INFLUENCE OF OVERLAYERS ON DEPTH OF IMPLANTED-HETEROJUNCTION RECTIFIERS

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ABSTRACT

In this paper we compare distributions of concentrations of dopants in an implanted-junction rectifiers in a heterostructures with an overlayer and without the overlayer. Conditions for decreasing of depth of the considered p-n-junction have been formulated.

KEYWORDS

Heterostructures; implanted-junction rectifiers: overlayers.

1. INTRODUCTION

In the present time integration rate of elements of integrated circuits intensively increasing. At the same time dimensions of the above elements decreases. To increase integration rate of elements of integrated circuits and to increase dimensions of the same elements are have been elaborated different approaches [1-10]. In the present paper we consider a heterostructure with two layers: a substrate and an epitaxial layer. The heterostructure could include into itself a third layer: an overlayer (see Figs. 1). We assume, that type of conductivity of the substrate (*n* or *p*) is known. The epitaxial layer has been doped by ion implantation to manufacture required type of conductivity (*p* or *n*). Farther we consider annealing of radiation defects. Main aim of the present paper is comparison two ways of ion doping of the epitaxial layer: ion doping of epitaxial layer and ion doping of epitaxial layer through the overlayer.
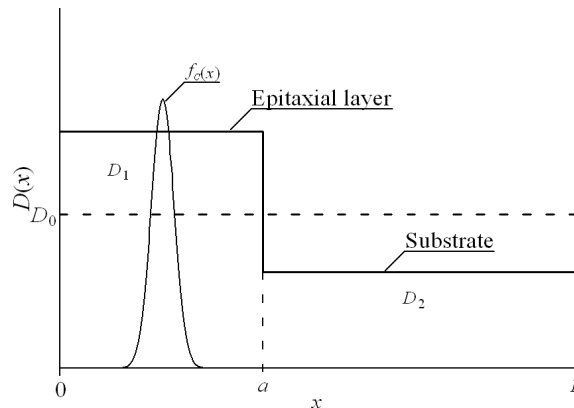

Fig. 1*a*. Heterostructure, which consist of a substrate and an epitaxial layer with initial distribution of implanted dopant
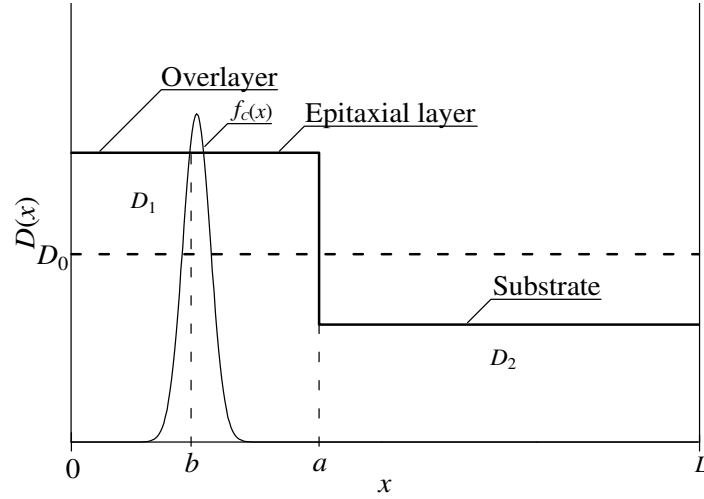
Fig. 1*b*. Heterostructure, which consist of a substrate, an epitaxial layer and an overlayer with initial distribution of implanted dopant

2. METHOD OF SOLUTION

To solve our aim we determine spatio-temporal distributions of concentrations of dopants. We calculate the required distributions by solving the second Fick's law in the following form [1]

$$\frac{\partial C(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x}\left[D_C \frac{\partial C(x,y,z,t)}{\partial x}\right] + \frac{\partial}{\partial y}\left[D_C \frac{\partial C(x,y,z,t)}{\partial y}\right] + \frac{\partial}{\partial z}\left[D_C \frac{\partial C(x,y,z,t)}{\partial z}\right]. \quad (1)$$

Boundary and initial conditions for the equations are

$$\frac{\partial C(x,y,z,t)}{\partial x}\bigg|_{x=0} = 0, \ \frac{\partial C(x,y,z,t)}{\partial x}\bigg|_{x=L_x} = 0, \ \frac{\partial C(x,y,z,t)}{\partial y}\bigg|_{y=0} = 0, \ \frac{\partial C(x,y,z,t)}{\partial y}\bigg|_{x=L_y} = 0,$$

$$\frac{\partial C(x,y,z,t)}{\partial z}\bigg|_{z=0} = 0, \ \frac{\partial C(x,y,z,t)}{\partial z}\bigg|_{x=L_z} = 0, \ C(x,y,z,0)=f(x,y,z). \quad (2)$$

Here the function $C(x,y,z,t)$ describes the distribution of concentration of dopant in space and time. D_C describes distribution the dopant diffusion coefficient in space and as a function of temperature of annealing. Dopant diffusion coefficient will be changed with changing of materials of heterostructure, heating and cooling of heterostructure during annealing of dopant or radiation defects (with account Arrhenius law). Dependences of dopant diffusion coefficient on coordinate in heterostructure, temperature of annealing and concentrations of dopant and radiation defects could be written as [11-13]

$$D_C = D_L(x,y,z,T)\left[1+\xi\frac{C^\gamma(x,y,z,t)}{P^\gamma(x,y,z,T)}\right]\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{\left(V^*\right)^2}\right]. \quad (3)$$

Here function $D_L(x,y,z,T)$ describes dependences of dopant diffusion coefficient on coordinate and temperature of annealing T. Function $P(x,y,z,T)$ describes the same dependences of the limit of solubility of dopant. The parameter γ is integer and usually could be varying in the following

interval $\gamma \in [1,3]$. The parameter describes quantity of charged defects, which interacting (in aver-age) with each atom of dopant. Ref.[11] describes more detailed information about dependence of dopant diffusion coefficient on concentration of dopant. Spatio-temporal distribution of concentration of radiation vacancies described by the function $V(x,y,z,t)$. The equilibrium distribution of concentration of vacancies has been denoted as V^*. It is known, that doping of materials by diffusion did not leads to radiation damage of materials. In this situation $\zeta_1 = \zeta_2 = 0$. We determine spatio-temporal distributions of concentrations of radiation defects by solving the following system of equations [12,13]

$$\frac{\partial I(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial x}\right] + \frac{\partial}{\partial y}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial y}\right] - k_{I,I}(x,y,z,T) \times$$
$$\times I^2(x,y,z,t) + \frac{\partial}{\partial z}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial z}\right] - k_{I,V}(x,y,z,T)\, I(x,y,z,t)\, V(x,y,z,t) \quad (4)$$

$$\frac{\partial V(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial x}\right] + \frac{\partial}{\partial y}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial y}\right] - k_{V,V}(x,y,z,T) \times$$
$$\times V^2(x,y,z,t) + \frac{\partial}{\partial z}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial z}\right] - k_{I,V}(x,y,z,T) I(x,y,z,t)\, V(x,y,z,t).$$

Boundary and initial conditions for these equations are

$$\left.\frac{\partial \rho(x,y,z,t)}{\partial x}\right|_{x=0} = 0, \quad \left.\frac{\partial \rho(x,y,z,t)}{\partial x}\right|_{x=L_x} = 0, \quad \left.\frac{\partial \rho(x,y,z,t)}{\partial y}\right|_{y=0} = 0, \quad \left.\frac{\partial \rho(x,y,z,t)}{\partial y}\right|_{y=L_y} = 0,$$

$$\left.\frac{\partial \rho(x,y,z,t)}{\partial z}\right|_{z=0} = 0, \quad \left.\frac{\partial \rho(x,y,z,t)}{\partial z}\right|_{z=L_z} = 0, \quad \rho(x,y,z,0) = f_\rho(x,y,z). \quad (5)$$

Here $\rho = I, V$. We denote spatio-temporal distribution of concentration of radiation interstitials as $I(x,y,z,t)$. Dependences of the diffusion coefficients of point radiation defects on coordinate and temperature have been denoted as $D_\rho(x,y,z,T)$. The quadric on concentrations terms of Eqs. (4) describes generation divacancies and diinterstitials. Parameter of recombination of point radiation defects and parameters of generation of simplest complexes of point radiation defects have been denoted as the following functions $k_{I,V}(x,y,z,T)$, $k_{I,I}(x,y,z,T)$ and $k_{V,V}(x,y,z,T)$, respectively.

