

# Geometry Teacher's Edition - Problem Solving

[CK-12 Foundation](#)

December 7, 2009

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# Chapter 1

## Geometry TE - Problem Solving

Geometry- Teacher's Edition- Problem Solving By Jen Kershaw

### 1.1 Basics of Geometry

#### Points, Lines and Planes

##### I. Section Objectives

- Understand the undefined terms point, line and plane.
- Understand the defined terms, including space, segment, and ray.
- Identify and apply basic postulates of points, lines and planes.
- Draw and label terms in a diagram.

##### II. Problem Solving Activity- Global Architecture

- Objective: The objective of this activity is to have students recognize the postulates connected with points, lines and planes in real life architecture.
- Here are the postulates.
  - Line Postulate- There is exactly one line through any two points.
  - Plane Postulate- There is exactly one plane that contains any three non- collinear points.
  - Postulate- A line connecting points in a plane also lies with the plane.
  - Postulate- the intersection of two distinct lines will be a single point.
  - Postulate- the intersection of two planes is a line.
- You can either print the pictures, use computer displays or slides.
- The students need to use the following postulates and identify examples of each postulate as displayed in the picture.

- Students should be encouraged to use mathematical language as they describe and write about each example of a postulate.
- Figure01.01.01- The Guggenheim Museum in Spain [http://en.wikipedia.org/wiki/Guggenheim\\_Museum\\_Bilbao](http://en.wikipedia.org/wiki/Guggenheim_Museum_Bilbao)
- Figure 01.01.02- Eiffel Tower [http://en.wikipedia.org/wiki/File:Tour\\_Eiffel\\_Wikimedia\\_Commons.jpg](http://en.wikipedia.org/wiki/File:Tour_Eiffel_Wikimedia_Commons.jpg)
- Figure01.01.03- St. Basil's Cathedral [http://en.wikipedia.org/wiki/Saint\\_Basil%27s\\_Cathedral](http://en.wikipedia.org/wiki/Saint_Basil%27s_Cathedral)

### III. Meeting Objectives

- Students found examples of points, lines and planes in each architectural figure.
- Students identified and applied the basic postulates associated with points, lines and planes.

### IV. Notes on Assessment

- There are several different illustrations of each postulate in the architecture provided. This is an activity where students are not only expected to be able to locate an example of each, but also where they need to use mathematical language to write about how each postulate is shown in the architecture.
- An example of this is in the picture of the Guggenheim. You can see that two of the planes intersect in exactly one line. The sun is even shining directly on the line.

## Segments and Distance

### I. Section Objectives

- Measure distances using different tools.
- Understand and apply the ruler postulate to measurement.
- Understand and apply the segment addition postulate to measurement.
- Use endpoints to identify distances on a coordinate grid.

### II. Problem Solving Activity- Town Design

- Students are assigned the task of using measurement to design their own town.
- Students will need rulers, pencils, colored pencils and chart paper.
- Each town needs to have the following buildings in it: a post office, a police station, a bank, a park, a school and some houses. The students can expand this list if they choose.
- Each town map needs a scale to determine the distances from one building to another building. This scale could be expanded to include standard and metric measurement.
- Each students needs to develop a key that shows the measurements from one building to another.

- Distances must be actual distances measured with rulers and matched to scale. This could be expanded to include both standard and metric measurement.
- After the design has been completed, the students need to write a series of directions and distances for someone to travel around their town. The distances should be clear enough for any other student to follow.
- Students pair up with a peer to check and make sure that student directions/measurements are accurate.
- Finally, allow students time to share their designs in small groups or with the entire class.

### III. Meeting Objectives

- Students were required to measure distance using different tools.
- Street measurement is a real life example of the ruler postulate.

### IV. Notes on Assessment

- Observe students as they work on this assignment.
- You can create a rubric where each piece of the assignment is worth points.
- For example, the scale is worth 4 points.
- Then student grading is calculated out of the total possible points.
- Check final work for accuracy.
- Are the directions/measurements clear and accurate?
- Does the town contain all of the essential buildings?
- Provide student feedback and grading according to a rubric.

## **Rays and Angles**

### I. Section Objectives

- Understand and identify rays.
- Understand and classify angles.
- Understand and apply the protractor postulate.
- Understand and apply the angle addition postulate.

### II. Problem Solving Activity- Angle Hunt

- Use Figure01.03.01 in this activity.
- Students are going to use the figure to apply each of the section objectives.
- Students will need rulers, colored pencils or markers and protractors.

- First, students take the drawing and find ten different angles.
- They need to use letters and label each of the following angles.
- Next, they make a list of each of the ten angles and classify each.
- Then, they apply the protractor postulate to measure each of the ten angles.
- Finally, they apply the angle addition postulate and create four different combinations of angles to calculate total measures.
- When finished, pair up students and have them check each other's work.
- Each student needs to provide their peer partner with verbal and written feedback.
- Then allow students time to share feedback in small groups.

### III. Meeting Objectives

- Students demonstrated understanding angles.
- Students demonstrated classifying angles.
- Students applied the protractor postulate.
- Students applied the angle addition postulate.

### IV. Notes on Assessment

- Create a rubric to grade each students work.
- Were ten angles labeled and identified?
- Are the measurements of each angle accurate?
- Did students successfully use the angle addition postulate?
- Provide students with feedback and grading on their work.

## **Segments and Angles**

### I. Section Objectives

- Understand and identify congruent line segments.
- Identify the midpoint of line segments.
- Identify the bisector of a line segment.
- Understand and identify congruent angles.
- Understand and apply the Angle Bisector Postulate.

### II. Problem Solving Activity-Revolving Door Design



- Use the two diagrams from this Wikipedia site. These are Figure01.04.01 and Figure01.04.02
- [http://en.wikipedia.org/wiki/Revolving\\_door](http://en.wikipedia.org/wiki/Revolving_door)
- Students are going to be assigned the task of designing their own revolving door.
- Point out that the diagram of the revolving door has four wings to it.
- The students are going to be assigned the task of designing a revolving door with at least six wings in it.
- Students will need rulers, protractors, pencils, and paper.
- They can choose to add more wings, but the revolving door needs to have at least six in it.
- Here are the specifics of the assignment:
  - Design a revolving door with at least six wings.
  - Each angle must be congruent.
  - Label each angle measure using a protractor.
  - Identify line segments that are bisected.
  - Identify the midpoint of each line segment.
  - Label each part of the revolving door and demonstrate congruency.

### III. Meeting Objectives

- Students will demonstrate an understanding of line segments, angles, congruency and bisecting angles in this lesson.
- Students will also demonstrate measuring angles and identifying angles.

### IV. Notes on Assessment

- Assess student work by thinking about each of the following points.
- Were the students successful in executing a design that matches the specifics of the assignment?
- Are the angles of the wings congruent?
- Are the angle measures labeled?
- Is it clear that students understand the concepts discussed in the lesson?

## **Angle Pairs**

### I. Section Objectives

- Understand and identify complementary angles.
- Understand and identify supplementary angles.

- Understand and utilize the Linear Pair Postulate.
- Understand and identify vertical angles.

## II. Problem Solving Activity- Visualize It

- Students are going to go on a search for different types of angle pairs. This can be done in the classroom, but it would be best to expand it to the entire school or outside.
- If possible, allow the use of digital cameras.
- If this is not possible, students can draw sketches of the places where they locate each type of angle pairs.
- Students can photograph or draw each.
- They will need rulers, pencils, chart paper, clip boards.
- Students need to locate three examples of each.
- They first find three examples of complementary angles.
- Three examples of supplementary angles.
- Three examples of vertical angles.
- Students must write a description of each example and explain why it is a complementary angle pair, supplementary angle pair or vertical angle pair.
- Print student pictures and create a display of student work.

## III. Meeting Objectives

- Students will identify and write about complementary angles. This demonstrates understanding.
- Students will identify and write about supplementary angles. This demonstrates understanding.
- Students will identify and write about vertical angles. This demonstrates understanding.

## IV. Notes on Assessment

- Have students work in groups to assess each other's work.
- Request that students read each description of the angle pair to be sure that it describes each angle pair in mathematical terms.
- You want to see that students are using measurements such as  $90^\circ$  for complementary angles, and that they are demonstrating that vertical angles are congruent.

# Classifying Triangles

## I. Section Objectives

- Define triangles.
- Classify triangles as acute, right, obtuse or equiangular.
- Classify triangles as scalene, isosceles or equilateral.

## II. Problem Solving Activity- Bicycle Design

- Begin by showing students the following short movie clip from this website.
- [http://www.thefutureschannel.com/dockets/hands-on\\_math/bicycle\\_design/](http://www.thefutureschannel.com/dockets/hands-on_math/bicycle_design/)
- Allow time for a student discussion about the short video.
- Ask students what they observed about bicycle design from the video.
- Write these notes on the board.
- Allow time for questions and then give students the assignment.
- Students are going to create their own bicycle design.
- They need to use at least two different types of triangles in the design.
- Students will need paper, rulers, pencils and colored pencils.
- Students can design their own bicycle.
- When finished, they need to label each type of triangle used and label it according to side length of and angles.
- Then students may decorate their design.
- Allow time for students to share their work.

## III. Meeting Objectives

- Students will define triangles by using them in their bicycle design.
- Students will classify the triangles used according to side length.
- Students will classify the triangles used according to angle measure.

## IV. Notes on Assessment

- Study each student design.
- Does the design have at least two different types of triangles in it?
- Are the triangles labeled according to side length?
- Are the triangles labeled according to angle measure?
- Provide students with feedback/corrections.

# Classifying Polygons

## I. Section Objectives

- Define polygons.
- Understand the difference between convex and concave polygons.
- Classify polygons by number of sides.
- Use the distance formula to find side lengths on a coordinate grid.

## II. Problem Solving Activity- Polygon Sort

- This activity requires students to sort polygons in three different ways.
  1. According to whether or not it is a polygon
  2. Convex or concave
  3. According to the number of sides
- To prepare this activity, you will need to create or copy different polygons. You want an assortment of polygons and non- polygons, convex polygons, concave polygons and regular polygons (i.e. quadrilaterals, hexagons, etc.). Then you can place these all around the room.
- When students begin the activity, they need to hunt for a specific number of figures. You could have each student find three different ones to work with. Then they can choose one for each exercise.
- Then you can do a sorting exercise.
- For example, “All of the polygons sit down. All of the non- polygons stand up.”
- Then you can ask for a few examples from each group to explain why they are or are not a polygon.
- Next, you can do another sort. Concave figures to the front of the room. Convex to the back of the room.
- Same thing- ask for students to demonstrate why the figure is concave or convex.

## III. Meeting Objectives

- Students will be required to define polygons.
- Students will demonstrate an understanding between concave and convex polygons.
- Students will classify polygons according to the number of sides.

## IV. Notes on Assessment

- Assess student understanding by checking each “sorting exercise”
- Also ask different student for feedback about why they “sorted” their polygons the way that they did.
- Allow time for feedback and student questions.

## Problem Solving in Geometry

### I. Section Objectives

- Read and understand given problem situations.
- Use multiple representations to restate problem situations.
- Identify problem- solving plans.
- Solve real- world problems using planning strategies.

### II. Problem Solving Activity- Camping Fire Expansion

- Use the diagram on page 65 of the text. This will be Figure01.08.01.
- Here is an expansion on the earlier problem.
- Students can work in groups on this problem.
- The fire has begun to spread. It had spread to a tent that is fifty feet north of her tent. It has also spread to additional tent that is twenty- five miles south of the river and fifty feet south of the original tent. Two other campers have begun helping with the fire problem. How can all three minimize their distances? What is the shortest distance any one of them can run to put out the fire?

### III. Meeting Objectives

- Be sure that the students have completed the work in problem 7 of the text before tackling this problem.
- If they have, then this problem should be a natural extension of the original one.
- Encourage students to follow the problem solving steps. There are two parts to this problem. Be sure that the students identify each part.
- Then have students draw a diagram to show the original tent and the two new tents as well. Students can label them A, B and C.
- Finally, using a scale, have students measure the distances.
- Who has the shortest distance to run?
- Ask students to show their work and to justify their thinking.

### IV. Notes on Assessment

- Walk around as students work on this problem.
- Offer assistance when necessary and remind the students of the problem solving steps.
- When finished, allow students an opportunity to share their work.
- If there are different answers, ask groups to justify their answer.

## 1.2 Reasoning and Proof

### Inductive Reasoning

#### I. Section Objectives

- Recognize visual patterns and number patterns.
- Extend and generalize patterns.
- Write a counterexample to a pattern rule.

#### II. Problem Solving Activity-Pascal's Triangle

- Students are going to work with a diagram of Pascal's Triangle for this activity.
- Pascal's Triangle is Figure 02.01.01
- [http://en.wikipedia.org/wiki/Pascal%27s\\_triangle](http://en.wikipedia.org/wiki/Pascal%27s_triangle)
- Students are going to problem solve to find a rule to Pascal's Triangle.
- Have students work in small groups.
- They can use color on the triangle to point out different patterns.
- Allow a lot of time for the students to explore the patterns of the triangle.
- Ask them to use the Wikipedia pattern to write the rule for the triangle.
- Once they have the rule, they need to write the next two rows of the triangle.
- Then demonstrate two ways that you know that your rule is accurate.
- Finally, write a conjecture and a counterexample for the rule.
- Allow time for student sharing.

#### III. Meeting Objectives

- Students will recognize visual patterns and number patterns.
- Students will be required to extend and generalize patterns in Pascal's Triangle.
- Students will write conjectures and counterexamples of their rule.

#### IV. Notes on Assessment

- Do some independent study on Pascal's Triangle prior to completing this activity.
- Ask leading questions if students are stuck, but refrain from offering solutions.
- Encourage students to help each other with the patterns if they are having difficulties.

## Conditional Statements

### I. Section Objectives

- Recognize if- then statements.
- Identify the hypothesis and conclusion of an if- then statement.
- Write the converse, inverse and contrapositive of an if- then statement.
- Understand a biconditional statement.

### II. Problem Solving Activity-Advertisements

- Students are going to use newspapers and magazines for this problem solving activity.
- Begin the activity by talking about how advertisers use conditional statements to lure people into purchasing their products. For example, a phone company will often offer a free phone for a cell phone plan.
- *Note: If you can find one such add it would be great to bring it in for a demonstration.*
- Tell students that their assignment is to use newspapers and magazines to find one such conditional advertisement.
- Then they are to take that advertisement and create a display using it to show the converse, inverse, contrapositive and biconditional statement of the advertisement.
- Students can decorate and design their display.
- Allow time for students to share their work when finished.

### III. Meeting Objectives

- Students will recognize if- then statements in advertisements.
- Students will write converse, inverse and contrapositive statements.
- Students will understand biconditional statements by writing them.
- Students will present their work to their peers.

### IV. Notes on Assessment

- Be sure that the students have selected a conditional statement in an advertisement.
- Check their work for accuracy when writing each of the different statements.
- Allow time for students to share their work.
- Include creativity in student evaluations.
- Students could receive a classwork or homework grade for this assignment.

## Deductive Reasoning

### I. Section Objectives

- Recognize and apply some basic rules of logic
- Understand the different parts that inductive reasoning and deductive reasoning play in logical reasoning
- Use truth tables to analyze patterns of reasoning

### II. Problem Solving Activity-Write It Out

- Students are going to write statements based on Figure 02.03.01 of vertical angles.
- Draw the figure on the board/overhead.
- Underneath it write the words “Vertical angles are congruent.”
- Put the students in groups.
- There are four “starters” on index cards. A starter card gives the students a beginning geometric statement. They then need to take this statement and complete it using the Law of Detachment or the Law of Syllogism.
- For example, if the starter is  $\angle 1 \cong \angle 2$ , then the students would need to write two other statements using the figure as a guide.
- Possible answers could be:  $\angle 1$  and  $\angle 2$  are vertical angles. Vertical  $\angle$ 's are congruent.
- Here are the three other starters:  $\angle 1$  and  $\angle 4$  are supplementary angles.
- $\angle 3$  and  $\angle 4$  each equal  $55^\circ$ .
- $\angle 2$  and  $\angle 3$  are adjacent angles.
- $\angle 3$  and  $\angle 4$  are congruent.
- Finally, when finished, ask students to identify whether the Law of Detachment or the Law of Syllogism was at work in each set of statements.

### III. Meeting Objectives

- Students will use the basic rules of logic.
- Students will understand the role of inductive and deductive reasoning.
- Students will apply this reasoning to geometric content.

### IV. Notes on Assessment

- Observe students as they work.
- If groups are struggling, refer them back to their notes on previously learned material.
- Refrain from offering suggestions.
- Give feedback based on content and accuracy.



