to calculate in high order theory of aberration of the systems which are included in any combination of the next elements: drift length, solenoid, dipole, quadrupole, sextupole, etc., without principal limits on a number of poles, both in sence of charged particle dynamics and in sence of spin motion for polarized particles. Obtained analyti-

cal expressions for aberration coefficients can be used for the next steps of optimization calculations.

With the help of this complex some problems have been solved in consequence with a design study of Moscow meson factory. Symbolical analysis has been done on ES-1055M, ES-1061 and IBM PC/AT computers.

USING **REDUCE** SYSTEM FOR CALCULATION OF RENORMALISED STRESS-ENERGY TENSOR FOR MATTER FIELDS IN CURVED SPACE-TIME

A.V.Nesteruk
Finacial and Economical Institute, Leningrad
S.A.Pritomanov
Chemical and Technological Institute, Dnepropetrovsk
USSR

It is well known that a background gravitational field can create particles and cause the vacuum polarisation of material massive and massless fields. Usually these quantum effects are described by the renormalised stress-energy tensor for corresponding fields.

Stress-energy tensor calculation is a very difficult and complicate task. Based on the n-wave regularisation method proposed by Zeldovich and Starobinsky[1] we elaborate an algorithm for calculation stress-energy tensor for scalar, spinor and vector fields in an isotropic and anisotropic spaces. The algorithm was realised as a complex of the programs for **REDUCE-3.2** system on personal computer compatible with IBM PC/XT,AT (with 512 or 640 K RAM). **Analytical** computations were performed for local and nonlocal parts of stress-energy tensor, conformal anomaly of scalar[2] and spinor fields in anisotropic homogeneous space-time Bianchi-I type. Our results completely coincide with the well-known ones calculated by other methods.

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INVESTIGATION OF A SYSTEM OF EQUATIONS TO CALCULATE THE CHARGED PARTICLE BEAM EMITTANCE

L. P. Lapteva, I. V. Malyovanny, A. V. Samojlov, B. S. Volkov

NRI, AS USSR, Moscow

To define the charged particle beam emittance with intensities, excluding the application of coincidence circuits, it is necessary to know all elements of the matrix of second moments (beam σ-matrix). Determination of emittance is very important for many various purposes, such as beam dynamic, experimental and applied problems. The methods of defininining a σ-matrix that can be realized in real time and without special procedures, which may demand some changes in transport system regimes, or supplementary equipment, are of great interest. Here we consider a method to determine a σ-matrix based on using beam profile monitors, which, as a rule, constitutes all parts of transport system. It is shown here that one can define the elements of the σ-matrix by simply solving this system of linear equations. The analysis of the linear system determinants was made using the computer algebras HECAS. REDUCE and MACSYMA. The advantages and disadvantages of these computer algebras are compared from the point of view of dealing with such problems. As a result of the investigation, the optical circuits for the location of the electric magnets and beam profile monitors, allowing to evaluate the emittance in real time, are presented.

THE BETATRON OSCILLATIONS IN THE VICINITY OF NONLINEAR RESONANCE IN CYCLIC ACCELERATORS INVESTIGATION WITH THE HELP OF ANALYTICAL COMPUTER CALCULATION

I.V.Amirkhanov, E.P.Zhidkov, I.E.Zhidkova

Joint Institute for Nuclear Research, Dubna

The modern physical installations are very complicated. As a consequence of multikilometer accelerators with the big period of beam accumulation, circulation and extraction, the

requirements for space and time electromagnetic fields performances have become the most stringent. This in turn has caused the development of mathematical methods applied to charge particle dynamics investigations.

Particle betatron oscillations in a cyclic accelerator are described by a system of two nonlinear second order different al equations with periodic coefficients.

The Krylov-Bogolubov averaging method in high (second and third) approximations with the help of analytical calculation system REDUCE-3.2 is used to investigate these equations.

The inquiry of betatron oscillations stability in the vicinity of nonlinear resonance is made with the use of the obtained averaging equations.

ALGEBRAIC-NUMERICAL CALCULATION OF SYMPLECTIC MAPS FOR PARTICLE TRACKING INSIDE ACCELERATORS

S.N.Andrianov

LENINGRAD UNIVERSITY, LENINGRAD, USSR

The problem of constraction of area-preserving maps is extremely important for long time particle tracking inside accelerators. Traditional numerical integration of particle trajectories doesn't provide conservative properties. The present paper gives a map constraction method for nonlinear aberrations of n-th order which is area-preserving (symplectic) up to n-th order accuracy. The use of Lie algebraic methods for analytical constraction of such symplectic maps simplifies the calculation of aberrations and their corrections. The realization of Lie algebraic tools and other methods on the REDUCE 3.2 computer algebra allows to invistigate the slow extraction system and to propose some variants of parasitic oscillations correction.[1]

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USE SAC "REDUCE" FOR CLASSIFYING THE STÄCKEL SPACES IN THEORY OF GRAVITY

V.G.Bagrov, K.E.Osetrin

Institute of High Current Electronics, USSR Academy of Sciences, Tomsk, USSR

By Stäckel space one understands the Riemannian space in which the Hamilton-Jacobi equation

$$g^{ij} S_{,i} S_{,j} - m^2 = 0 \qquad (1)$$

can be integrated by the complete separation of variables. The metrics of Stäckel spaces have been found earlier in papers [1]. These metrics are defined with an accuracy to arbitrary functions, each of which depends only on one coordinate. When substituting these metrics into gravitational field equations (e.g. into the Einstein's equations in General Relativity, or into the Brans-Dicke equations in the scalar-tensor theory) they are transformed into the functional equations of the following view:

$$\sum_j \prod_i H_i^j(x^i) = 0, \qquad (2)$$

where $H_i^j(x^i)$ - are some expressions involving only the functions from metric. The integration of equations of the (2) type results in a system of algebraic equations. The derivation of equations (2) themselves as well as the classification and verification of the solutions obtained is also a very complicated problem. However the main part of such calculations can be automatized using SAC "Reduce". The classification of metrics was made following the Petrov technique. For illustration we refer the reader to [2], when all the Mill-Stäckel electrovac metrics have been found.

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THE ROTATIONAL EVOLUTION OF MACLAURIN ELLIPSOIDS WHICH ACCRETE MATTER THROUGH A DISC: AN APPLICATION OF COMPUTER ALGEBRA

I.L.Andronov[1], K.S.Kölbig[2], Z.T.Kostova
Joint Institute for Nuclear Research, Dubna

The rotational evolution of incompressible axial-symmetric self-gravitating bodies onto which matter falls from the outer space through an accretion disc is studied. The viscosity forces are assumed to be sufficiently strong to bring the outer and inner layers to synchronous rotation. No other dissipative forces as, for example, tidal distortion by a companion star, are taken into account. At initial time t=0, the object is supposed to be at rest and spherically-symmetric.

This restricted problem can be solved analytically for the main physical parameters, i.e., mass m, equatorial radius (major semi-axis) a, polar radius (minor semi-axis) c, eccentricity e, angular velocity w, and angular momentum l. By using a package written in REDUCE for manipulating power series, the leading coefficients in such series for these parameters as functions of the eccentricity e and the mass m are derived.

