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Determining Efficient Solutions to Multiple Objective Linear Programming Problems

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Abstract

A new posteriori method namely, **moving optimal method** is proposed for solving multiple objective linear programming problems. It provides efficient solution line segments to the problem with the percentage level of satisfaction of each of the objective at each point on it which is very much useful to decision makers for choosing an efficient solution according to their level of satisfaction on the objective functions. Illustrative example is presented to clarify the idea of the proposed approach.

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1. Introduction

In the classical linear programming model, a single linear objective function with linear constraints is only considered. In practice, many constrained linear models may involve more than one objective with conflicting nature. Such problems are termed as multiple objective linear programming (MOLP) problems. The MOLP models can arise in the fields of Science, Engineering and Management Science. The main aim of a MOLP problem is to optimize 'k' different linear objective functions, subject to a set of linear constraints where $k \geq 2$. In the MOLP problem, optimizing all objective functions at the same time is not possible because of the conflicting nature of the objectives. The concept of optimality in the MOLP problem is replaced with that of efficiency [1,10] .

In the literature, a variety of algorithms [6, 2, 15, 14, 7, 11, 13, 4, 9, 8, 3, 12] for finding efficient solutions to multi-objective optimization problems have been proposed. For getting good solution from a set of efficient solutions, a decision maker is needed to provide additional preference information and to identify the most satisfactory solution. Hwang and Masud [6] classified the methods for solving multi-objective optimization problems into three categories namely, priori methods, interactive methods and posteriori methods. The posteriori methods provide the whole picture of solutions space of the problem. Hence, these methods are most preferred one for decision makers. considered the fuzzy approach to solve the MOLP problem.

In this paper, we propose a new posteriori method for finding efficient solutions and efficient solution line segments to the MOLP problem, called. moving optimal method. In the proposed method, solving the jth objective related problem using the optimal solution of kth objective related problem as an initial solution by the simplex method and collecting all feasible solutions obtained by each of iteration of the simplex method; this process is repeated for all possible pairs of the objective related problems ; collecting all efficient solutions from the set of the solutions obtained and the percentage level of satisfaction of each of the objective function for each efficient solution is provided for the benefit of decision makers. Hence, the moving optimal method is based on the simplex method only which differs totally from utility function method, goal programming approach, min-max method, two phase method, fuzzy programming technique, genetic approach and evolutionary approach. Numerical example is given for better understanding the procedures of the proposed method. Efficient solution line segments to MOLP models provided by the moving optimal method are very much helpful for the decision makers to evaluate the economical activities and make self satisfied managerial decisions when they are handling MOLP models.

2. Preliminaries

Consider the following MOLP problem :

(P) Maximize $Z(X) = (Z_1(X), Z_2(X), ..., Z_k(X))$
subject to $AX \le B$, $X \ge 0$

where $Z_i : R^n \to R$ ($i = 1,2,...,k$) is linear , A is an $m \times n$ real matrix, $B = (b_1,...,b_m) \in R^m$ and $X \in R^n$, an n-dimensional Euclidian space.

Let $P = \{U \in R^n; AU \leq B \text{ and } U \geq 0\}$ be the set of all feasible solution to the problem (P).

Now, we need the following definitions which can be found in [1,2,3]

Definition 2.1 A feasible point $X^\circ \in P$ is said to be an efficient solution for (P) if there exists no other feasible point X of the problem (P) such that $Z_i(X) \geq Z_i(X^\circ)$, $i = 1,2,...,k$ and $Z_r(X) > Z_r(X^\circ)$, for some $r \in \{1,2,\ldots,k\}$. That is, an efficient solution is the solution that cannot be improved in one objective function without deteriorating their performance in at least one of the rest.

For simplicity, $(Z_1(X^\circ), Z_2(X^\circ),..., Z_k(X^\circ))$ is called a solution to the problem (P) if X° is a solution to the problem (P).

Now, the problem (P) can be separated into ‘ k ’ number of single objective LP problems as follows:

(P_t) Maximize $f_t(X)$

subject to $AX \leq B$, $X \geq 0$,

where $t = 1,2,...,k$.

Definition 2.2 : Let X_t° be the optimal solution to the problem (P_t), $t = 1,2,...,k$. Then, the value of the objective function $(Z_1(X_1^\circ), Z_2(X_2^\circ),..., Z_k(X_k^\circ))$ is called the ideal solution to the MOLP problem (P).