## Algebraic Properties

### I. Section Objectives

- Identify and apply properties of equality
- Recognize properties of congruence “inherited” from the properties of equality
- Solve equations and cite properties that justify the steps in the solution
- Solve problems using properties of equality and congruence

### II. Problem Solving Activity-Match It Up

- Students are going to create a matching game that they can then play in small groups.
- Each small group needs to create a pair for each of the properties. One card will have the name of the property on it, and the match will be a numerical example, and or a geometric example.
- Be sure that the students write out an actual example of the property and not just variables as they did in class.
- This will help them to take the lesson in the text to a new level.
- Be sure that the students have index or small cards, pens, rulers, etc.
- After the cards have all been created, have one group exchange with another group and play that team’s game.
- When finished, ask the teams to give each other feedback on the examples used.
- Here are the properties to use:
  - Reflexive Property
  - Symmetric Property
  - Transitive Property
  - Substitution Property
  - Addition Property of Equality
  - Multiplication Property of Equality
  - Reflexive Property of Congruence with segments and angles
  - Symmetric Property of Congruence with segments and angles
  - Transitive Property of Congruence with segments and angles

### III. Meeting Objectives

- Students will identify and apply properties of equality.
- Students will solve equations and cite properties in their examples.
- Students solve problems using the properties.

### IV. Notes on Assessment

- This activity has two parts. The first part is to observe the students as they work on creating the game.
- The second part is to watch them play it.
- Because they are switching game cards with another team, any errors will quickly come to light.
- Be sure to allow time for feedback/correction.

## Diagrams

### I. Section Objectives

- Provide the diagram that goes with a problem or proof.
- Interpret a given diagram.
- Recognize what can be assumed from a diagram and what can not be
- Use standard marks for segments and angles in diagrams.

### II. Problem Solving Activity-Name That Postulate!

- This is a game. The students will create the game cards and then a “Jeopardy” kind of game can be played in the large class or in small groups.
- Students are assigned the task of creating an index card with a diagram that represents each postulate.
- Students should use diagrams and also standard marks for segments and angles in their examples.
- There are eleven postulates, so if there are twenty- two students in the class, each postulate would be represented by two different diagrams. You need to assign the students the postulates to avoid too many repeats.
- Allow time for the students to create their diagrams and then use peers to check each other’s work for accuracy.
- When finished, collect the cards and play the game with the students.

### III. Meeting Objectives

- Students create diagrams to better understand postulates.
- Students interpret given diagrams when playing the game.
- Students use standard marks for segments and angles when creating their game cards.

### IV. Notes on Assessment

- Assessment is easier with this lesson because the students will be playing the game. You will be able to see who understands the postulates and who doesn’t.
- Also, having students check each other’s work before playing the game will definitely help to catch any errors.
- You can help add any corrections when playing the game and looking at each game card.

## Two- Column Proof

### I. Section Objectives

- Draw a diagram to help set up a two- column proof.
- Identify the given information and statement to be proved in a two- column proof.
- Write a two- column proof.

### II. Problem Solving Activity-Wind Generators

- Use a figure like this one of a wind generator. This is Figure 02.06.01
- [www.blaineschools.org/Schools/WRMS/Tech/Zsupportin\\_documents/Images/wind\\_generator.jpg](http://www.blaineschools.org/Schools/WRMS/Tech/Zsupportin_documents/Images/wind_generator.jpg)
- Here is the problem.
- “Mike Eisele did an experiment for his science project to figure out which angle of degree on a propeller of a wind generator would be the most efficient. He figured out that  $75^\circ$  was the most efficient. Your task is to take this given information and write a proof to using geometric principles. We’ll call one angle of the propeller angle 1 and the other angle 2.”
- Show students the diagram of the wind generator. Point out the two angles that you are working with and then write this information on the board.
- *On Board:*

Given:  $m\angle 1 = 75^\circ$

$\angle 1 \cong \angle 2$

Prove:  $m\angle 2 = 75^\circ$

### III. Meeting Objectives

- Students will use a diagram to help set up a two- column proof.
- Students can draw a diagram of a wind generator and label the given angles.
- Students will write a two- column proof.

### IV. Notes on Assessment

- Here is a possible answer for the given proof.

Table 1.1

Statements	Reasons
$m\angle 1 = 75^\circ$	Given
$\angle 1 \cong \angle 2$	Definition of Congruent Angles
$m\angle 1 = m\angle 2$	Substitution
$75 = m\angle 2$	Symmetric Property
$m\angle 2 = 75$	

- Observe students while they work. Offer assistance when necessary.
- If you want to learn more about Mike Eisele and his experiment, see the Enrichment section of this Teacher's Edition. The website about Mike and his experiment is [www.share3.esd105.wednet.edu/mcmillend/02SciProj/](http://www.share3.esd105.wednet.edu/mcmillend/02SciProj/)

## Segment and Angle Congruence Theorems

### I. Section Objectives

- Understand basic congruence properties.
- Prove theorems about congruence.

### II. Problem Solving Activity-Angle or Segment?

- For this problem solving activity, you will need to prepare a set of cards with angle statements and a set of cards with segment statements.
- Use numbers or letters to mark each card. Then you will know which statements the students were working with.
- Write each angle statement as reflexive, symmetric or transitive.
- Write each segment statement as reflexive, symmetric or transitive.
- Students are going to each be given a card with either an angle statement or a segment statement on it.
- Remind students to label their work with a letter or number that matches the card that they have been given.
- Then the students need to write out the property for the card and draw a diagram that illustrates the statement on the card.
- Students will need rulers, pencils and protractors for this assignment.
- All work should be accurate and measured.
- Allow a certain amount of time for this first card, when finished, ask the students to pass the card to a neighbor and repeat this assignment. You want the students to each work on two different angle cards and two different segment cards.
- This will help to secure student understanding.
- Have students share their work in small groups when finished.

### III. Meeting Objectives

- Students will understand basic congruence properties.
- Students will prove basic congruence properties through diagrams and group discussions.

### IV. Notes on Assessment

- Collect student work.
- Check each student's work for accuracy and offer written feedback.

## Proofs about Angle Pairs

### I. Section Objectives

- State theorems about special pairs of angles.
- Understand proofs of the theorems about special pairs of angles.
- Apply the theorems in problem solving.

### II. Problem Solving Activity-Judges Table

- Before explaining the activity, select four students to serve as judges.
- Explain that the students are going to need to use theorems and proofs to “PROVE” each statement.
- The judges will be deciding if the students have successfully proved their statement.
- Students should work in groups of three for this assignment.
- The judges are also going to need to complete the work for all of the statements that way they know whether or not students have successfully proven their statement.
- Use Figure 02.08.01- provide each group with a copy of the diagram.
- Here are some possible statements:
  - $\overline{AD} \cong \overline{BC}$
  - Given that  $\angle 1 \cong \angle 2$ , which other angles are congruent?
  - $m\angle 5$  and  $m\angle 6 = 90^\circ$
- You can create as many different statements as you would like.
- Allow time for the students to work and then they present their case to the judges.
- The judges accept it or decline it. If accepted, students can work on another statement. If declined, the students need to go back and try again.

### III. Meeting Objectives

- Students will state theorems about special pairs of angles.
- Students will understand how to prove theorems about special pairs of angles.
- Students will apply the theorems in problem solving.

### IV. Notes on Assessment

- Sit on the panel with the judges.
- Listen to the statements and offer feedback.
- The students can be given extra credit for the number of statements that they are able to prove.
- Students could also be given a classwork grade for this assignment.

## 1.3 Parallel and Perpendicular Lines

### Lines and Angles

#### I. Section Objectives

- Identify parallel lines, skew lines, and perpendicular lines
- Know the statement of and use the Parallel Line Postulate.
- Know the statement of and use the Perpendicular Line Postulate.
- Identify angles made by transversals.

#### II. Problem Solving Activity-School Map

- Students are going to use parallel lines, perpendicular lines and skew lines to create a map of the school.
- You can begin this lesson by using a fire exit map of the school to assist students in getting started.
- The students are going to work in groups of three to create a map of the school.
- Students will need chart paper, rulers, yard sticks, colored pencils.
- Take students on a walk around the school to begin taking notes for their design.
- Note: If your school is very large, you can either challenge students with the whole school or select a floor to design. This could create a complete school map in the end with groups putting their “floors” together.
- Have students draw and design their map.
- When finished with the map, students must identify two sets of parallel lines and two sets of perpendicular lines.
- Extension- you can extend this activity even further by asking the students to identify an example of the Parallel Line Postulate and an example of the Perpendicular Line Postulate on their map.
- Allow time at the end for the students to share their work.
- Create a class display of student maps.

#### III. Meeting Objectives

- Students will identify parallel lines.
- Students will identify skew lines.
- Students will identify perpendicular lines.

#### IV. Notes on Assessment

- Use each map to assess student work.

- Do the students have a good understanding of parallel lines?
- Of perpendicular lines?
- Of skew lines?
- Are students able to identify an example of each postulate?

## Parallel Lines and Transversals

### I. Section Objectives

- Identify angles formed by two parallel lines and a non- perpendicular transversal.
- Identify and use the Corresponding Angles Postulate.
- Identify and use the Alternate Interior Angles Theorem.
- Identify and use the Alternate Exterior Angles Theorem.
- Identify and use the Consecutive Interior Angles Theorem.

### II. Problem Solving Activity-Airport Map

- For this problem solving activity, students are going to use an aerial map of Logan International Airport in Boston, Massachusetts.
- Figure 03.02.01 can be found at [www.en.wikipedia.org/wiki/File:KBOS\\_Aerial\\_NGS.jpg](http://www.en.wikipedia.org/wiki/File:KBOS_Aerial_NGS.jpg)
- Students need to use the picture to draw their own version of the map.
- Then using color, they need to identify the following:
  - In red, two parallel lines and a non- perpendicular transversal.
  - In blue- two corresponding angles
  - In green- two alternate interior angles
  - In orange- two alternate exterior angles
  - In purple- two consecutive interior angles
- When finished, you can extend this by having the students use a protractor to determine angle measures.
- Allow time for students to share their work.

### III. Meeting Objectives

- Students will identify the different angles created by parallel lines and transversals.
- Students will also identify the types of angles identified in the different theorems.

### IV. Notes on Assessment

- Assess student understanding through student sharing.
- Look at each student's diagram.
- Is it labeled correctly?
- Did the students follow directions?
- Is there anything missing?
- Does the diagram demonstrate that students understand the different angle pairs?

## Proving Lines Parallel

### I. Section Objectives

- Identify and use the Converse of the Corresponding Angles Postulate.
- Identify and use the Converse of Alternate Interior Angles Theorem.
- Identify and use the Converse of Alternate Exterior Angles Theorem.
- Identify and use the Converse of Consecutive Interior Angles Theorem.
- Identify and use the Parallel Lines Property.

### II. Problem Solving Activity-Forest Tower

- Students are going to design a tower to be used in National Park by a Forest Ranger.
- The students are going to need to design the tower so that the posts of the tower are parallel and are connected or braced by a transversal.
- They will need to demonstrate how the angles of the transversal prove that the tower poles are parallel.
- Begin the lesson by explaining, "Today you are going to design a tower to be used in a National Park by a Forest Ranger."
- To design the tower, you must create four parallel poles to place your platform on top of. The poles must be connected by a supporting transversal.
- You must use what you have learned about angles to design the tower and prove that the poles are parallel.
- Students will need protractors, rulers, chart paper and pencils.
- All angles must be labeled and measured.
- Look at Figure 03.03.01 to get an idea of a possible example.
- When finished, allow students time to share their work.

### III. Meeting Objectives

- Students will show interior and exterior angles in their diagram.



- Students will show corresponding angles in their diagram.
- Students will apply what they have learned in a real- life example.
- Students will demonstrate the Parallel Lines Property.

#### IV. Notes on Assessment

- Look at each student diagram and assess student work.
- Is the diagram labeled?
- Are the angle measures correct?
- Were the students able to label the diagram to demonstrate that the poles are parallel?
- Offer students corrections and feedback.

## Slopes of Lines

### I. Section Objectives

- Identify and compute slope in the coordinate plane.
- Use the relationship between slopes of parallel lines.
- Use the relationship between slopes of perpendicular lines.
- Plot a line on a coordinate plane using different methods.

### II. Problem Solving Activity-Wheelchair Ramps

- Students are going to use what they have learned about slope to design a wheelchair ramp.
- A wheelchair ramp has a slope of  $\frac{1}{12}$  ft. - [www.newdisability.com/wheelchairramp.htm](http://www.newdisability.com/wheelchairramp.htm)
- Here is the problem.
- A new home has a front door that is  $3\frac{1}{2}$  feet off of the ground. A wheelchair ramp needs to have a slope of  $\frac{1}{12}$  ft. Based on this fact and on the height of the door, design a wheelchair ramp that will work for this new home. Show all of your work in your diagram.
- Allow time for the students to work on this dilemma.
- Then tell students that every unit on a coordinate grid represents one foot. Ask them to draw their wheelchair ramp on coordinate grid.
- When finished, allow time for students to share their work.

### III. Meeting Objectives

- Students will use what they have learned about slope to determine the rise and run of the ramp.
- Students will demonstrate an understanding of how the slope of a line impacts the rise and run of the line.

#### IV. Notes on Assessment

- Examine student work.
- Is the rise and run of the ramp accurate?
- According to the slope of  $\frac{1}{12}$ , the rise is 3.5 ft and the run is 42 inches.
- Is this drawn accurately?
- Is it graphed correctly on the coordinate grid?
- Allow time for students to share their work and offer feedback and correction when necessary.

### **Equations of Lines**

#### I. Section Objectives

- Identify and write equations in slope- intercept form.
- Identify equations of parallel lines.
- Identify equations of perpendicular lines.
- Identify and write equations in standard form.

#### II. Problem Solving Activity-The Park Path

- To work on this problem, students are going to be designing a path for a park using Parallel and Perpendicular Lines.
- Here is the problem:
- “Maria is on a team that is designing paths through a local park. The team has cleared some of the brush and has created one path through the park. Here is the path graphed on the coordinate grid.”
- Figure03.05.01
- “The equation for this path is  $y = 3x + 2$ .”
- “The team needs to draw in two more paths. The first one will be parallel to this one, and the next one will be perpendicular to this one.”
- “Use what you have learned to draw these three paths on a coordinate grid. Use your problem solving skills to name the equation of each line. Be sure to write your equations in slope- intercept form.”

#### III. Meeting Objectives

- Students will write equations in slope- intercept form.
- Students will identify the equations of parallel lines.
- Students will identify the equations of perpendicular lines.
- Students will graph these lines on the coordinate grid.

#### IV. Notes on Assessment

- The line parallel to  $y = 3x + 2$  can be several different things. The key is that it must have the same slope. As long as the line has the same slope, then it is parallel to this first line.
- The perpendicular line must have a slope that is the reciprocal of the  $\frac{3}{1}$ . Therefore, any line with a slope of  $\frac{1}{3}$  will be perpendicular to this first line.
- Allow time for student questions.
- Check student work for accuracy.

### Perpendicular Lines

#### I. Section Objectives

- Identify congruent linear pairs of angles.
- Identify the angles formed by perpendicular intersecting lines.
- Identify complementary adjacent angles.

#### II. Problem Solving Activity-Rug Angles

- In this problem solving activity, students are going to identify the angles formed by perpendicular lines.
- Use this rug design as Figure03.06.01 [www.homedecorators.com/detail.php?parentid=26996&aid=bzrt](http://www.homedecorators.com/detail.php?parentid=26996&aid=bzrt)
- Ask students to use this figure to draw their own rug design.
- They can use color and different sizes of polygons on the rug. The big point is to be sure that they are using perpendicular lines on the rug.
- In the design, the students need to include two linear pairs.
- They need to include two sets of perpendicular intersecting angles.
- They need to show one set of complementary adjacent angles.
- These rugs can be designed on chart paper or graph paper- you can leave it up to the students on the size of the rug.
- Allow time for the students to share their work when finished.

#### III. Meeting Objectives

- Students will identify and draw congruent linear pairs of angles.
- Students will identify and draw angles formed by perpendicular intersecting lines.
- Students will identify and draw complementary adjacent angles.

#### IV. Notes on Assessment

- Did the students accomplish this task?
- Are all of the angles present?
- Are all of the perpendicular lines accurate?
- Is there anything missing in the design?
- Offer feedback to students during group discussions.

## Perpendicular Transversals

### I. Section Objectives

- Identify the implications of perpendicular transversals on parallel lines.
- Identify the converse theorems involving perpendicular transversals and parallel lines.
- Understand and use the distance between parallel lines.

### II. Problem Solving Activity-Magazine Hunt

- For this problem solving activity, you will need an assortment of magazines.
- Students are going to go on a magazine hunt for parallel lines and their transversals.
- Students are to find a picture that illustrates an example of parallel lines and perpendicular transversals.
- Then, students need to cut out the picture.
- Label all of the angles.
- Label all of the measures of all of the angles.
- Finally, ask students to explain how the converse theorems impact the perpendicular lines in each picture.
- If time allows, you can request that students work with more than one picture.
- You can also extend this activity to include a presentation piece, so that students are required to verbalize what they have learned through the activity.

