Reappraisal of the Reference Dose Distribution in the UNSCEAR 1977 Report

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Abstract. This paper provides the update of the reference dose distribution proposed by G.A.M. Web and D. Beninson in Annex E to the UNSCEAR 1977 Report. To demonstrate compliance with regulatory obligations regarding doses to individuals, they defined it with the following properties: (1) The distribution of annual doses is log-normal, (2) The mean of the annual dose distribution is 5mGy (10% of the ICRP 1977 dose limit), (3) The proportion of workers exceeding 50mGy is 0.1%. The concept of the reference dose distribution is still important to understand the inherent variation of individual doses to workers controlled by source-related and individual-related efforts of best dose reduction. In the commercial nuclear power plant, the dose distribution becomes the more apart from the log-normal due to the stronger ALARA efforts and the revised dose limits. The monitored workers show about 1mSv of annual mean and far less than 0.1% of workers above 20mSv. The updated models of dose distribution consist of log-normal (no feedback on dose X) $\ln(X)\sim N(\mu,\sigma^2)$, hybrid log-normal (feedback on higher X by ρ) $hyb(\rho X)=\rho X+\ln(\rho X)\sim N(\mu,\sigma^2)$, hybrid S_B (feedback on higher dose quotient $X/(D-X)$ not close to D by ρ) $hyb[\rho X/(D-X)]\sim N(\mu,\sigma^2)$ and Johnson's S_B (limit to D) $\ln[X/(D-X)]\sim N(\mu,\sigma^2)$. These models afford interpreting the degree of dose control including dose constraint/limit to the reference distribution. Some of distributions are examined to characterize the variation of doses to members of the public with uncertainty.

KEYWORDS: reference dose distribution; log-normal; hybrid log-normal; Johnson's S_B; hybrid S_B; worker; member of the public.

1. Introduction

The distribution of doses incurred by workers has been studied by many scientists since works of Gale (1965) and Brodsky, et al. (1976) [1]. The milestone is the annex E "Doses from occupational exposure", of the 1977 UNSCEAR report [2], provided one of the bases on setting the dose limit of 50mSv in the ICRP 1977 recommendations [3], in terms of reflecting actual distributions of doses to workers via worldwide collections of occupational dose statistics compiled by relevant authorities. Then the log-normal distribution has been established as a typical model of worker dose distribution.

The log-normal first suggested by Galton [4] as a distribution of the product of independent positive random variables was reappraised by Kapteyn [5] using an analogous machine for generating a skew frequency curve, like the Galton's apparatus of generating the bell-shape Gaussian frequency curve. Gibrat [6] found the log-normal distribution useful for representing the distribution of size for various economic quantities. Aitchison and Brown [7] have been providing the excellent text for the practical usage of the log-normal distribution since 1957. This distribution, however, has been widely used only since the latter half of the 20th century.

Doses to workers are controlled under the system of radiological protection. In order to interpret the effect of the system, there are models of dose distribution [1]: the hybrid log-normal (Kumazawa and Numakunai, 1980), the mixed log-normal (Warman et al., 1981), the Johnson's S_B (Kendall, et al., 1982), and the Weibull (Darby, et al., 1982). Thus the distribution of doses incurred by workers should be modelled as reflecting the result of constraints imposed by the nature of jobs, by the skill of workers, by the safety culture of management/regulation.

The dose distribution, however, should be discussed, not only occupational exposure issue (dose constraint) but also public exposure issue (representative person), in terms of the reference distribution.

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2. The reference distribution

2.1 Definition and application of the reference distribution by UNSCEAR

In the 1977 UNSCEAR report, the reference distribution was defined to characterize the aspects of a dose distribution which contain relevant information for the objectives relating to source justification, relative cost-benefit assessment, evaluation of trends, and indication of worker's risk level: (a) The distribution of annual dose is log-normal; (b) The mean of the annual dose distribution is 50 mGy (one tens of the ICRP maximum permissible annual whole-body dose); (c) The proportion of workers exceeding the maximum permissible annual dose of 50mGy is 0.1 per cent.

The 1977 report stated "it appears to the Committee that a distribution with properties (a) to (c) would comply well with the intent of the ICRP dose limitation system for persons exposed to radiation in the course of their work". The 1982 report [9], however, stated "It was not the intent of the Committee that this reference distribution be considered an ideal or optimal distribution of doses and it should not be so interpreted".

The 1993 and 2000 reports [10,11] stated "The Committee is principally interested in comparing dose distributions and in evaluating trends", and showed three characteristics of dose distributions instead of the reference distribution: (a) the average annual effective dose, $\bar{E}$, related to the average level of individual risk; (b) the annual collective dose, S, related to the impact of the practice; (c) the collective dose distribution ratio, $SR_E=S(>E)/S$, where $S(>E)$ is the sum of annual doses >E mSv, relating to an indication of the collective dose fraction of workers exposed to higher levels of individual risk.