It turns out that the values of a, e, and l increase with mass m, whereas c reaches its maximum at m=1.035 (e=0.133) and then decreases to zero while e→1, and m→∞, a→∞, thus producing a flat self-gravitating disc. Under the above assumptions, this object, which consists of incompressible fluid, can therefore accrete an infinite amount of matter, in contrast to the case of a compressible gas.

The incompressible fluid approximation corresponds to the value n=0 in the polythropic equation of state $P=K_\gamma \rho^\gamma$, $\gamma = 1+1/n$, where P and ρ are the local pressure and density, respectively, and K_γ is a constant. It may also be applied to asteroids consisting of rocks and dust, but not to stellar objects, for which the effective value of n varies from 1.5 to 3, or to other objects for which the equation of state is more complicated. For these cases, numerical methods are more appropriate. This will be discussed elsewhere.

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[1]Dept. of Astronomy, Odessa State University, T.G.Shevchenko Park, Odessa, USSR, 270014.

[2]Visitor from CERN, European Laboratory for Particle Physics, CH-1211 Geneva 23, Switzerland.

DERIVING THE ANALYTICAL EXPRESSIONS FOR THE LYAPUNOV QUANTITIES AND STUDING THEM BY COMPUTERS

G.A.Dolgov , S.D.Shchuko

Research Institute for Applied Mathematics and Cybernetics of Gorky State University
USSR

There is considered some dynamic system of the kind

$$\dot{x}=\lambda x-y+\sum_{j=2}^{n} P_j(x,y),\quad \dot{y}=x+\lambda y+\sum_{j=2}^{n} Q_j(x,y),$$

where $P_j(x,y)$, $Q_j(x,y)$ are the homogeneous j-exponent polynomials.

The problem of distinguishing a centre and a focus in some specific point $O(0,0)$ and the related problem of estimating the number of limit cycles within a small neighborhood of this point at various values of parameters usually results in the subsequent obtaining and reduction to zero of Lyapunov quantities

Lyapunov quantities are the polynomials with exponents still increasing with respect to parameters of the system under consideration. Each current Lyapunov quantity is derived by transforming the polynomial-element matrices ,all preceding the Lyapunov quantities being at this assumed equal to zero . The polynomial coefficients are ratios of integers without admittance of any approximate calculation for them .To solve the declared problem of qualitative theory of differential equations,the factorization of the polynomials obtained has been carried out .

To derive Lyapunov quantities there has been proposed some specialized polynomial algebra described in the Fortran-Dubna language for the Soviet BESM-6 computers .It helped to have derived and studied Lyapunov quantities of some classes we deal with.

For the systems of the kind

$$\dot{x}=\lambda x-y,\quad \dot{y}=x+\lambda y+Q_3(x,y)$$

the problem of distinguishing a centre and a focus and that of estimating the number of limit cycles within neighborhood of a specific point through computer-based analytical transformations has been solved .

The application of analytical calculation system (ACS) **ALGEBRA** – 0.5 for the ES computers makes it possible not only to derive and analyse the Lyapunov quantity analytical expressions but also to obtain their numerical values having the system s parameters assigned .

COMPUTER ALGEBRA. INTEGRABLE SYSTEMS AND ALGEBRAIC CURVES

N.A. Kostov

Joint Institute for Nuclear Research. Dubna

In the present paper we study the family of commuting differential operators with elliptic coefficients of the Halphen type and also with spectral parameter. The integrability condition of the pair of equations of Halphen type gives the large class of nonlinear differential equations of Lax-Novikov type. The algorithmic solution of the problem of constructing Baker-Akhiezer function associated to N-sheeted coverings of the elliptic curve is given. This algorithm is implemented using computer algebra system REDUCE 3. Many examples, including the construction of new elliptic solutions of completely integrable dynamical systems related to one dimensional Schrodinger equation with Lame potential are presented.

AUTOMATIC GENERATION OF FINITE-VOLUME AND FINITE-DIFFERENCE CODE

S. Steinberg and P.J. Roache

Department of Mathematics and Statistics, University of New Mexico
Ecodynamics Research Associates, Inc., Mexico

MACSYMA is used to automatically write finite-volume code for solving general symmetric elliptic partial differential equations in general coordinates. Most of the MACSYMA code is written for n-dimensional problems, and then used to write code in one, two, or three dimensions. The three-dimensional code is about 1400 lines long.

It is assumed that a physical problem is presented in a complex geometry, and then transformed (using a general, non-orthogonal transformation) to a unit box in logical space (where the computations are done). The transformation is typically done using numerical transformations (MACSYMA is used to write this code). In this abstract, finite-volume means that control volumes in logical space are used to derive a finite-difference scheme for approximating the problem. Much of the programming complexity is caused by the fact that different quantities are computed at different points in the grid: the coordinates in physical space are computed at cell corners; the solution of the PDE is computed (as a cell-averaged quantity) at the cell centers; and the fluxes are computed a cell face centers. Thus, MACSYMA must create loops that run over cell corners, cell edge centers, cell face centers, and cell centers along with the relevant algebraic formulas.

Both the two-dimensional and three-dimensional codes have been thoroughly tested and will soon be incorporated into production code at Ecodynamics. Amazingly enough, this process produces essentially error-free code (only one error has been detected in each of the codes).

INTEGRATION OF SOME CLASSES OF LINEAR ORDINARY DIFFERENTIAL EQUATIONS

L.M.Berkovich*, V.P.Gerdt, Z.T.Kostova, M.L.Nechaevsky*

Joint Institute for Nuclear Research, Dubna

Various aspects of using computer algebra are considered for solution of classic Kummer problem: reduction of second order linear ordinary differential equations (LODE) to a given form. The most general type of the point local variable transformation preserving the order and linearity of equation is applied. Very important cases of the mentioned problem are such ones when the LODE under consideration is reduced either to an equation with constant coefficients (the most attention is paid to that case), or to LODE determining one of the known special functions. Obviously, if the explicit form of the corresponding transformation is found then the solution of the initial LODE is expressed in quadratures or special functions. The necessary and sufficient conditions of mutual transformation of LODE are given on the base of factorisation of differential operators. A specific structure of variable transformation which includes known substitutions as special case and allows to treat a very wide class of equations is of theoretical and applied significance.

The developed method gives an opportunity to approach the problem of solving LODE on the other hand as well-modifying ("multiplying") integrable potentials.In the paper an original procedure is presented of constructing sequences of 4-parameter families of LODE being integrable in terms of an initial (generating) equation.

On the base of the described approach which is constructive in its character, the corresponding algorithms for integration of LODE are worked out and implemented in computer algebra system REDUCE. Several examples of physical significance are presented.