Now let us calculate distributions of concentrations of divacancies $\Phi_V(x,y,z,t)$ and diinterstitials $\Phi_I(x,y,z,t)$ in space and time by solving the following system of equations [12,13]

$$\frac{\partial \Phi_I(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x}\left[D_{\Phi I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial x}\right] + \frac{\partial}{\partial y}\left[D_{\Phi I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial y}\right] +$$
$$+ \frac{\partial}{\partial z}\left[D_{\Phi I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial z}\right] + k_{I,I}(x,y,z,T)I^2(x,y,z,t) - k_I(x,y,z,T)I(x,y,z,t) \quad (6)$$

$$\frac{\partial \Phi_V(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x}\left[D_{\Phi V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial x}\right] + \frac{\partial}{\partial y}\left[D_{\Phi V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial y}\right] +$$
$$+ \frac{\partial}{\partial z}\left[D_{\Phi V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial z}\right] + k_{V,V}(x,y,z,T)V^2(x,y,z,t) - k_V(x,y,z,T)V(x,y,z,t).$$

Boundary and initial conditions for these equations are

$$\left.\frac{\partial\Phi_\rho(x,y,z,t)}{\partial x}\right|_{x=0}=0, \ \left.\frac{\partial\Phi_\rho(x,y,z,t)}{\partial x}\right|_{x=L_x}=0, \ \left.\frac{\partial\Phi_\rho(x,y,z,t)}{\partial y}\right|_{y=0}=0, \ \left.\frac{\partial\Phi_\rho(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,$$

$$\left.\frac{\partial\Phi_\rho(x,y,z,t)}{\partial z}\right|_{z=0}=0, \ \left.\frac{\partial\Phi_\rho(x,y,z,t)}{\partial z}\right|_{z=L_z}=0, \ \Phi_I(x,y,z,0)=f_{\Phi I}(x,y,z), \ \Phi_V(x,y,z,0)=f_{\Phi V}(x,y,z). \ (7)$$

The functions $D_{\Phi\rho}(x,y,z,T)$ describe dependences of the diffusion coefficients of the above complexes of radiation defects on coordinate and temperature. The functions $k_I(x,y,z,T)$ and $k_V(x,y,z, T)$ describe the parameters of decay of these complexes on coordinate and temperature.

To determine spatio-temporal distribution of concentration of dopant we transform the Eq.(1) to the following integro-differential form

$$\frac{x\,y\,z}{L_xL_yL_z}\int_{L_x}\int_{L_y}\int_{L_z}C(u,v,w,t)\,dw\,dv\,du=\int_0^t\int_{L_y}\int_{L_z}D_L(x,v,w,T)\left[1+\varsigma_1\frac{V(x,v,w,\tau)}{V^*}+\varsigma_2\frac{V^2(x,v,w,\tau)}{(V^*)^2}\right]\times$$

$$\times\left[1+\xi\frac{C^\gamma(x,v,w,\tau)}{P^\gamma(x,v,w,T)}\right]\frac{\partial C(x,v,w,\tau)}{\partial x}\,d\tau\frac{y\,z}{L_yL_z}+\int_0^t\int_{L_x}\int_{L_z}D_L(u,y,w,T)\left[1+\xi\frac{C^\gamma(u,y,w,\tau)}{P^\gamma(x,y,z,T)}\right]\times$$

$$\times\left[1+\varsigma_1\frac{V(u,y,w,\tau)}{V^*}+\varsigma_2\frac{V^2(u,y,w,\tau)}{(V^*)^2}\right]\frac{\partial C(u,y,w,\tau)}{\partial y}\,d\tau\frac{x\,z}{L_xL_z}+\int_0^t\int_{L_x}\int_{L_y}D_L(u,v,z,T)\times$$

$$\times\left[1+\varsigma_1\frac{V(u,v,z,\tau)}{V^*}+\varsigma_2\frac{V^2(u,v,z,\tau)}{(V^*)^2}\right]\left[1+\xi\frac{C^\gamma(u,v,z,\tau)}{P^\gamma(x,y,z,T)}\right]\frac{\partial C(u,v,z,\tau)}{\partial z}\,d\tau\frac{x\,y}{L_xL_y}+$$

$$+\frac{x\,y\,z}{L_xL_yL_z}\int_{L_x}\int_{L_y}\int_{L_z}f(u,v,w)\,dw\,dv\,du. \tag{1a}$$

Now let us determine solution of Eq.(1a) by Bubnov-Galerkin approach [14]. To use the approach we consider solution of the Eq.(1a) as the following series

$$C_0(x,y,z,t)=\sum_{n=0}^N a_{nC}\,c_n(x)\,c_n(y)\,c_n(z)\,e_{nC}(t).$$

Here $e_{nC}(t)=\exp\left[-\pi^2n^2D_{0C}t\left(L_x^{-2}+L_y^{-2}+L_z^{-2}\right)\right]$, $c_n(\chi)=cos(\pi n\,\chi/L_\chi)$. Number of terms N in the series is finite. The above series is almost the same with solution of linear Eq.(1) (i.e. for $\xi=0$) and averaged dopant diffusion coefficient D_0. Substitution of the series into Eq.(1a) leads to the following result

$$\frac{x\,y\,z}{\pi^2}\sum_{n=1}^N\frac{a_C}{n^3}s_n(x)\,s_n(y)\,s_n(z)\,e_{nC}(t)=-\frac{y\,z}{L_yL_z}\int_0^t\int_{L_y}\int_{L_z}\left\{1+\left[\sum_{n=1}^N a_{nC}\,c_n(x)\,c_n(v)\,c_n(w)\,e_{nC}(\tau)\right]^\gamma\times\right.$$

$$\left.\times\frac{\xi}{P^\gamma(x,v,w,T)}\right\}\left[1+\varsigma_1\frac{V(x,v,w,\tau)}{V^*}+\varsigma_2\frac{V^2(x,v,w,\tau)}{(V^*)^2}\right]D_L(x,v,w,T)\sum_{n=1}^N a_{nC}\,s_n(x)\,c_n(v)\times$$

$$\times n\,c_n(w)\,e_{nC}(\tau)d\tau-\frac{x\,z}{L_xL_z}\int_0^t\int_{L_x}\int_{L_z}\left\{1+\left[\sum_{m=1}^N a_{mC}\,c_m(u)\,c_m(y)\,c_m(w)\,e_{mC}(\tau)\right]^\gamma\frac{\xi}{P^\gamma(u,y,w,T)}\right\}\times$$

$$\times D_L(u,y,w,T)\left[1+\varsigma_1\frac{V(u,y,w,\tau)}{V^*}+\varsigma_2\frac{V^2(u,y,w,\tau)}{(V^*)^2}\right]\sum_{n=1}^{N}n\,c_n(u)\,s_n(y)\,c_n(w)\,e_{nC}(\tau)\,d\tau\times$$

$$\times a_{nC}-\frac{x\,y}{L_xL_y}\int_0^t\int_{L_x}^x\int_{L_y}^y D_L(u,v,z,T)\left\{1+\frac{\xi}{P^\gamma(u,v,z,T)}\left[\sum_{n=1}^{N}a_{nC}c_n(u)c_n(v)c_n(z)e_{nC}(\tau)\right]^\gamma\right\}\times$$

$$\times\left[1+\varsigma_1\frac{V(u,v,z,\tau)}{V^*}+\varsigma_2\frac{V^2(u,v,z,\tau)}{(V^*)^2}\right]\sum_{n=1}^{N}n\,a_{nC}c_n(u)\,c_n(v)\,s_n(z)\,e_{nC}(\tau)\,d\tau+\frac{x\,y\,z}{L_xL_yL_z}\times$$

$$\times\int_{L_x}^x\int_{L_y}^y\int_{L_z}^z f(u,v,w)\,dw\,dv\,du,$$

where $s_n(\chi)=sin(\pi n\chi/L_\chi)$. We used condition of orthogonality to determine coefficients a_n in the considered series. The coefficients a_n could be calculated for any quantity of terms N. In the common case the relations could be written as

$$-\frac{L_x^2L_y^2L_z^2}{\pi^5}\sum_{n=1}^{N}\frac{a_{nC}}{n^6}e_{nC}(t)=-\frac{L_yL_z}{2\pi^2}\int_0^t\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}D_L(x,y,z,T)\left\{1+\left[\sum_{n=1}^{N}a_{nC}c_n(x)c_n(y)c_n(z)e_{nC}(\tau)\right]^\gamma\times\right.$$

