Impurity Transport and Plasma Flow in a Mixed Collisionality Stellarator Plasma

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Abstract:

Neoclassical accumulation of impurities in the core of hot stellarator plasmas is a known problem. The complexity of neoclassical transport in stellarators means that few analytic studies are available to support numerical modelling efforts, and a robust understanding of the parameter dependence of the impurity flux is still lacking. Therefore we present an extension of the existing analytic treatment for highly collisional plasmas, into the experimentally relevant mixed collisionality regime – where a dominant heavy, collisional, impurity is present in a collisionless bulk plasma, taken here to be in the $1/\nu$ regime. We find that temperature screening of the impurity flux by the bulk ion temperature gradient will arise. We also determine the bulk ion flow in the flux surface, and thus the effect of the impurity on the bulk ion contribution to the bootstrap current.

1 Introduction

The analytic treatment of neoclassical transport in a hot stellarator plasma is a difficult problem, due to the complex magnetic field structure. Many numerical codes tackle this, but supporting theoretical work is limited. With the commissioning of the W7-X stellarator, the extension of the existing analytic work into the low-collisionality regimes relevant to hot stellarator plasmas is of current interest [1]. The neoclassical accumulation of impurities in the core of stellarator plasmas is a known problem, but a robust understanding of the parameter dependence of the impurity flux is still lacking.

A flux-friction formalism was introduced in [2] and used to determine the radial flux of a single impurity, present in a background plasma. However, both the impurity and bulk ion species were taken to be highly collisional and the applicability of such a regime is restricted to edge plasmas. We have extended that work into the experimentally relevant mixed collisionality regime, where a dominant heavy, highly charged and so collisional, impurity species z is present in a collisionless bulk plasma. The bulk ion species i is taken here to be in the $1/\nu$ regime, so the effect of the radial electric field on the particle trajectories is neglected. We evaluate the radial impurity flux, expressing it in terms of neoclassical transport coefficients, which give the response to the usual radial driving gradients and consider the factors affecting the direction of the flux. We also calculate the bulk ion parallel flow, which gives the bulk ion contribution to the bootstrap current in this regime, accounting for the effect of the impurities.

2 Formulation

In this section, we outline the formalism used to calculate the radial impurity flux, following [2]. The drift kinetic equation is expanded in the magnetisation parameter $\delta_a = \rho_a/L$ as usual [3], where ρ_a is the gyroradius of species a, mass m_a and charge $Z_a e$, and L is a characteristic length scale, perpendicular to the background magnetic field. We assume $Z_z > 1$, but not so large as to require that δ_z is higher order with respect to δ_i. Excluding the very low collisionality stellarator regimes, we take the geometry to be sufficiently well optimised that the $1/\nu$ regime can be produced, and the leading order piece of the expanded distribution function $f_a = f_{a0} + f_{a1} + \ldots$ is Maxwellian, $f_{a0} = \left(n_a/\pi^{3/2}v_{Ta}^3\right)\exp\left(-v^2/v_{Ta}^2\right)$, where v is velocity and the thermal velocity $v_{Ta} = \sqrt{2T_{a0}/m_a}$. The background temperatures T_{a0} of the species equalise, and quasineutrality requires that the leading order potential $\phi_0 = \phi_0(\psi)$ and density n_{a0} are flux functions, where ψ is the flux surface label. The first order drift kinetic equation for the distribution function f_{a1} then takes the form $C_a\left(f_{a1}\right) = v_\parallel \nabla_\parallel f_{a1} + \mathbf{v}_{da}\cdot\nabla f_{a0} + (Z_a e/T_{a0})v_\parallel f_{a0}\nabla_\parallel\phi_1$. The independent velocity space coordinates are $\epsilon_a = m_a v^2/2 + Z_a e\phi_0$ and $\mu_a = m_a v_\perp^2/2B$, and parallel and perpendicular are taken with respect to the background magnetic field direction. The linearised, gyroaveraged, collision operator for species a is denoted by $C_a = \sum_b C_{ab}$ where the sum is over the ion species present. The effect of friction against electrons, e, is small in the electron-ion mass ratio, so is neglected throughout.