COMPUTER ALGEBRA APPLICATIONS FOR SOLVABILITY OF BOUNDARY VALUE PROBLEMS

A.N.Rumyantsev

Perm University, USSR

Linear and nonlinear boundary value problems for systems of functional-differential equtions (with ordinary derivatives) are considered. Such problems include boundary value problems for delay differential systems with integral boundary conditi-

*) Dept. of Algebra and Geometry, Kuibyshev State University, USSR.

ons. The descriptions of constructive methods for investigating of solvability of the problems is presented.[1,2] The details of realization of the methods using computer algebra are discussed (FORMAC, μMATH). The illustrative examples are presented.

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APPLICATION OF COMPUTER ALGEBRA TO ANALYSIS OF NECESSARY INTEGRABILITY CONDITIONS FOR POLYNOMIAL-NONLINEAR EVOLUTION EQUATIONS

V.P.Gerdt, N.V.Khutornoy

Joint Institute for Nuclear Research, Dubna

Alexey Yu. Zharkov

Saratov state University

USSR

We use the symmetry approach to establish an efficient program [1] in REDUCE for verifying necessary integrability conditions for polynomial-nonlinear evolution equations and systems in one-spatial and one-temporal dimensions. These conditions follow from the existence of higher infinitesimal symmetries and conservation law densities. In the description of our algorithms and their implementation in REDUCE we present in particular the basic algorithm for reversing the operator of the total derivative with respect to the spatial variable. One of the most interesting application of the present program is the problem of classification when the complete list of integrable equations from a given multiparametric family is needed. In this case the program generates necessary integrability conditions in form of a system of nonlinear algebraic equations in the parameters present in the initial equations. In spite of their often complicated structure, there are systems for which the solution can be found in exact form [2]. We present three examples of evolution equations for which this system in fact is solved: a seventh order family of KdV-like scalar equations, a seventh-order family of MKdV-like scalar equations, and a third order family of coupled KdV-like systems. The corresponding algebraic systems have infinitely many solutions. These solutions have been found with the help of the computer algebra system REDUCE by applying the Groebner basis method.

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DETERMINATION OF DISTRIBUTION PARAMETERS FOR W_n^2 AND W_n^3 TEST STATISTICS BY THE COMPUTER ALGEBRA APPLICATION

P.V. ZRELOV, V.V. IVANOV

Joint Institute for Nuclear Research, Dubna

The goodness-of-fit criteria are used for testing agreement between experimental distribution of some physical value and theoretical hypothesis. The so-called nonparametric goodness-of-fit criteria, which allow one to analyse nonhistogrammed experimental data, take a special place among them. The omega-squared criterion (w_n^2), effectively applied for an experimental high energy physics data handling, as well as the omega-cube criterion (w_n^3) proposed by the authors /1/, relate to the class of criteria mentioned above, in particular.

Efficiency of the criterion employed is determined by its power function. Determination of sharp lower and upper bounds of power functions for alternative hypotheses, which are at a fixed "distance" from a null-hypothesis, is used by D.Chapman /2/ to compare some goodness-of-fit tests. In this method, in particular, some parameters of test statistics distribution in case that empirical sample belongs to alternative hypotheses with the distribution functions ensuring the minimization or maximization of power function, are determined. The exact expressions of such parameters for the w_n^2-statistics are cumbersome and for the w_n^3-statistics they are practically impossible to be obtained in the ordinary way.

In this work the computer algebra systems REDUCE and muMATH are used to determine the mean value and the variance of the alternative distributions for the w_n^2-statistics, which allows one to get more exact expressions of the power functions than those obtained by D.G. Chapman approximately. Moreover, the corresponding parameters are obtained for the w_n^3-statistics as well, which enables the minimum and maximum power functions for the new omega-cube criterion to be determined.

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THE EVALUATION OF INTEGRALS OF HYPERGEOMETRIC FUNCTIONS

V.S.Adamchik, Yu.F.Luchko, O.I.Marichev
Byelorussian University, Minsk
USSR

The report contains the summary of algorithm and description of programming packet for the evaluation of definite and indefinite integrals of elementary and special functions of hypergeometric type. This algorithm is very bulky: it spreds over 80 per cent of integrals, which are contained in the world reference literature and unbounded sets of new integrals too.

The algorithm was worked out in the monograph[1] and was more precisely specified in the following papers of the author. This algorithm was broadly used during the compilation of three volumes reference book[2], since it is the most effective and universal method of exact evaluation of integrals of wide classes.

The idea of the algorithm consists in the following. The initial integral is transformed to contour integral from ratio of products of Gamma-functions by means of Mellin's transform and Parseval's equality. The residue theorem is used for the evaluation of the last integral, which due to the strict rules results in sums of hypergeometric series. The values of integral itself and the integrand functions are the special cases of the known Meijer's G-function[2,3].

Programming packet is realized in programming languages PASCAL and REDUCE. It also offers the opportunity of finding the values for some integral transform: (Laplace, Hankel, Fourier, etc.). The REDUCE's part of packet contains the main properties of the wellknown special functions, such as the Bessel and Gamma-functions, and the ones which are related to them, Anger function, Weber function, Whittaker functions, generalized hypergeometric functions. Special place in the packet is occupied by Meijer's G-function, for which the main properties such as finding the particular cases and representation by means of hypergeometric series are realized.

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ON CALCULATION OF INTEGER-VALUED CHARACTERISTICS OF REPRESENTATIONS OF SIMPLE LIE ALGEBRAS

A.A.Zolotykh

Moscow State University, Moscow, USSR

The aim of this paper is to announce the package of applied programs that was worked out by the author.

The package means to be used for calculation of some integer-valued characteristics of simple Lie algebras and their irreducible finite-dimensional representations. This package gives us possibilities to calculate the following characteristics:

- integer-valued characteristics of Lie algebras (such as the dimension of algebras, the order of Weil groups, dimensions of basic representations, the list of all positive roots and so on);
- lists of minimal representations of Lie algebras (all representations are ordered by their dimensions);
- dimensions of Lie algebra representations;
- characters of Lie algebra representations;
- decompositions of tensor products of representations into irreducible components.

The package is written in FORTRAN-77 and contains about 5000 operators. At present it can be used on computers of PDP types (or SM in the USSR).

The author plans in the nearest future:

- to adapt the package for personal computers;
- to extend the package for semisimple Lie algebras;
- to add the reduction program connecting representations of Lie algebras and their subalgebras.

More detailed information can be received in the laboratory of computational methods of the mechanical-mathematical department of Moscow State University.

CAS USE FOR NONLINEAR PARTICLE DYNAMIC ANALYSIS

A.S.Dubrovski, L.V.Provorov, V.G.Shironosov
Physical - Technical Institute of the USSR
Science Academy Ural Branch (Izhevsk)
Central Airhygrodynamic Institute (Zhukovski-3)
USSR

A Program package "OPUS" (i.e. searching (O) of the periodic (P) solution equation (U) based on the least action principle (S)) is presented in the paper. "OPUS" has been written by means of the computer algebra system "ALCOR" in Refal language [1] and is based on method generalization for the periodic solutions of the differential equations using the critical points of the S action function [2]. The package allows to perform the particles confinement in the resonance range [3] as well as out of it [4].