Definition 2.3 : An efficient solution $X^\circ \in P$ is said to be the best compromise solution to the problem (P) if the distance between the ideal solution and the objective value at X°, $(Z_1(X^\circ), Z_2(X^\circ),..., Z_k(X^\circ))$ is minimum among the efficient solutions to the problem (P).

3. Moving Optimal Method

Now, we define the following new terms namely, efficient solution line segment and the percentage level of satisfaction of the kth objective of the MOLP problem for the given solution to the problem (P) which are used in the proposed method.

Definition 3.1: Let $(Z_1(X^\circ), Z_2(X^\circ), ..., Z_k(X^\circ))$ and $(Z_1(U^\circ), Z_2(U^\circ), ..., Z_k(U^\circ))$ be the objective value of the problem (P) at the efficient solutions X° and U° respectively. The line segment joining the points $(Z_1(X^\circ), Z_2(X^\circ), ..., Z_k(X^\circ))$ and $(Z_1(U^\circ), Z_2(U^\circ), ..., Z_k(U^\circ))$ is said to be **an efficient solution line segment** if each point on the line segment is an efficient solution to the problem (P).

Definition 3.2: The percentage level of satisfaction of the objective of the problem (P_t) for the solution U to the problem (P), $LS(Z_t;U)$ is defined as follows:

$$LS(Z_t;U) = \begin{cases} \left(\dfrac{Z_t(U)}{Z_t(X_t^\circ)}\right)\times 100 & \text{if the problem } (P_t) \text{ is maximization type} \\ \left(\dfrac{2Z_t(X_t^\circ) - Z_t(U)}{Z_t(X_t^\circ)}\right)\times 100 & \text{if the problem } (P_t) \text{ is minimization type} \end{cases} .$$

Now, we introduce a new method namely, moving optimal method for finding efficient solutions and efficient solution line segment with their percentage level of satisfaction to the MOLP problem.

The proposed method proceeds as follows:

Step 1: Construct ‘ k’ number of single objective LP problems from the given problem (P) as follows:

(P_t) Maximize $Z_t(X)$

subject to $AX \le B$, $X \ge 0$,

where $t = 1,2,...,k$.

Step 2: Compute the optimal solution to the problem (P_t) using the simplex method for $t = 1,2,...,k$. Let the optimal solution be X_t°, $t = 1,2,...,k$. Then, obtain the ideal solution to the given problem (P) .

Step 3: **(a)** Take $t \in \{1,2,...,k\}$.

(b) Compute all feasible solutions to the problem (P_s), $s = 1,2,...,k, u \ne t$ up to its optimal solution considering the optimal solution to the problem (P_t), X_t° as an initial basic feasible solution to the problem (P_s) using the simplex method.

Step 4: Repeat the Step 3. for each value of t in $\{1,2,...,k\}$ and collect all solutions to the given problem (P) with its objective values.

Step 5: Collect all efficient solutions to the given problem (P) from the Step 4. and Compute the distance between the ideal solution to the given problem (P) and the objective value of the given problem at each of the efficient solution, that is, each efficient solution to the given problem (P).

Step 6. Draw a polygon connecting the efficient solutions to the given problem (P) such that atleast one of the co-ordinates is in ascending order and mark the efficient solution line segments which are the sides of the polygon .

Step 7: Find the best compromise solution to given problem (P) and construct the satisfaction level table for the efficient solutions of the problem (P).

Step 8: The values of the decision variables of the problem (P) for a point on the efficient solution line segment can be obtained using the objective values at the point and the matrix method.

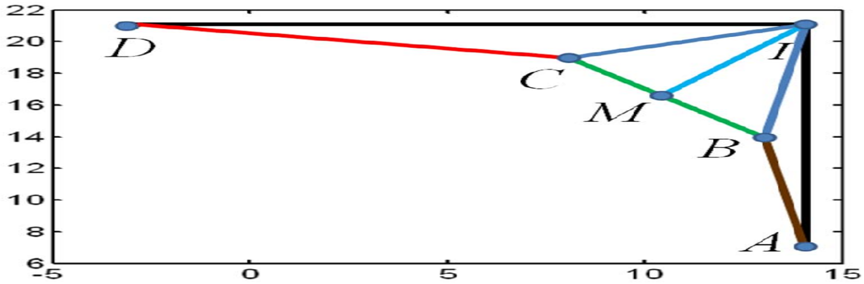
Remark 3.1: In general, the proposed method provides an infinite number of efficient solutions to the given MOLP problem which are very much useful to decision makers for selecting a solution according to their satisfaction.