Flux-friction relation The radial flux of a species is given by the total drift, magnetic plus $E\times B$, and can be written as [2]

$$\langle \mathbf{\Gamma}_a \cdot \nabla\psi\rangle = \left\langle \int f_a \mathbf{v}_{da}^{\rm tot}\cdot\nabla\psi d^3v\right\rangle = \frac{1}{Z_a e}\left\langle uBR_{a\parallel} + \left(p_{a\parallel} - p_{a\perp}\right)\frac{\nabla_\parallel(uB^2)}{2B}\right\rangle. \quad (1)$$

The equilibrium function $u = j_{0\parallel}/p_0' B$ is given in terms of the equilibrium pressure gradient p_0', where a prime denotes the derivative with respect to ψ, and the equilibrium parallel current $j_{0\parallel}$, with B the magnitude of the magnetic field. Angled brackets $\langle\ldots\rangle$ denote the flux surface average of a quantity. The drive due to the total parallel friction $R_{a\parallel} = m_a\int v_\parallel C_a(f_{a1})d^3v$ may be compared to that due to the species' pressure anisotropy $p_{a\parallel} - p_{a\perp}$. The parallel friction between unlike species drives the flux, and for the case of disparate mass ions considered here we will have $R_{zi\parallel} \sim m_i n_i\left(V_{i\parallel} - V_{z\parallel}\right)\nu_{iz}$, in terms of the bulk ion $V_{i\parallel}$ and impurity $V_{z\parallel}$ parallel flows. In a collisionality expansion of the first-order drift kinetic equation the pressure anisotropy will appear in first order for the collisional species [2]. We then find that the pressure anisotropy drive will be small when the collisionalities satisfy

$$\frac{1}{\nu_{*iz}} \ll \frac{n_i Z_z^2}{n_z}\sqrt{\frac{m_i}{m_z}}\nu_{*zz}, \tag{2}$$

where $\nu_{*ab} = \nu_{ab}/\omega_{ta} = L_\parallel/\lambda_{mfp}^{ab}$, ω_{ta} is the transit frequency, $L_\parallel$ is a characteristic parallel lengthscale and the mean free path $\lambda_{mfp}^{ab} = v_{Ta}/\nu_{ab}$. We assume this ordering is satisfied and will take the dominant drive here to come from the parallel friction. In the mixed collisionality case considered, with $Z_z \gg 1$ and $Z_i \sim 1$, this imposes a lower limit on the bulk ion collisionality.

Our task is then to calculate the interspecies friction. Momentum conservation in collisions allows us to write the impurity flux in terms of the bulk ion-impurity parallel friction $R_{zi\parallel} = -R_{iz\parallel}$, and we then use the common disparate mass form of the collision operator $C_{iz}(f_{i1}) = \nu_D^{iz}(v)\left(\mathcal{L}(f_{i1}) + m_i v_\parallel V_{z\parallel} f_{i0}/T_{i0}\right)$ to give

$$R_{zi\parallel} = m_i \int \nu_D^{iz}(v) v_\parallel f_{i1} d^3v - \frac{m_i n_{i0}}{\tau_{iz}} V_{z\parallel}. \tag{3}$$

Here we have noted that the pitch angle scattering operator $\mathcal{L} = (1/2)\partial_\xi \left(1-\xi^2\right)\partial_\xi$ is self-adjoint, $\xi = \cos\theta = v_\parallel/v$ is the cosine of the particle pitch angle, the normalised velocity $x_a = v/v_{Ta}$, the deflection frequency $\nu_D^{iz}(v) = 3\pi^{1/2}/4\tau_{iz}x_i^3 = \hat{\nu}_D^{iz}/x_i^3$ and the collision time is $\tau_{iz} = 3(2\pi)^{3/2}\sqrt{m_i}T_{i0}^{3/2}\epsilon_0^2/n_{z0}Z_z^2 Z_i^2 e^4 \ln\Lambda$. We thus require the bulk ion distribution to first order and the impurity parallel flow.