For n-approximation (n=1,2,..) having an analytical form "OPUS" allows:

- to calculate the stable weighting zones of different bodies and particles (from elementary to macro) in and out of resonance range,
- to find the periodic solutions of the differential equations, not containing an explicit small parameter, for Lagrange's systems, where dissipation and disturbance are taken into consideration,
- to analyze nonlinear systems stability near the periodical solutions,
- to optimize system dynamics using problem parameters,
- to determine spectrum and conditions of chaos beginning for nonlinear systems.

By using "OPUS" on IBM PC/AT you can solve the problems of charged participle confinement in the traps. An example of it is represented in the paper.

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GROUP CLASSIFICATION OF NONLINEAR DIFFERENTIAL SYSTEMS USING PERSONAL COMPUTER

A.A.Barmin, V.L.Topunov, D.L.Shishkov

Mech.-mat. MSU, MDPI, Moscow, USSR

In this contribution we present a software meant for investigation of structure of symmetry group of nonlinear systems depending on the form of free functions and parameter values, figuring on the system. The algorithm is based on the exterior form calculus for building the defining equations of the symmetry group. A passive form of the defining system is build using a Groebner base techniques. Further investigation consists of stepwise utilization of relations binding the free functions end parameters of the original system. As an application of this scheme a modal of ellectric curent flow in thermoconductive plasma has been investigated, the model is described by the following equations:

$$Tt = k(T)(Txx+Tyy)+s(T)\,grad(f)^2 \;;\; div(s(T)\,grad(f))=0.$$

Here T is the temperature, f is the field potential, k(T) is the termoconductivity, s(T) is the electroconductivity of plasma. Such a model is widely used for describing heat discharge. A classification of the symmetry groups depending on the form of the k(T) and s(T) terms has been built. Exact analytic solutions, corresponging to the symmetry group have been found. These solutions have been used for simulating nonstationary electric flow during a heat discharge.

AN ALGORITHM FOR REDUCING A BILINEAR ALGEBRAIC SYSTEM INTO TRIANGULAR FORM

Z.T.Kostova, R.M.Yamaleev

Joint Institut for Nuclear Research, Dubna

The method of elimination for bilinear algebraic systems is presented.We propose an algorithm, based on vector linearization method /1/.The initial nonlinear system of equations is reduced to

linear system with noncommutative coefficients. Using analytical technique, which is analogical to direct and inverse Gauss method of solving linear equations we reduce the initial bilinear system to triangular form. At the top of the triangle we have polynomial in one variable and after that the linear system of equations in the rest of variables. The last system of equations is equivalent to initial one , i.e. solutions of initial nonlinear system remain unchanged. The algorithm is realized on computer algebra REDUCE-3.2 for personal computers. This algorithm may be generalised to the systems of arbitrary degree.

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ON CA APPLICATION IN SOLVING SOME STATISTICAL DYNAMICAL PROBLEMS

V.V. Malanin, I.E. Poloskov
The Perm State University, USSR

There has recently been an increased applications of computer algebra (CA) systems and separate symbolic manipulation programs for the random vibrations analysis in mechanical objects.

The probability density $\overline{p}(x,t)$ of the phase vector $\overline{x}$ of a nonlinear system subject to random fluctuations is known to satisfy the Fokker-Plank-Kolmogorov equation

$$\frac{\partial \rho}{\partial t}=-\sum_{i=1}^{n}\frac{\partial}{\partial x_i}\left[a_i(\bar{x},t)\,\rho\right]+\frac{1}{2}\sum_{i,j=1}^{n}\frac{\partial^2}{\partial x_i \partial x_j}\left[b_{ij}(\bar{x},t)\rho\right],\quad \rho(\bar{x},t_0)=\rho_0(\bar{x}). \qquad (1)$$

To approximate the problem solution (1) the following algorithms have been implemented by CA FORMAC: a combination of the iteration method and density expansion into a series in terms of Hermit functions; the above but combined with the Galerkin method for the stationary probability density determination and others.

There is a growing interest in stochastic sensitivity theory problems and estimation of random parameters effects in nonlinear systems in particular. To solve the problems one has to differentiate the right-hand parts of the equations of motion. For this procedure automatization the FORMAC program has been developed which differentiates sequences of arithmetic statements written in terms of FORTRAN-4 (e.g. part of a source program text) with due account of subroutine CALL statements. The program input is a list of names of independent variables (phase coordinates and random parameters), information on subroutines called, output variables and derivatives identifiers. It results in a sequence of FORTRAN 4 statements taking the form of a subroutines and evaluating derivatives.

SOME COMPUTER REALIZATIONS OF THE REDUCE-3 CALCULATIONS FOR EXCLUSIVE PROCESSES

Ya.Z.Darbaidze, Z.V.Merebashvili
High Energy Physics Institute of Tbilissi
State University, Georgia, USSR

V.A.Rostovtsev
Joint Institute for Nuclear Research
Laboratory of Computer Techniques and Automation
Dubna, USSR

The REDUCE-3 algorithm for the calculation of the squared gauge invariant set of tree diagrams is given in the α^3 order of the perturbation theory. The necessity of using such program packages as Factorizator, "COLOR"-Factor and so on is shown. The correctness of calculation for the infrared radiation corrections as compared with manual calculations is discussed. An example of applying the programs for the matrix and noncommutative algebras is presented when the well-known supersymmetric commutative relation is proved.

GROUP-CLASSIFIED POLYNOMIALS OF PHASE SPACE AND OPERATIONS AMONG THEM

KURT BERNARDO WOLF and GUILLERMO KRÖTZSCH
INSTITUTO DE INVESTIGACIONES EN MATEMÁTICAS APLICADAS Y EN SISTEMAS
UNIVERSIDAD NACIONAL AUTÓNOMA DE MÉXICO
Cuernavaca, México

Polynomial functions of the (classical) phase variables p^2, $p \cdot q$ and q^2 are useful in various fields of physics, including geometric aberration optics. They serve as homogeneous spaces for various group actions, and as realizations of the group elements themselves. The groups include the linear symplectic groups, and nonlinear aberration groups associated to these[1]. The operations in which the polynomials participate may be linear combinations, products, Poisson brackets, and Baker-Campbell-Hausdorff compounding embodying the group composition law.

We are interested in handling the polynomials through structures that are efficient under the above operations, by their coefficients in various bases. The *monomial* basis is one obvious choice that has been refined impressively[2]. Not-so obvious improvements, for Poisson brackets and aberration-group compounding, is the *symplectic* basis; it uses solid spherical harmonics in three variables, basically. The task then remains to find the *tables*[3] embodying the operations up to some polynomial degree, i.e., aberration order.