### III. Meeting Objectives

- Students will identify perpendicular transversals and parallel lines.
- Students will identify the angles associated with perpendicular transversals and parallel lines.
- Students will identify the converse theorems associated with parallel lines and perpendicular transversals.

### IV. Notes on Assessment

- Have the students selected an appropriate picture?

- Does this picture show parallel lines and perpendicular transversals?
- Are all of the angles accurately labeled?
- Have the students made notes of the converse theorems involved?
- Assess student understanding through written work and presentations.

## Non- Euclidean Geometry

### I. Section Objectives

- Understand non- Euclidean geometry concepts.
- Find taxicab distances.
- Identify and understand taxicab circles.
- Identify and understand taxicab midpoints.

### II. Problem Solving Activity- Taxicab Geometry

- In this activity, students are going to work on the following problem. This can be done individually or in pairs.
- Here is the problem.
- “Juan rides his bike 2800 feet from his home to the park to play baseball. Using a scale of 1 unit: 200 feet, draw a possible path for Juan on a coordinate grid.”
- Students will need to work backwards to solve this problem.
- There are several different solutions to this problem.
- The important thing to note is that Juan travels 14 units according to the scale.
- Any combination of Juan traveling a combination of 14 units is correct.
- You will get many different pictures of this problem.
- Allow time for the students to share their work and explain how they got their answers.
- Extension of this is to say that Juan travels 5,820 feet to school. This is the equivalent of one mile. Give the students the same scale and see what they can do with it.
- Have students draw a diagram.
- The solution is that Juan walks 29.1 units to school. There are many different diagrams that could come out of this problem. Watch for the .1 in the diagram too.

### III. Meeting Objectives

- Students will work to understand non- Euclidean geometry.
- Students will find taxicab distances.

- Students will draw diagrams to show taxicab distances.
- Students will explain their work to their peers.

#### IV. Notes on Assessment

- Do not provide the students with much direction in this lesson.
- Provide them with the scale and the problem and let them go to work.
- Using what they have learned in the text, the students should be able to work backwards and figure out that Juan walks 14 units to the park.
- Then you can assess their diagrams.
- Are the diagrams accurate?
- Are the students able to explain their process of finding the solution to the problem?
- Provide correction/feedback when necessary.

## 1.4 Congruent Triangles

### Triangle Sums

#### I. Section Objectives

- Identify interior and exterior angles in a triangle.
- Understand and apply the Triangle Sum Theorem.
- Utilize the complementary relationship of acute angles in a right triangle.
- Identify the relationship of the exterior angles in a triangle.

#### II. Problem Solving Activity-Triangle Sums

- Provide students with a copy of Figure04.01.01
- Students are going to work in pairs with this figure.
- Part one- students need to figure out the angle measures for each of the missing angles.
- Part two- students are going to write down the theorem that helped them to figure out the measure of each angle.
- Allow time for the students to explain their work in small groups.
- Solution:  $a = 140^\circ$  Exterior angles in a Triangle Theorem

$b = 100^\circ$  Triangle Sum Theorem

$c = 60^\circ$  Acute Angles Theorem

$d = 30^\circ$  Triangle Sum Theorem or Exterior angles in a Triangle Theorem

Extension- have students draw their own puzzle for others to solve.

### III. Meeting Objectives

- Students will identify interior and exterior angles in a triangle.
- Students will apply the Triangle Sum Theorem.
- Students will apply the Exterior Angles Theorem.
- Students will apply the Acute Angles Theorem.
- Students will demonstrate understanding through diagrams and verbal explanations.

### IV. Notes on Assessment

- Is the student's work accurate?
- Can the student talk about why each theorem is appropriate for each angle measure?
- Ask questions and provide feedback when necessary.

## **Congruent Figures**

### I. Section Objectives

- Define congruence in triangles.
- Create accurate congruence statements.
- Understand that if two angles of a triangle are congruent to two angles of another triangle, the remaining angles will also be congruent.
- Explore properties of triangle congruence.

### II. Problem Solving Activity-Congruent Triangles Game

- This game involves a bit of prep by the teacher.
- First, you are going to divide the class numbers in thirds. Then, you design three cards.
- Card 1- a congruent triangle.
- Card 2- it's matching triangle
- Card 3- a congruence statement about the two triangles.
- Create enough cards so that each student receives one. Be sure that your triangles have tic marks on them.

- Add some challenge by using repeated letters in the diagrams and congruence statements. Students will need to pay close attention to the order of the angles in the congruence statement to find the correct matches.
- Then shuffle and hand out the cards.
- Students need to move around the room and find their matches. Each group should have three students in each group when finished.
- This is a fun activity that has a lot of movement in it.

### III. Meeting Objectives

- Students will define congruence in triangles.
- Students will understand congruent angles in a triangle.
- Students will use triangle congruence statements.

### IV. Notes on Assessment

- Are the groups correct?
- Have students been able to find their team mates?
- Listen to student conversation as they work, are the students talking about properties of congruence?
- Allow time for the students to share the strategies that they used to find each other.

## Triangle Congruence Using SSS

### I. Section Objectives

- Use the distance formula to analyze triangles on a coordinate grid.
- Understand and apply the SSS postulate of triangle congruence.

### II. Problem Solving Activity- Triangle Measurement

- Students are going to use the distance formula to analyze triangles on a coordinate grid.
- Here is the problem:
- “Maria is designing a triangular garden in her yard. Her neighbor wants an identical triangle garden in her yard too. Maria is going to use the same dimensions to build her neighbor a garden. To plan her work, Maria plots her garden out using a coordinate grid. Here are the coordinates for Maria’s triangular garden.
- $(-3, 7)(-1, 5)(-4, 1)$
- “If Maria uses these dimensions, and moves the plot five units to the left and three units down, what will the coordinates of her neighbor’s garden be?”



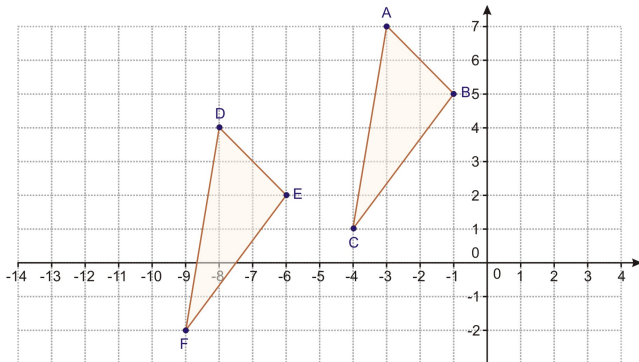
- Students can work on this problem in pairs or individually.
- Have the students draw out the coordinate grid with the two triangles on them.
- Allow time for the students to share their work when finished.
- Extension- students can transfer these triangles to measurement. For example, they could use 1 foot for every one unit on the coordinate grid.

### III. Meeting Objectives

- Students will use the distance formula to plot the triangle on the coordinate grid.
- Students will draw Congruent Triangles on the coordinate grid.
- Students will use the distance formula when converting units to feet in the extension.

### IV. Notes on Assessment

- Were the students able to follow the directions accurately?
- Is the triangle in the correct location?
- Here is the solution graph. It is Figure04.03.01



## Triangle Congruence Using ASA and AAS

### I. Section Objectives

- Understand and apply the ASA Congruence Postulate.
- Understand and apply the AAS Congruence Postulate.
- Understand and practice two- column proofs.
- Understand and practice flow proofs.

### II. Problem Solving Activity- Double Triangles

- For this activity, there is the preparation of each student drawing a small triangle to work with.
- Then have the students work in groups. The students need to choose two of the triangles that have been drawn to work with.
- Then, the students need to draw two triangles that are congruent to the selected triangles.
- This means that each group will have two pairs of Congruent Triangles.
- Each team will have two pairs of students on it. Each pair selects one set of Congruent Triangles to work with.
- Now, the teams quiz each other.
- Each team needs to prove that their triangles are congruent.
- The other team can question and challenge any claims that the teammates use to justify congruency.
- When the team has successfully proven that their triangles are congruent, the team can draw either a flow proof or a two- column proof with their statements and reasons in it.

### III. Meeting Objectives

- Students will apply the ASA Congruence Postulate.
- Students will apply the AAS Congruence Postulate.
- Students will use these postulates to prove that their triangles are congruent.

### IV. Notes on Assessment

- Question each team, were they successful in proving that their triangles are congruent?
- Is the proof written to show SSS, ASA, or AAS?
- Allow time for the students to share their work.
- Offer corrections when necessary.

## **Proof Using SAS and HL**

### I. Section Objectives

- Understand and apply the SAS Congruence Postulate.
- Identify the distinct characteristics and properties of right triangles.
- Understand and apply the HL Congruence Theorem.
- Understand that SSA does not necessarily prove triangles are congruent.

### II. Problem Solving Activity- The Wall Design

- Here is the problem.

- “Harry has a square wall in his bedroom. He knows it is square because he has measured it. He decides to paint a bold red diagonal down the wall. Harry starts at the upper right corner and measuring carefully, paints a red diagonal from the upper right corner to the bottom left corner of the wall. He stands back to admire his work. His brother comes in, looks at the wall and says, “It’s great, but your triangles aren’t even.” Harry says that they are.”
- Your task is to draw a diagram and write a two- column proof showing that Harry’s triangles are congruent. Because Harry has not given us the dimensions of his wall, use the knowledge that it is square and decide your own dimensions.
- Allow time for the students to work on this task.
- If stuck, remind students that the triangles can be proved congruent using SAS. Refer them back to the text for clarification and for right triangle properties.
- Students need to show the measure of two sides, the measure of two angles to compare- these measurements should be labeled in their diagrams.

### III. Meeting Objectives

- Students will apply the SAS Congruence Postulate.
- Students will identify the distinct characteristics and properties of right triangles.
- Students will apply the HL Congruence Theorem.

### IV. Notes on Assessment

- Check student diagrams.
- Is the work accurately labeled with measurements?
- Did the students make note of the two triangles?
- Which postulate or theorem did the student use to prove congruency?
- Is the student aware of the properties of right triangles?
- Are the right triangles noted in the student’s work?

## Using Congruent Triangles

### I. Section Objectives

- Apply various triangles congruence postulates and theorems.
- Know the ways in which you can prove parts of a triangle congruent.
- Find distances using Congruent Triangles.
- Use construction techniques to create Congruent Triangles.

### II. Problem Solving Activity-The Kite Design

- Students will need chart paper, compasses, rulers and colored pencils.
- Here is the problem.
- “Jonas is working on designing a kite. He knows that for his kite to fly well, that it needs to be created with four Congruent Triangles. Jonas starts by drawing a line segment that will serve as the center line of his kite. Now he needs to draw in the four triangles. He can’t remember how to do it.
- Your task is to use a compass and a straightedge to help Jonas design his kite.
- Remember to measure your work so that it is accurate.
- Here is the line segment that Jonas started with.
- Figure04.06.01
- Allow time for the students to work.
- When finished, the students can decorate their kites.

### III. Meeting Objectives

- Students will apply various triangle congruence postulates and theorems.
- Students will use construction techniques to create Congruent Triangles.
- Students will show their work in a design.

### IV. Notes on Assessment

- Leave students alone to work on this problem.
- If students are having trouble, refer them back to the text to follow the steps for the construction.
- Assess student work for accuracy and creativity.
- Provide feedback/correction when students are finished.
- Create a class display with the designs.

## **Isosceles and Equilateral Triangles**

### I. Section Objectives

- Prove and use the Base Angles Theorem.
- Prove that an equilateral triangle must also be equiangular.
- Use the converse of the Base Angles Theorem.
- Prove that an equiangular triangle must also be equilateral.

### II. Problem Solving Activity-Triangle Proofs

- Provide students with the definition for the Base Angles Theorem.

- Base Angles Theorem- If two sides of a triangle are congruent, then their opposite sides are congruent, and the angles opposite the congruent sides are congruent.
- Next, assign students the task of proving that this is correct.
- The students need to draw a diagram involving isosceles triangles to demonstrate this theorem.
- Then, they need to write a two- column proof or a flow proof.
- When finished, the students are going to present their work in small groups.
- Allow time for the other students in the group to provide feedback.
- Extension- repeat the exercise with the students working with the converse of the Base Angles Theorem.
- Possibility- you can also assign half the class the Base Angles Theorem and the other half the converse of the Base Angles Theorem.

### III. Meeting Objectives

- Students will draw a design using isosceles triangles.
- The design is to prove the Base Angles Theorem.
- Students will demonstrate understanding through a design and through a verbal explanation.

### IV. Notes on Assessment

- Did the students complete the design?
- Does it use isosceles triangles?
- Did the students show the bisection of the triangle?
- Does the diagram show the angles and sides that are congruent?
- Does the proof have the correct statements and reasons?
- Is the student able to explain his/her thinking in words?
- Provide comments/feedback when necessary.

## **Congruence Transformations**

### I. Section Objectives

- Identify and verify congruence transformations.
- Identify coordinate notation for translations.
- Identify coordinate notation for reflections over the axes.
- Identify coordinate notation for rotations about the origin.

### II. Problem Solving Activity-Transformation Game

- In this game, students are going to manipulate one figure to represent the clues that the team has been given. You can play this as a whole class and make it about the speed with which a team can complete the transformation.
- Each team will need a triangle (made of cardstock) and a large coordinate grid. You can do this on chart paper.
- Each team starts with their triangle in the first quadrant. The team can decide where to put it. They draw it in or trace it in as the starting point.
- Now the team has a moveable triangle and a drawn one.
- Then call out the first command, “Rotate  $90^\circ$ ”
- The team rotates the triangle using the moveable triangle and places the triangle in a new place to show the rotation. When the team has completed the task they raise their hands.
- Teams that are correct and finish first get a point.
- Then begin again. The team moves the triangle back to the starting point and starts again. Call out a new command.
- Example “Translate 5 right and 3 down.”
- Then the game repeats.

### III. Meeting Objectives

- Students will identify and use congruence transformations.
- Students will identify coordinate notation for translations.
- Students will identify coordinate notation for reflections.
- Students will identify coordinate notation for rotations.
- Students will practice all of these in a creative game.

### IV. Notes on Assessment

- Assessment is completed through observation as each team is playing the game.
- Make note of teams that are having difficulty and offer assistance after class.
- If necessary, stop the game and offer corrections or review.

## 1.5 Relationships within Triangles

### Midsegments of a Triangle

#### I. Section Objectives

- Identify the midsegment of a triangle.

- Apply the Midsegment Theorem to solve problems involving side lengths and midsegments of triangles.
- Use the Midsegment Theorem to solve problems involving variable side lengths and midsegments of triangles.

## II. Problem Solving Activity- Midsegment Mystery

- Here is the problem.
- “Sari is working on a design for the roof of a dog house. Here is her design.”
- Figure05.01.01
- “Sari has forgotten her notes, but she knows that  $x$  is equal to 7. What is her measurement for  $y$  if  $x$  is equal to 7?”
- Use what you have learned about midsegments and triangles to help her work this out.
- Allow students time to work. Ask them to show all of their work when finished.

## III. Meeting Objectives

- Students will identify the midsegment of a triangle.
- Students will apply the Midsegment Theorem to solve problems involving side lengths and midsegments.
- Students will demonstrate understanding by showing all of their work.

## IV. Notes on Assessment

- Here is the solution to the problem.
- $x = 7$   
 $3x + 5 = \frac{1}{2}y$   
 $3(7) + 5 = \frac{1}{2}y$   
 $26 = \frac{1}{2}y$   
 $y = 52$
- Be sure that student work is labeled and that students show all of their work.
- Provide feedback when necessary.

# Perpendicular Bisectors in Triangles

## I. Section Objectives

- Construct the perpendicular bisector of a line segment.
- Apply the Perpendicular Bisector Theorem to identify the point of concurrency of the perpendicular bisectors of the sides (the circumcenter).
- Use the Perpendicular Bisector Theorem to solve problems involving the circumcenter of triangles.

## II. Problem Solving Activity-The Camping Dilemma

- Here is the problem.
- “Boy Scout Troop 462 is going on a camping trip. When they arrive at the campground, they are given a huge triangular field to set up in. The boys decide to place their tents along the perimeter of the triangle and to put the campfire in the center of the triangle. To do this, they will need to use perpendicular bisectors to identify the circumcenter.”
- Your task is to use the following diagram to help the boy scouts. Draw in the perpendicular bisectors and label the place where the campfire should go.
- Figure05.02.01
- Students can work individually or in pairs on this problem.
- Allow students time to share their work when they are finished.

## III. Meeting Objectives

- Students will construct perpendicular bisectors of line segments.
- Students will use the Perpendicular Bisector Theorem to identify the circumcenter of the triangles.
- Students will solve problems by helping the boy scouts with their campfire location.

## IV. Notes on Assessment

- Did the students draw in all of the perpendicular bisectors accurately?
- Is the circumcenter in the correct location?
- Did the students identify where the campfire should be?
- Are students able to verbalize how they went about solving the problem?
- Offer feedback and support as students are working.