$$\times\frac{\xi}{P^\gamma(x,y,z,T)}\left.\right\}\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\sum_{n=1}^{N}\frac{a_{nC}}{n}s_n(2x)c_n(y)c_n(z)e_{nC}(\tau)\times$$

$$\times\left\{y\,s_n(y)+\frac{L_y}{\pi n}[c_n(y)-1]\right\}\left\{z\,s_n(z)+\frac{L_z}{\pi n}[c_n(z)-1]\right\}dz\,dy\,dx\,d\tau-\int_0^t\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}D_L(x,y,z,T)\times$$

$$\times D_L(x,y,z,T)\left\{1+\left[\sum_{n=1}^{N}a_{nC}c_n(x)c_n(y)c_n(z)e_{nC}(\tau)\right]^\gamma\frac{\xi}{P^\gamma(x,y,z,T)}\right\}\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\right.$$

$$+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\left.\right]\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\sum_{n=1}^{N}\left\{x\,s_n(x)+\frac{L_x}{\pi n}[c_n(x)-1]\right\}\frac{a_{nC}}{n}\times$$

$$\times\frac{L_yL_z}{2\pi^2}c_n(x)s_n(2y)c_n(z)e_{nC}(\tau)\left\{z\,s_n(z)+\frac{L_z}{\pi n}[c_n(z)-1]\right\}dz\,dy\,dx\,d\tau-\frac{L_xL_y}{2\pi^2}\times$$

$$\times\int_0^t\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\left\{1+\left[\sum_{n=1}^{N}a_{nC}c_n(x)c_n(y)c_n(z)e_{nC}(\tau)\right]^\gamma\frac{\xi}{P^\gamma(x,y,z,T)}\right\}\left[1+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}+\right.$$

$$+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}\left.\right]D_L(x,y,z,T)\sum_{n=1}^{N}\frac{a_{nC}}{n}c_n(x)c_n(y)s_n(z)\left\{x\,s_n(x)+\frac{L_x}{\pi n}[c_n(x)-1]\right\}\times$$

$$\times\left\{y\,s_n(y)+\frac{L_y}{\pi n}[c_n(y)-1]\right\}e_{nC}(\tau)dz\,dy\,dx\,d\tau+\sum_{n=1}^{N}\int_0^{L_x}\left\{x\,s_n(x)+\frac{L_x}{\pi n}[c_n(x)-1]\right\}\times$$

$$\times\int_0^{L_y}\left\{y\,s_n(y)+\frac{L_y}{\pi n}[c_n(y)-1]\right\}\int_0^{L_z}\left\{z\,s_n(z)+\frac{L_z}{\pi n}[c_n(z)-1]\right\}f(x,y,z)dz\,dy\,dx.$$

As an example for $\gamma=0$ we obtain

$$a_{nC}=\int_0^{L_x}\int_0^{L_y}\left\{y\,s_n(y)+\frac{L_y}{\pi n}[c_n(y)-1]\right\}\int_0^{L_z}\left\{z\,s_n(z)+\frac{L_y}{\pi n}[c_n(z)-1]\right\}f(x,y,z)dz\,dy\{x\,s_n(x)+$$

$$\times [c_n(x)-1]\frac{L_x}{\pi n}\Big\}d\,x\left(\frac{n}{2}\left\{\int_0^t\int_0^{L_x}s_n(2x)\int_0^{L_y}c_n(y)\left\{y\,s_n(y)+\frac{L_y}{\pi n}[c_n(y)-1]\right\}\int_0^{L_z}D_L(x,y,z,T)\times\right.\right.$$

$$\times\left\{z\,s_n(z)+\frac{L_y}{\pi n}[c_n(z)-1]\right\}\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\left[1+\frac{\xi}{P^\gamma(x,y,z,T)}\right]\times$$

$$\times c_n(z)d\,z\,d\,y\,d\,x\,e_{nC}(\tau)d\,\tau+\int_0^t e_{nC}(\tau)\int_0^{L_x}c_n(x)\left\{x\,s_n(x)+\frac{L_y}{\pi n}[c_n(x)-1]\right\}\int_0^{L_y}s_n(2y)\int_0^{L_z}c_n(z)\times$$

$$\times\left\{z\,s_n(z)+\frac{L_y}{\pi n}[c_n(z)-1]\right\}\left[1+\frac{\xi}{P^\gamma(x,y,z,T)}\right]\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\times$$

$$\times D_L(x,y,z,T)\,d\,z\,d\,y\,d\,x\,d\,\tau+\int_0^t e_{nC}(\tau)\int_0^{L_x}c_n(x)\left\{x\,s_n(x)+\frac{L_x}{\pi n}[c_n(x)-1]\right\}\int_0^{L_y}c_n(y)\left\{s_n(y)\times\right.$$

$$\times y+\frac{L_y}{\pi n}[c_n(y)-1]\right\}\int_0^{L_z}s_n(2z)\,D_L(x,y,z,T)\left[1+\frac{\xi}{P^\gamma(x,y,z,T)}\right]\left[1+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}+\right.$$

$$\left.\left.+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}\right]d\,z\,d\,y\,d\,x\,d\,\tau\right\}-\frac{L_z^2 L_z^2 L_z^2}{\pi^5 n^6}e_{nC}(t)\right)^{-1}.$$

For $\gamma=1$ one can obtain the following relation to determine required parameters

$$a_{nC}=-\frac{\beta_n}{2\alpha_n}\pm\sqrt{\beta_n^2+4\alpha_n\int_0^{L_x}c_n(x)\int_0^{L_y}c_n(y)\int_0^{L_z}c_n(z)f(x,y,z)d\,z\,d\,y\,d\,x}\,,$$

where $\alpha_n=\dfrac{\xi L_y L_z}{2\pi^2 n}\int_0^t e_{nC}(\tau)\int_0^{L_x}s_n(2x)\int_0^{L_y}c_n(y)\int_0^{L_z}c_n(z)\left[1+\varsigma_1\dfrac{V(x,y,z,\tau)}{V^*}+\varsigma_2\dfrac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\times$

$$\times\frac{D_L(x,y,z,T)}{P(x,y,z,T)}\left\{y\,s_n(y)+\frac{L_y}{\pi n}[c_n(y)-1]\right\}\left\{z\,s_n(z)+\frac{L_z}{\pi n}[c_n(z)-1]\right\}d\,z\,d\,y\,d\,x\,d\,\tau+\frac{\xi L_x L_z}{2\pi^2 n}\times$$

$$\times\int_0^t e_{nC}(\tau)\int_0^{L_x}c_n(x)\left\{x\,s_n(x)+\frac{L_x}{\pi n}[c_n(x)-1]\right\}\int_0^{L_y}\int_0^{L_z}c_n(z)\frac{D_L(x,y,z,T)}{P(x,y,z,T)}\left\{z\,s_n(z)-\frac{L_z}{\pi n}[c_n(z)-1]\right\}\times$$

$$\times\frac{D_L(x,y,z,T)}{P(x,y,z,T)}\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]d\,z\,s_n(2y)d\,y\,d\,x\,d\,\tau+\frac{\xi L_x L_y}{2\pi^2 n}\times$$

$$\times\int_0^t e_{nC}(\tau)\int_0^{L_x}c_n(x)\int_0^{L_y}c_n(y)\int_0^{L_z}s_n(2z)\frac{D_L(x,y,z,T)}{P(x,y,z,T)}\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\times$$

$$\times\left\{x\,s_n(x)+\frac{L_x}{\pi n}[c_n(x)-1]\right\}\left\{y\,s_n(y)+\frac{L_y}{\pi n}[c_n(y)-1]\right\}d\,z\,d\,y\,d\,x\,d\,\tau,$$

$$\beta_n=\frac{L_y L_z}{2n\pi^2}\int_0^t e_{nC}(\tau)\times$$

$$\times \int_0^{L_x} s_n(2x) \int_0^{L_y} c_n(y) \left\{ y s_n(y) + \frac{L_y}{\pi n}[c_n(y)-1] \right\} \int_0^{L_z} c_n(z) \left[1 + \varsigma_1 \frac{V(x,y,z,\tau)}{V^*} + \varsigma_2 \frac{V^2(x,y,z,\tau)}{(V^*)^2} \right] \times$$

$$\times D_L(x,y,z,T) \left\{ z s_n(z) + \frac{L_z}{\pi n}[c_n(z)-1] \right\} dz\,dy\,dx\,d\tau + \frac{L_x L_z}{2n\pi^2} \int_0^t e_{nC}(\tau) \int_0^{L_x} c_n(x) \int_0^{L_y} s_n(2y) \times$$