The figure of E was 15mSv in the 1993 report, but it became several (15, 10, 5, 1 mSv) in the 2000 report, because of the change of dose limit (ICRP,1990) and the variety of distributions by workforce. Both reports defined another ratio, termed the individual dose distribution ratio, $NR_E=N(>E)/N$, where N is the total monitored or exposed workforce and $N(>E)$ is the number of workers (>E mSv), in order to calculate the ratio SR_E where it is not reported.

In the 2000 report, to compare dose distributions and to evaluate trends, the Committee considered the following characteristics of dose distribution: (a) $\bar{E}$; (b) S; (c) SR_E and (d) NR_E, for 15, 10, 5, 1 mSv. These characteristics are used to examine the result of many constraints imposed by the nature of work itself, by management, by the workers and by legislation. Doses to workers might be very low in some jobs, but those might have to be high routinely in other jobs. Management controls act as feedback mechanism, especially when individual doses approach the annual dose limit, or some proportion of it. Thus the observed dose distributions could bring the consequences on the characteristics of (a) to (d).

2.2 Probability distributions suitable for the reference distribution

The probability distribution for the reference distribution of doses controlled by feedback should fit observed dose distributions well. Here the feasible probability distributions are given.

2.2.1 Log-normal (LN): defined as $\ln(X) \sim N(\mu,\sigma^2)$

The genesis of the LN distribution is inherent in the law of proportionate effects on the magnitude of dose. The incremental dose ΔX randomly depends on the previous dose X in the uncertainty of exposure rate and time. We simply put $\Delta X=\varepsilon X$, where ε is the random variable of exposure intensity.

Suppose the exposure process of dose X_0 to X_T for a period $[0,T]$ can be divided by n hypothetical steps, the sum $A_n(t)$ of the random variable ε_i from $i=1$ to $[nt|T]$ relates to the dose X_T at T as follows:

$$A_n(t) = \Sigma^*\varepsilon_i = \Sigma^*(1/X_{i-1})\Delta X_i \xrightarrow{n\to\infty} \int_{X_0}^{X_T}(1/X)\,dX = \ln(X_T) - \ln(X_0). \quad (1)$$

When we assume (a) max ε_i is almost everywhere convergent to zero as n increases and (b) the expectation $M_n(t)$ and the variance of $A_n(t)$, respectively, converge to constant functions $M(t)$ and $V(t)$ in probability, $A_n(t)–M_n(t)$ is a martingale (it approaches 0 when $n\rightarrow\infty$). According to the martingale central limiting theorem, the limiting distribution of $A_n(t)$ becomes normally distributed. Thus, from Equation (1), $\ln(X_T)$ also becomes normally distributed, that is, X_T becomes log-normally distributed.

2.2.2 Hybrid Log-normal (HLN): defined as $\rho X+\ln(\rho X) \sim N(\mu,\sigma^2)$

The genesis of the HLN distribution is inherent in the law of proportionate effects on the magnitude of dose and feedback control by the incremental dose. The incremental dose $\Delta X=\varepsilon X$ is reduced to be $\Delta X=\varepsilon X/(1+\rho X)$ by the feedback control of exposure intensity as $\varepsilon' = \varepsilon - \rho\Delta X$ and $\Delta X=\varepsilon' X$. The dose reduction efforts are to set the shielding or to change the process of works, etc.

Suppose the exposure process of X_0 to X_T in $[0,T]$ by n hypothetical steps, the sum $A_n(t)$ is relative to the dose X_T at T by using the hybrid function, $\mathrm{hyb}(x) = x + \ln(x)$ (>0), as follows:

$$A_n(t) = \Sigma^* \varepsilon_i = \Sigma^* (1/X_{i-1} + \rho)\Delta X_i \xrightarrow{n\to\infty} \int_{X_0}^{X_T} (1/X + \rho)\, dX = \mathrm{hyb}(\rho X_T) - \mathrm{hyb}(\rho X_0) \qquad (2)$$

Using the martingale central limiting theorem, the limiting distribution of $A_n(t)$ and also $\mathrm{hyb}(\rho X_T)$ in Equation (2) becomes normally distributed, that is, X_T becomes hybrid log-normally distributed.

2.2.3 Johnson's S_B *(JSB): defined as* $\ln[(X–a)/(b–X)] \sim N(\mu,\sigma^2)$

The genesis of the JSB distribution is inherent in the law of proportionate effects on the dose quotient of $Y=(X–a)/(b–X)$, where individual doses are controlled in the range of $a<X<b$. When X goes close to b, Y steeply increases toward the infinity. To limit the dose below b inevitably requires the tight dose control, replacing the worker likely to exceed b by personal alarm dosimeter. Then the increment ΔY randomly depends on the previous value of Y, as $\Delta Y=\varepsilon Y$, where ε is the random variable of exposure intensity in dose quotient.