# Angle Bisectors in Triangles

## I. Section Objectives

- Construct the bisector of an angle.
- Apply the Angle Bisector Theorem to identify the point of concurrency of the perpendicular bisectors of the sides (the incenter).
- Use the Angle Bisector Theorem to solve problems involving the incenter of triangles.

## II. Problem Solving Activity- Inscribing Circles

- This activity will focus on the students inscribing circles into already designed triangles.



- To prepare the activity, have several triangles drawn out for the students.
- Divide the students into groups.
- Each group needs to receive three different triangles.
- The students are completing these tasks at the same time. Each student completes his/her part of inscribing the triangle.
- The first student draws in one angle bisector and passes the triangle to the right.
- The next student draws in the next angle bisector and passes the triangle to the right.
- The next student draws in the perpendicular bisectors.
- The final student uses a compass to inscribe the circle.
- When finished, the students will have completed this task for three different triangles.

### III. Meeting Objectives

- Students will construct the bisectors of angles in a triangle.
- Students will draw in perpendicular bisectors of the angles in a triangle.
- Students will use a compass to inscribe a circle into a triangle.
- Students will share their work with peers.

### IV. Notes on Assessment

- Check student work for accuracy.
- Are the angles bisected correctly?
- Are the perpendicular bisectors correct?
- Have the students located the circumcenter?
- Is the circle correctly inscribed into the triangle?
- Can the students explain how and why they completed each piece the way that they did?
- Offer feedback/correction when necessary.

## **Medians in Triangles**

### I. Section Objectives

- Construct the medians of a triangle.
- Apply the Concurrency of Medians Theorem to identify the point of concurrency of the medians of the triangle (the centroid).
- Use the Concurrency of Medians Theorem to solve problems involving the centroid of triangles.

## II. Problem Solving Activity-Napoleon's Theorem

- Begin by sharing Napoleon's Theorem with the students from the Wikipedia site. Use the diagram as well this is Figure05.04.01
- [http://en.wikipedia.org/wiki/Napoleon%27s\\_theorem](http://en.wikipedia.org/wiki/Napoleon%27s_theorem)
- Students are going to prove Napoleans Theorem.
- Tell students that they are going to create a design three levels in complexity to prove Napoleans Theorem.
- Show them that the Wikipedia diagram is three levels of complexity.
- Students can use chart paper, colored pencils, and rulers.
- They need to be prepared to show, through their diagram, how Napoleon's Theorem is accurate and true.
- Each student works on his or her own design, but you may want to allow them to work in groups to help each other.
- Extension- does this continue to prove true if more triangles are added? Where can they be added?

## III. Meeting Objectives

- Students will construct the medians of a triangle.
- Students will show how Napoleans Theorem works.
- Students will explain their thinking through a presentation.
- Students will demonstrate understanding through each person's design.

## IV. Notes on Assessment

- Is student work accurate?
- Does it show accurate equilateral triangles?
- Are the students able to explain their work to prove Napoleon's Theorem?
- What happens when the pattern is extended beyond three levels? Are there more equilateral triangles to be found?
- Offer feedback and support as students work.

## **Altitudes in Triangles**

### I. Section Objectives

- Construct the altitude of a triangle.
- Apply the Concurrency of Altitudes Theorem to identify the point of concurrency of the altitudes of the triangle (the orthocenter).

- Use the Concurrency of Altitudes Theorem to solve problems involving the orthocenter of triangles.

## II. Problem Solving Activity-Drawing Triangles

- In this activity, students are going to demonstrate that they understand the concepts associated with altitude by constructing different triangles.
  1. Students need to construct an acute triangle.
    - Label the altitude of the triangle.
    - Label the orthocenter.
  2. Students need to construct an obtuse triangle.
    - Label the altitude of the triangle.
    - Label the orthocenter of the triangle.
  3. Students need to construct a right triangle.
    - Label the altitude of the triangle.
    - Label the orthocenter of the triangle.
- Allow students time to share their work in small groups.

## III. Meeting Objectives

- Students will construct the altitude of a triangle.
- Students will locate and label the orthocenter of a triangle.
- Students will demonstrate understanding through their constructions.

## IV. Notes on Assessment

- Look at each student's triangles.
- Is the orthocenter of the acute triangle inside the triangle?
- Is the orthocenter of the obtuse triangle outside the triangle?
- Is the orthocenter of the right triangle at the vertex of the right triangle?
- Assess student understanding by assessing each individual triangle.
- Offer feedback/correction when necessary.

## Inequalities in Triangles

### I. Section Objectives

- Determine relationships among the angles and sides of a triangle.
- Apply the Triangle Inequality Theorem to solve problems.

### II. Problem Solving Activity-Prove the Theorem

- In this problem, students need to use the information given to draw a diagram and prove the theorem.
- Students are going to prove the theorem that states that the angle opposite the longest side of a triangle with unequal sides will have the greatest measure.
- Here is the problem.
- “In triangle  $ABC$ ,  $\overline{AB} < \overline{BC}$ , the measure of angle  $A$  is  $80^\circ$ . The measure of angle  $C$  is  $40^\circ$ , and the measure of angle  $B$  is  $60^\circ$ . Given the theorem on side lengths and angle measures, can this be a true statement? Why or why not?”
- Provide students time to work on this problem.
- Students need to provide a diagram, a written explanation and a verbal explanation to explain their thinking.
- When finished, allow students time to present their work to the class.

### III. Meeting Objectives

- Students will determine relationships among the angles and sides of a triangle.
- Students will apply the Triangle Inequality Theorem to solve problems.
- Students will demonstrate understanding through diagrams, written explanations and verbal explanations.

### IV. Notes on Assessment

- Here is the solution to the problem.
- This is a true statement because the length of  $BC$  is longer than  $AB$ . The angle opposite  $BC$  is the greatest of the three angles in the triangle. It measures  $80^\circ$ . Therefore, the theorem is accurate and proven through this statement.
- Be sure that the students have diagram that resembles Figure05.06.01
- Offer feedback and correction when necessary.

## Inequalities in Two Triangles

### I. Section Objectives

- Determine relationships among the angles and sides of two triangles.
- Apply the SAS and SSS Triangle Inequality Theorems to solve problems.

### II. Problem Solving- Name that Inequality

- This problem solving activity is a game.
- Preparation is to prepare two Congruent Triangles for each group to work with.
- Students play it in groups of four. In the groups of four, they split up into pairs.
- Each pair is a team that plays against each other.
- When the students play, they are trying to “stump” the other party.
- The play begins like this, one team comes up with a problem for the other team to solve.
- For example, “If I lengthen side  $AB$  what inequality compares side  $AB$  to  $CD$ ?”
- Then the other team has to answer it.
- If they answer it correctly, the team receives a point.
- If not, the other team gets a point.
- Then they repeat the process by switching team positions.
- Both teams play until time is up.
- Students need to be encouraged to use the SAS Triangle Inequality Theorem and the SSS Triangle Inequality Theorem as well as the converse Theorems.
- Students can create as many different types of questions as they would like.
- Students can be very creative in their approach to writing questions.

### III. Meeting Objectives

- Students will determine relationships among the angles and sides of two triangles.
- Students will apply the SAS and SSS Triangle Inequality Theorems to solve problems.
- Students will demonstrate their knowledge and understanding through the quiz game.

### IV. Notes on Assessment

- Walk around as students play and assist students when necessary.
- Offer suggestions and challenge students to create difficult questions.
- Notice which students are having difficulty with the assignment and offer assistance and coaching.

## Indirect Proofs

### I. Section Objectives

- Reason indirectly to develop proofs of statement.

### II. Problem Solving Activity-Draw it Out!

- Assign students the task of drawing an example of a geometric proof and an algebraic proof.
- Tell the students that they are going to use indirect reasoning to develop these proofs of statements.
- Here is one possible example for an algebraic problem.
- “Marcy is selling candy bars for the school band. She starts out selling five bars. But in the end, she sells three times as many as her friend John does. The band teacher congratulates her on selling over forty candy bars. If John sold less than 12 bars, Marcy did not sell more than forty bars.”
- Students need to write a proof to show this statement is true.
- Here is the answer:
  - $3x + 5 = 41$
  - $3(11) + 5 = 38$
  - $38 \neq 41$
- Since the band teacher congratulated Marcy on selling more than forty bars, we can have our equation equal 41.
- John sold less than 12 bars, so we can use 11 as a possible answer for  $x$ .
- This shows that our statement that Marcy did not sell more than forty bars is a true statement.
- Then have students write their own statement and proof of statement for a geometric proof.
- You could provide them with a diagram or have them draw in their own.

### III. Meeting Objectives

- Students will reason indirectly to write proofs of statements.
- Students will work with both algebraic and geometric examples.

### IV. Notes on Assessment

- Look at student examples.
- Is the work accurate?
- Does the reasoning make sense?
- Are the students applying the correct theorems to prove their statements?
- Offer feedback and correction when necessary.

## 1.6 Quadrilaterals

### Interior Angles

#### I. Section Objectives

- Identify the interior angles of convex polygons.
- Find the sums of interior angles in convex polygons.
- Identify the special properties of interior angles in convex quadrilaterals.

#### II. Problem Solving Activity-Pentagon Cleaning

- Figure 06.01.01 Pentagon
- Here is the problem.
- “Washers Cleaning Company is in charge of cleaning the entire pentagon. This cleaning company is unique because it is made up of people who love math. Because of this, they clean the Pentagon in triangular sections, figuring out how much has been completed and how much is left based on degrees. If they have cleaned  $\frac{2}{3}$  of the Pentagon, how many degrees are left to clean?”
- Steps to solving this problem:
- To solve this problem, the students will need to figure out some things.
- 1. How many triangles are there in a pentagon?
- 2. How many degrees are there in a pentagon?
- 3. If  $\frac{2}{3}$  is clean, than  $\frac{1}{3}$  is left to be cleaned.
- 4. How many degrees are in the  $\frac{1}{3}$ ?
- 5. Finally, use all of this information to figure out the solution to the problem.
- Students need to use diagrams to demonstrate their solution as well as writing.

#### III. Meeting Objectives

- Students will identify the interior angles of convex polygons.
- Students will find the sums of interior angles in convex polygons.
- Students will demonstrate understanding through diagrams and writing.

#### IV. Notes on Assessment

- Does student work show all of the information in steps 1 – 5?
- Did the students figure out that the missing number of degrees is  $180^\circ$ ?
- Listen to students explain their solutions.
- Offer suggestions and feedback when necessary.

## Exterior Angles

### I. Section Objectives

- Identify the exterior angles of convex polygons.
- Find the sums of exterior angles in convex polygons.

### II. Problem Solving Activity- The Garden Dilemma

- Here is the problem.
- “Johanna has designed a garden in the shape of a hexagon for the local botanical garden. She is very proud of her work. Each vertex of the hexagon has one garden path that extends from it making the garden a central feature of the botanical garden. The carpenter who works at the botanical garden wants to design an edging for each angle of the pentagon. To do this, he needs your help. He knows the measure for three of the exterior angles and he needs to find the measure of the other two. He knows that these two angles are congruent. Use the figure to find the measure of the two missing angles.”
- Figure 06.02.01
- Students need to draw a picture to illustrate this problem.
- Students need to write about how they solved the problem.
- Students need to explain their work to the class.

### III. Meeting Objectives

- Students will identify the exterior angles of convex polygons.
- Students will find the sums of exterior angles in convex polygons.
- Students will show their work in a diagram or drawing.
- Students will write a written explanation of their work.

### IV. Notes on Assessment

- Here is the simple solution to the problem.
- $50 + 110 + 80 = 240$
- $360 - 240 = 120$
- Because the two remaining angles are congruent, we can divide this number by two.
- This leaves us with each remaining angle being equal to  $60^\circ$ .
- $a = 60^\circ$
- $b = 60^\circ$
- Check student work to be sure that the students have included all of the important components of solving this problem.
- Listen as students explain their work and offer suggestions/feedback when needed.



# Classifying Quadrilaterals

## I. Section Objectives

- Identify and classify a parallelogram.
- Identify and classify a rhombus.
- Identify and classify a rectangle.
- Identify and classify a square.
- Identify and classify a kite.
- Identify and classify a trapezoid.
- Identify and classify an isosceles trapezoid.
- Collect the classifications in a Venn diagram.
- Identify how to classify shapes on a coordinate grid.

## II. Problem Solving Activity-Quad Design

- This is a creative design activity.
- The students are going to need colored pencils, crayons, markers, rulers, large blank sheets of paper.
- The task is to create a design that has each of the seven figures in it.
- Students need to create a color key to identify the seven figures in the design.
- They can include any other shapes/color that they would like, as long as the seven figures are in the design.
- Here are the figures that must be in the design: parallelogram, rhombus, rectangle, square, kite, trapezoid, isosceles trapezoid
- Allow students the entire class to work.
- Tell students that creativity does count in the activity.

## III. Meeting Objectives

- Students will identify and classify a parallelogram.
- Students will identify and classify a rhombus.
- Students will identify and classify a rectangle.
- Students will identify and classify a square.
- Students will identify and classify a kite.
- Students will identify and classify a trapezoid.
- Students will identify and classify an isosceles trapezoid.

#### IV. Notes on Assessment

- Check each design to be sure that all of the seven figures are in it.
- Be sure that there is a key that is clear and easy to read.
- Include color and creativity in the assessment of each student's work.
- Create a bulletin board to display student work.

### Using Parallelograms

#### I. Section Objectives

- Describe the relationships between opposite sides in a parallelogram.
- Describe the relationship between opposite angles in a parallelogram.
- Describe the relationship between consecutive angles in a parallelogram.
- Describe the relationship between the two diagonals in a parallelogram.

#### II. Problem Solving Activity-Parallelogram Exploration

- Walk the students through this activity.
- Allow time for students to share their responses after each step of the activity.
- Students will need paper, pencils and rulers to complete this task.
- First, ask the students to draw two sets of intersecting parallel lines on a piece of paper.
- When finished, ask them to label the vertices  $A, B, C, D$
- Ask "What do you notice about this figure?"
- Some students may recognize the parallelogram right away.
- If so, help the students to expand their thinking to see the other properties of the figure as well.
- Ask "What do you notice about the opposite sides of the figure?"
- Answer- They are congruent and parallel.
- Ask "What do you notice about the opposite angles?"
- Answer- They are also congruent.
- Ask- "Which angles are supplementary?"
- For this answer, have the students demonstrate the answer by using a protractor.
- Ask- "How can you use the diagonals of the shape to figure out the number of degrees in this figure?"
- Students should be using triangles for this.

#### III. Meeting Objectives

- Students will describe the relationships between opposite sides in a parallelogram.
- Students will describe the relationship between opposite angles in a parallelogram.
- Students will describe the relationship between consecutive angles in a parallelogram.
- Students will describe the relationship between the two diagonals in a parallelogram.

#### IV. Notes on Assessment

- Assessment for this lesson is completed as the students work through each step of the activity.

## Proving Quadrilaterals are Parallelograms

### I. Section Objectives

- Prove a quadrilateral is a parallelogram given congruent opposite sides.
- Prove a quadrilateral is a parallelogram given congruent opposite angles.
- Prove a quadrilateral is a parallelogram given that the diagonals bisect each other.
- Prove a quadrilateral is a parallelogram if one pair of sides is both congruent and parallel.

### II. Problem Solving Activity-Is it really a Parallelogram?

- For this activity, students will need to cut two strips of paper that are the same length and two strips that aren't.
- Then have the students attach the four strips of paper together at the ends with fasteners.
- This will form a quadrilateral.
- Explain to the students that some quadrilaterals are parallelograms and some aren't.
- Then divide the students into groups.
- Students need to come up with ways to demonstrate the following points using their moveable figures.
- These points will help students to see how to prove that a quadrilateral is a parallelogram or that the shape that they have created is NOT a parallelogram.
  1. Prove a point about opposite sides.
  2. Prove a point about opposite angles.
  3. Demonstrate the Supplement Theorem.
  4. Show how the number of degrees in a quadrilateral is the same as a parallelogram by using the Triangle Sum Theorem.
- When students are finished working in their groups, give them time for each group to demonstrate one point to the rest of the class.
- Ask students to write what they have learned in their notebooks.

### III. Meeting Objectives

- Students will prove a quadrilateral is a parallelogram given congruent opposite sides.
- Students will prove a quadrilateral is a parallelogram given congruent opposite angles.
- Students will prove a quadrilateral is a parallelogram given that the diagonals bisect each other.
- Students will prove a quadrilateral is a parallelogram if one pair of sides is both congruent and parallel.

### IV. Notes on Assessment

- Check student work for accuracy. Offer feedback during presentations.
- Notice how the students demonstrate each point in their presentations.

## **Rhombi, Rectangles, and Squares**

### I. Section Objectives

- Identify the relationship between the diagonals in a rectangle.
- Identify the relationship between the diagonals in a rhombus.
- Identify the relationship between the diagonals and opposite angles in a rhombus.
- Identify and explain biconditional statements.