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A PROGRAM FOR THE ANALYTICAL CONSTRUCTION OF THE BIRKHOFF-GUSTAVSON NORMAL FORM

S.I. Vinitsky, V.A. Rostovtsev
Joint Institute for Nuclear Research
V.Yu. Gonchar, N.A. Chekanov
Kharkov Physical and Technical Institute
Ukrainian Academy of Sciences
USSR

Transformation of an initial classical Hamiltonian to a simpler form, we shall call the normal form, represents an universal method of the analysis and integration of equations of motion. If the Hamiltonian can be represented as a power series so that its quadratic terms are summs of Hamiltonians of noncoupled harmonic oscillators with incommensurable frequences than, as Birkhoff has shown, there exists a

canonical transformation such that the Hamiltonian in terms of new variables is a power series in the variables of action only. Gustavson modified the Birkhoff method to the case of commensurable frequencies. The Hamiltonian thus obtained is the Birkhoff-Gustavson normal form. Transformation of the Hamiltonian to the normal form may be accomplished by a sequence of canonical transformations, each of which transforms a non-normalized term of a lower order to the normal form.

We have elaborated and presented here two routines on the REDUCE system, GLTA and GITN for the analytical construction of the Birkhoff-Gustavson normal form. The range of validity of the Birkhoff-Gustavson normal form is discussed.

QUALITATIVE INVESTIGATIONS OF SYSTEMS WITH COMPUTER ALGEBRA USAGE

A.V.Banshchikov, L.A.Burlakova, V.D.Irtegov
Irkutsk computer center of the SB of the USSR AS, Irkutsk, USSR

In papers [1,2] the functional contents of the package with MECHANIC symbolic calculations, intended for constructing of the differential equations of a motion of systems of interconnected rigid and deformable bodies and their qualitative investigation is described. Apart from many developments on derivation of differential equations of a motion of systems bodies, where the symbolic calculations are used at the stage of problem preparation for numerical integration, then in MECHANIC that part is being developed, which deals with the qualitative analysis of solutions of obtained equations of a motion in symbolic form, which allows to use computer algebra in investigating the field of mechanics and stability of a motion.

In this report a number of algorithms, put in the basis for the package development is described. The means for obtaining the first integrals which are second-order and linear by velocities (by quasivelocities), and also of the generalized Jacobian integral and cyclic ones is discussed. The algorithms are based on the construction of invariant correlations of differential equations. The first integrals and invariant correlations are used for investigation of stability or instability of stationary solution by means of Lyapunov function method in accordance with Routh- Lyapunov theorems, Lyapunov theorems, Chetaev theorems and others. In particular, this approach may be applied to the systems of the form:

$$A\ddot{X} + B\dot{X} + I\dot{X} + PX + CX = Q(\dot{X},X),$$

where A,B,C are symmetric, P, I are skew-symmetric constant matrices, Q is the vector of nonlinear components; $\ddot{X},\dot{X},X$ are the vectors of accelerations, velocities, coordinates (in deviations from the unperturbed motion), for investigation of various forces influence on stability.

Moreover, the problems of stability of invariant variety of stationary motions in existing of constantly acting perturbations of certain classes are also discussed. Package abilities are illustrated at the example of second-order system. The package is realized at PL/1(F) and PL/1(O) in operating systems MVS, and VM at ES.

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USE OF SYMBOLIC CALCULATIONS IN PROBLEMS OF MECHANICS

V.V. Leonov, V.M. Rudenko, A.I. Zhurov

Institute for Problems of Mechanics, Academy of Sciences of USSR, Prospect Vernadskogo 101, 117526 Moscow, USSR.

Procedures written by means of the symbolic calculation system REDUCE are considered in the paper. With them one can carry out the following problems of mechanics:

deducing the kinetic and the potential energies of a mechanical system;

working out the Lagrangian of the mechanical system;

obtaining the equations of motion, if we know the Lagrangian, with the 2-nd kind method by Lagrange;

reducing the equations to the Cashy normal form rewriting them to solve for the highest derivatives;

linearizing the equations in the vicinity of the equilibrium condition;

constructing the solutions in a form of Taylor series, to a needed accuracy, with respect to the independent variable.

The mentionned stages may be carried out independently from each other. For example, if the Lagrangian of the mechanical system is known the 1-st and 2-nd steps may be omitted.

The working of the procedures is illustrated by example of a gyroscope with a gymbal suspension which rotation axes of the inside and outside rings aren't perpendicular to each other.

There is a program-package "Polymech-symbol" which includes these procedures as its part and is implemented on machines ES-1055, ES-1055M and other computers which can support the REDUCE system.

The use of computer methods for calculating the quantum chemistry integrals

L.V. Artamonov, L.V. Mozdor, A.B. Roitsin, Yu.S. Fishman[1]

The Semiconductors and Cybernetics[1] Institutes of the Ukr.Sod Academy of Sciences, Kiev, USSR

The method[1-4] of exact analytical calculation of the multicentered quantum chemistry integrals and its realization by means of the language "Analytic-79" are described. It is shown that the multicentered integrals over the Slater orbitals can be represented through the linear combinations of the elementary and special functions. For the first time the exact expressions for the most complicated quantum chemistry integrals - the four-centered integrals - are obtained. The analytical expressions are compared with the results of direct computer calculations of the initial expressions for multicentered integrals. The analytical expressions for multicentered integrals are considered in the aspect of their application for calculating the energy structure of the multiatomic systems from the first principles[5-7] and, in particular, for calculating the parameters of exchange interactions of different nature. The prospects of computer algebra applications for the quantum chemistry problem are discussed.

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SYMBOLIC COMPUTATIONS IN DISCRETE GROUPS

V.V. Bludov, E.V. Khlamov

Irkutsk State University

USSR

Possibilities of a software system DISCOTEG, developed by the auter, are discussed. This system is destined for symbolic computations in discrete groups. The system DISCOTEG does not pretend to a breadth of a well-known system CAYLEY[1], but is not inferior to it in an effectiveness of realized methods, and is undoubtedly more accessible for a soviet user.

The system DISCOTEG is destined for investigations in combinatorial group theory. Two classical methods [2]: Todd-Coxeter and Reidemeister-Schreier are the algorithmic base of the system.

An interactive regime is provided in the system. It gives a possibility to control a process of a problem solution searching. An initial data language closes to combinatorial group theory one. The effectiveness of the system depends on a used memory space, therefore a real version of the system is developed for ES 1061 computer, as it is a most spread in the USSR computer with a memory greater than 10 MBytes. Questions of a system transfer to PC are investigated.

Fibonacci groups structure, nilpotent groups torsion questions, and questions related with a solubility of equations over cyclic groups have been investigated with the system DISCOTEG[3].

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A BOUND OF DEGREE OF IRREDUCIBLE EIGENPOLYNOMIAL OF SOME DIFFERENTIAL OPERATOR

A.V.Astrelin

Moscow State University, Moscow, USSR

The following problem is considered: for differential operator $D = P\cdot\frac{\partial}{\partial x} + Q\cdot\frac{\partial}{\partial y}$ to find such integer, that any irreducible polynomial f dividing Df has degree deg f ≤ K.

An algorithm to solve this problem, when P and Q are homogeneous polynomials of equal degrees, i.e., $P(x,y) = x^n \cdot p(\frac{y}{x})$, $Q(x,y) = x^n \cdot q(\frac{y}{x})$, is proposed.