Suppose the exposure process of dose X_0 ($>a$) to X_T ($<b$) or Y_0 to Y_T in $[0,T]$ by n hypothetical steps, the sum $A_n(t)$ relates to the dose quotient Y_T at T, by substituting Y for X in Equation (1). According to the martingale central limiting theorem, $\ln(Y_T)$ becomes normally distributed and X_T becomes distributed as the JSB distribution. The JSB distribution reflects the distribution of doses controlled so tightly below a given level.

2.2.4 Hybrid S_B *(HSB model): defined as* $\rho(X–a)/(b–X)+\ln[\rho(X–a)/(b–X)] \sim N(\mu,\sigma^2)$

The genesis of the HSB distribution is inherent in the law of proportionate effects on the dose quotient and feedback control by the incremental dose quotient. $\Delta Y=\varepsilon Y$ is reduced as $\Delta Y=\varepsilon Y/(1+\rho Y)$ by the feedback control of exposure intensity in dose quotient as $\varepsilon' = \varepsilon - \rho\Delta Y$ and $\Delta Y = \varepsilon' Y$. The measure of dose reduction is to set the temporal shielding or to partially change the process of works, etc.

Suppose the n-step exposure process of dose X_0 to X_T or Y_0 to Y_T in $[0,T]$, the sum $A_n(t)$ is relative to the dose quotient Y_T at T, by substituting Y for X in Equation (2). According to the martingale central limiting theorem, $\mathrm{hyb}(\rho Y_T)$ becomes normally distributed and X_T becomes distributed as the HSB distribution. The HSB distribution reflects the distribution of individual doses controlled strongly but not tightly, by excising a prudent management of dose without the replacement of worker.

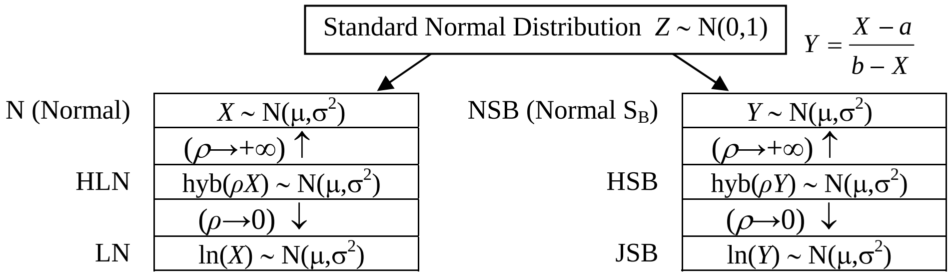
2.2.5 A system of probability distributions for the extended reference distribution

All probability distributions here can be derived from the standard normal distribution (see Figure 1). The LN and HLN distributions are suitable for the reference distribution defined in $0<X<+\infty$. The JSB and HSB distributions are suitable for the reference distribution defined in $0<a<X<b$.

The LN and HLN distributions provide the reference distribution so that it should limit the probability of exposure exceeding a given upper level, while the JSB and HSB distributions provide the reference distribution so that it should surely avoid exposure exceeding the upper level.

The HLN distribution ranges from the LN to the normal distribution according to the magnitude of ρ ($0<\rho<+\infty$). The HSB distribution also ranges from the JSB to the normal S_B distribution according to the magnitude of ρ ($0<\rho<+\infty$). As $Y=(X-a)/(b-X)$ is nearly equal to X/b for $a\rightarrow 0$ and $X/b \ll 1$, the JSB and HSB distributions approximate the LN and HLN distributions, respectively. Thus these distributions make a system of probability distributions.

Figure 1: A system of probability distribution for the extended reference distribution



2.3 The extended reference distribution

The extended reference distribution is defined in the same way as the reference distribution in the 1977 UNSCEAR report by setting the arithmetic mean of dose and the probability of individual dose exceeding the prescribed dose constraint, where other parameters are based on the past dose statistics.

The LN model is suitable for the reference distribution of doses far below the dose constraint. The HLN model is good for the reference distribution, where some doses become large enough to need the feedback dose control not approaching the dose constraint. The HSB model is fit for the reference distribution, where some doses become very large to need the strong feedback control based on dose quotient $(X-a)/(b-X)$ with no replacement of skilled worker. The JSB model provides the reference distribution, where some doses become extremely large enough to need the tight control of replacing skilled workers likely to exceed the upper level b.

2.3.1 LN model

The mean and percentage point of the LN distribution are $\exp(\mu+\sigma^2/2)$ and $\exp(\mu+z_Q\sigma)$, respectively, where z_Q is the standard normal variate corresponding to the upper probability Q. The parameters μ and σ of the reference distribution on the LN model are obtained for $\bar{x}$ and x_Q as shown in Table 1.