### II. Problem Solving Activity-Can you prove it?

- To prepare this activity, you will need to draw either a rectangle or a rhombus on a coordinate grid. You can have some be accurate and some close.
- The students are going to need to figure out if the figure is a rectangle or a rhombus or does it just look like one.
- Students will be using the principles that they learned in the text to determine whether the figure is really a rectangle or a rhombus.
- Students can work in pairs or small groups on this activity.
- In a rectangle, the students should be pointing out that or proving that the diagonals are congruent.
- In a rhombus, the students should be proving or pointing out that the diagonals intersect at a right angle.
- Students can also use the angles of both figures and the relationship between the angles.
- Allow time for the students to investigate and prepare to prove what their figure is or is not.
- Then allow time for each group to present their discovery.

### III. Meeting Objectives

- Students will use the relationship of the diagonals in a rectangle to prove whether a figure is a rectangle or not.
- Students will use the relationship of the diagonals in a rhombus to prove whether a figure is a rhombus or not.
- Students will explain their thinking to their peers.

#### IV. Notes on Assessment

- Listen to each group prove their figure.
- Challenge their thinking by asking questions.
- Be sure that student answers are clear and precise.
- Offer correction/feedback when needed.

## Trapezoids

### I. Section Objectives

- Understand and prove that the base angles of isosceles trapezoids are congruent.
- Understand and prove that if base angles in a trapezoid are congruent, it is an isosceles trapezoid.
- Understand and prove that the diagonals in an isosceles trapezoid are congruent.
- Understand and prove that if the diagonals in a trapezoid are congruent, the trapezoid is isosceles.
- Identify the median of a trapezoid and use its properties.

### II. Problem Solving Activity- Trapezoidal Towers

- In this problem, the students are going to have to use what they have learned about trapezoids to justify why or why not a tower characteristic is an isosceles trapezoid or a non- isosceles trapezoid.
- Here is the problem.
- “Jonas is studying architecture. He is really interested in unique geometric architecture. Jonas decides to share two of his favorite buildings with his friend Sam. One is of the Shanghai World Financial Center and the other is of Sutton Place in NYC. Jonas looks at the two towers and says, “It’s a shame that the hole in the top of the Shanghai World Financial Center isn’t an isosceles trapezoid like Sutton Place that would be really cool.” Sam looks puzzled.”
- Figure 06.07.01 [http://en.wikipedia.org/wiki/Shanghai\\_World\\_Financial\\_Center](http://en.wikipedia.org/wiki/Shanghai_World_Financial_Center)
- Figure 06.07.02 <http://www.thecityreview.com/sutton/rivtow.html>
- Look at each building. Why does Jonas think that the hole in the Shanghai World Financial Center is not an isosceles triangle? Use what you have learned to help Sam understand.
- Students need to use a written explanation and diagrams to complete this problem.

### III. Meeting Objectives

- In completing this problem, the students will use the properties of isosceles and non- isosceles trapezoids.
- Students will use the Base Angles Theorem.
- Students will write out and justify their answers.

### IV. Notes on Assessment

- Read student answers and look at student diagrams.
- Were they able to see that the base angles of Sutton Place are congruent?
- Were they able to see that the opposite sides of the Shanghai building are not congruent?
- Is student writing clearly written?
- Are there diagrams to illustrate student understanding?

## Kites

### I. Section Objectives

- Identify the relationship between diagonals in kites.
- Identify the relationship between opposite angles in kites.

### II. Problem Solving Activity-Let's Go Fly a Kite!

- For this activity, use the kite dimensions found in the text under the exercises on page 398-399. This is Figure 06.08.01.
- Give students material for making a kite.
- A great material is Tyvek which is used in housing for wall coverage. You can complete about 100 kites with one roll.
- If cost is an issue, then use paper or plastic, but the Tyvek works the best.
- Also, use wooden dowels for the supports of the kite.
- Ask the students to use the angle measurements in the exercises to design a kite.
- They can make it as large or small as they would like as long as the angle measures are the same.
- *Note: This will be used again in the next chapter on Similarity. Because the angle measures are the same, the small and large kites will be similar.*
- Then let the students work.
- You can do a whole project on this too complete with flying the kites on a windy day.
- Students will LOVE it!!

### III. Meeting Objectives

- Students will identify the relationship between diagonals in kites.
- Students will identify the relationship between opposite angles in kites.
- Students will use their knowledge to construct a kite.

### IV. Notes on Assessment

- Assess student work on three different levels.
- 1. Did the students correctly measure the angles to construct an accurate kite?
- 2. Did the students construct their kite?
- 3. Was student work accurate and completed on time?
- Create a rubric for grading students on each element of the kite.
- Include creativity in your grading scale.

## 1.7 Similarity

### Ratios and Proportions

#### I. Section Objectives

- Write and simplify ratios.
- Formulate proportions.
- Use ratios and proportions in problem solving.

#### II. Problem Solving Activity- Ratio/Proportion Relay

- Since this lesson is mostly review, use this fun game to review the concepts in the lesson.
- To prepare, bring in a bunch of assorted items from home. Make these random household items, but have more than one of each type of items. For example, three hairbrushes or five apples.
- Then put the students are four teams.
- This activity is timed.
- Students come up one at a time.
- Students have 15 – 20 seconds to look at the table and write as many ratios as they can.
- Then buzz it and the next person comes up.
- The last person has to take the ratios of the first three and create as many proportions as he/she can in two minutes.

- Students are not allowed to coach each other in their work.
- The team with the most ratios earns points.
- The team with the most correct proportions wins.
- You can repeat this more than once. Students will love it.

### III. Meeting Objectives

- Students will write and simplify ratios.
- Students will write proportions.
- Students will use ratios and proportions in problem solving.

### IV. Notes on Assessment

- How many ratios did each team write?
- Are the proportions accurate?
- Assess the time constraints, did the students need more or less time- adjust as necessary.

## **Properties of Proportions**

### I. Section Objectives

- Prove theorems about proportions.
- Recognize true proportions.
- Use proportions theorem in problem solving.

### II. Problem Solving Activity-Proportional Teamwork

- For this activity, students are going to draw two proportional triangles.
- Be sure to include measurements for each side of each triangle.
- Then, when instructed, students are going to pass the triangles to the person to their right. The person on the end of the room passes across the room to the “first” person.
- Then each person must write one proportion that represents the two triangles.
- Then they pass them again.
- Once again this repeats and the next student writes one proportion about the two triangles.
- Now there are two proportions on the paper under the triangles.
- Next it is passes again, and the last student writes the one remaining proportion.
- Finally, it is given back to the starting person (who drew the triangles).



- That student needs to correct the work of the other three.
- This student needs to write at least one theorem that is represented by these proportions and explain why he/she selected that theorem.

### III. Meeting Objectives

- Students will prove theorems about proportions.
- Students will recognize true proportions.
- Students will use proportions theorem in problem solving.

### IV. Notes on Assessment

- Collect all student work at the end of the class.
- First, check the triangles. Are they proportional?
- Next, check all of the proportions that were written about the triangles.
- Finally, check the notes/corrections and the theorem used.
- Is it explained well?
- Is it correct?
- Provide students with feedback and comments.

## Similar Polygons

### I. Section Objectives

- Recognize similar polygons.
- Identify corresponding angles and sides of similar polygons from a statement of similarity.
- Calculate and apply scale factors.

### II. Problem Solving Activity- Let's Go Fly a Kite- Part Two

- For this activity, students are going to revisit the work they did on kites.
- Particularly, students are going to use their work on creating kite designs from pg. 398-399.
- Ask students to identify what makes these kites similar. Brainstorm a list and write them on the board.
- Students should be commenting that because the angle measures are all the same, that the side lengths will be proportional.
- Then have students pair up.
- These two students need to compare their kite designs.

- They need to write four different proportions comparing the side lengths of the kites.
- Then have students design a statement of similarity that best compares their kites.
- If they can figure out the scale factor between the two kites have them include that in their work.
- Allow time for the student to share their work when finished.

### III. Meeting Objectives

- Students will recognize similar polygons in their kite designs.
- Students will identify and explain similar side lengths of the kite design.
- Students will write proportions to show similarity.
- Students will calculate scale factors.

### IV. Notes on Assessment

- Assess student work by walking around and observing students as they work.
- Then assess student understanding during student presentations.
- Help students to make connections about similar polygons by asking questions and providing feedback.

## Similarity by AA

### I. Section Objectives

- Determine whether triangles are similar.
- Understand AAA and AA rules for similar triangles.
- Solve problems about similar triangles.

### II. Problem Solving Activity- Thales and the Pyramids

- This is a great lesson to use Thales and the simple way that Thales measured the pyramids.
- You can use this website to find out more information about Thales.
- <http://educ.queensu.ca/~fmc/april2002/Pyramids.htm>
- Essentially, Thales figured out the height of the pyramids by using his own height. He waited until his shadow equaled his height. Then he measured the height of the pyramid and he knew that the height of the pyramid was equal to the pyramid's height.
- Here is the problem.
- "Stan is 6 feet tall. When he goes outside, his shadow is only nine feet long. The shadow of the tree in his yard is eighteen feet long. Based on these numbers, what is the height of the tree?"
- Use a drawing to figure out the answer to this problem.

- Have students write a paragraph to explain their process and how they arrived at their answer.
- When finished, have the students share their work in small groups.

### III. Meeting Objectives

- Students will determine similar triangles.
- Students will understand AAA and AA for similar triangles.
- Students will use AAA and AA with indirect measurement to solve problems.
- Students will share their understanding in their written work.
- Students will explain their thinking in small groups.

### IV. Notes on Assessment

- Assess each diagram to assess student understanding.
- Are the triangles similar?
- Are they labeled correctly?
- Was a proportion used to solve for the height of the tree?
- Then assess student writing.
- Is student thinking clear?
- Offer notes/feedback.

## **Similarity by SSS and SAS**

### I. Section Objectives

- Use SSS and SAS to determine whether triangles are similar.
- Apply SSS and SAS to solve problems about similar triangles.

### II. Problem Solving Activity-Triangle Jeopardy

- To play this game, divide students into small groups.
- To prepare this game, use a set of index cards and write one of the ways to prove similarity among triangles on each card.
- You should have cards that say SSS, SAS, AA and AAA
- Be sure that you have several of each card and mix them up.
- Each student in the group takes a turn.
- The student selects a card.

- Then he/she must come up with an example that illustrates the way to prove similar triangles.
- Each team can have 1 helpful hint- that is from you, and 1 lifeline from their group.
- If the student completes the challenge correctly, the team receives a point.
- You can play this game for quite a while.
- Some variations can include scale factor or diagrams on the board and then the group needs to show how the triangles are similar.

### III. Meeting Objectives

- Students will use and apply the SSS and SAS when determining whether triangles are similar.
- Students will use and apply the SSS and SAS to solve problems about similar triangles.
- Students will explain their work verbally and through diagrams.

### IV. Notes on Assessment

- Assessment comes through the process of the game.
- Because students work individually, you will have a really good idea of who understands about similar triangles and who needs more assistance.
- Provide coaching/feedback when necessary through “helpful hints.”

## **Proportionality Relationships**

### I. Section Objectives

- Identify proportional segments when two sides of a triangle are cut by a segment parallel to the third side.
- Divide a segment into any given number of congruent parts.

### II. Problem Solving Activity-Midsegment Match- up

- To prepare this activity, you will need to draw a bunch of triangles and cut them along the midsegment line.
- Then pass out one part of a triangle to each student.
- Students need to measure their part of the triangle.
- Then, they need to find the student who has their match.
- Students walk around the room and find a match for their triangle.
- When finished, all of the triangles should be complete.
- Once students have found a match, or think that they have, they need to write proportions to justify their thinking.

- Allow time for the students to share their work when finished.

### III. Meeting Objectives

- Students will identify proportional segments when two sides of a triangle are cut by a segment parallel to the third side.

### IV. Notes on Assessment

- Observe students as they walk around finding the match for their triangle part.
- Notice if students are using measurements or not.
- If not, remind students to look for the proportionality in the measurements of each section of the triangle.
- Be sure to listen as students share their work.
- Are they able to articulate why the triangles are similar?
- Is the match that they selected an accurate match?
- Offer feedback/correction as needed.

## Similarity Transformations

### I. Section Objectives

- Draw a dilation of a given figure.
- Plot the image of a point when given the center of dilation and scale factor.
- Recognize the significance of the scale factor of a dilation.

### II. Problem Solving Activity- T- Shirt Dilation

- Use the picture of Mickey Mouse for this problem- Figure 07.07.01
- [http://en.wikipedia.org/wiki/Mickey\\_mouse](http://en.wikipedia.org/wiki/Mickey_mouse)
- Provide students with a copy of the picture.
- Then have students measure the height of Mickey Mouse according to the picture.
- Have them also measure the width of Mickey's arms.
- Tell them to keep this measurement as they will need it to solve the problem.
- Here is the problem.
- "Sasha is creating a t-shirt for her sister with Mickey Mouse on it. She takes a picture of Mickey Mouse from Wikipedia and measures it. Now she knows that she needs to triple the size of the picture for it to be perfect on the t-shirt. Given this information, help Sasha by creating proportions that show how the original picture compares to the one on the t- shirt."

- Students need to use a diagram to show their work.
- Students need to show all of their problem solving.
- Remind them to show all work.
- Have students share their work when finished.

### III. Meeting Objectives

- Students will draw a dilation of a given figure.
- Students will use a scale factor to show a dilation.
- Students will explain their thinking through writing and through diagrams.
- Students will share their work to demonstrate understanding.

### IV. Notes on Assessment

- Is the diagram accurate?
- Is the dilation correct?
- Have the students shown all of the measurements?
- Are the proportions correct?
- Have the students shown all of their work?
- Is the student able to verbally explain their work?

## **Self- Similarity (Fractals)**

### I. Section Objectives

- Appreciate the concept of self- similarity.
- Extend the pattern in a self- similar figure.

### II. Problem Solving Activity-Fractal Fun

- If you have access to technology, then use this site to have students watch the different fractals being formed.
- <http://en.wikipedia.org/wiki/Fractal>
- Then tell students that they are going to create their own fractal.
- They can begin with a triangle, a pentagon or a hexagon.
- Students need to show four levels of the fractal.
- Remind them of the steps on how the fractal is created.

- You can use the steps in the text to assist the students.
- Then provide them with rulers, colored pencils and paper to create their fractals.
- Hint: There are bound to be errors at first. Remind the students to use the text to support their work. They can also revisit the website to help them to brainstorm ideas and create an exciting fractal.
- Leave students alone as much as possible.
- Allow students time to think and struggle a bit. It will help them to come to a clearer understanding of the concepts in the lesson.

### III. Meeting Objectives

- Students will appreciate the concept of self- similarity.
- Students will extend the pattern in a self- similar figure.
- Students will create their own fractal.

### IV. Notes on Assessment

- Walk around and help students who are really struggling.
- Be sure to allow students some time to work before you jump in as it is always better for students to try to solve problems on their own.
- Create a rubric to help you in grading each fractal.
- First, does the fractal work?
- Is their work complete?
- Does it show four levels?
- Make notes and share corrections/feedback with students.

## 1.8 Right Triangle Trigonometry

### The Pythagorean Theorem

#### I. Section Objectives

- Identify and employ the Pythagorean Theorem when working with right triangles.
- Identify common Pythagorean triples.
- Use the Pythagorean Theorem to find the area of isosceles triangles.
- Use the Pythagorean Theorem to derive the distance formula on a coordinate grid.

#### II. Problem Solving Activity-The Ramp Dilemma

- For this problem, students will be using the Pythagorean Theorem to figure out whether or not the following dimensions work for a bike ramp. They will be using the concept of Pythagorean triples in their work.
- Here is the problem.
- “Jonas is building a bike ramp. He has a pattern for a small model of a bike ramp and he wants to build a larger version of the bike ramp for his yard. His pattern uses measurements in inches. The pattern says that the dimensions of the bike ramp model will be 9 – 12 – 15. Jonas wants to enlarge this pattern. Use what you have learned about the Pythagorean Theorem and Pythagorean Triples to design a bike ramp that is not larger than 21 feet – 28 feet – 34 feet.”
- Solution Notes:
- This problem requires several steps. The first thing that the students are going to need to do is to convert feet into inches. Then they will know how many inches the ramp design needs to be.
- Next, the students need to begin to build proportions that work for triples.
- By multiplying by three, the students will find that they can build several different ramps.
- Here are some options:
- 27 – 36 – 45, 81 – 108 – 135
- The largest one without going over is 243 – 324 – 405.
- Students need to draw a design with a scale to show their ramp.
- Then they need to write the final dimensions in feet.
- In feet-  $20\frac{1}{4}$  – 27 –  $33\frac{3}{4}$  feet.

### III. Meeting Objectives

- Students will use the Pythagorean Theorem in designing a bike ramp.
- Students will use their knowledge of Pythagorean triples in designing their ramp.
- Students will draw a design to show their work.
- The design will be drawn to scale.

### IV. Notes on Assessment

- See the problem solving activity section for notes and solutions.