Let us take rational function $\frac{p(z)}{q(z)-z \cdot p(z)}$. If its denominator equals 0, answer is K=1. Otherwise represent this function as a sum of partial fractions. If this decomposition has the form $\sum \frac{r_i}{z-c_i}$ with rational r_i, define $r_0 = 1 - r_1 - \ldots - r_k$ and define N as least common multiple of denominators of $r_0, \ldots, r_k$. In this case the answer will be

$$\max \left(\sum_{r_i>0} N \cdot r_i \, , \, - \sum_{r_i<0} N \cdot r_i \right)$$

Irreducible solutions f of equation $Df = A \cdot f$, where $A(x,y)$ is any polynomial, will be the divisors of polynomial

$$\prod_{r_i>0} l_i^{N \cdot r_i} - c \prod_{r_i<0} l_i^{-N_i r}, \text{ where } l = \begin{cases} x, & \text{when } i=0 \\ y - c \cdot x, & \text{when } i>0 \end{cases}$$

for some c. If the decomposition has no that form the answer is 1.

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RESEARCH OF EIGEN FREQUENCIES OF MECHANICAL SYSTEM BY MEANS OF REDUCE SYSTEM

V.M. Rudenko

Institute for Problems of Mechanics, USSR Academy of Sciences, Prospect Vernadskogo 101, 117526 Moscow, USSR

Possibilities of use of the symbolic calculation system REDUCE for analysis of eigen frequencies of the mechanical systems are considered in the paper. An algorithm, implemented by means of the REDUCE system, for determining formulae of computation of the eigen frequencies is presented. The method and the features of the algorithm operating are illustrated by an example of a gyroscope with an elastic gimbal suspension. The research of the eigen frequencies includes both the numerical and the symbolic analysis of the mechanical system. A simplicity (compactness) and a small relative error in the

frequency calculating are the main criterion in deducing these formulae. This aim is achieved at the expense of use of the physical features of the mechanical system and at the expense of representation of the characteristic determinant in form of a rational function.

Plots and computation for the eigen frequencies of the gyroscope are presented in the paper. These illustrate quality of the expression obtained and help to make correct choice of the maine parameter of approximation.

There is a program-package "Polymech-symbol" that includes the procedures under consideration as its part and is implemented on machines ES-1055, ES-1055M and other computers which can support the REDUCE system and the FORTRAN language.

INVESTIGATIONS OF STATIONARY MOTIONS OF THE SOLID BODY WITH GROEBNER BASES METHOD

S. A. Gutnik

Institute of Computer Aided Design, Academy of Sciences, USSR,
Moscow, USSR

In this study the motion of the solid body about its center of mass in central newtonian gravitational field under the action of a constant torque, the gyrostatic torque and drag force torque is investigated. Equilibrium positions of the solid body are in general case the solutions of the system of 9 algebraic equations of the second degree with 9 unknowns. The principal moments of inertia and projections of the disturbing moments enter this system as parameters. This polynomial system is solved by the Groebner Bases method, a generalization of the Gauss exclusion method. Algebraic equation from the reduced Groebner Basis with respect to one unknown specifies all the equilibrium positions of the solid body in the space of parameters.

Groebner Bases are calculated for the following cases : the action of the constant moment or gyrostatic moment only. Various combinations of disturbing moments are considered.

NUMERICAL - ANALYTICAL ALGORITHM OF THE SOLUTION OF MATHEMATICAL PHYSICS BOUNDARY VALUE PROBLEMS

M.G.Dmitriev,[*] K.O.Annaniyazov[**]

[*]The Physical-Technical Institute of Turkmen Academy of Sciences

[**]Scientific Industrial Corporation "Sun" of Turkmen Academy of Sciences
Ashkhabad,USSR

This paper deals with the combined method of the solution of optimal control problems and mathematical physics boundary value illustrating some possibilities of REDUCE system.

Some parts of this method were used by D.R. Stoutemyer , for solving nonlinear program by means of MACSYMA and J.P.Geer, C.M.Andersen ,while solving of boundary value problems on the base of combination of Galerkin method and the method of pertubation theory. We combine all the above-mentioned ideas and in addition use the method of penalty functions for elimination restrictions in appearing minimization problems. The inverse penalty value used as symbol along with other symbols of the problem allows one to govern the accuracy of fulfilment of restrictions and the accuracy of the solution of the auxiliary minimisation problems.

The method is decribed by using two examples: on the linear problem of stabilization of the temperature field in the thin round plate and on the nonlinear problem, describing the behaviour of the boundary layer of the plate, steamlined in the longitudinal direction.

APPLICATION OF COMPUTER ALGEBRA FOR SOLVING SOME MAGNET-OPTICAL PROBLEMS

I.V.Malyovanny, A.V.Samoilov, B.S.Volkov

Nuclear Research Institute, AS USSR
Moscow, USSR

The beam formed by the transport system must have quite certain beam extent, divergence and correlation between the particle coordinates and angles (beam phase space) in accordance with several dynamic, experimental and applied requirements. The necessary transformations of the beam phase space are fulfilled by the transport

systems, consisting of many quadrupoles which are very expensive and powerful electric magnets. Minimization of the number of quadrupoles and their power is very actual question that constantly arises when calculating and projecting transport systems.

By the use of computer algebra there were obtained relations proving that any transformation of the in phase space of the charged particle beam can be made by the quadrupole transport system. The conditions when such statement is true are shown.

SYMBOLIC-NUMERICAL METHODS FOR THE COMPUTER-AIDED STABILITY INVESTIGATION OF DIFFERENCE SCHEMES

E.V.Vorozhtsov and S.I.Mazurik

Institute of Theoretical and Applied Mechanics of the U.S.S.R. Academy of Sciences Siberian Division, Novosibirsk
USSR

At present more and more attention is paid to the problem of the application of symbolic manipulations on a computer for the automation of a process of the stability investigation of difference schemes. An impossibility of a complete formalization in a symbolic form of different stages of the above process is a limiting factor in the application of the above-mentioned means. In its turn, this circumstance does not enable one to carry out an analysis of the schemes completely in an analytic form on a computer. In this connection we discuss in our report the questions of an interface between symbolic manipulations and numerical computations with the purpose of a complete computer-aided automation of the stability investigation of difference schemes. The analysis technique is based on using the Fourier method and various algebraic criteria of the Routh-Hurwitz and Schur-Cohn theories. The application of the computer algebra means, in our case of the apparatus developed in the REFAL language, in accordance with the above-mentioned approaches, enables us to reduce the process of the stability analysis of schemes to the localization of the characteristic polynomial zeros or to the solution of some nonlinear system of inequalities with respect to the parameters sought for. For the automation of the latter stages we have used in particular various numerical methods of the optimization theory and of the digital image processing. To avoid the errors which may be introduced by a man while organizing the symbolic-numerical interface the computer generates at the symbolic stages of the methods a special FORTRAN subroutine. This subroutine is then used at the numerical stages of the methods for the determination of the locus of points of the sta-

bility domain boundaries of the schemes. To reduce the computational errors at the numerical stages we employ various techniques of the scaling of the elements of the arising matrices as well as the arithmetics of stored orders. The application of these techniques enabled us to substantially increase the accuracy of computations and the reliability of the results obtained. In addition, the application of the digital pattern recognition techniques makes it possible to construct the boundaries of multiply connected stability domains of difference schemes, and the computer graphics means enable us to rapidly visualize the obtained results of the analyses of schemes.