The proposed method for solving the MOLP problem is illustrated by the following example.

Example 3.1: Consider the following MOLP problem:

(P) Maximize $Z(X) = (Z_1(X), Z_2(X)) = (-x_1 + 2x_2, 2x_1 + x_2)$
subject to
$-x_1 + 3x_2 \le 21$; $x_1 + 3x_2 \le 27$; $4x_1 + 3x_2 \le 45$; $3x_1 + x_2 \le 30$; $x_1, x_2 \ge 0$.

Now, by the Step 1. to Step 6. of the proposed method, the ideal solution $I = (14,21)$ and the efficient solutions $X_1 = (0,7)$, $X_2 = (3,8)$, $X_3 = (6,7)$ and $X_4 = (9,3)$ and their corresponding objective values are A $= (14, 7)$, B $= (13, 14)$, C $= (8, 19)$ and D $= (-3, 21)$ respectively, are obtained.
By the Step 7, the line segments DC, CB and BA, that is, the sides of the polygon ABCD are efficient solution segments.
Now, by the Step 8., $M = (10, 17)$ is the point on CB such that the distance between M and I is minimum. So, $M = (10, 17)$ is the best compromise solution to the given problem.



Now, by the Step 9, the point corresponding to $M = (10, 17)$ is (4.8, 7.4) and the level of satisfaction table is given below:

S.No.	Efficient solution (X)	$Z(X)$	$LS(Z_1; X)$	$LS(Z_2; X)$
1	$X_1 = (0,7)$	$A = (14, 7)$	100	33.33
2	$G_1 = (3t_1, 7 + t_1)$; $t_1 \in [0,1]$	$Z(G_1)$	$LS(Z_1; G_1)$	$LS(Z_2; G_1)$
3	$X_2 = (3,8)$	$B = (13, 14)$	92.86	66.67
4	$G_2 = (3 + 3t_2, 8 - t_2)$; $t_2 \in [0,1]$	$Z(G_2)$	$LS(Z_1; G_2)$	$LS(Z_2; G_2)$
5	$X_3 = (6,7)$	$C = (8, 19)$	57.14	90.48
6	$G_3 = (6 + 3t_3, 7 - 4t_3)$; $t_3 \in [0,1]$	$Z(G_3)$	$LS(Z_1; G_3)$	$LS(Z_2; G_3)$
7	$X_4 = (9,3)$	$D = (-3, 21)$	-21.4	100
8	$X_5 = (4.8, 7.4)$	$M = (10,17)$	71.43	80.95

where

$Z(G_1) = (14 - t_1, 7 + 7t_1)$; $Z(G_2) = (13 - 5t_2, 14 + 5t_2)$; $Z(G_3) = (8 - 11t_3, 19 + 2t_3)$;

$LS(Z_1; G_1) = \left(\frac{14 - t_1}{14}\right) \times 100$; $LS(Z_2; G_1) = \left(\frac{7 + 7t_1}{21}\right) \times 100$;

$LS(Z_1; G_2) = \left(\frac{13 - 5t_2}{14}\right) \times 100$; $LS(Z_2; G_2) = \left(\frac{14 + 5t_2}{21}\right) \times 100$;

$LS(Z_1; G_3) = \left(\frac{8 - 11t_3}{14}\right) \times 100$ and $LS(Z_2; G_3) = \left(\frac{19 + 2t_3}{21}\right) \times 100$.

Remark 3.2: For the Example 4.1., the best compromise solution given by De and Bharti Yadav [3] is (8,19), but by the proposed method, we provide three efficient solution line segments, $Z(G_1)$, $Z(G_2)$ and $Z(G_3)$ including the best compromise solution (10,17) which is better than (8,19).

4. Conclusion

In this paper, the moving optimal method for finding efficient solutions to the MOLP problem is proposed. The efficient solution line segments with level of satisfaction provided by the moving optimal method enables the decision makers to choose an appropriate solution depending on their financial position and their level of satisfaction of objectives. In near future, we extend the moving optimal method to multiple objective integer linear programming problems and multiple objective fuzzy linear programming problems.

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