## Converse of the Pythagorean Theorem

### I. Section Objectives

- Understand the converse of the Pythagorean Theorem.
- Identify acute triangles from side measures.



- Identify obtuse triangles from side measures.
- Classify triangles in a number of different ways.

## II. Problem Solving Activity-Figure Those Triangles

- This is a game that has the students use number cards to figure out whether triangles are right triangles, acute triangles or obtuse triangles.
- To prepare for this game, use index cards and write one number on each card. Number the cards 2 – 25. Each group will need a set of cards to play the game.
- Divide the students into groups.
- Students are going to use the number cards to create as many different types of triangles as they can.
- Begin with right triangles.
- Students are given a short period of time to use the number cards to create as many different combinations of numbers that will equal right triangles. For example, using 3 – 4 – 5, the students will have a right triangle because  $3^2 + 4^2 = 5^2$ .
- Students work together to write out as many as possible using the numbers 2 – 25.
- Then move on to acute triangles. Remind students that  $a^2 + b^2 > c^2$ .
- Then move on to obtuse triangles. Remind students that  $a^2 + b^2 < c^2$ .
- Finally have students share their combinations.
- Winning teams can be determined based on the number of combinations created and on accuracy.

## III. Meeting Objectives

- Students will use number combinations to determine side lengths of right triangles.
- Students will use number combinations to determine side lengths of acute triangles.
- Students will use number combinations to determine side lengths of obtuse triangles.

## IV. Notes on Assessment

- Walk around as students play the game.
- Offer assistance when needed.
- Have students share their combinations to determine accuracy.

## Using Similar Right Triangles

### I. Section Objectives

- Identify similar triangles inscribed in a larger triangle.
- Evaluate the geometric mean of various objects.
- Identify the length of an altitude using the geometric mean of a separated hypotenuse.
- Identify the length of a leg using the geometric mean of a separated hypotenuse.

### II. Problem Solving Activity-The Camping Question

- Students are going to use the geometric mean to figure out the altitude of a triangle.
- Here is the problem.
- “Mariah is going on a teen wilderness trip. She will be using a tent that is triangular in shape and she has been told that the height of the tent can’t exceed nine feet. For this to be possible, the tent pole can not be greater than nine feet. She has a diagram of her tent. Use what you have learned about the geometric mean and the given the dimensions, to figure out the height of the tent. Will Mariah’s tent work for her trip or does she need to get a different tent?”
- Figure08.03.01
- Students first need to convert inches to feet. This will show them that the tent is divided into two 8 foot sections.
- Solution:  $8 \times 8 = 64$
- $\sqrt{64} = 8$
- The height of the tent is 8 feet, so Mariah’s tent will fit the specifications of the trip.

### III. Meeting Objectives

- Students will evaluate the geometric mean of various objects.
- Students will identify the length of an altitude using the geometric mean of a separated hypotenuse.
- Students will justify their answers.

### IV. Notes on Assessment

- Check student work for accuracy.
- Did the students convert inches to feet correctly?
- Did they remember to multiply the dimensions of the divided hypotenuse?
- Did they come up with a solution of 8 feet for the altitude of the tent?
- Offer feedback/correction when needed.

## Special Right Triangles

### I. Section Objectives

- Identify and use the ratios involved with right isosceles triangles.
- Identify and use the ratios involved with 30 – 60 – 90 triangles.
- Identify and use ratios involved with equilateral triangles.
- Employ right triangle ratios when solving real- world problems.

### II. Problem Solving Activity-Triangle Tiling

- For this activity, you will need to cut out a bunch of squares of different sizes. Each square needs to have a diagonal dividing it into two right triangles.
- Figure 08.04.01
- Each student is going to receive a square to work with.
- Tell students that they need to measure their square and figure out the length of the hypotenuse of each square.
- The measurements must be labeled on the back of the design.
- Then they can decorate the square however they would like.
- Finally, the students are going to create a square tiling on a piece of paper.
- They must be sure that the dimensions of each square is accurate (many students will trace the initial square).
- Creativity counts for this assignment.
- Students could repeat this assignment using 30 – 60 – 90 triangles too.

### III. Meeting Objectives

- Students will use ratios involved in right isosceles triangles.
- Students will see how a square and a right triangle are related.
- Students will understand the properties of 45 – 45 – 90 triangles.
- Students will demonstrate understanding by creating their tiling.
- Students will employ right triangle ratios with real- world problems.

### IV. Notes on Assessment

- Create a rubric to grade student work.
- Share the rubric grading scale with the students before beginning.
- Be sure that student measurements are accurate.
- Encourage students to use color and creativity in their designs.
- Create a wall display of student work.

## Tangent Ratios

### I. Section Objectives

- Identify the different parts of right triangles.
- Identify and use the tangent ratio in a right triangle.
- Identify complementary angles in right triangles.
- Understand tangent ratios in special right triangles.

### II. Problem Solving Activity-Ladder Tangents

- Students will use what they have learned about tangents to solve the following problem.
- Here is the problem.
- “A building is 20 feet tall. Mark and his roofing company are going to replace the roof on the building. The ladder that they are using 10 feet from the building creating an angle of  $55^\circ$ . Use this information and the diagram to solve the following questions.”
- Figure08.05.01
- Write these questions on the board.
- 1. What is the tangent of angle  $x$ ?
- 2. What is the length of the ladder?
- Solution:
- $\tan x = \frac{3}{5}$
- Hypotenuse:  $a^2 + b^2 = c^2$
- $12^2 + 10^2 = c^2$
- $144 + 100 = c^2$
- $244 = c^2$
- $\sqrt{244} = c$
- 15.6 feet =  $c$

### III. Meeting Objectives

- Students will understand the different parts of right triangles.
- Students will use the tangent ratio in a right triangle.
- Students will identify complementary angles in right triangles.
- Students will understand tangent ratios in special right triangles.

### IV. Notes on Assessment

- Walk around as the students are working.
- Notice which students are having difficulty and offer support.
- Allow students time to share their work when finished.

## Sine and Cosine Ratios

### I. Section Objectives

- Review the different parts of right triangles.
- Identify and use the sine ratio in a right triangle.
- Identify and use the cosine ratio in a right triangle.
- Understand sine and cosine ratios in special right triangles.

### II. Problem Solving Activity-Dion's Yard Design

- Here is the problem.
- “Dion has a square yard. He wants to divide the yard on the diagonal by planting a beautiful row of fruit trees. The yard measures  $16 \times 16$ . If this is the case, what is the measurement of the diagonal? Draw a diagram of the yard and the diagonal to also answer the following questions about the ratios in the yard.”
- Students begin by drawing a diagram of the yard.
- Using the Pythagorean Theorem, the diagonal is 22.6 feet long.
- Now use these measurements to answer the following questions.
- 1. What is  $\sin a$ ?
- 2. What is  $\sin b$ ?
- 3. What is  $\cos a$ ?
- 4. What is  $\cos b$ ?
- Solutions:
- 1. 1.1425
- 2. .7
- 3. .7
- 4. .7
- Allow time for the students to share their work at the end.
- Extension: Would the ratios be the same if the diagonal extended the opposite way as well?

### III. Meeting Objectives

- Students will review the different parts of right triangles.
- Students will identify and use the sine ratio in a right triangle.
- Students will identify and use the cosine ratio in a right triangle.

#### IV. Notes on Assessment

- Check the student diagram first.
- Is it labeled the correct way?
- Do the students understand the difference between the adjacent, the opposite and the hypotenuse?
- Are the sin ratios accurate?
- Are the cosine ratios accurate?
- Listen for understanding and clarity when students share their work.

### **Inverse Trigonometric Ratios**

#### I. Section Objectives

- Identify and use the arctangent ratio in a right triangle.
- Identify and use the arcsine ratio in a right triangle.
- Identify and use the arccosine ratio in a right triangle.
- Understand the general trends of trigonometric ratios.

#### II. Problem Solving Activity-Teaching Time

- The students are going to be teaching the class about how to find trigonometric ratios and inverse trigonometric ratios.
- Divide the students into six groups.
- Assign each of the groups one of the following topics.
  - 1. Tangents
  - 2. Sines
  - 3. Cosines
  - 4. Arctangents
  - 5. Arcsines
  - 6. Arccosines
- Then tell students that they need to prepare a five minute presentation on their topic to teach the others in the class about it.
- Students with the arc tangents can use calculators.
- Each presentation must have a diagram, an example and a problem for the class to solve.
- Allow time for the students to work.
- Then have each group present their topic.

### III. Meeting Objectives

- Students will identify and use the arctangent ratio in a right triangle.
- Students will identify and use the arcsine ratio in a right triangle.
- Students will identify and use the arccosine ratio in a right triangle.

### IV. Notes on Assessment

- Create a checklist for each point that needs to be in the student presentation.
- Evaluate each group as they present their topic.
- Is the presentation complete?
- Are there any missing pieces?
- Do the students have a good understanding of the material?
- Are the students able to speak clearly about it?

## **Acute and Obtuse Triangles**

### I. Section Objectives

- Identify and use the Law of Sines.
- Identify and use the Law of Cosines.

### II. Problem Solving Activity-Teaching Time

- For this activity, the students will be divided into groups of 4 – 6. Then in this larger group, divide the group into smaller sections of 2 – 3 people.
- Each team will be assigned one of the Laws.
- Each team has the assignment of teaching the other team about their Law.
- First, the team must design a problem that they think best illustrates the Law of Sines or the Law of Cosines.
- Then they must prepare their presentation.
- Finally, the teams present their work to the other teams.
- Each team needs to evaluate the other.
- You can use a rubric for evaluations.
- Some possible points might be:
  - 1. Clarity of explanation
  - 2. Is the work accurate?

- 3. Did you learn something new about the law?
- Allow time for students to share about their experience when finished.

### III. Meeting Objectives

- Students will identify and use the Law of Sines.
- Students will identify and use the Law of Cosines.
- Students will demonstrate understanding through their presentation.

### IV. Notes on Assessment

- Observe each group as it presents.
- You won't get to see all of it, but do your best to get an idea of how each group is doing. Make notes on the presentations.
- Collect all notes from each group.
- Then compare student evaluations with your evaluations.
- Assign each group a final grade.

## 1.9 Circles

### About Circles

#### I. Section Objectives

- Distinguish between radius, diameter, chord, tangent, and secant of a circle.
- Find relationships between congruent and similar circles.
- Examine inscribed and circumscribed polygons.
- Write the equation of a circle.

#### II. Problem Solving Activity- Circle Rhymes

- Create a rhyme about circles.
- Students can make it a rap, a song or a poem.
- Here are the components of the rhyme.
- Each must have:
  - Radius
  - Diameter



- Chord
- Tangent
- Secant
- Defined in it.
- Students can choose to include other elements, but these key ones must be in there.
- In addition, if students want to mention pi, they can, just be sure that they have the other parts of a circle involved.
- Allow students time to work on this poem.
- Students can perform/present it in small groups.
- If you have a really creative group that likes to perform, you might have them do it for the whole class. If this is the case, then presentations may take a whole class period.

### III. Meeting Objectives

- Students will distinguish between radius, diameter, chord, tangent, and secant of a circle.
- Students will present their material in a creative fun way.

### IV. Notes on Assessment

- Be sure that all of the important components are present.
- Students really can't do this wrong.
- Sit back and enjoy watching students use their creativity to bring circles to life.

## **Tangent Lines**

### I. Section Objectives

- Find the relationship between a radius and a tangent to a circle.
- Find the relationship between two tangents draw from the same point.
- Circumscribe a circle.
- Find equations of concentric circles.

### II. Problem Solving Activity-Lawn Sprinklers

- Here is the problem.
- “Tomas is putting in a sprinkler system in his back yard. He has divided the yard into six square sections. Inside the center of each square he has planted a sprinkler. The sprinkler spray extends to a distance of 56 feet. If this is the case, how much area will Tomas cover with his six sprinklers? Here is a diagram of one of the square plots to help you out.”

- Figure 09.02.01
- Tell students to show all of their work in their answer.
- Solution:
- 56 is the radius, so 112 is the diameter of the circle of the spray.
- This is also the side length of one of the square.
- Since we are looking for area, the formula for area of a square is  $s^2$
- $A = 112^2 = 12,544$  feet.
- For six squares we multiply this number  $\times 6 = 75,264$  feet.

### III. Meeting Objectives

- Students will use circles in problem solving.
- Students will use what they have learned about inscribed circles in problem solving.

### IV. Notes on Assessment

- Look at all student work.
- Is the work accurate?
- Did the students solve for area and not for perimeter?
- Did the students figure the distance for the diameter?
- Did the students arrive at the correct answer?
- Provide students with feedback/correction.

## **Common Tangents and Tangent Circles**

### I. Section Objectives

- Solve problems involving common internal tangents of circles.
- Solve problems involving common external tangents of circles.
- Solve problems involving externally tangent circles.
- Solve problems involving internally tangent circles.
- Common tangent

### II. Problem Solving Activity-The Gear Problem

- Here is the problem.

- “Ron is constructing a board with moveable gears for his little brother to play with. He constructs the first circle gear and puts the second gear right next to the first one. He wants to figure out the distance between the centers of the two gears so that he can use the correct size band to tie the two together. Use the diagram to help Ron figure out the distance of  $\overline{AB}$ .”
- Figure 09.03.01
- Students need to include a diagram and a written explanation of their solution.
- Here is the solution:
- Figure 09.03.02
- The distance is approximately 14.4.

### III. Meeting Objectives

- Students will solve problems involving common external tangents of circles.
- Students will solve problems involving externally tangent circles.

### IV. Notes on Assessment

- Collect both diagrams and written work from students.
- Does the written work match the diagram?
- Is the diagram clear?
- Is the written explanation clear?
- Did the students arrive at the correct answer?
- Provide students with correction/feedback.

## Arc Measures

### I. Section Objectives

- Measure central angles and arcs of circles.
- Find relationships between adjacent arcs.
- Find relationships between arcs and chords.

### II. Problem Solving Activity-The Pie Dilemma

- Here is the problem.
- “Jessica has made a delicious blueberry pie. Unfortunately, she cut the pie into uneven pieces.”
- Figure 09.04.01
- “Given the diagram, help Jessica figure out the dimensions of each section of the pie based on her uneven cuts.”

- Find the measure of arc  $CD$ , arc  $AC$  and arc  $BDCA$ .
- Students need to use a diagram, a written explanation and a worksheet showing all work.
- Solution:
- The angle of arc  $AB$  is  $80^\circ$ .
- Since  $AB$  and  $CD$  are congruent, then  $CD$  is also  $80^\circ$ .
- Arc  $AC = 180 - 80 = 100^\circ$
- Arc  $BDCA = 100 + 180 = 280^\circ$

### III. Meeting Objectives

- Students will find the measure central angles and arcs of circles.
- Students will find relationships between adjacent arcs.
- Students will find relationships between arcs and chords.

### IV. Notes on Assessment

- Collect student work.
- Check student work for accuracy.
- Offer feedback/notes as needed.

## Chords

### I. Section Objectives

- Find the lengths of chords in a circle.
- Find the measure of arcs in a circle.

### II. Problem Solving Activity-Circle Dilemma

- Here is the problem.
- “Mrs. Watson decided to teach her class about chords in a new way. She began by taking them out to the elementary school playground where there is a huge circle painted on the ground. Then she drew a straight line across and marked the center of the circle. Mrs. Watson gave her students a tape measure and asked them to figure out the hypotenuse of the right triangle.”
- “Jeff was puzzled, what right triangle? But Maria wasn’t. She took the tape measure and went to work. Here is what she found.”
- Figure 09.05.01
- “Help Maria finish the problem. What is the measure of the hypotenuse?”

- Have students show their work through a diagram and written explanation.
- Solution:
- Use the Pythagorean Theorem.
- $5^2 + 10^2 = c^2$
- $25 + 100 = c^2$
- $125 = c^2$
- $11.2 \approx c$

### III. Meeting Objectives

- Students will find the lengths of chords in a circle.
- Students will find the lengths of the sides of a right triangle.
- Students will use this information in problem solving.

### IV. Notes on Assessment

- Collect student work.
- Check student work for accuracy.
- Does the diagram accurately show the given information?
- Is the arithmetic correct?
- Does the written explanation make sense?
- Offer correction/feedback as needed.

## Inscribed Angles

### I. Section Objectives

- Find the measure of inscribed angles and the arcs they intercept.

### II. Problem Solving Activity- Find the Inscribed Angles

- For this activity, students are going to work in small groups to write a problem.
- Each problem must have a circle with a triangle or a quadrilateral inscribed in it.
- Students are going to use a protractor to measure the angles of the inscribed angles.
- Students are assigned the task of writing a word problem for another student to solve.
- Students must include a diagram and have an answer key to check student work.
- Allow time for students to write their problems.