DERIVING THE VARIATIONAL FORMULATION AND CANONIAL EQUATIONS FOR MOTION OF ELECTROPOLARIZED SUPERFLUID HELIUM WITH SAC REDUCE

V.V.Bikov, V.B.Gorsky
Gorky State University, USSR

The variational formulation of some physical field model is a useful alternative form enabling to use a well-developed mechanism of variational calculation to study the model .There exist ,however, no sophisticated algorithms allowing for any field model to find a related functional . Very important are also the Hamiltonian canonial equations of motion allowing for the study of specific medium to use the classical theory of excitations,to reveal common regulations in wave interactions,to gradually go over to quantum generalization . Deriving the canonial equations makes up at this a difficult and poorly predicted problem /1/ .

In this paper ,taking as an example the motion of electropolarized superfluid helium-II the variational formulation and canonial equations are obtained through the system of analytical calculations REDUCE-2 (SAC) /2,3/. Assigning some approximate form for the desired functional based on Hamiltonian principle and using the SAC to derive stationary conditions ,upon some iterations we have found a necessary density of functional :

$$\mathcal{L}=\frac{1}{2}\rho v_s^2+SAv_n-U+\varphi[\partial\rho/\partial t+div(\rho v_s+SA)]+\alpha(\partial S/\partial t+div Sv_n)+$$
$$+\beta(\partial q/\partial t+div qv_n)-\gamma(\partial a/\partial t+v_n\nabla a)+\Phi(div\mathcal{D}-q);\ SA\equiv\rho_n(v_n-v_s),\ \mathcal{D}\equiv\mathcal{E}(\rho,T,E)E\ ,$$

where v_s , v_n are the superfluid and normal speeds; ρ , ρ_n ,the full and normal densities of helium; S ,the entropy ; U ,the internal energy ; q ,the electrical charge density ; a ,the Lagrange coordinate of liquid particle; $\mathcal{D}$,the electrical induction ; E ,the electrical voltage; $\mathcal{E}$,the electrical permeance; t ,the time; φ , α , β , γ , Φ ,the Lagrange multipliers . Having applied to $\mathcal{L}$ the Legendre transformation with density of full energy assumed to be a Hamiltonian h and having used the SAC ,we have derived the canonial equations of motion :

$$\dot\rho=\delta h/\delta\varphi\equiv-div(\rho v_s+SA),\ \dot\varphi=-\delta h/\delta\rho\equiv-\mu-\frac{1}{2}v_s^2,\ \dot S=\delta h/\delta\alpha\equiv-div Sv_n,\ \dot\alpha=-\delta h/\delta S\equiv-v_n\nabla\alpha-T,$$
$$\dot q=\delta h/\delta\beta\equiv-div qv_n,\ \dot\beta=-\delta h/\delta q\equiv-v_n\nabla\beta-\Phi,\ \dot\gamma=\delta h/\delta a\equiv-div\gamma v_n,\ \dot a=-\delta h/\delta\gamma\equiv-v_n\nabla a,$$

where μ is a chemical potential; T, the temperature; $\delta/\delta\varphi$, the variational derivative; and $\dot{p} \equiv \partial p/\partial t$.

Note that the variational description and canonical equations proposed by us for helium motion in some specific case of ordinary liquid go over to the analogous results of electrodynamics /4/.

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DERIVING THE ANALYTICAL FORM OF CONDITIONS FOR SIGN DEFINITION

M.A.Choubarov
Research Institute for Applied Mathematics and Cybernetics of Gorky State University,Gorky, USSR

The requirement of sign definition for multi-variable polynomials arises in various applications,for example,when applying the Lyapunov second method to the analysis of stability of non-excited motions,or in studying the absolute stability of Lurje-class dynamic system with the Gurvitz matrix ,or the problems of synthesizing the RLC-circuits . Below proposed is the way of deriving the conditions of polynomial positivity through forming the polynomial remain sequence (PRS). The polynomial remain sequence $R_\nu(P_1(z), P_2(z))=(P_1, P_2, \ldots, P_\nu)$ in the ring $G[z]$ is found as a result of pseudodivision of the kind

$$\alpha_i^{(\nu)} P_{i-2} = q_i P_{i-1} + S_i ; \; P_i = S_i / \beta_i^{(\nu)}, \; i = 3, \ldots, \nu;$$

$$\alpha_i^{(\nu)}, \beta_i^{(\nu)} \subset G, \; P_i \subset G[z]$$,and $P_{\nu+1} \equiv 0$.

Through C_i and e_i let us denote the highest and lowest coefficients of P_i ,and introduce the values $C_i = C_i \Phi_i^{(\nu)}$ and $E_i = e_i \Phi_i^{(\nu)}$,where $\Phi_1^{(\nu)} = 1$, $\Phi_i^{(\nu)} = \prod_{k=3}^{i-1} (-\alpha_k^{(\nu)} / \beta_k^{(\nu)})^{i}$,for $i \geq 2$;

and $V(x_1, x_2) = (1 - sgn(x_1 x_2))/2$,for $x_1 x_2 \neq 0$.

Theorem . The necessary and sufficient conditions for positivity of polynomial $f(z) = \sum f_i z^{n-i}$ $(n>0)$ will be $f_n > 0$ and one of the following conditions :

1) for the PRS $R_\nu(f, f_z')$ in the area of its definition

$$\sum_{i=1}^{\nu-1} V(E_i, e_{i+1}) = \sum_{i=1}^{\nu-1} V(C_i, c_{i+1});$$

2) for the PRS $R_\nu(f(z\bar{z}), z f_z'(z^2))$ in the area of its definition

$$\sum_{i=1}^{\nu-1} V(C_i, c_{i+1}) = \frac{\nu-1}{2}.$$

For specific PRSs such as simplified PRS or Hermite PRS/1/, the expressions for values C_i and E_i will be considerably simplified .

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HIERARCHICALLY PERFORMED SYMBOLIC-NUMERIC SOLUTION OF ONE ELLIPSOMETRY TASK

Ch. N. Kazasov

Sofia University, Bulgaria

In the given note the solving of the task of finding the first and second derivatives of the ellipsometric angles Ψ and Δ for arbitrary multilayer structures following two steps is discussed: obtaining of a symbolical solution in CAS REDUCE-3, by which a complete FORTRAN subroutine is created, and a subsequent execution of the latter, with the purpose of computing a numerical solution. It is stressed that it is impossible to replace completely all the subexpressions for obtaining a closed formula of the functions Ψ and Δ and then to find the derivatives in the REDUCE system due to the very complicated expressions.