Table 1: The reference distribution on the LN model, $\ln(X) \sim N(\mu,\sigma^2)$

Equations	Solutions (a)
$\bar{x} = \exp\left(\mu + \sigma^2/2\right)$ $x_Q = \exp\left(\mu + \sigma \cdot z_Q\right)$	$\sigma = z_Q - \sqrt{z_Q{}^2 - 2\ln\left(x_Q/\bar{x}\right)}$ $\mu = \ln\left(x_Q\right) - \sigma \cdot z_Q$

(a) select the smaller one of two solutions for σ (see Appendix I, Annex E of [2])

2.3.2 HLN model

The mean and percentage point of the HLN distribution are calculated by the numerical function of $\bar{x}$ =HLNmean(ρ,μ,σ) and ρx_Q=cyb($\mu+z_Q\sigma$), where the function cyb(x) is the inverse function of the hybrid function hyb(x). The parameters μ and σ of the reference distribution on the HLN model are obtained for $\bar{x}$ and x_Q as shown in Figure 2, by setting a value of ρ based on the past dose statistics.

Figure 2: The reference distribution on the HLN model, hyb(ρX) ~ N(μ,σ^2)

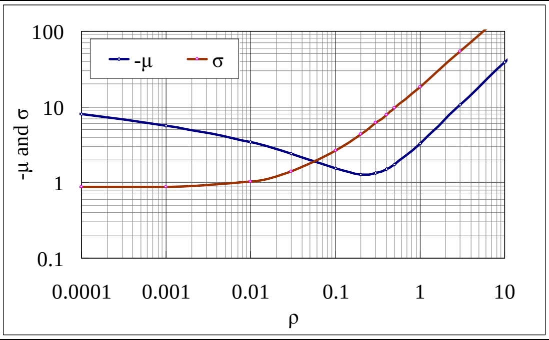
Equations

$$\rho\bar{x} = HLNmean(1, \mu, \sigma)$$
$$\rho x_Q = \mathrm{cyb}(\mu + \sigma \cdot z_Q)$$

Solutions

μ, σ: one solution for a given value of ρ

Right figure for $\bar{x}$ =5mSv, x_Q=50mSv, Q=0.1% (UNSCEAR,1977 [2])



2.3.3 HSB model

The mean and percentage point of the HSB distribution are calculated by the numerical function of HSBmean(a,b,ρ,μ,σ) and x_Q=b–(b–a)/{1+ρ^{-1}cyb(μ+$z_Q\sigma$)}. The parameters μ and σ of the reference distribution on the HLN model are obtained for $\bar{x}$ and x_Q as shown in Figure 3, selecting a value of ρ for a fixed value of b and a=0 based on the past dose statistics.

Figure 3: The reference distribution on the HSB model, hyb[$\rho(X–a)/(b–X)$] ~ N(μ,σ^2)

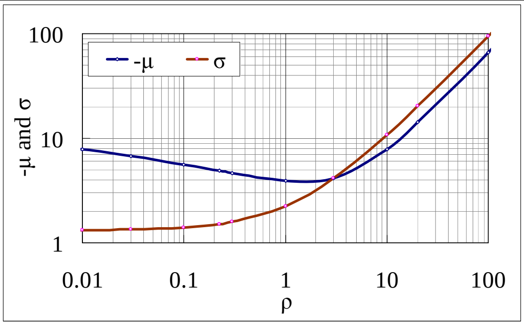
Equations

$$\bar{x} = HSBmean(0, b, \rho, \mu, \sigma)$$
$$x_Q = b - b/\{1 + \rho^{-1}\,\mathrm{cyb}(\mu + \sigma \cdot z_Q)\}$$

Solutions, setting the upper level b

μ, σ: one solution for a given value of ρ

Right figure for $\bar{x}$ =5mSv, x_Q=50mSv, Q=0.1%, b: 72.5 mSv estimated from LWRs in 1986 [12]



2.3.4 JSB model

The mean and percentage point of the JSB distribution are calculated by the numerical function of JSBmean(a,b,μ,σ) and x_Q=b–(b–a)/{1+exp(μ+$z_Q\sigma$)}. The parameters μ and σ of the reference distribution on the JSB model are obtained for $\bar{x}$ and x_Q as shown in Table 2, selecting a value of b based on the past dose statistics.

Table 2: The reference distribution on the JSB distribution, ln[$\rho(X–a)/(b–X)$] ~ N(μ,σ^2)

Equations	Solutions
$\bar{x} = JSBmean(0, b, \mu, \sigma)$ $x_Q = b - b/\{1 + \exp(\mu + \sigma \cdot z_Q)\}$	e.g. $\sigma = 3.0962$, $\mu = -4.5936$ for a=0mSv, b=50.3457mSv estimated from LWRs in 1986 [12]

2.3.5 The comparison of reference distributions among models

The reference distribution of the HLN model for $\bar{x}$ =5mSv and x_Q=50mSv changes from that of the LN model to that of the N model according to the value of ρ (see Figure 4). LN_N(x)/N is the reference distribution of the LN model in the 1977 UNSCEAR report, and LN_S(x)/S is the collective dose distribution obtained by calculating the first moment distribution of this reference distribution.