- If students are having difficulty, have them brainstorm things with circles and other things in them- like a circus ring with a stand in it, an amphitheater, a raft in a lake, etc.
- When students are finished, have them switch papers with another group and solve each other's problem.
- Allow time for students to share when finished.
- There is a learning curve for this and don't be surprised if some of the initial attempts at writing a problem are missing essential components. If this is the case, have the students try again and rewrite the problem.
- Be sure that diagrams are accurate- this will make a huge difference.

### III. Meeting Objectives

- Students will find the measure of inscribed angles and the arcs they intercept.
- Students will use what they have learned to write a word problem.
- Students will use what they have learned in problem solving.

### IV. Notes on Assessment

- Walk around and observe students as they work.
- If students are having difficulty, offer suggestions.
- Help students to understand how to solve the problems.
- Help students to be sure that the problems that they are writing are complete.
- Offer suggestions/feedback as needed.

## **Angles of Chords, Secants and Tangents**

### I. Section Objectives

- Find the measures of angles formed by chords, secants and tangents.

### II. Problem Solving Activity-Folding It Out

- Students are going to be examining different chords, secants and tangents as found with a circle model.
- Begin by having each students cut out a circle from a sheet of colored paper.
- Then ask students to fold the circle in half.
- Then fold it in thirds.
- Then fold it in half again.
- This way you have a circle that has been divided.

- Finally, trace each line of the folds of the circle so that they can be seen.
- Then glue the circle with its lines to a piece of white paper.
- Have the students choose one pair of lines to extend outside the circumference of the circle.
- Then have students work in small groups to figure out the measures of each of the angles created by the different folds of paper.
- Some students may have folded their circles differently.
- This is alright as long as the students are going to work together.
- Ask them to work together on figuring out the angle measures.
- Allow students time to work and then to share their work when finished.

### III. Meeting Objectives

- Students will find the measures of angles formed by chords, secants and tangents.
- Students will work in groups to figure out the different angle measures.
- Students will share their work when finished.

### IV. Notes on Assessment

- Walk around as students work and offer assistance as needed.
- If students are having difficulty, remind them to refer back to the text.
- Collect student work when finished, and assess student understanding based on the accuracy of student work.
- Offer feedback and correction as needed.

## Segments of Chords, Secants and Tangents

### I. Section Objectives

- Find the lengths of segments associated with circles.

### II. Problem Solving Activity-Criss- Cross Applesauce

- Here is the problem.
- “Jack and Andy decided to play a game called Criss Cross Applesauce in the school yard. They each took a jump rope and ran across the circular playground. Their ropes crossed at one point. They wondered who had a longer rope. Using this diagram, help them to figure this out.”
- Figure 09.08.01
- Allow time for students to share their work when finished.

- Solution:
- The product of one line segment is equal to the product of the other line segment.
- Therefore, we can solve for the missing segment of the line by writing an equation and multiplying.
- $12x = (6)(10)$
- $12x = 60$
- $x = 5$
- One rope is 16 feet long.
- One rope is 17 feet long.
- The boy with the 17 foot long rope has the longer rope.
- Extension- What happens when an 18 foot rope is added to the mix- where does this rope cross the circle? Does it intersect with the other two ropes?

### III. Meeting Objectives

- Students will find the lengths of segments associated with circles.
- Students will use this information in problem solving.
- Students will share their work with a peer.

### IV. Notes on Assessment

- Were the students able to arrive at the correct answer?
- Can they explain how they got their answer?
- Is their work accurate?
- Listen in as students work and offer feedback and correction as needed.

## 1.10 Perimeter and Area

### Triangles and Parallelograms

#### I. Section Objectives

- Understand the basic concepts of the meaning of area.
- Use formulas to find the area of specific types of polygons.

#### II. Problem Solving Activity-Flooring Dilemma

- Here is the problem.



- “Ron is working for a flooring company for the summer. He is assigned the task of figuring out the area of a strangely shaped room at the library. Here is a diagram of the room that Ron needs to measure.”
- Figure 10.01.01
- “Ron thinks that the room looks like two rectangles and one triangle. He has some of the measurements from his diagram, but needs to figure out some of the other measurements. Is Ron correct? Can the room be divided into two rectangles and a triangle? What are the missing measurements? What is the area of the room?”
- Students will work in pairs to figure out these answers.
- Here are the solutions.
- The length of  $x$  is 8 feet because  $12 - 4 = 8$ .
- The area of the first rectangle is  $6 \times 12 = 72$  feet.
- The area of the second rectangle is  $8 \times 8 = 64$  feet.
- The area of the triangle is  $= \frac{1}{2}(6)(6) = 18$  feet
- Total area  $= 72 + 64 + 18 = 154$  feet

### III. Meeting Objectives

- Students will understand the basic concepts of the meaning of area.
- Students will use formulas to find the area of specific types of polygons.
- Students will use this understanding in problem solving.

### IV. Notes on Assessment

- Be sure that the students answered all of the questions in the problem.
- Are the measurements accurate?
- Did they divide the figure into two rectangles and a triangle?
- Did they find a different variation that works?
- Check all work and offer correction/feedback when necessary.

## Trapezoids, Rhombi and Kites

### I. Section Objectives

- Understand the relationships between the areas of two categories of quadrilaterals: basic quadrilaterals and special quadrilaterals.
- Derive area formulas for trapezoids, rhombi and kites.
- Apply the area formula for these special quadrilaterals.

## II. Problem Solving Activity-Jeffrey's Kite

- Students will use what they have learned in the lesson to find the area of Jeffrey's kite.
- Here is the problem.
- "Jeffrey has designed a kite to be flown in the city parade. He has made a large version of a small kite. His kite fits perfectly in a rectangular box that is 2' × 3'. In inches, what is the area of Jeffrey's kite?"
- Students need to use what they have learned to solve this problem. They can draw an interpretation of the kite design if they wish to.
- Students will need to convert feet to inches to begin with.
- Solution:
- 2 feet = 24 inches
- 3 feet = 36 inches
- The rectangle is 24 × 36
- Use the formula for finding the area of a kite.
- $\frac{1}{2}d_1d_2 = \frac{1}{2}(24)(36)$
- $\frac{1}{2}(864)$
- Area = 432 inches = 36 feet
- Conclusion: Jeffrey has created a HUGE kite.

## III. Meeting Objectives

- Students will use the formula for area to find the area of a kite.
- Students will learn to compare rectangles and the area of a kite.
- Students will demonstrate understanding through problem solving.

## IV. Notes on Assessment

- Be sure that the students have solved the problem accurately.
- Did the students convert the measurement units?
- Is the problem labeled correctly?
- Did the students draw a diagram to explain their answer?
- Offer correction/feedback when necessary.

## Area of Similar Polygons

### I. Section Objectives

- Understand the relationship between the scale factor of similar polygons and their areas.
- Apply scale factors to solve problems about areas of similar polygons.
- Use scale models or scale drawings.

### II. Problem Solving Activity-Mt. Rushmore

- You can use the following website for reference.
- [http://library.thinkquest.org/04oct/00398/ThinkquestHome\\_files/Thinkquest/Lia%20Web%20Page.htm](http://library.thinkquest.org/04oct/00398/ThinkquestHome_files/Thinkquest/Lia%20Web%20Page.htm)
- “Jessica is designing a model of Mt. Rushmore. Mt. Rushmore is 500 feet high and 400 feet wide. Based on a scale of 1” = 25 feet, what are the dimensions of Jessica’s model?”
- “When you are sure that you have the correct measurement, use grid paper and draw a diagram of Jessica’s model. Be sure to draw your diagram to scale.”
- Solution:
- 1” = 25 feet
- 500 feet high = 20” high
- 400 feet wide = 16” wide
- The students can draw this free hand using a ruler and large sheets of paper, or they can use grid paper to create another scale using the units on grid paper and then draw a diagram there.
- Allow time for the student to share their work when finished.

### III. Meeting Objectives

- Students will use scale models or scale drawings.
- Students will apply what they have learned about scale models and drawing when problem solving.

### IV. Notes on Assessment

- Is student work accurate?
- Does the diagram accurately represent the measurement of Mt. Rushmore?
- Have the students used the scale and proportion to come up with the correct measurements?
- Provide students with coaching and feedback.

## Circumference and Arc Length

### I. Section Objectives

- Understand the basic idea of a limit.
- Calculate the circumference of a circle.
- Calculate the length of an arc of a circle.

### II. Problem Solving Activity- The Cul-de- sac Question

- Here is the problem.
- Figure 10.04.01
- “Janine lives on a cul-de-sac. The diameter of the cul- de- sac is twenty- four feet. There is one road into the cul- de- sac and the measure of that arc is  $60^\circ$ . Janine would like to plant flowers along the edge of the cul-de-sac, but she does not need to plant them where the road is. How many feet can Janine plant?”
- Solution:
- Students will need to do several steps to figure this one out.
- First, find the circumference.
- The circumference is 75.36 feet.
- Next figure out the arc length.
- The measure of the arc is  $60^\circ$ .
- The arc length is 6.28 feet.
- Now subtract the arc length from the circumference.
- $75.36 - 6.28 = 69.08$
- Janine will plant approximately 69 feet.

### III. Meeting Objectives

- Students will calculate the circumference of a circle.
- Students will calculate the length of an arc of a circle.
- Students will use what they have learned in problem solving.

### IV. Notes on Assessment

- Check student work for accuracy.
- Have the students correctly figured the circumference?
- Have the students correctly figured the arc length?
- Is the answer clearly labeled?
- Offer support/correction when needed.

## Circles and Sectors

### I. Section Objectives

- Calculate the area of a circle.
- Calculate the area of a sector.
- Expand understanding of the limit concept.

### II. Problem Solving Activity-Circle Shading

- Students will use what they have learned about area and circles to figure out the area of the shaded region of the figure.
- Figure 10.04.01
- Here is the problem:
- “Use what you have learned about circles and area to find the area of the shaded region of the figure. Show all of your work in your answer.”
- Solution:
- The solution to this problem has two parts.
- First, students need to figure out the area of the circle. Since the length of the side of the square is ten inches, that is also the diameter of the circle since the circle fills almost the entire square. Therefore, the radius is five inches.
- $A = \pi r^2$
- $A = (3.14)(25)$
- $A = 78.5 \text{ in}^2$
- Next, the students need to find the area of the square.
- $A = s^2$
- $A = 10^2$
- $A = 100 \text{ in}^2$
- Now since the shaded region is what we are looking to find, we can subtract the area of the circle from the area of the square, and the remaining inches will indicate the shaded region.
- $100 - 78.5 = 21.5 \text{ inches}$

### III. Meeting Objectives

- Students will calculate the area of a circle.
- Students will use area in problem solving.

### IV. Notes on Assessment

- Be sure that each of the pieces of the solution is in the student’s answer.
- Offer correction/feedback when necessary.

## Regular Polygons

### I. Section Objectives

- Recognize and use the terms involved in developing formulas for regular polygons.
- Calculate the area and perimeter of a regular polygon.
- Relate area and perimeter formulas for regular polygons to the limit process in prior lessons.

### II. Problem Solving Activity-Teaching Time

- In this activity, students are going to teach the other students in the class.
- Divide students into small groups.
- Assign each group one of the following.
  1. Explain what makes a polygon a regular polygon
  2. Parts and terms of a regular polygon
  3. Figuring Perimeter- one group gets  $P = ns$
  4. Figuring Perimeter- one group gets  $P = 2nx$
  5. Calculating Area using  $A = \frac{1}{2} Pa$
- Once each group is given their assignment, their work is to demonstrate/teach their topic to the other students.
- Each group needs to use a diagram.
- Each group needs to have an example that demonstrates.
- Each group needs to have an example for the rest of the class to work with.
- Allow time for students to work and then present their topics.

### III. Meeting Objectives

- Students will recognize and use the terms involved in developing formulas for regular polygons.
- Students will calculate the area and perimeter of a regular polygon.
- Students will demonstrate understanding by teaching this material to their peers.

### IV. Notes on Assessment

- Take notes on each group's presentation.
- Is the material clearly presented?
- Have the students successfully used a diagram?
- Are the students able to answer questions from others in the class?
- Offer correction/feedback as needed.

## Geometric Probability

### I. Section Objectives

- Identify favorable outcomes and total outcomes.
- Express geometric situations in probability terms.
- Interpret probabilities in terms of lengths and areas.

### II. Problem Solving Activity-What are the Odds?

- Write the following list of things on the board. Students are going to play a game with the different topics.
- 6 bananas
- 5 apples
- 4 oranges
- 10 pineapples
- 12 lemons
- 8 bunches of grapes
- 7 melons
- To prepare the game, you need to create cards with statements that compare these items- for example, oranges to apples. Prepare a bunch of cards.
- The students play in pairs or teams of four.
- They play against each other.
- The students flip the top card over.
- Then whoever can say the probability first wins the point.
- It is a noisy game, but fun.
- Students play this similar to the card game “War”

### III. Meeting Objectives

- Students will identify total outcomes and favorable outcomes.
- Students will practice these skills while playing a fun game.

### IV. Notes on Assessment

- Walk around and observe students as they play the game.
- Offer feedback when needed.
- Extension of this activity- play the same game again but this time use a list of geometric measurements.

## 1.11 Surface Area and Volume

### The Polyhedron

#### I. Section Objectives

- Identify polyhedral.
- Understand the properties of polyhedral.
- Use Euler's formula to solve problems.
- Identify regular (Platonic) polyhedral.

#### II. Problem Solving Activity-Using Euler

- During this activity, students are going to use Euler's formula to determine the number of vertices, faces and edges for the pyramid.
- These pictures can be printed or if technology is available students can use a website to find the picture.
- Here is a picture of a pyramid with a rectangular base. This is Figure 11.01.01
- [http://en.wikipedia.org/wiki/File:01\\_khafre\\_north.jpg](http://en.wikipedia.org/wiki/File:01_khafre_north.jpg)
- Students need to create a three dimensional diagram of the pyramid.
- Then they can label the faces, vertices and edges of the pyramid.
- Finally, the students need to use Euler's formula and demonstrate the accuracy of their answers.
- Extension is for students to draw the pyramid design to scale. This would involve more research as students would need to search for the dimensions of the pyramid.
- Then allow time for the students to share their work.

#### III. Meeting Objectives

- Students will identify polyhedrons.
- Students will understand the properties of polyhedra.
- Students will use Euler's formula to solve problems.
- Students will demonstrate understanding through a diagram and presentation.

#### IV. Notes on Assessment

- Walk around as students are working on this assignment.
- Students may have difficulty picturing the rest of the pyramid given they can only see it from one perspective.
- Offer support by referring students back to the text.
- Another option is to provide students with a solid of a pyramid for them to work with.
- During presentations, listen for whether or not students have a grasp of Euler's formula.
- Provide coaching/feedback as needed.



## Representing Solids

### I. Section Objectives

- Identify isometric, orthographic, cross- sectional views of solids.
- Draw isometric, orthographic, cross- sectional views of solids.
- Identify, draw and construct nets for solids.

### II. Problem Solving Activity-Solids

- If you used the activity in the Differentiated Instruction lesson of the Teacher's Edition, you can move right on to the second part of this activity.
- If not, then begin with this part.
- Part One- Give students a geometric solid to work with.
- Then students are going to create four different things.
  - 1. Create an orthographic projection of their solid.
  - 2. Create a cross- section of the solid.
  - 3. Create a net for the solid.
  - 4. Use the net to create an actual model of the solid.
- Part Two- students are going to use the net that they created to design a mobile of solids.
- For this, students can create the same solid in different sizes, or they can create nets for different solids.
- Encourage students to use color and creativity in their mobile.
- Students can connect all of the created solids together with string and wooden dowels.
- Have materials on hand for students to complete their mobiles when finished.
- You can create a great display of mobiles and hang them all around the classroom.

### III. Meeting Objectives

- Students will identify isometric, orthographic, cross- sectional views of solids.
- Students will draw isometric, orthographic, cross- sectional views of solids.
- Students will identify, draw and construct nets for solids.
- Students will show their work in a mobile.

### IV. Notes on Assessment

- Look at each mobile and assess it.
- Are the nets created correctly?
- Did the student use different sized versions of the same solid or different solids?
- Is the mobile finished?
- Did the student use color and creativity in his/her design?

## Prisms

### I. Section Objectives

- Use nets to represent prisms.
- Find the surface area of a prism.
- Find the volume of a prism.

### II. Problem Solving Activity-Prism Pair- Up

- This activity requires some preparation.
- First, you will need a bunch of boxes.
- Then you will need to figure out the surface area of each of the boxes.
- Write each surface area on an index card.
- You will need as many boxes as you have students in one class.
- Then you bring all of the boxes to the classroom.
- To complete the activity, each of the students is given a ruler/tape measure and an index card with a surface area on it.
- Students will need to take notes as they work- remind them to do this.
- Also encourage students to think of strategies before beginning the task.
- Does a large box have a large surface area or a small surface area?
- Then the students need to figure out which box has their correct surface area.
- This will take some time.
- Don't rush the students, but encourage them to work together.
- Once they have found their box, the students need to create a diagram of it.
- Label the surface area and find the volume of the prism.
- Allow time for the students to share their work when finished.