$$\times \left\{ x s_n(x) + \frac{L_x}{\pi n}[c_n(x)-1] \right\} \int_0^{L_z} D_L(x,y,z,T) c_n(z) \left[1 + \varsigma_1 \frac{V(x,y,z,\tau)}{V^*} + \varsigma_2 \frac{V^2(x,y,z,\tau)}{(V^*)^2} \right] \times$$

$$\times \left\{ z s_n(z) + \frac{L_z}{\pi n}[c_n(z)-1] \right\} dz\,dy\,dx\,d\tau + \frac{L_x L_y}{2n\pi^2} \int_0^t e_{nC}(\tau) \int_0^{L_x} \left\{ x s_n(x) + \frac{L_x}{\pi n}[c_n(x)-1] \right\} \times$$

$$\times c_n(x) \int_0^{L_y} \left\{ y s_n(y) + \frac{L_y}{\pi n}[c_n(y)-1] \right\} \int_0^{L_z} D_L(x,y,z,T) \left[1 + \varsigma_1 \frac{V(x,y,z,\tau)}{V^*} + \varsigma_2 \frac{V^2(x,y,z,\tau)}{(V^*)^2} \right] \times$$

$$\times s_n(2z)\,dz\,c_n(y)\,dy\,dx\,d\tau - L_x^2 L_y^2 L_z^2 e_{nC}(t) \Big/ \pi^5 n^6 .$$

The same approach could be used for calculation parameters a_n for different values of parameter γ. However the relations are bulky and will not be presented in the paper. Advantage of the approach is absent of necessity to join dopant concentration on interfaces of heterostructure.
The same Bubnov-Galerkin approach has been used for solution the Eqs.(4). Previously we transform the differential equations to the following integro- differential form

$$\frac{xyz}{L_x L_y L_z} \int_{L_x}^x \int_{L_y}^y \int_{L_z}^z I(u,v,w,t)\,dw\,dv\,du = \frac{yz}{L_y L_z} \int_0^t \int_{L_y}^y \int_{L_z}^z D_I(x,v,w,T) \frac{\partial I(x,v,w,\tau)}{\partial x} dw\,dv\,d\tau +$$

$$+ \frac{xz}{L_x L_z} \int_0^t \int_{L_x}^x \int_{L_z}^z D_I(u,y,w,T) \frac{\partial I(u,y,w,\tau)}{\partial x} dw\,du\,d\tau - \frac{xyz}{L_x L_y L_z} \int_{L_x}^x \int_{L_y}^y \int_{L_z}^z k_{I,V}(u,v,w,T)\,I(u,v,w,t) \times$$

$$\times V(u,v,w,t)\,dw\,dv\,du + \frac{xy}{L_x L_y} \int_0^t \int_{L_x}^x \int_{L_y}^y \frac{\partial I(u,v,z,\tau)}{\partial z} D_I(u,v,z,T)\,dv\,du\,d\tau - \frac{xyz}{L_x L_y L_z} \times$$

$$\times \int_{L_x}^x \int_{L_y}^y \int_{L_z}^z k_{I,I}(u,v,w,T) I^2(u,v,w,t)\,dw\,dv\,du + \frac{xyz}{L_x L_y L_z} \int_{L_x}^x \int_{L_y}^y \int_{L_z}^z f_I(u,v,w)\,dw\,dv\,du \quad (4a)$$

$$\frac{xyz}{L_x L_y L_z} \int_{L_x}^x \int_{L_y}^y \int_{L_z}^z V(u,v,w,t)\,dw\,dv\,du = \frac{yz}{L_y L_z} \int_0^t \int_{L_y}^y \int_{L_z}^z D_V(x,v,w,T) \frac{\partial V(x,v,w,\tau)}{\partial x} dw\,dv\,d\tau +$$

$$+ \frac{xz}{L_x L_z} \int_0^t \int_{L_x}^x \int_{L_z}^z D_V(u,y,w,T) \frac{\partial V(u,y,w,\tau)}{\partial x} dw\,du\,d\tau + \frac{xy}{L_x L_y} \int_0^t \int_{L_x}^x \int_{L_y}^y \frac{\partial V(u,v,z,\tau)}{\partial z} \times$$

$$\times D_V(u,v,z,T)\,dv\,du\,d\tau - \frac{xyz}{L_x L_y L_z} \int_{L_x}^x \int_{L_y}^y \int_{L_z}^z k_{I,V}(u,v,w,T) I(u,v,w,t) V(u,v,w,t)\,dw\,dv\,du -$$

$$- \frac{xyz}{L_x L_y L_z} \int_{L_x}^x \int_{L_y}^y \int_{L_z}^z k_{V,V}(u,v,w,T) V^2(u,v,w,t)\,dw\,dv\,du + \frac{xyz}{L_x L_y L_z} \int_{L_x}^x \int_{L_y}^y \int_{L_z}^z f_V(u,v,w)\,dw\,dv\,du .$$

We determine spatio-temporal distributions of concentrations of point defects as the same series

$$\rho_0(x,y,z,t) = \sum_{n=1}^{N} a_{n\rho}\, c_n(x)\, c_n(y)\, c_n(z)\, e_{n\rho}(t).$$

Parameters $a_{n\rho}$ should be determined in future. Substitution of the series into Eqs.(4a) leads to the following results

$$\frac{x\,y\,z}{\pi^3}\sum_{n=1}^{N}\frac{a_{nI}}{n^3}s_n(x)s_n(y)s_n(z)e_{nI}(t) = -\frac{y\,z\,\pi}{L_xL_yL_z}\sum_{n=1}^{N}a_{nI}\int_0^t\int_{L_y}^y c_n(y)\int_{L_z}^z c_n(z)D_I(x,v,w,T)\,dw\,dv\times$$

$$\times e_{nI}(\tau)\,d\tau\,s_n(x) - \frac{x\,z\,\pi}{L_xL_yL_z}\sum_{n=1}^{N}a_{nI}s_n(y)\int_0^t e_{nI}(\tau)\int_{L_x}^x c_n(x)\int_{L_z}^z c_n(z)D_I(u,y,w,T)\,dw\,du\,d\tau -$$

$$-\frac{x\,y\,\pi}{L_xL_yL_z}\sum_{n=1}^{N}a_{nI}s_n(z)\int_0^t e_{nI}(\tau)\int_{L_x}^x c_n(x)\int_{L_y}^y c_n(y)D_I(u,v,z,T)\,dv\,du\,d\tau - \int_{L_x}^x\int_{L_y}^y\int_{L_z}^z k_{I,I}(u,v,v,T)\times$$

$$\times\left[\sum_{n=1}^{N}a_{nI}c_n(u)c_n(v)c_n(w)e_{nI}(t)\right]^2 dw\,dv\,du\,\frac{x\,y\,z}{L_xL_yL_z} - \frac{x\,y\,z}{L_xL_yL_z}\int_{L_x}^x\int_{L_y}^y\int_{L_z}^z\sum_{n=1}^{N}a_{nI}c_n(u)c_n(v)c_n(w)\times$$

$$\times e_{nI}(t)\sum_{n=1}^{N}a_{nV}c_n(u)c_n(v)c_n(w)e_{nV}(t)k_{I,V}(u,v,v,T)\,dw\,dv\,du + \int_{L_x}^x\int_{L_y}^y\int_{L_z}^z f_I(u,v,w)\,dw\,dv\,du\times$$

$$\times x\,y\,z/L_xL_yL_z$$

$$\frac{x\,y\,z}{\pi^3}\sum_{n=1}^{N}\frac{a_{nV}}{n^3}s_n(x)s_n(y)s_n(z)e_{nV}(t) = -\frac{y\,z\,\pi}{L_xL_yL_z}\sum_{n=1}^{N}a_{nV}\int_0^t\int_{L_y}^y c_n(y)\int_{L_z}^z c_n(z)D_V(x,v,w,T)\,dw\,dv\times$$

$$\times e_{nV}(\tau)\,d\tau\,s_n(x) - \frac{x\,z\,\pi}{L_xL_yL_z}\sum_{n=1}^{N}a_{nV}s_n(y)\int_0^t e_{nV}(\tau)\int_{L_x}^x c_n(x)\int_{L_z}^z c_n(z)D_V(u,y,w,T)\,dw\,du\,d\tau -$$

$$-\frac{x\,y\,\pi}{L_xL_yL_z}\sum_{n=1}^{N}a_{nV}s_n(z)\int_0^t e_{nV}(\tau)\int_{L_x}^x c_n(x)\int_{L_y}^y c_n(y)D_V(u,v,z,T)\,dv\,du\,d\tau - \int_{L_x}^x\int_{L_y}^y\int_{L_z}^z k_{V,V}(u,v,v,T)\times$$