Impurities By considering density conservation, using the $v_\parallel/B$ moment of the first order drift kinetic equation written in conservative form and noting the equilibrium current function satisfies $\nabla_\parallel u = -B^{-1}\nabla\cdot(B^{-1}\mathbf{b}\times\nabla\psi)$, we obtain a general form for the impurity flow

$$V_{z\parallel} = \left(\frac{1}{n_{z0}Z_z e}\frac{dp_{z0}}{d\psi} + \frac{d\phi_0}{d\psi}\right) uB + \frac{K_z(\psi)\,B}{n_{z0}}. \tag{4}$$

$K_z(\psi)$ is the flux surface function resulting from integration along a field line. To treat the collisional impurity species, the first order drift kinetic equation is expanded in the small parameter $1/\nu_{*zz}$. Beginning at order -1, we find $C_z(f_{z1}^{(-1)}) = 0$, that is the impurity distribution has the form of a perturbed Maxwellian. We can then constrain the parallel impurity flow by considering momentum conservation through the first two orders. In leading order it requires $R_{zi\parallel}^{(1)} = 0$: by definition $V_{z\parallel}^{(-1)} \sim \delta_z v_{Tz}\nu_{*zz}$, but eq. (3) implies $V_{z\parallel}^{(-1)} \sim \delta_i v_{Ti}$. As we assume the bulk ion are collisionless $\nu_{*iz} < 1$, allowing only a finite impurity content $Z_{\text{eff}} - 1 = O(1)$ or explicitly $\nu_{*iz}n_z Z_z/n_i Z_i^2 < 1$, requires for consistency $V_{z\parallel}^{(-1)} = 0$, where $n_e Z_{\text{eff}} = \sum_{a=i,z} n_a Z_a^2$. Thus $R_{zi\parallel}^{(0)}$ is found to be the leading order friction driving the particle flux. The flux surface function K_z is constrained by parallel momentum conservation at zeroth order in the collisional expansion. The kinetic equation to this order becomes a Spitzer-type problem for $f_{z1}^{(0)}$ [3], and the flux surface average of the $Bmv_\parallel$ moment gives $\left\langle BR_{zi\parallel}^{(0)}\right\rangle = 0$. We find that this form, with eq. (3), is sufficient and we will not need to determine the function K_z explicitly.