### III. Meeting Objectives

- Students will find the surface area of a prism.
- Students will find the volume of a prism.
- Students will show their work in a diagram.

### IV. Notes on Assessment

- Did the students match up the correct prism with their measurements?
- Is the diagram labeled accurately?
- Is the volume of the prism correct?
- Offer correction/feedback when necessary.

# Cylinders

## I. Section Objectives

- Find the surface area of cylinders.
- Find the volume of cylinders.
- Find the volume of composite three- dimensional figures.

## II. Problem Solving Activity-Cylinder Sizing

- For this activity, the students are going to need a variety of different size cylinders to work with.
- This activity is going to combine our work with cylinders and scale factor together.
- Students can work in groups.
- Each group selects a cylinder to work with.
- First, the students need to figure out the surface area and volume of that cylinder.
- Next, the students are going to create a cylinder that is three times as large as the one that they have in their hands.
- Students will need large pieces of paper, scissors, rulers, string, tape or glue.
- The students will have to figure out the dimensions of the new cylinder.
- They will need to figure the surface area and the volume of the new cylinder as well.
- Then they will draw a net of the cylinder.
- Finally, they can cut the net out and build the actual cylinder.
- When finished, each group should have two cylinders constructed.
- Allow time for the students to share their work.

## III. Meeting Objectives

- Students will find the surface area of cylinders.
- Students will find the volume of cylinders.
- Students will use their work in problem solving.

## IV. Notes on Assessment

- Check student work for accuracy.
- Are the cylinders in proportion?
- Did the students calculate the surface area correctly?
- Did the students calculate the volume correctly?
- Did the students construct the second cylinder?
- Listen for student understanding of surface area and volume during student presentations.

# Pyramids

## I. Section Objectives

- Identify pyramids.
- Find the surface area of a pyramid using a net or formula.
- Find the volume of a pyramid.

## II. Problem Solving Activity-Create Your Own Pyramid

- For this activity, the students are going to design their own pyramids.
- First, they need to decide which polygon to use as a base.
- The project has two parts to be completed- the first is a diagram of the pyramid done with a scale, the second is an actual model of the pyramid.
- Students need to decide how large they want their pyramid.
- They can use these measurements to design the actual pyramid.
- Once they know how large they want to actually build a pyramid, they can draw a design to show their work.
- After the design has been completed, the students must figure out the surface area and the volume of the pyramid.
- The final stage is to draw a net (in actual size) and cut it out to construct the pyramid.
- Allow students time to share their work.
- Some students will build large pyramids, others small- either is fine as long as the work is accurate.

## III. Meeting Objectives

- Students will identify pyramids.
- Students will find the surface area of a pyramid using a net or formula.
- Students will find the volume of a pyramid.
- Students will demonstrate their understanding through a design and a model.

## IV. Notes on Assessment

- Check student work for accuracy.
- The design should be drawn to scale- so the design should not be the same size as the model unless the students are making a small model. Then that should be indicated on the diagram.
- Listen during presentations for student understanding.
- Can the student explain what they did and why they did it?
- Is the surface area measurement accurate?
- Is the volume accurate?
- Offer correction/feedback as needed.

# Cones

## I. Section Objectives

- Find the surface area of a cone using a net or formula.
- Find the volume of a cone.

## II. Problem Solving Activity-Ice Cream Cones

- Here is the problem.
- “George works at an ice cream parlor. He wants to design a new cone for them to put ice cream in. The standard cone that they use now has a radius of 2” and a height of 4”. This cone easily holds 2 scoops of ice cream. George wants to design a cone that holds 6 scoops of ice cream. Using these dimensions, figure out what George’s cone will look like.”
- To complete your work:
  1. Draw a design of the new cone. Be sure to include measurements.
  2. Figure out the surface area of the cone.
  3. Figure out the volume of the cone.
  4. Decide whether this design is even possible for an ice cream cone- justify your answer.
- Solution:
- Based on the ratios- the new cone would have a radius of 6” and a height of 12”.
- The surface area =  $C = 37.68$
- $A = 113.04$
- $SA = 2,556” \approx 213$  feet
- Volume = 452 units
- This is a ridiculous size for a cone!!!

## III. Meeting Objectives

- Students will figure out the surface area of a cone.
- Students will figure out the volume of a cone.
- Students will use what they have learned while problem solving.

## IV. Notes on Assessment

- Check student work.
- Is the surface area calculated correctly?
- Is the volume calculated correctly?
- What conclusions did the students come to regarding the size of the cone?
- Did they justify their answer?

# Spheres

## I. Section Objectives

- Find the surface area of a sphere.
- Find the volume of a sphere.

## II. Problem Solving Activity- Measuring Spheres

- Bring in a bunch of different size balls.
- The students will need string, measuring tape, rulers, paper and pencils to complete this assignment.
- Students may complete this assignment in small groups.
- First, divide up the groups and have the students select one ball to work with.
- Using their tools, and any other given information on the ball, students need to figure out the surface area and the volume of this first ball.
- Then they select a ball that appears to be proportional to this first ball.
- Again, students are going to figure out the surface area and the volume for the second ball.
- Then students need to draw a diagram comparing the two and write about any conclusions that they can determine about the two balls.
- Allow time for students to share their work when finished.

## III. Meeting Objectives

- Students will use tools to calculate the dimensions of a sphere.
- Students will find the surface area of a sphere.
- Students will find the volume of the sphere.
- Students will compare two different spheres.
- Students will draw a diagram explaining their work.
- Students will write about their conclusions.
- Students will explain and share their work with the class.

## IV. Notes on Assessment

- Be sure that each step of the objectives is accurate.
- Listen for student understanding during the presentations.
- What conclusions did the students draw about the two balls?

## Similar Solids

### I. Section Objectives

- Find the volumes of solids with bases of equal areas.

### II. Problem Solving Activity-Building Dimensions

- Here is the problem.
- “Kara has a sister who is an architect. Her sister is designing a building which is in the shape of a rectangular prism. When Kara visits, she sees her sister is building a model of the building. Her sister says that the model will be 2 feet high, 1.5 inches wide and 4.2 inches deep. She tells Kara that it has a scaling factor of  $200/1$ ”.
- Based on this information, help Kara to figure out the dimensions of the real building.”
- Solution:
- First- the students will need to have all of the measurements in inches since the scaling factor is in inches.
- That means that the model will be 36 inches high, 1.5 inches wide and 4.2 inches deep.
- They can multiply these by 200.
- Then convert back to feet by dividing by 12– since most buildings aren’t measured in inches.
- The final dimensions are: 300 feet high, 25 feet wide and 70 feet deep.

### III. Meeting Objectives

- Students will use the scaling factor with similar solids.
- Students will problem solve using this information.
- Students will share their work with their peers.

### IV. Notes on Assessment

- Is student work accurate?
- Did the students convert the original dimensions to inches?
- Is their final answer in feet?
- What conclusions did the students draw about the building?

## 1.12 Transformations

### Translations

#### I. Section Objectives

- Graph a translation in a coordinate plane.
- Recognize that a translation is an isometry.
- Use vectors to represent a translation.

#### II. Problem Solving Activity-Translation Teamwork

- For this activity, the students will be creating a game.
- Divide students onto teams of two.
- The first step is for students to design translations on a coordinate grid. They will need to create four different translations.
- Then have them write the coordinates for each translation on a separate index card.
- Finally, they need to write directions indicating the spacing of the translation. For example, three up and four to the right.
- Now they can put the whole thing together.
- Each team joins another team.
- They put all of the game cards together and they have to match up the correct diagram with the card coordinates and the directions.
- This can have a time limit for students as well.

#### III. Meeting Objectives

- Students will graph translations on a coordinate plane.
- Students will recognize that a translation is an isometry.

#### IV. Notes on Assessment

- When students are finished, collect each teams “playing cards”
- Check student work for accuracy.
- Offer correction/feedback when necessary.



# Matrices

## I. Section Objectives

- Use the language of matrices.
- Add matrices.
- Apply matrices to translations.

## II. Problem Solving Activity-Matrix Translations

- Students will use what they have learned about matrices and translations and apply the two together.
- First, the students need to be given a sheet of graph paper, a blank paper, a ruler and colored pencils.
- Tell students to begin by drawing a quadrilateral on the coordinate grid.
- They don't need to do anything else.
- Then have them pass their papers to the right.
- Next write 2 up and four down on the board.
- The students need to use this information and the diagram in front of them to write a matrix and use that matrix to figure out the coordinates of a new triangle.
- Tell them to draw this new triangle in a new color.
- Have students sign their name in the color of the triangle they drew.
- Then pass papers again.
- Students should go back to the original triangle.
- Next write 3 down and 2 up on the board.
- Students use a matrix again to design a new triangle.
- Ask the students to draw this one in with a different color.
- You can repeat this as many times as you wish- the students will a translation collection when finished.

## III. Meeting Objectives

- Students will use the language of matrices.
- Students will add matrices.
- Students will apply matrices to translations.

## IV. Notes on Assessment

- Collect all student work when finished.
- Check student work for accuracy.
- Offer correction/feedback when necessary.

## Reflections

### I. Section Objectives

- Find the reflection of a point in a line on a coordinate plane.
- Multiple matrices.
- Apply matrix multiplication to reflections.
- Verify that a reflection is an isometry.

### II. Problem Solving Activity-Reflection Matrices

- First, the students will need graph paper, rulers and colored pencils for this activity.
- Tell students to begin by drawing a quadrilateral on a coordinate grid.
- They can use color if they wish to.
- Then tell them that they will be drawing a reflection of this quadrilateral where  $y = x$ .
- If students need more help- tell them that they will be using  $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$  to multiply and find the coordinates of the new polygon.
- Students write a matrix and show all matrix multiplication.
- Then the students draw in the new quadrilateral on the coordinate grid.
- Finally, have students exchange papers and complete a peer check.
- Each person needs to check the other person's work.
- Allow time for the students to share their reflections in small groups when finished.

### III. Meeting Objectives

- Students will find the reflection of a point in a line on a coordinate plane.
- Students will multiple matrices.
- Students will apply matrix multiplication to reflections.
- Students will verify that a reflection is an isometry.
- Students will use what they have learned to draw a reflection of a quadrilateral.
- Students will demonstrate understanding by explaining their work in small groups.

### IV. Notes on Assessment

- Walk around and observe students as they work.
- Which strategies are the students using?
- Listen in on small group discussions.
- Is the student understanding lines of reflection?
- Offer assistance/feedback as needed.

## Rotations

### I. Section Objectives

- Find the image of a point in a rotation in a coordinate plane.
- Recognize that a rotation is an isometry.
- Apply matrix multiplication to rotations.

### II. Problem Solving Activity-Rotating Triangles

- For this problem, the students need to practice drawing rotations using matrices.
- Students will need graph paper, rulers and colored pencils.
- Tell students to use color to show the different rotations that they are going to be drawing on the coordinate grid.
- First, have the students draw a triangle on the coordinate grid.
- Then ask the students to write a matrix to describe this triangle.
- Next, they use matrix multiplication to rotate the triangle  $180^\circ$ .
- Finally, they draw in the triangle.
- Then using the same original triangle, students are going to use matrix multiplication to rotate a triangle  $90^\circ$ .
- After multiplying, they draw in the triangle.
- When finished, have students exchange papers so that a peer can check their work.
- Staple the work sheet with the matrix multiplication to the back of the sheet with the coordinate grid drawings.

### III. Meeting Objectives

- Students will find the image of a point in a rotation in a coordinate plane.
- Students will recognize that a rotation is an isometry.
- Students will apply matrix multiplication to rotations.
- Students will draw in new rotations on a coordinate grid.

### IV. Notes on Assessment

- Collect student work.
- Compare the matrix multiplication to the triangles on the coordinate grid.
- Does the work match up?
- If not, what is missing?
- Offer correction/feedback as needed.
- This could be graded as a classwork or homework assignment.

## Composition

### I. Section Objectives

- Understand the meaning of composition.
- Plot the image of a point in a composite transformation.
- Describe the effect of a composition on a point or polygon.
- Supply a single transformation that is equivalent to a composite of two transformations.

### II. Problem Solving Activity-Glide Rotations

- Students are going to draw a glide rotation of their own choosing.
- Tell students that the figure is going to be reflected in the x axis.
- Then tell students to draw in their figure.
- Next, they need to use matrix multiplication to design a reflection of that figure.
- Finally, they can move it 4 units to the right and one up.
- Students draw in the final figure.
- Allow time for them to share their work when finished.
- Solution:
- Because the figure is reflected in the  $x$  axis, students will multiply the coordinates of the figure by  $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
- Then they use  $(4, 1)$  to add to the product of the first two steps.
- Finally, the students can draw in the figure.

### III. Meeting Objectives

- Students will understand the meaning of composition.
- Students will plot the image of a point in a composite transformation.
- Students will describe the effect of a composition on a point or polygon.
- Students will supply a single transformation that is equivalent to a composite of two transformations.
- Students will share their work with their peers.

### IV. Notes on Assessment

- Collect student work.
- Be sure to collect the diagram and the written work.
- Check it for accuracy.
- Provide feedback/correction as needed.

# Tessellations

## I. Section Objectives

- Understand the meaning of tessellation.
- Determine whether or not a given shape will tessellate.
- Identify the regular polygons that will tessellate.
- Draw your own tessellation.

## II. Problem Solving Activity-Designing Tessellations

- For this lesson, the students are going to design a tessellation that uses two different polygons.
- Students first need to identify which polygons will tessellate and which won't.
- Then the students can work on their design.
- Each student will need colored pencils, rulers, paper, cardstock, and large paper.
- Explain to the students that they will be graded on their work.
- Each student needs to create a pattern to demonstrate that the tessellation that they have suggested does work.
- Students need to have a peer check their work.
- Students need to have an instructor check their work.
- When the pattern has been approved, students may begin work on their final design.
- This will help students problem solve and eliminate redoing work.
- Allow time for the students to present their work when finished.

## III. Meeting Objectives

- Students will understand the meaning of tessellation.
- Students will determine whether or not a given shape will tessellate.
- Students will identify the regular polygons that will tessellate.
- Students will draw their own tessellations.
- Students will share their work when finished.

## IV. Notes on Assessment

- Design a rubric to grade student work.
- Share the rubric with the students ahead of time.
- Possible sections include focus, creativity, design, accuracy, etc.
- Provide written feedback to students on their work.

# Symmetry

## I. Section Objectives

- Understand the meaning of symmetry.
- Determine all the symmetries for a given plane figure.
- Draw or complete a figure with a given symmetry.
- Identify planes of symmetry for three- dimensional figures.

## II. Problem Solving Activity-Magazine Symmetry Hunt

- For this activity, students are going to hunt through magazines of example of symmetry.
- You will need to prepare and bring a stack of magazines to class. Architecture and nature magazines will be very helpful.
- They are going to create a collage of the pictures that they find.
- Students will need paper, glue, scissors, a ruler and a pencil.
- Students can use pictures that show symmetry of a plane figure and symmetry of a three- dimensional figure.
- When the students select a given picture, they need to cut it out and attach it to the collage.
- Then they draw in the lines of symmetry.
- Students do this until their collage is complete.
- Then have the students write a paragraph explaining their collage- any themes and the types of symmetry found in the collage.
- Allow time for students to share their work when finished.

## III. Meeting Objectives

- Students will understand the meaning of symmetry.
- Students will determine all the symmetries for a given plane figure.
- Students will draw or complete a figure with a given symmetry.
- Students will identify planes of symmetry for three- dimensional figures.
- Students will find real- life examples of symmetry.

## IV. Notes on Assessment

- Look at student collages.
- Read student descriptions of their collages.
- Develop a grading rubric for the assignment.

- Use observation as a piece of the rubric.
- Were the students organized?
- Were students focused?
- Offer feedback on student work.

## Dilations

### I. Section Objectives

- Use the language of dilations.
- Calculate and apply scalar products.
- Use scalar products to represent dilations.

### II. Problem Solving Activity-Dilation Design

- This activity is an extension of the work that was done in the Differentiation Lesson of the Teachers Edition.
- Students will create two dilations in this activity.
- Students will create a large dilation and a small dilation.
- Tell students that they will be working with the scale factors of  $\frac{1}{3}$  and 3.
- Next, students will need a coordinate grid to work with.
- Students begin by selecting a polygon that they would like to work with.
- The students can draw this polygon anywhere that they would like to on the coordinate grid.
- Once this is done, have the students write the coordinates out as a matrix.
- Then tell students to use scalar multiplication and the given scale factors to design two new polygons.
- Have students complete the work with the matrices on a separate sheet of paper.
- When finished, draw the two new polygons on the coordinate grid.
- Allow time for students to share their work when finished.

### III. Meeting Objectives

- Students will use the language of dilations.
- Students will calculate and apply scalar products.
- Students will use scalar products to represent dilations.

### IV. Notes on Assessment

- Collect both the coordinate grid with the dilated polygons and the matrix worksheet.
- Then check student work for accuracy.
- Offer correction/feedback as needed.