$$\times\left[\sum_{n=1}^{N}a_{nV}c_n(u)c_n(v)c_n(w)e_{nI}(t)\right]^2 dw\,dv\,du\,\frac{x\,y\,z}{L_xL_yL_z} - \frac{x\,y\,z}{L_xL_yL_z}\int_{L_x}^x\int_{L_y}^y\int_{L_z}^z\sum_{n=1}^{N}a_{nI}c_n(u)c_n(v)c_n(w)\times$$

$$\times e_{nI}(t)\sum_{n=1}^{N}a_{nV}c_n(u)c_n(v)c_n(w)e_{nV}(t)k_{I,V}(u,v,v,T)\,dw\,dv\,du + \int_{L_x}^x\int_{L_y}^y\int_{L_z}^z f_V(u,v,w)\,dw\,dv\,du\times$$

$$\times x\,y\,z/L_xL_yL_z\,.$$

We used orthogonality condition of functions of the considered series framework the heterostructure to calculate coefficients $a_{n\rho}$. The coefficients a_n could be calculated for any quantity of terms N. In the common case equations for the required coefficients could be written as

$$-\frac{L_x^2L_y^2L_z^2}{\pi^5}\sum_{n=1}^{N}\frac{a_{nI}}{n^6}e_{nI}(t) = -\frac{1}{2\pi L_x}\sum_{n=1}^{N}\frac{a_{nI}}{n^2}\int_0^t\int_0^{L_x}[1-c_n(2x)]\int_0^{L_y}\left\{L_y+y\,s_n(2y)+\frac{L_y}{2\pi n}[c_n(2y)-1]\right\}\times$$

$$\times\int_0^{L_z}D_I(x,y,z,T)\left\{z\,s_n(z)+\frac{L_z}{2\pi n}[c_n(z)-1]\right\}dz\,dy\,dx\,e_{nI}(\tau)\,d\tau - \frac{1}{2\pi L_y}\sum_{n=1}^{N}\frac{a_{nI}}{n^2}\int_0^t\int_0^{L_x}\left\{x\,s_n(2x)+\right.$$

$$\left.+L_x+\frac{L_x}{\pi n}[c_n(2x)-1]\right\}\int_0^{L_y}\int_0^{L_z}D_I(x,y,z,T)\left\{L_z+z\,s_n(2z)+\frac{L_z}{2\pi n}[c_n(2z)-1]\right\}dz[1-c_n(2y)]\times$$

$$\times d\,y\,d\,x\,e_{nI}(\tau)\,d\tau\int_0^{L_z}D_I(x,y,z,T)\left\{L_z+z\,s_n(2z)+\frac{L_z}{2\pi n}[c_n(2z)-1]\right\}dz\,d\,y\,d\,x\,e_{nI}(\tau)\,d\tau-$$

$$-\frac{1}{2\pi L_z}\sum_{n=1}^{N}\frac{a_{nI}}{n^2}\int_0^t\int_0^{L_x}\left\{L_x+xs_n(2x)+\frac{L_x}{2\pi n}[c_n(2x)-1]\right\}\int_0^{L_y}\left\{L_y+ys_n(2y)+\frac{L_y}{2\pi n}[c_n(2y)-1]\right\}\times$$

$$\times\int_0^{L_z}[1-c_n(2z)]D_I(x,y,z,T)\,dz\,dy\,dx\,e_{nI}(\tau)\,d\tau-\sum_{n=1}^{N}a_{nI}^2e_{nI}(2t)\int_0^{L_x}\left\{L_x+\frac{L_x}{2\pi n}[c_n(2x)-1]+\right.$$

$$\left.+xs_n(2x)\right\}\int_0^{L_y}\left\{L_y+ys_n(2y)+\frac{L_y}{2\pi n}[c_n(2y)-1]\right\}\int_0^{L_z}k_{I,I}(x,y,z,T)\left\{L_z+\frac{L_z}{2\pi n}[c_n(2z)-1]+\right.$$

$$\left.+zs_n(2z)\right\}dz\,dy\,dx-\sum_{n=1}^{N}a_{nI}a_{nV}e_{nI}(t)e_{nV}(t)\int_0^{L_x}\left\{L_x+xs_n(2x)+\frac{L_x}{2\pi n}[c_n(2x)-1]\right\}\int_0^{L_y}\left\{L_y+\right.$$

$$\left.+ys_n(2y)+\frac{L_y}{2\pi n}[c_n(2y)-1]\right\}\int_0^{L_z}k_{I,V}(x,y,z,T)\left\{L_z+zs_n(2z)+\frac{L_z}{2\pi n}[c_n(2z)-1]\right\}dz\times$$

$$\times dy\,dx+\sum_{n=1}^{N}\int_0^{L_x}\left\{xs_n(x)+\frac{L_x}{\pi n}[c_n(x)-1]\right\}\int_0^{L_y}\left\{ys_n(y)+\frac{L_y}{\pi n}[c_n(y)-1]\right\}\int_0^{L_z}f_I(x,y,z,T)\times$$

$$\times\left\{L_z+zs_n(2z)+\frac{L_z}{2\pi n}[c_n(2z)-1]\right\}dz\,dy\,dx$$

$$-\frac{L_x^2L_y^2L_z^2}{\pi^5}\sum_{n=1}^{N}\frac{a_{nV}}{n^6}e_{nV}(t)=-\frac{1}{2\pi L_x}\sum_{n=1}^{N}\frac{a_{nV}}{n^2}\int_0^t\int_0^{L_x}[1-c_n(2x)]\int_0^{L_y}\left\{L_y+ys_n(2y)+\frac{L_y}{2\pi n}[c_n(2y)-1]\right\}\times$$

$$\times\int_0^{L_z}D_V(x,y,z,T)\left\{zs_n(z)+\frac{L_z}{2\pi n}[c_n(z)-1]\right\}dz\,dy\,dx\,e_{nV}(\tau)\,d\tau-\frac{1}{2\pi L_y}\sum_{n=1}^{N}\frac{a_{nV}}{n^2}\int_0^t\int_0^{L_x}\left\{xs_n(2x)+\right.$$

$$\left.+L_x+\frac{L_x}{\pi n}[c_n(2x)-1]\right\}\int_0^{L_y}\int_0^{L_z}D_V(x,y,z,T)\left\{L_z+zs_n(2z)+\frac{L_z}{2\pi n}[c_n(2z)-1]\right\}dz[1-c_n(2y)]\times$$

$$\times dy\,dx\,e_{nV}(\tau)\,d\tau\int_0^{L_z}D_V(x,y,z,T)\left\{L_z+zs_n(2z)+\frac{L_z}{2\pi n}[c_n(2z)-1]\right\}dz\,dy\,dx\,e_{nV}(\tau)\,d\tau-$$

$$-\frac{1}{2\pi L_z}\sum_{n=1}^{N}\frac{a_{nV}}{n^2}\int_0^t\int_0^{L_x}\left\{L_x+xs_n(2x)+\frac{L_x}{2\pi n}[c_n(2x)-1]\right\}\int_0^{L_y}\left\{L_y+ys_n(2y)+\frac{L_y}{2\pi n}[c_n(2y)-1]\right\}\times$$

$$\times\int_0^{L_z}[1-c_n(2z)]D_V(x,y,z,T)\,dz\,dy\,dx\,e_{nV}(\tau)\,d\tau-\sum_{n=1}^{N}a_{nV}^2e_{nV}(2t)\int_0^{L_x}\left\{L_x+\frac{L_x}{2\pi n}[c_n(2x)-1]+\right.$$

$$\left.+xs_n(2x)\right\}\int_0^{L_y}\left\{L_y+ys_n(2y)+\frac{L_y}{2\pi n}[c_n(2y)-1]\right\}\int_0^{L_z}k_{V,V}(x,y,z,T)\left\{L_z+\frac{L_z}{2\pi n}[c_n(2z)-1]+\right.$$

$$\left.+zs_n(2z)\right\}dz\,dy\,dx-\sum_{n=1}^{N}a_{nI}a_{nV}e_{nI}(t)e_{nV}(t)\int_0^{L_x}\left\{L_x+xs_n(2x)+\frac{L_x}{2\pi n}[c_n(2x)-1]\right\}\int_0^{L_y}\left\{L_y+\right.$$

$$\left.+ys_n(2y)+\frac{L_y}{2\pi n}[c_n(2y)-1]\right\}\int_0^{L_z}k_{I,V}(x,y,z,T)\left\{L_z+zs_n(2z)+\frac{L_z}{2\pi n}[c_n(2z)-1]\right\}dz\times$$

$$\times dy\,dx+\sum_{n=1}^{N}\int_0^{L_x}\left\{xs_n(x)+\frac{L_x}{\pi n}[c_n(x)-1]\right\}\int_0^{L_y}\left\{ys_n(y)+\frac{L_y}{\pi n}[c_n(y)-1]\right\}\int_0^{L_z}f_V(x,y,z,T)\times$$

$$\times \left\{ L_z + z\, s_n(2z) + \frac{L_z}{2\pi n}[c_n(2z)-1] \right\} d\,z\,d\,y\,d\,x.$$