Bulk ions The collisionless bulk ions are treated by expanding their first order drift kinetic equation in the small parameter ν_{*ii}. The piece of the distribution driven directly by the potential perturbation ϕ_1 gives no contribution here, so all such terms and the corresponding piece of the drift kinetic equation are neglected in what follows. In leading order $v_\parallel \nabla_\parallel f_{i1}^{(-1)} = 0$ along the zeroth order particle orbit, so $f_{i1}^{(-1)}$ is a function of the constants of motion $(\psi, \epsilon, \mu, \sigma)$, and must be even with respect to $\sigma = v_\parallel/|v_\parallel|$ in the trapped region of velocity space. The orbit average of the zeroth order in the collisionality expansion, $v_\parallel \nabla_\parallel f_{i1}^{(0)} = C_i\left(f_{i1}^{(-1)}\right) - \mathbf{v}_{di} \cdot \nabla f_{i0}$, constrains the form of $f_{i1}^{(-1)}$, when weighted by $B/v_\parallel$. In the passing region, the orbit average can be interpreted as a flux surface average on irrational flux surfaces, and by continuity also on rational surfaces. The drift term gives no contribution to the constraint, as the averaged radial drift of the passing particles vanishes, which can be seen by using the conservative form of the magnetic drift $\mathbf{v}_{da} = (v_\parallel/\Omega_a)\nabla \times \left(v_\parallel \mathbf{b}\right)$, where $\mathbf{b} = \mathbf{B}/B$, so $\left\langle (B/v_\parallel)\left(\mathbf{v}_{di} \cdot \nabla f_{i0}\right)\right\rangle = \left\langle \nabla \cdot \left((v_\parallel/\Omega_i)\mathbf{B} \times \nabla\psi\right)\right\rangle \partial_\psi f_{i0} = 0$. The passing region constraint is thus $\left\langle (B/|v_\parallel|) C_i\left(f_{i1,p}^{(-1)}\right)\right\rangle = 0$. In the trapped region, denoting consecutive bounce points by l_1 and l_2, the orbit average constraint reduces to $\sum_\sigma \int_{l_1}^{l_2} dl \left[\mathbf{v}_{di} \cdot \nabla f_{i0} - C_i\left(f_{i1,t}^{(-1)}\right)\right]/|v_\parallel| = 0$, where we have imposed the boundary condition that the number of co-moving particles at each bounce point l_j is equal to the number of counter-moving ones [3], $f\left(l_j, \sigma > 0\right) = f\left(l_j, \sigma < 0\right)$. At this point, we introduce a momentum conserving model operator to describe bulk ion self-collisions [4], which allows the form of the bulk ion distribution to be obtained explicitly: $C_{ii}\left(f_{i1}\right) = \nu_D^{ii}(v)\left(\mathcal{L}\left(f_{i1}\right) + m_i v_\parallel \mathcal{V}_{i\parallel} f_{i0}/T_{i0}\right)$, where the full energy dependent deflection frequency $\nu_D^{ii}(v) = \hat{\nu}_D^{ii}\left(\phi(x) - G(x)\right)/x_i^3$, $\hat{\nu}_D^{ii}$ is defined in analogy to $\hat{\nu}_D^{iz}$, $\nu_D^i(v) = \nu_D^{ii} + \nu_D^{iz}$, $\phi(x)$ is the error function and $G(x)$ is the Chandrasekhar function. The momentum restoring coefficient $\mathcal{V}_{i\parallel}$ is set by requiring momentum conservation in bulk ion self-collisions, $\int v_\parallel C_{ii}\left(f_{i1}\right) d^3v = 0$. We see from the constraint equations above that the momentum restoring term would try to drive a piece of the distribution $f_{i1}^{(-1)}$ which is odd in $v_\parallel$. In the trapped region we have noted the odd response must be zero, and as the leading order contributions to $R_{zi\parallel}$ and $V_{z\parallel}$ are zeroth order in the impurity collisionality expansion, we require for consistency $\mathcal{V}_{i\parallel}^{(-1)} = 0$. Therefore, $f_{i1,p}^{(-1)} = 0$, leaving $f_{i1}^{(-1)}$ here even in $v_\parallel$ and non-zero only in the trapped region.

Flow Finally, integrating the zeroth order equation along the leading order particle orbit, starting from a point l_0 on the flux surface, gives directly

$$f_{i1}^{(0)} = \int_{l_0}^{l} \left[C_i\left(f_{i1}^{(-1)}\right) - \mathbf{v}_{di} \cdot \nabla f_{i0}\right] \frac{dl'}{v_\parallel} + \mathcal{C}_0\left(\epsilon, \mu\right). \tag{5}$$

The odd piece of this function will give the parallel friction and current contribution of interest here. The integration constant is determined by the parallel momentum constraint arising at next order in the collisionality expansion, that is $\left\langle (B/|v_\parallel|) C\left(f_{i1}^{(0)}\right)\right\rangle = 0$ in the passing region and $\sum_\sigma \int_{l_1}^{l_2} dl C_i\left(f_{i1}^{(0)}\right)/|v_\parallel| = 0$ in the trapped region. In the latter, as $f_{i1,t}^{(-1)}$ is even, along with $\mathbf{v}_{di}$, the $\sum_\sigma$ will annihilate everything but an odd contribution from $\mathcal{C}_0$. However, $\mathcal{C}_0$ must be even in the trapped region, as it must vanish at the bounce points to satisfy the continuity condition, so $\mathcal{C}_0$ must be zero here in the trapped region. Introducing the velocity space coordinate $\lambda = v_\perp^2/v^2B$, where $\nabla_\parallel\big|_{\epsilon,\mu}\lambda = 0$, in the passing region, $\lambda < 1/B_{max}$, where B_{max} is the maximum magnetic field strength on a flux surface. Inserting the explicit form for $f_{i1}^{(0)}$ in the passing constraint, the drift term in conservative form generates $\partial_\lambda \int_{l_0}^{l} dl'\mathbf{v}_{di}\cdot\nabla f_{i0}/v_\parallel = -(\sigma v m_i \partial_\psi f_{i0}/2Z_i e)\int_{l_0}^{l} dl'\left(\hat{\mathbf{b}}\times\nabla\psi\right)\cdot\nabla\left(1/\sqrt{1-\lambda B}\right)$ and the passing region constraint on $\mathcal{C}_0$ takes the form