The graphs of HLN_ρ for N(<x)/N and S(<x)/S are the reference dose distributions and the collective dose distributions for ρ (0.001, 0.1, 1, 100). The graph of the HLN reference distribution ranges from HLN_0.001 close to the LN one to HLN_100 close to the normal one. This flexibility of the HLN graph can express the effect of feedback control on large doses, resulted in departure from the LN graph, in terms of the reference distribution. Table 3 shows distribution ratios of SR_E and NR_E for E of 1, 5, 10 and 15mSv obtained from Figure 4. Thus the collective distribution ratio, e.g., SR_{15}, varies largely by the degree of feedback control (ρ) despite the same collective dose to a workforce.

Figure 5 shows the extended reference distribution ($\bar{x}$ =2.6mSv, x_Q=50mSv, z_Q=4.473) of annual dose statistics of 1986 US LWRs [12], with the best fit conditions for HLN (ρ=2.96), HSB (ρ=3.35, a=0, b=72.5) and JSB (a=0, b=50.3), plotted as LN_N, HLN_N, HSB_N and JSB_N by model.

Figure 4: Reference distributions of the HLN model by changing ρ (0.001, 0.1, 1 and 100).

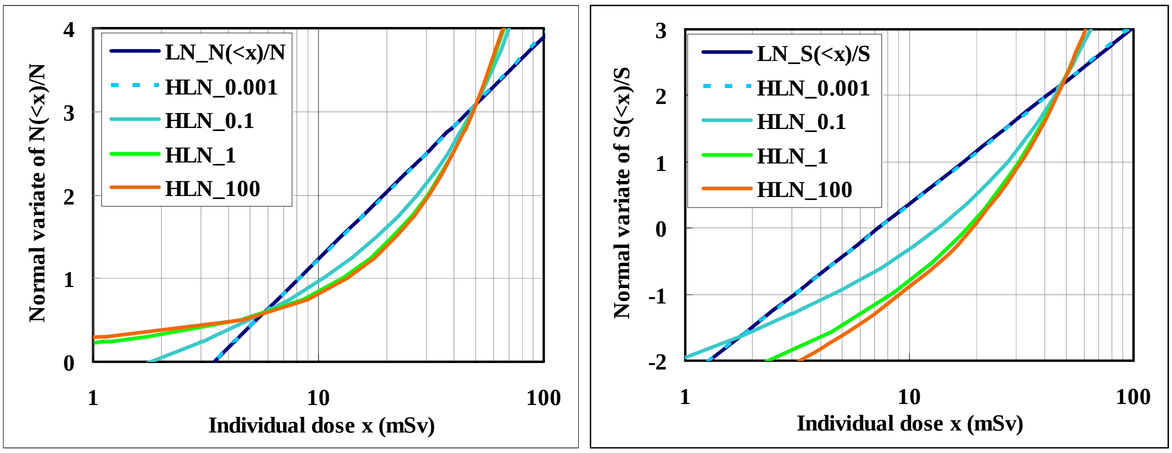
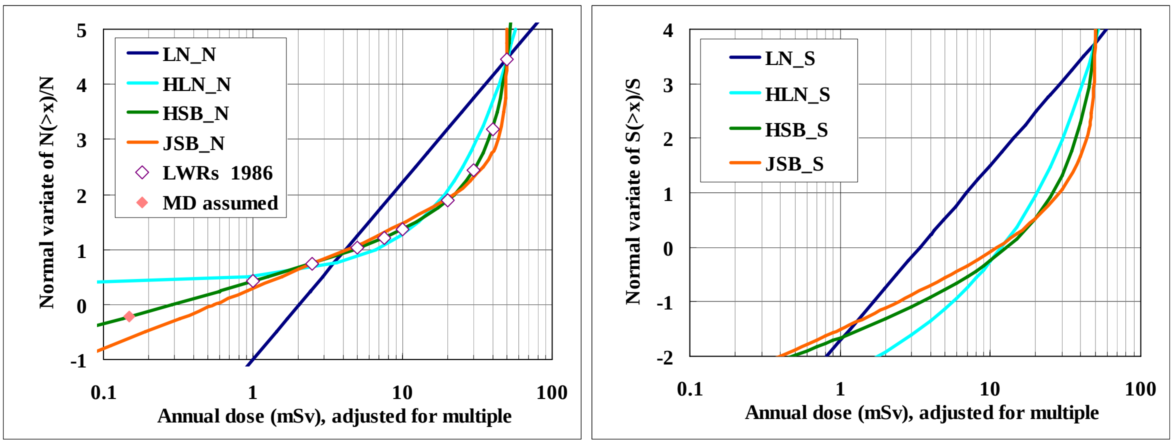


Table 3: Distribution ratios SR_E and NR_E obtained from above graphs

SR_E or NR_E		SR_E=S(>E)/S , collective dose				NR_E=N(>E)/N, individual dose			
E (mSv) (a)		1	5	10	15	1	5	10	15
ρ (mSv^{-1})	0(LN)	0.989	0.668	0.357	0.202	0.923	0.332	0.109	0.044
	0.001	0.988	0.671	0.363	0.207	0.918	0.332	0.110	0.046
	0.1	0.975	0.824	0.626	0.449	0.600	0.306	0.168	0.096
	1	0.994	0.931	0.788	0.611	0.409	0.297	0.200	0.129
	100	0.998	0.948	0.813	0.633	0.385	0.301	0.210	0.137

(a) E is selected by the 2000 UNSCEAR report [11].