In the final form relations for required parameters could be written as

$$a_{nI} = -\frac{b_3+A}{4b_4} \pm \sqrt{\frac{(b_3+A)^2}{4} - 4b_4\left(y + \frac{b_3 y - \gamma_{nV}\lambda_{nI}^2}{A}\right)}, \quad a_{nV} = -\frac{\gamma_{nI}a_{nI}^2 + \delta_{nI}a_{nI} + \lambda_{nI}}{\chi_{nI}a_{nI}},$$

where $\gamma_{n\rho} = e_{n\rho}(2t)\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z} k_{\rho,\rho}(x,y,z,T)\left\{ L_x + x\,s_n(2x) + \frac{L_x}{2\pi n}[c_n(2x)-1] \right\}\left\{ y\,s_n(2y) + L_y + \right.$

$\left. + \frac{L_y}{2\pi n}[c_n(2y)-1] \right\}\left\{ L_z + z\,s_n(2z) + \frac{L_z}{2\pi n}[c_n(2z)-1] \right\} d\,z\,d\,y\,d\,x, \quad \delta_{n\rho} = \frac{1}{2\pi L_x n^2}\int\limits_0^t e_{n\rho}(\tau)\times$

$\times \int\limits_0^{L_x}\int\limits_0^{L_y}\left\{ y\,s_n(y) + \frac{L_y}{2\pi n}[c_n(y)-1] \right\}\int\limits_0^{L_z}\left\{ z\,s_n(z) + \frac{L_z}{2\pi n}[c_n(z)-1] \right\} D_\rho(x,y,z,T)\,d\,z\,d\,y\,[1-$

$-c_n(2x)]\,d\,x\,d\,\tau + \frac{1}{2\pi L_y n^2}\int\limits_0^t e_{n\rho}(\tau)\int\limits_0^{L_x}\left\{ L_x + x\,s_n(2x) + \frac{L_x}{\pi n}[c_n(2x)-1] \right\}\int\limits_0^{L_y}[1-c_n(2y)]\int\limits_0^{L_z}\{ L_z +$

$+ z\,s_n(2z) + \frac{L_z}{2\pi n}[c_n(2z)-1] \} D_\rho(x,y,z,T) d\,z\,d\,y\,d\,x\,d\,\tau + \frac{1}{2\pi L_z n^2}\int\limits_0^t e_{n\rho}(\tau)\int\limits_0^{L_x}\{ x\,s_n(2x) +$

$+ L_x + \frac{L_x}{\pi n}[c_n(2x)-1] \}\int\limits_0^{L_y}\left\{ L_y + y\,s_n(y) + \frac{L_y}{2\pi n}[c_n(y)-1] \right\}\int\limits_0^{L_z}[1-c_n(2z)]D_\rho(x,y,z,T)\,d\,z\,\times$

$\times d\,y\,d\,x\,d\,\tau - \frac{L_x^2 L_y^2 L_z^2}{\pi^5 n^6}e_{n\rho}(t), \quad \chi_{nIV} = \int\limits_0^{L_x}\left\{ x\,s_n(x) + \frac{L_x}{\pi n}[c_n(x)-1] \right\}\int\limits_0^{L_y}\left\{ L_y + \frac{L_y}{2\pi n}[c_n(2y)-1] + \right.$

$\left. + y\,s_n(2y) \right\}\int\limits_0^{L_z} k_{I,V}(x,y,z,T)\left\{ L_z + z\,s_n(2z) + \frac{L_z}{2\pi n}[c_n(2z)-1] \right\} d\,z\,d\,y\,d\,x\,e_{nI}(t)\,e_{nV}(t),$

$\lambda_{n\rho} = \int\limits_0^{L_x}\left\{ x\,s_n(x) + \frac{L_x}{\pi n}[c_n(x)-1] \right\}\int\limits_0^{L_y}\left\{ y\,s_n(y) + \frac{L_y}{\pi n}[c_n(y)-1] \right\}\int\limits_0^{L_z}\left\{ z\,s_n(z) + \frac{L_z}{\pi n}[c_n(z)-1] \right\}\times$

$\times f_\rho(x,y,z,T)d\,z\,d\,y\,d\,x, \quad b_4 = \gamma_{nV}\gamma_{nI}^2 - \gamma_{nI}\chi_{nI}^2, \quad b_3 = 2\gamma_{nV}\gamma_{nI}\delta_{nI} - \delta_{nI}\chi_{nI}^2 - \delta_{nV}\chi_{nI}\gamma_{nI},$

$A = \sqrt{8y + b_3^2 - 4b_2}, \quad b_2 = \gamma_{nV}\delta_{nI}^2 + 2\lambda_{nI}\gamma_{nV}\gamma_{nI} - \delta_{nV}\chi_{nI}\delta_{nI} + (\lambda_{nV} - \lambda_{nI})\chi_{nI}^2, \quad b_1 = 2\lambda_{nI}\times$

$\times \gamma_{nV}\delta_{nI} - \delta_{nV}\chi_{nI}\lambda_{nI}, \quad y = \sqrt[3]{\sqrt{q^2 + p^3} - q} - \sqrt[3]{\sqrt{q^2 + p^3} + q} - \frac{b_3}{3b_4}, \quad p = \frac{3b_2 b_4 - b_3^2}{9b_4^2},$

$q = (2b_3^3 - 9b_2 b_3 + 27b_1 b_4^2)/54b_4^3.$

We determine distributions of concentrations of simplest complexes of radiation defects in space and time as the following functional series

$$\Phi_{\rho 0}(x,y,z,t) = \sum_{n=1}^N a_{n\Phi\rho}\,c_n(x)\,c_n(y)\,c_n(z)\,e_{n\rho}(t).$$

Here $a_{n\Phi\rho}$ are the coefficients, which should be determined. Let us previously transform the Eqs. (6) to the following integro-differential form

$$\frac{xyz}{L_xL_yL_z}\int_{L_x}^{x}\int_{L_y}^{y}\int_{L_z}^{z}\Phi_I(u,v,w,t)\,dw\,dv\,du=\int_{0}^{t}\int_{L_y}^{y}\int_{L_z}^{z}D_{\Phi I}(x,v,w,T)\frac{\partial\Phi_I(x,v,w,\tau)}{\partial x}\,dw\,dv\,d\tau\times$$

$$\times\frac{yz}{L_yL_z}+\frac{xz}{L_xL_z}\int_{0}^{t}\int_{L_x}^{x}\int_{L_z}^{z}D_{\Phi I}(u,y,w,T)\frac{\partial\Phi_I(u,y,w,\tau)}{\partial y}\,dw\,du\,d\tau+\frac{xy}{L_xL_y}\int_{0}^{t}\int_{L_x}^{x}\int_{L_y}^{y}D_{\Phi I}(u,v,z,T)\times$$

$$\times\frac{\partial\Phi_I(u,v,z,\tau)}{\partial z}\,dv\,du\,d\tau+\frac{xyz}{L_xL_yL_z}\int_{L_x}^{x}\int_{L_y}^{y}\int_{L_z}^{z}k_{I,I}(u,v,w,T)I^2(u,v,w,\tau)\,dw\,dv\,du-\ (6a)$$

$$-\frac{xyz}{L_xL_yL_z}\int_{L_x}^{x}\int_{L_y}^{y}\int_{L_z}^{z}k_I(u,v,w,T)\,I(u,v,w,\tau)\,dw\,dv\,du+\frac{xyz}{L_xL_yL_z}\int_{L_x}^{x}\int_{L_y}^{y}\int_{L_z}^{z}f_{\Phi I}(u,v,w)\,dw\,dv\,du$$

$$\frac{xyz}{L_xL_yL_z}\int_{L_x}^{x}\int_{L_y}^{y}\int_{L_z}^{z}\Phi_V(u,v,w,t)\,dw\,dv\,du=\int_{0}^{t}\int_{L_y}^{y}\int_{L_z}^{z}D_{\Phi V}(x,v,w,T)\frac{\partial\Phi_V(x,v,w,\tau)}{\partial x}\,dw\,dv\,d\tau\times$$