$$\frac{\partial \mathcal{C}_0(v,\lambda)}{\partial\lambda} = -\frac{\sigma v}{v_{Ti}^2}\frac{f_{i0}}{\left\langle\sqrt{1-\lambda B}\right\rangle}\left(\frac{T}{e_i}\mathcal{G}\frac{\partial \ln f_{i0}}{\partial\psi} + \frac{\left\langle\left(\nu_D^{ii}\mathcal{V}_{i\parallel} + \nu_D^{iz}V_{z\parallel}\right)B\right\rangle}{\nu_D^i(v)}\right). \tag{6}$$

We have introduced the geometry function $\mathcal{G}$ here, equivalent to $\langle g_4\rangle$ in previous work [5]

$$\mathcal{G} = \left\langle\sqrt{1-\lambda B}\int_{l_0}^{l}\left(\hat{\mathbf{b}}\times\nabla\psi\right)\cdot\nabla\left(\frac{1}{\sqrt{1-\lambda B}}\right)dl'\right\rangle. \tag{7}$$

With the expressions for $f_{i1}^{(0)}$ and $V_{z\parallel}$ in hand, we can now evaluate the moment required to give the contribution to the impurity flux, $\langle uBR_{zi\parallel}\rangle$. The same piece of the distribution gives the bulk ion flow, and thus contribution to the bootstrap current, via the integral $\left\langle J_\parallel^i B\right\rangle = Z_i e\left\langle B\int v_\parallel f_{i1}^{(0)} d^3v\right\rangle$. To clarify the evaluation with momentum conservation in an impure plasma, we take a simplified energy dependence in the bulk ion self-collision frequency, so $\nu_D^{iz}/\nu_D^{ii} \approx \tau_{ii}/\tau_{iz} = n_z Z_z^2/n_i Z_i^2 \equiv \zeta$. The parameter ζ usefully represents the impurity content and this approximation reproduces the correct limits for $Z_{\text{eff}} \to 1$ and $Z_{\text{eff}} \to \infty$. Note that neglecting the momentum conserving terms retained here in C_i produces the usual result [5, 6] obtained with only a pitch angle scattering collision operator, $\left\langle J_\parallel^i B\right\rangle^{PAS} = p_{i0}A_{1i}\left[f_s(\mathcal{G}) + \langle uB^2\rangle\right]$. We have introduced the function f_s to describe the trapping effect of the magnetic geometry, $f_s(y) = (3/4)\langle B^2\rangle\int_0^{1/B_{max}} d\lambda\lambda y/\left\langle\sqrt{1-\lambda B}\right\rangle$, and made the identification $\langle g_2\rangle = -\langle uB^2\rangle$ for the function commonly occurring in this context: $g_2 = B^2\int_{l_0}^{l} dl'\left(\hat{\mathbf{b}}\times\nabla\psi\right)\cdot\nabla B^{-2}$.

3 Transport Coefficients

The final expressions for the impurity flux and bulk ion bootstrap current may usefully be written in terms of a set of transport coefficients $\mathcal{L}$, giving the effectiveness of the radial driving gradients in the system, defined as

$$A_{1a} = \frac{d\ln p_{a0}}{d\psi} + \frac{Z_a e}{T_{a0}}\frac{d\phi_0}{d\psi}, \qquad A_{2a} = \frac{d\ln T_{a0}}{d\psi}. \tag{8}$$

Although the effect of the radial electric field on the ion orbits was neglected, a finite drive from ϕ_0 formally remains at large aspect ratio [3]. We find

$$\langle\Gamma_z\cdot\nabla\psi\rangle = \frac{m_i p_{i0}}{Z_z Z_i e^2\tau_{iz}}\left[\mathcal{L}_{11}^{zz}A_{1z} + \mathcal{L}_{11}^{zi}A_{1i} + \mathcal{L}_{12}^{zi}A_{2i}\right], \tag{9}$$

with

$$\mathcal{L}_{11}^{zi} = \left[\left\langle u^2 B^2 \right\rangle - \frac{\left\langle uB^2 \right\rangle^2}{\left\langle B^2 \right\rangle}\right], \qquad \mathcal{L}_{12}^{zi} = -\frac{3}{2}\mathcal{L}_{11}^{zi}, \qquad \mathcal{L}_{11}^{zz} = -\frac{1}{Z_z}\mathcal{L}_{11}^{zi}. \tag{10}$$