Figure 5: A comparison of the extended reference distribution among models (US LWRs [12])



The plots (◊) are the observed distribution of annual doses adjusted for multiple reporting of transient individuals among the full year operation reactors in 1986 [12] and the measurable dose (MD) assumed as 0.15mSv. The data reported is the total collective dose of 423.86 person·Sv of 161,656 monitored workers with the mean of 2.6mSv or 93,979 measurable workers with the mean of 4.5mSv, where the dose is defined as the sum of external and internal effective dose. Graphs are $\ln(X) \sim N(0.714, 0.715^2)$, $\text{hyb}(2.96X) \sim N(-15.022, 37.579^2)$, $\text{hyb}[3.35X/(72.5\text{-}X)] \sim N(-4.299, 3.075^2)$, and $\ln[X/(50.3-X)] \sim N(-4.507, 2.120^2)$ for the LN, HLN, HSB and JSB models, respectively.

The reference distribution HSB_N fits the best among four models, to the observed distribution. The reference distribution LN_N shows the largest departure from the data (◊), that is, to imply the effect of strong control of doses below 50mSv. All reference distributions in Figure 5 have the same collective dose and the same upper probability of dose above 50mSv, but they show that the actual radiation works should required the relatively high exposure in order of several tens in mSv incurred by some skilled workers. The collective dose distribution ratio is also quite different between LN_S and others (HLN_S, HSB_S and JSB_S) for E of 1, 5, 10 and 15.

The HSB model gives a best interpretation of the actual dose control how to constrain the individual doses below the dose of 50mSv in the necessary order of total collective dose as mentioned above. Recently the actual dose statistics often shows the HSB or the JSB dose distribution. The LN and HLN distributions are, however, necessary for the analysis of actual dose distributions to evaluate the difference of dose control. Thus the all extended reference distributions are inevitable for us to quantify the degree of dose management adequately by the comparison.

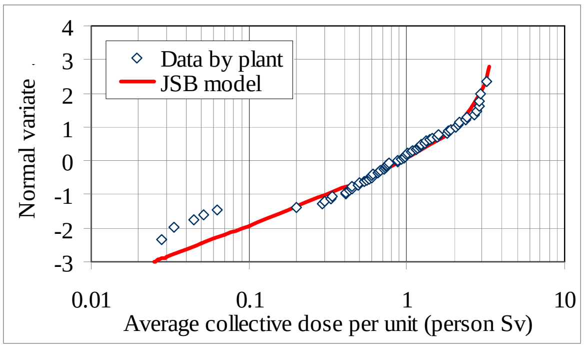
3. Examples of the extended reference distribution applied to actual dose statistics

3.1 Occupational exposure

The optimization is directly excised in the different strata of dose management, ranging from job-by-job to plant-by-plant. It is, however, more indirectly or obscurely excised over the industry or the national level. The distribution of doses to workers in a plant may reflect implicitly the effect of dose control in collective dose and individual dose. The dose distribution varies by plant because of the difference in operating year, past maintenance history, radiation protection program, etc. As a national level, the dose distribution is affected by the change of regulatory dose standards or dose limits by ICRP.

Figure 6 shows the distribution of average collective dose per unit by plant among 66 plants in 2006. Except for five smallest data, the JSB model fits well the distribution of collective doses per unit among 66 plants, ranging from 0.2 to 3.2 person·Sv. The person·Sv per unit is roughly proportional to the number of measurable workers by plant. In Figure 7, each of the observed dose distribution by plant (1~66) lies roughly along the distribution of doses (◊ all) pooled over 66 plants, on the hybrid probability plot. The bold straight line is the HLN fit to the pooled data (◊ all). Except some distributions of low doses, the HLN model fits well the data of dose distributions by plant.

Figure 6: Distribution of average annual collective doses per unit by plant of 66 plants in 2006 [12].