$$\times\frac{yz}{L_yL_z}+\frac{xz}{L_xL_z}\int_{0}^{t}\int_{L_x}^{x}\int_{L_z}^{z}D_{\Phi V}(u,y,w,T)\frac{\partial\Phi_V(u,y,w,\tau)}{\partial y}\,dw\,du\,d\tau+\frac{xy}{L_xL_y}\int_{0}^{t}\int_{L_x}^{x}\int_{L_y}^{y}D_{\Phi V}(u,v,z,T)\times$$

$$\times\frac{\partial\Phi_V(u,v,z,\tau)}{\partial z}\,dv\,du\,d\tau+\frac{xyz}{L_xL_yL_z}\int_{L_x}^{x}\int_{L_y}^{y}\int_{L_z}^{z}k_{V,V}(u,v,w,T)V^2(u,v,w,\tau)\,dw\,dv\,du-$$

$$-\frac{xyz}{L_xL_yL_z}\int_{L_x}^{x}\int_{L_y}^{y}\int_{L_z}^{z}k_V(u,v,w,T)\,V(u,v,w,\tau)\,dw\,dv\,du+\frac{xyz}{L_xL_yL_z}\int_{L_x}^{x}\int_{L_y}^{y}\int_{L_z}^{z}f_{\Phi V}(u,v,w)\,dw\,dv\,du\,.$$

Substitution of the previously considered series in the Eqs.(6a) leads to the following form

$$-xyz\sum_{n=1}^{N}\frac{a_{n\Phi I}}{\pi^3n^3}s_n(x)\,s_n(y)\,s_n(z)\,e_{nI}(t)=-\frac{yz\pi}{L_xL_yL_z}\sum_{n=1}^{N}n\,a_{n\Phi I}s_n(x)e_{nI}(t)\int_{0}^{t}\int_{L_y}^{y}\int_{L_z}^{z}c_n(v)c_n(w)\times$$

$$\times D_{\Phi I}(x,v,w,T)\,dw\,dv\,d\tau-\frac{xz\pi}{L_xL_yL_z}\sum_{n=1}^{N}a_{n\Phi I}\int_{0}^{t}\int_{L_x}^{x}\int_{L_z}^{z}c_n(u)c_n(w)\,D_{\Phi I}(u,v,w,T)\,dw\,du\,d\tau\times$$

$$\times n\,s_n(y)e_{n\Phi I}(t)-\frac{xy\pi}{L_xL_yL_z}\sum_{n=1}^{N}n\,a_{n\Phi I}s_n(z)e_{n\Phi I}(t)\int_{0}^{t}\int_{L_x}^{x}\int_{L_y}^{y}c_n(u)c_n(v)\,D_{\Phi I}(u,v,z,T)\,dv\,du\,d\tau+$$

$$+\frac{xyz}{L_xL_yL_z}\int_{L_x}^{x}\int_{L_y}^{y}\int_{L_z}^{z}k_{I,I}(u,v,w,T)I^2(u,v,w,\tau)\,dw\,dv\,du+\int_{L_x}^{x}\int_{L_y}^{y}\int_{L_z}^{z}f_{\Phi I}(u,v,w)\,dw\,dv\,du\times$$

$$\times\frac{xyz}{L_xL_yL_z}-\frac{xyz}{L_xL_yL_z}\int_{L_x}^{x}\int_{L_y}^{y}\int_{L_z}^{z}k_I(u,v,w,T)\,I(u,v,w,\tau)\,dw\,dv\,du$$

$$-xyz\sum_{n=1}^{N}\frac{a_{n\Phi V}}{\pi^3n^3}s_n(x)\,s_n(y)\,s_n(z)\,e_{nV}(t)=-\frac{yz\pi}{L_xL_yL_z}\sum_{n=1}^{N}n\,a_{n\Phi V}s_n(x)e_{nV}(t)\int_{0}^{t}\int_{L_y}^{y}\int_{L_z}^{z}c_n(v)c_n(w)\times$$

$$\times D_{\Phi V}(x,v,w,T)\,dw\,dv\,d\tau-\frac{xz\pi}{L_xL_yL_z}\sum_{n=1}^{N}n\int_{0}^{t}\int_{L_x}^{x}\int_{L_z}^{z}c_n(u)c_n(w)\,D_{\Phi V}(u,v,w,T)\,dw\,du\,d\tau\times$$

$$\times a_{n\Phi V}s_n(y)e_{n\Phi V}(t)-\frac{xy\pi}{L_xL_yL_z}\sum_{n=1}^{N}n\,s_n(z)e_{n\Phi V}(t)\int_{0}^{t}\int_{L_x}^{x}\int_{L_y}^{y}c_n(u)c_n(v)\,D_{\Phi V}(u,v,z,T)\,dv\,du\,d\tau\times$$

$$\times\, a_{n\Phi V} + \frac{x\,y\,z}{L_x L_y L_z} \int\limits_{L_x L_y L_z}^{x\ y\ z} k_{V,V}(u,v,w,T)V^2(u,v,w,\tau)\,dw\,dv\,du + \int\limits_{L_x L_y L_z}^{x\ y\ z} f_{\Phi V}(u,v,w)\,dw\,dv\,du\,\times$$

$$\times\,\frac{x\,y\,z}{L_x L_y L_z} - \frac{x\,y\,z}{L_x L_y L_z}\int\limits_{L_x L_y L_z}^{x\ y\ z} k_V(u,v,w,T)\,V(u,v,w,\tau)\,dw\,dv\,du\,.$$

We used orthogonality condition of functions of the considered series framework the heterostructure to calculate coefficients $a_{n\Phi\rho}$. The coefficients $a_{n\Phi\rho}$ could be calculated for any quantity of terms N. In the common case equations for the required coefficients could be written as

$$-\frac{L_x^2 L_y^2 L_z^2}{\pi^5}\sum_{n=1}^{N}\frac{a_{n\Phi I}}{n^6}e_{n\Phi I}(t) = -\frac{1}{2\pi L_x}\sum_{n=1}^{N}\int\limits_0^t\int\limits_0^{L_x}[1-c_n(2x)]\int\limits_0^{L_y}\left\{L_y+y\,s_n(2y)+\frac{L_y}{2\pi n}[c_n(2y)-1]\right\}\times$$

$$\times\frac{a_{n\Phi I}}{n^2}\int\limits_0^{L_z}D_{\Phi I}(x,y,z,T)\left\{z\,s_n(z)+\frac{L_z}{2\pi n}[c_n(z)-1]\right\}dz\,dy\,dx\,e_{n\Phi I}(\tau)\,d\tau - \frac{1}{2\pi}\sum_{n=1}^{N}\int\limits_0^t\int\limits_0^{L_x}\{x\,s_n(2x)+$$

$$+L_x+\frac{L_x}{2\pi n}[c_n(2x)-1]\}\int\limits_0^{L_y}[1-c_n(2y)]\int\limits_0^{L_z}D_{\Phi I}(x,y,z,T)\left\{z\,s_n(z)+\frac{L_z}{2\pi n}[c_n(z)-1]\right\}dz\,dy\,dx\,\times$$

$$\times a_{n}\frac{e_{n\Phi I}(\tau)}{n^2 L_y}d\tau - \frac{1}{2\pi L_x}\sum_{n=1}^{N}\frac{a_{n\Phi I}}{n^2}\int\limits_0^t\int\limits_0^{L_x}\left\{x\,s_n(x)+\frac{L_x}{2\pi n}[c_n(x)-1]\right\}\int\limits_0^{L_y}\left\{y\,s_n(2y)+\frac{L_y}{2\pi n}[c_n(2y)-1]+\right.$$

$$+L_y\}\int\limits_0^{L_z}[1-c_n(2y)]D_{\Phi I}(x,y,z,T)\,dz\,dy\,dx\,e_{n\Phi I}(\tau)d\tau + \frac{1}{\pi^3}\sum_{n=1}^{N}\frac{a_{n\Phi I}}{n^3}\int\limits_0^t e_{n\Phi I}(\tau)\int\limits_0^{L_x}\left\{\frac{L_x}{2\pi n}[c_n(x)-1]+\right.$$

$$+x\,s_n(x)\}\int\limits_0^{L_y}\left\{y\,s_n(y)+\frac{L_y}{2\pi n}[c_n(y)-1]\right\}\int\limits_0^{L_z} I^2(x,y,z,t)k_{I,I}(x,y,z,T)\left\{\frac{L_z}{2\pi n}[c_n(z)-1]+\right.$$