The drive from the electric field, appearing through both A_{1z} and A_{1i}, formally cancels. The Schwartz inequality shows that the geometric coefficient, appearing in square brackets in $\mathcal{L}_{11}^{zi}$, is positive definite. This gives rise to the outward flux due to A_{1z} required by entropy considerations, but notably, from $\mathcal{L}_{11}^{zi}$ and $\mathcal{L}_{12}^{zi}$, produces a net outward impurity flux due to the bulk ion temperature gradient - that is temperature screening, analogous to that occurring in tokamaks. This is independent of the details of the magnetic geometry or impurity content. This contrasts to the result found previously at high collisionality, where the impurity flux was driven inward [2]. The bulk ion contribution to the bootstrap current is

$$\left\langle J_{\|}^{i} B \right\rangle = p_{i0}\left[\mathcal{L}_{31}^{ii} A_{1i} + \mathcal{L}_{32}^{ii} A_{2i}\right], \tag{11}$$

with

$$\mathcal{L}_{31}^{ii} = \frac{1}{1 - f_s(1)}\left[f_s(\mathcal{G}) + \left\langle uB^2 \right\rangle\right], \qquad \mathcal{L}_{32}^{ii} = -\frac{f_s(1)}{1+\zeta}\left(\zeta + \frac{2\eta_2}{3\eta_1}\right)\frac{3}{2}\mathcal{L}_{31}^{ii}. \tag{12}$$

The coefficients $\eta_1 = \sqrt{2} - \ln(1+\sqrt{2})$ and $\eta_2 = -(1/\sqrt{2}) + (5/2)\eta_1$ arise from the velocity space integrals determining the bulk ion momentum restoring coefficient $\mathcal{V}_{i\|}$. In the pure plasma limit, the form of $\left\langle J_{\|}^{i} B \right\rangle$ here reduces to that obtained previously accounting for momentum conservation in self-collisions [6], with numerical differences in the coefficients due to the different collision operators used.

Indications of temperature screening of the impurity flux with decreasing collisionality were seen in a previous numerical study with the SFINCS transport code [7], which solves the drift kinetic equation for an arbitrary number of species, using the full linearised collision operator in arbitrary stellarator geometry. We have therefore begun a dedicated comparison. Taking the standard configuration W7-X equilibrium presented in [7], we have evaluated the radial profile of the impurity flux (formally neglecting the $d\phi_0/d\psi$ drive) and the effect of including momentum conservation, in the presence of the impurities, on the bootstrap current profile. We have assumed a reduced temperature, specifically $T_i/4$ to satisfy the collisionality restrictions required here. We take a constant $Z_{\text{eff}} = 1.5$, assumed to be due to fully ionised C^{6+} impurity. The radial profiles of the predicted impurity flux coefficients obtained from SFINCS are shown in FIG. 1a, normalised to compare to the predicted value of $\mathcal{L}_{11}^{zi}$, which is positive, as expected. We find very good agreement. In FIG. 1b, the profile of the predicted bulk ion bootstrap current coefficients are compared to those from SFINCS, and that obtained when only pitch angle scattering is retained in the collision operator, as given in Sec. 2. The well-known effect of resonances [9] which arise when evaluating the geometry coefficient $[f_s(\mathcal{G}) + \langle uB^2 \rangle]$ can be seen.