Further, a method for performance of the symbolic step of the solution of the task is offered, expressed in placing of the subexpressions for computation of Ψ and Δ into hierarhical levels, obtaining of the derivatives for each level bottom-up, and finding of the required toplevel derivatives using the already known derivatives of the lower level subexpressions. Also the common characteristics of the given task, wich reguire hierarchical planning on the stage of symbolic step, and the advantages of this method are mentioned.

ANALYSER OF FORMULAE FOR THE AUTOMATED TRAINING SYSTEM

V.I. Demchenko, R.N. Fedorova

Joint Institute for Nuclear Research, Dubna

Some problems of application of computer algebra to constructing the automated training systems (ATS) for courses with a high mathematization level (mathematics, physics, engineering, etc.) are considered.

The main difficulty in construction of ATS mathematical courses is the correctness control of trainee's answers. The questions to trainees and the answers to them as a rule have a form of analytical expressions, formulae and other mathematical constructions. The analysis of the answers is impossible without executing such typical analytical operations as simplification, comparison of expressions, substitutions, pattern matching, etc.

On the basis of SAC FORMAC-PL/1 special program "Formulae Analyser" has been developed. In a definite class of expressions the algorithm for proving the equivalence of two analytical expressions (trainee's answer and system pattern) has been realized. The program is prepared as an external dynamical function of the system ATS-VUZ and requires 250-300K operating memory of ES computer.

APPLICATION OF ANALYTIC COMPUTER CALCULATIONS FOR SYNTHESIS OF FAST-ACTING DEVICES FOR USEFUL EVENTS SELECTION IN HIGH ENERGY PHYSICS SPECTROMETERS

K.E.Kozubsky, N.M.Nikityuk

Joint Institute for Nuclear Research, Dubna

The aspects of the application of analytic computer calculations for logical devices and special-purpose processors synthesis in high energy physics spectrometers are considered. For these purposes an original approach based on the Algebraic Coding Theory and algebraical methods of signal processing is used. The variables representing coordinates and multiplicity of events registered in multi-channel detectors by charged particles are the Galois field GF(2^m) elements.

This representation has some advantages:

1) Since algebraical operations are implemented over Galois field elements, the problem of minimization and formal representation of switch functions, is simplified;

2) The inputs and outputs condition of combinative scheme is encoded by Galois field elements; and the next inputs and outputs condition is represented as polynomial function of present inputs and outputs condition;

3) With the large number of variables, analytic computer calculations can be used.

Symmetries and First Integrals of Chiral Skyrme Model

A.R.Ptukha, V.I.Sanyuk

Peoples Friendship University, Moscow, USSR

In this contribution we discuss some ways of finding a particular form of symmetry transformations and corresponding first integrals for the equation on the skyrmion profile functions $\theta(x)$ (for details one may consult lectures [1])

$$\theta'' = f(x,\theta,\theta') \equiv \frac{1}{x^2+4\sin^2\theta}\left[\sin 2\theta\left(1-2\theta'^2+\frac{2\sin^2\theta}{x^2}\right)-2x\theta'\right]. \quad (1)$$

Standard group-theoretical methods [2], including those which have been realized in computer algebra systems give us the possibility mostly to derive the determining equations which are solvable only under rather substantial limitations on the form of the function $f(x,\theta,\theta')$. In our case to get a solution of these determining equations is as difficult as to solve the initial eq. (1). But from the corresponding to eq. (1) overdetermined system of equations in partial derivatives it is possible to find out that the first integral $F(x,\theta,\theta')$ one can represent as a polynomial in $\theta'(x)$:

$$F(x,\theta,\theta') = A_n(x,\theta)\,\theta'^n + \ldots + A_1(x,\theta)\,\theta' + A_0(x,\theta) \quad (2)$$

with unknown coefficients $A_i(x,\theta)$.

It is possible to define those coefficients from the condition of the Poisson brackets triviality: $\{H,F\}=0$, where H - the Hamiltonian one can get from the skyrmion energy functional with the help of Poincare's trick [3]. The solvable cases for the corresponding system of equations are discussed in this report. Finally an effective algorithm of finding symmetry transformations and first integrals for Newton's type equations (1) arising in a number of problems in theoretical and mathematical physics is proposed.

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ON APPLICATION OF THE METHOD OF ANALYTIC CONTINUATION

S.P.Suetin
Steklov Mathematical Institute, Moscow, USSR

For the well-known (see [1]) Emden (Lane-Emden) differential equation

$$(*) \quad xy''(x) + 2y'(x) + xy^{\alpha}(x) = 0, \quad y(0) = 1, \quad y'(0) = 0,$$

$0 \le \alpha \le 5$, the exact solutions have been found only for α equal to 0,1 and 5. For that reason the main methods to investigate the equation (*) are numerical methods [2].

One question related to equation (*) is to find the first positive zero $\rho(\alpha)$ of the solution of (*). It is well known[1] that there is a formal power series solution of equation (*) and there are recurrence relations for the coefficients of the power series. The number of coefficients is computed in REDUCE system as polynomials in α. Since for small $\alpha \neq 0$ the point $\rho(\alpha)$ is a singular point nearest to the origin of the solution of equation (*), we use the D'Alambert ratio test and compute the number of coefficients of the formal power series of $\rho(\alpha)$ in α [3]. The Taylor polynomials approximate the function $\rho(\alpha)$ only for the small α. We use the Pade-approximants method to construct the analytic continuation. The corresponding rational fractions approximate the function $\rho(\alpha)$ on the interval $0 \le \alpha < 1$.

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On A Reduction Algorithm For Chiral Lagrangians

V.I.Sanyuk
Peoples Friendship University, Moscow, USSR

On the base of rather popular Skyrme model [1] a reduction scheme is presented, which gives a possibility to reduce the problem of studying (3 + 1) - dimensional chiral field models functionals to the problem of investigating dynamic systems with one and a half degrees of freedom.

As a first step we find out in what form the model Lagrangian should contain time derivatives of field functions, providing the realization of the functional minima on static fields. By the definition chiral functionals are invariant under transformations from the chiral group $G_L \otimes G_R$ as they are constructed out of chiral invariants. With this in mind we are to find out a maximal compact group G for the functional of interest following the standard procedure and then up to the Coleman-Palais theorem to search for the minima in the class of G-invariant fields. So far the next step in the reduction scheme is to use the algorithm for explicit form of G invariant fields derivation and to get the so-called anzats. In what follows we are to study the problem of the functional absolute minima, which may be realizable on this anzats for a given value of the topological invariant of the model (the topological charge). Here we are to use methods of direct minimization of functional with an expanding of the initial phase space of the model and the spherical rearrangement method in order to find the absolute minima of the functional. At all of those stages we discuss possible limitations of the given reduction scheme.

As distinct from some previous publications the reduction scheme is presented in the algorithmic form, so that it may be useful for realization as a computer algebra system. We hope that it will help to create a rather effective instrument for studying a number of models in particle physics, in condensed matter physics, in physics of magnetics and so on.

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