$$+z\,s_n(z)\}\,dz\,dy\,dx - \frac{1}{\pi^3}\sum_{n=1}^{N}\frac{a_{n\Phi I}}{n^3}\int\limits_0^t e_{n\Phi I}(\tau)\int\limits_0^{L_x}\left\{x\,s_n(x)+\frac{L_x}{2\pi n}[c_n(x)-1]\right\}\int\limits_0^{L_y}\left\{\frac{L_y}{2\pi n}[c_n(y)-1]+\right.$$

$$+y\,s_n(y)\}\int\limits_0^{L_z}\left\{z\,s_n(z)+\frac{L_z}{2\pi n}[c_n(z)-1]\right\}k_I(x,y,z,T)I(x,y,z,t)\,dz\,dy\,dx + \frac{1}{\pi^3}\sum_{n=1}^{N}\frac{a_{n\Phi I}}{n^3}\times$$

$$\times\int\limits_0^t e_{n\Phi I}(\tau)\int\limits_0^{L_x}\left\{x\,s_n(x)+\frac{L_x}{2\pi n}[c_n(x)-1]\right\}\int\limits_0^{L_y}\left\{y\,s_n(y)+\frac{L_y}{2\pi n}[c_n(y)-1]\right\}\int\limits_0^{L_z}\left\{\frac{L_z}{2\pi n}[c_n(z)-1]+\right.$$

$$+z\,s_n(z)\}f_{\Phi I}(x,y,z)\,dz\,dy\,dx$$

$$-\frac{L_x^2 L_y^2 L_z^2}{\pi^5}\sum_{n=1}^{N}\frac{a_{n\Phi V}}{n^6}e_{n\Phi V}(t) = -\frac{1}{2\pi L_x}\sum_{n=1}^{N}\int\limits_0^t\int\limits_0^{L_x}[1-c_n(2x)]\int\limits_0^{L_y}\left\{L_y+y\,s_n(2y)+\frac{L_y}{2\pi n}[c_n(2y)-1]\right\}\times$$

$$\times\frac{a_{n\Phi V}}{n^2}\int\limits_0^{L_z}D_{\Phi V}(x,y,z,T)\left\{z\,s_n(z)+\frac{L_z}{2\pi n}[c_n(z)-1]\right\}dz\,dy\,dx\,e_{n\Phi V}(\tau)\,d\tau - \frac{1}{2\pi}\sum_{n=1}^{N}\int\limits_0^t\int\limits_0^{L_x}\{x\,s_n(2x)+$$

$$+L_x+\frac{L_x}{2\pi n}[c_n(2x)-1]\}\int\limits_0^{L_y}[1-c_n(2y)]\int\limits_0^{L_z}D_{\Phi V}(x,y,z,T)\left\{z\,s_n(z)+\frac{L_z}{2\pi n}[c_n(z)-1]\right\}dz\,dy\,dx\,\times$$

$$\times a_{n\Phi V}\frac{e_{n\Phi V}(\tau)}{n^2 L_y}d\tau - \frac{1}{2\pi L_x}\sum_{n=1}^{N}\frac{a_{n\Phi V}}{n^2}\int\limits_0^t\int\limits_0^{L_x}\left\{x\,s_n(x)+\frac{L_x}{2\pi n}[c_n(x)-1]\right\}\int\limits_0^{L_y}\left\{y\,s_n(2y)+\frac{L_y}{2\pi n}[c_n(2y)-1]+\right.$$

$$+L_y\bigg\}\int\limits_0^{L_z}[1-c_n(2y)]D_{\Phi V}(x,y,z,T)\,d\,z\,d\,y\,d\,x\,e_{n\Phi V}(\tau)d\,\tau+\frac{1}{\pi^3}\sum_{n=1}^N\frac{a_{n\Phi V}}{n^3}\int\limits_0^t e_{n\Phi V}(\tau)\int\limits_0^{L_x}\bigg\{\frac{L_x}{2\pi n}[c_n(x)-1]+$$

$$+x\,s_n(x)\bigg\}\int\limits_0^{L_y}\bigg\{y\,s_n(y)+\frac{L_y}{2\pi n}[c_n(y)-1]\bigg\}\int\limits_0^{L_z}V^2(x,y,z,t)\,k_{V,V}(x,y,z,T)\bigg\{\frac{L_z}{2\pi n}[c_n(z)-1]+$$

$$+z\,s_n(z)\}d\,z\,d\,y\,d\,x-\frac{1}{\pi^3}\sum_{n=1}^N\frac{a_{n\Phi V}}{n^3}\int\limits_0^t e_{n\Phi V}(\tau)\int\limits_0^{L_x}\bigg\{x\,s_n(x)+\frac{L_x}{2\pi n}[c_n(x)-1]\bigg\}\int\limits_0^{L_y}\bigg\{\frac{L_y}{2\pi n}[c_n(y)-1]+$$

$$+y\,s_n(y)\}\int\limits_0^{L_z}\bigg\{z\,s_n(z)+\frac{L_z}{2\pi n}[c_n(z)-1]\bigg\}k_V(x,y,z,T)V(x,y,z,t)d\,z\,d\,y\,d\,x+\frac{1}{\pi^3}\sum_{n=1}^N\frac{a_{n\Phi V}}{n^3}\times$$

$$\times\int\limits_0^t e_{n\Phi V}(\tau)\int\limits_0^{L_x}\bigg\{x\,s_n(x)+\frac{L_x}{2\pi n}[c_n(x)-1]\bigg\}\int\limits_0^{L_y}\bigg\{y\,s_n(y)+\frac{L_y}{2\pi n}[c_n(y)-1]\bigg\}\int\limits_0^{L_z}\bigg\{\frac{L_z}{2\pi n}[c_n(z)-1]+$$

$$+z\,s_n(z)\}f_{\Phi V}(x,y,z)\,d\,z\,d\,y\,d\,x.$$

3. DISCUSSION

In this section we compare spatial distributions of concentrations of dopant in a heterostructure with overlayer and without the overlayer. Fig. 2 shows result of the above comparison. One can find from the figure, that using the overlayer gives a possibility to manufacture more shallow *p-n-* junction in comparison with *p-n*-junction in the heterostructure without the overlayer. At the same time analysis of redistribution of radiation defects shows, that implantation of ions of dopant through the overlayer gives a possibility to decrease quantity of radiation defects in the epitaxial layer.
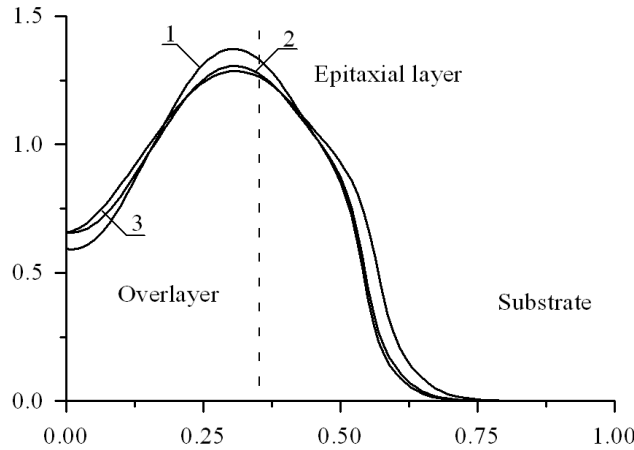

Fig.2. Distributions of concentration of implanted dopant in the considered heterostructure.

Curves 1 and 3 describe distributions of concentration of implanted dopant in the heterostructure with overlayer. Curve 1 describes distribution of concentration of implanted dopant for the case, when dopant diffusion coefficient in the overlayer is larger, than in the epitaxial layer. Curve 2 describes distribution of concentration of implanted dopant for the case, when dopant diffusion coefficient in the overlayer is smaller, than the epitaxial layer. Curve 2 describes distribution of concentration of implanted dopant in the heterostructure without overlayer.

It should be noted, that manufacturing the considered implanted-junction rectifier near interface between layers of heterostructure gives a possibility to increase sharpness of the *p-n*-junction and at the same time to increase homogeneity of concentration of the dopant in the enriched area [15-19]. In this situation it is practically to choose appropriate thickness of epitaxial layer.

4. CONCLUSIONS

In this paper we analyzed distributions of concentration of implanted dopant in heterostructure with overlayer and without overlayer during manufacture an implanted -heterojunction rectifier. We determine conditions, which correspond to decrease depth of the *p-n*-junction.

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