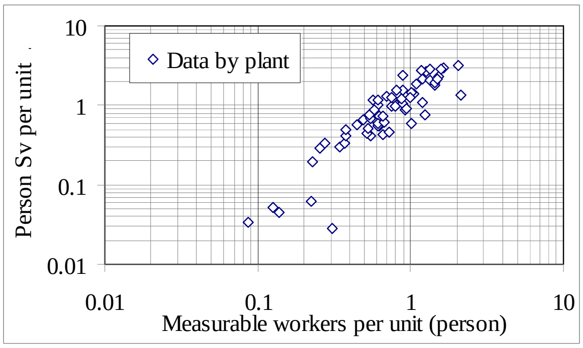
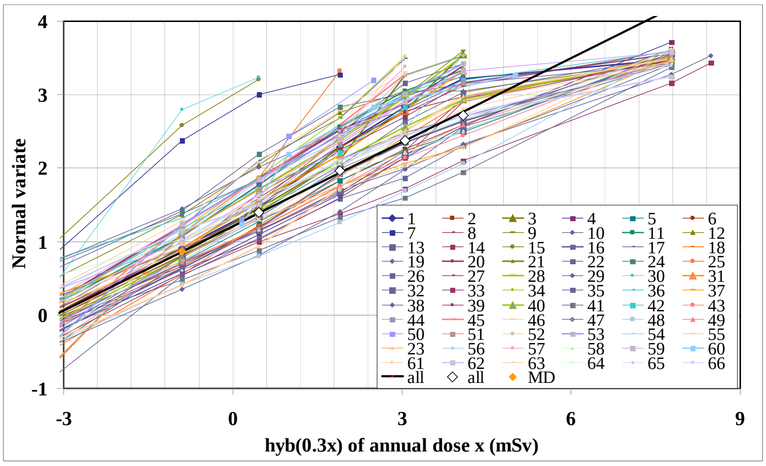
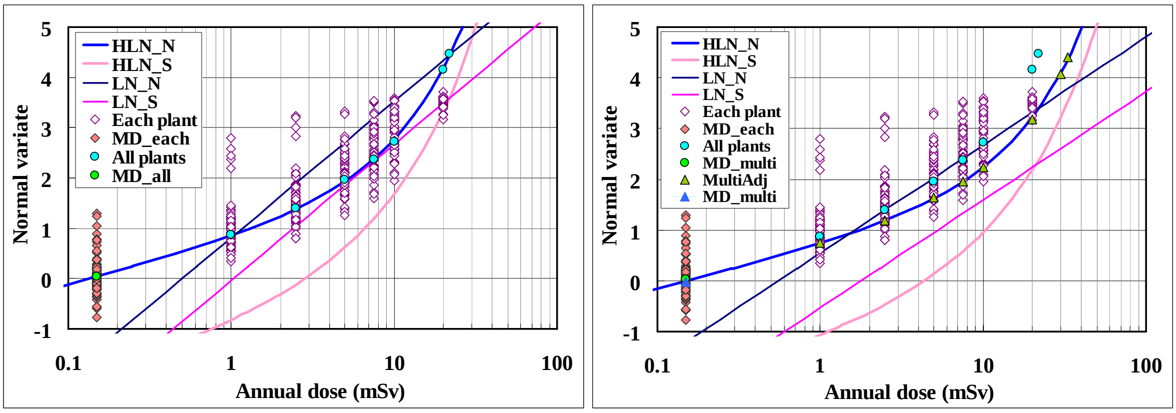


Figure 7: Distributions of annual doses by each of 66 plants in 2006 [12] on hybrid probability paper



The reference distribution HLN_N fits well the dose data (●) for pooled all plants on the left panel of Figure 8. To compare with the reference distribution LN_N, the downward curvature of HLN_N indicates the strong control of annual doses below about 25mSv. The reference distribution fitted to data (▲) on the right panel, however, shows the less dose control than that on the left panel, where the data (▲) is the distribution of annual doses adjusted for the multiple reporting in 2006.

Figure 8: HLN reference distributions for workforce pooled all plants and adjusted for the multiple.



The data (▲) adjusted for multiple reporting is suitable to evaluate the compliance with regulatory limits and to analyze the resultant effects of dose control as the national level, while the effect of dose control would not be understood explicitly. The data (●) is also not so clear about the effect of dose control because of just pooled one over all 66 dose distributions by plant. The data (◊) for each plant is likely more clear about the effect of actual dose management. However the extended reference distribution analysis results in dose control indication and the good distribution fit to the actual data.

In compliance with the ICRP recommendations about the optimization of protection (ALARA) and the satisfaction of dose constraint, the extended reference distributions are essential to project the dose distribution for the future jobs and to evaluate the resultant dose distributions. For the design of adequate allocation of collective dose and individual dose, the approach of the extended reference distribution would be useful in various aspects of occupational exposure by plant, country, year, etc.

3.2 Public exposure

The public exposure is low enough to require no individual monitoring in general, except for the high indoor radon exposure, which could be likely in the same order of occupational exposure.

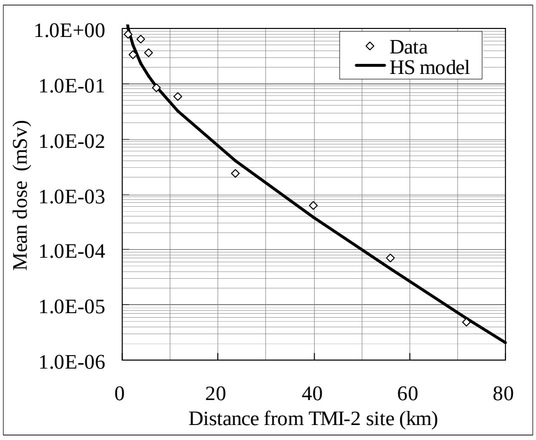
Such exposure needs the voluntary control of dose reduction as far as practicable. Also the public exposure due to the accidents should be reduced according to the magnitude of projected dose by evacuation or others. These countermeasures reduce the residual dose in the larger degree when the projected dose is the higher. The exposure control depending on the magnitude of dose might modify the distribution of doses to the population.