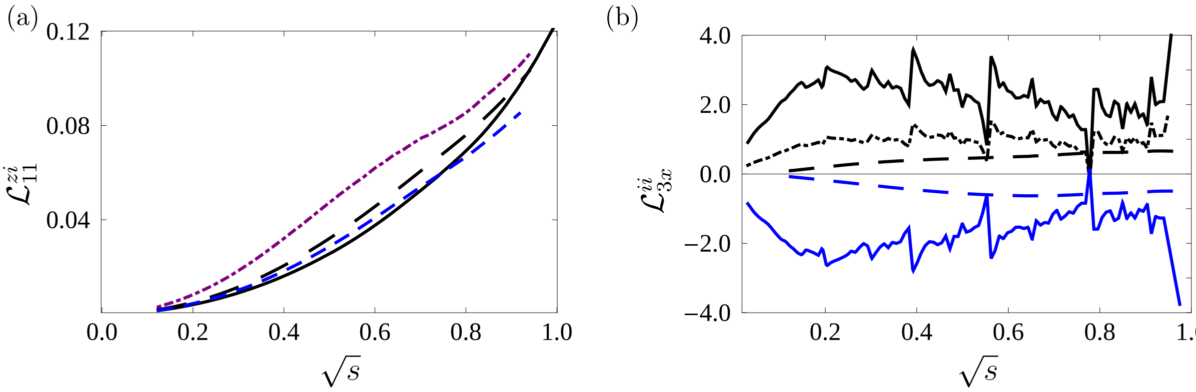

FIG. 1: Radial profiles, against square root of the normalised toroidal flux, of a) the predicted geometry factor $\mathcal{L}_{11}^{zi}$ (solid black) compared to the normalised impurity flux coefficients from SFINCS: $\mathcal{L}_{11}^{zi}$ (dash black), $-2\mathcal{L}_{12}^{zi}/3$ (dash blue), $-Z_z\mathcal{L}_{11}^{zz}$ (dot-dash purple) and b) the bootstrap current coefficients $\mathcal{L}_{31}^{ii}$ (black), $\mathcal{L}_{32}^{ii}$ (blue) predicted (solid) and from SFINCS (dash). With only pitch angle scattering ion collision operator, $\mathcal{L}_{32}^{ii} = 0$, dot-dash black predicted $\mathcal{L}_{31}^{ii}$. Note SFINCS output $\mathcal{L}_{12}^{zz}$, $\mathcal{L}_{31}^{iz}$, $\mathcal{L}_{32}^{iz}$ too small to be visible.

4 Discussion

The understanding and control of impurity transport in stellarators remains an open issue, with impurity accumulation typically predicted. The analysis of stellarator impurity transport in terms of a general flux-friction relation was introduced previously to treat collisional plasmas. Here we have extended this treatment into the more experimentally relevant mixed collisionality regime, in which a heavy, highly charged, collisional impurity is present in a collisionless, hydrogenic, bulk plasma in the $1/\nu$ regime. When the bulk ion collisionality against the impurities is sufficient, the impurity flux is dominated by the drive from the friction, and the effect of the weak impurity pressure anisotropy can be neglected. This requirement sets the range of validity of the analysis here, which can potentially be satisfied in W7-X type equilibria, during cooler phases of operation. The notable result is that in this mixed collisionality limit impurity temperature screening can arise, and it is independent of the details of the system geometry or impurity content. Such behaviour was indicated in a previous numerical study with the SFINCS code [7] and has been confirmed in the dedicated comparison started here.

The calculation of the radial flux by a flux-friction relation requires the piece of the bulk ion distribution which is odd in the parallel velocity. As this also gives the parallel ion flow, we have evaluated the bulk ion contribution to the bootstrap current in this regime, accounting for the presence of impurities and momentum conservation in collisions. The geometry dependent factors which appear are unchanged from those in existing calculations for a pure plasma, and our results reduce to these expressions in the appropriate limit. We have also evaluated the radial profile of the predicted ion contribution to the bootstrap current numerically for a W7-X-like equilibrium, and find that the correction to the profile predicted using only a pitch angle scattering collision operator is modest.

The main limitation of the analysis presented here is the neglect of the effect of the radial electric field on the bulk ion trajectories. Whilst this is not an unreasonable assumption for the collisional impurity species, we expect that the bulk ions will usually be in one of the lower collisionality regimes during hot operational phases, where the electric field is necessary to ensure their confinement. The light electrons will typically remain in the $1/\nu$ regime, and so can be described by a similar formalism to that presented. An extension of the calculation to such a regime is under consideration, which would be needed to determine the value of the ambipolar electric field.

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