Many experts use the LN distribution to express the distribution of doses to the population. This is actually right in most cases, but we need the extended dose distribution to the public exposure because of the stronger control on the higher exposure subgroup of people. In addition, the event of exposure occurred at one point or site is apt to bring the similar geometric structure of the point kernel model; $D \propto B \cdot \exp(-\mu R)/(4\pi R^2)$ where D: dose, R: source-receptor distance, μ: linear attenuation coefficient and B: buildup factor depended on μR. This model brings the equation $\ln(D)=\alpha + \beta \cdot \mathrm{hyb}(\rho R)$, conveniently calling the hybrid scale (HS) model.

Figure 9 shows the mean doses to the population from the TMI-2 accident [14] could be well fitted by the HS model. By combining the mean dose and the population by distance, the distribution of doses to the population is obtained within 50 miles (80km) from the site, as plotted (◊) in Figure 10. The graphs LN_N and HSB_NLN are, respectively, the reference dose distribution of the LN and HSB models. The extended reference distribution HSB_N of the HSB model shows the good fit to the data (◊). The model of $\mathrm{hyb}[\rho(X-a)]\sim N(\mu,\sigma^2)$ also fits well to the data(◊) [15].

Thus the reference dose distribution is applicable for the distribution of doses to the population to predict the characteristics of individual dose distribution or reasonably to minimize the fraction of collective dose exceeding a certain dose level by analysing the collective dose distribution HSB_S.

Figure 9: Mean doses to the population in a function of distance from the TMI-2 accident site [14,15].



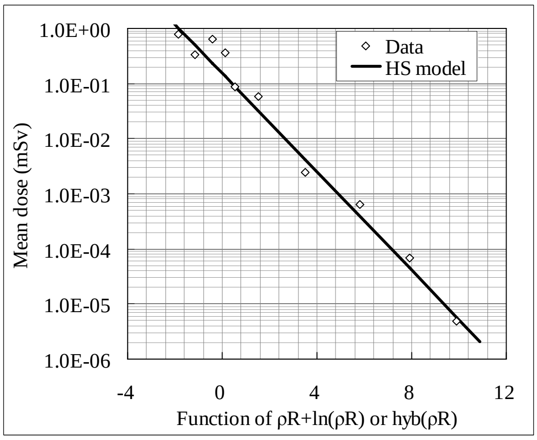
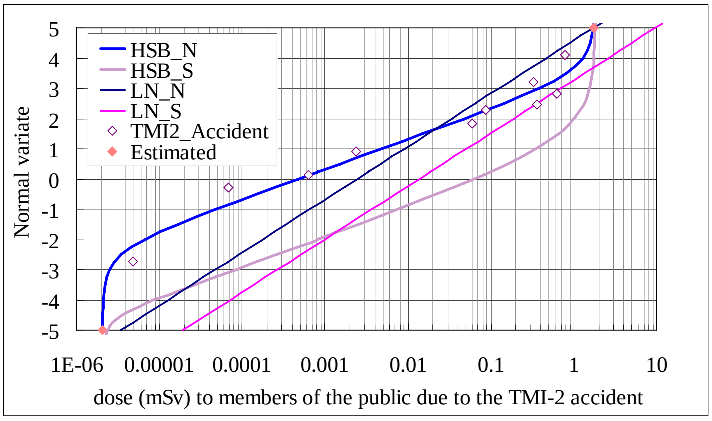


Figure 10: Reference distributions of doses to the population within 50 miles from the site [14].



4. Conclusion

The reference distribution defined in the 1977 UNSCEAR report was presented by extending the probability distributions to afford the degree of dose control on the decrease of mean dose (collective dose) and the feedback reduction of higher exposure below dose constraints. The models are the log-normal (no feedback of dose reduction), hybrid log-normal (feedback control on dose *X*), hybrid S_B (feedback control on dose quotient *X/(D-X)*) and Johnson's S_B (tight dose control limiting below *D*). The method of constructing the extended reference distribution was shown for each of four models. Some examples were presented to use the extended reference distribution for the actual dose statistics. It proved that the extended reference distribution can provide the interpretation of the reasonable control on individual dose under a fixed collective dose with the excellent fit to the actual data. It showed that the extended reference distribution is useful for the predicting the distribution of doses to the public and to minimize the fraction of collective dose due to exposure exceeding a certain dose level. There are still issues (collective dose due to low dose, selection of the upper probability for the public) on the application of the extended reference distribution. The further works might be expected.

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