Comme Appelé du Néant—As If Summoned from the Void: The Life of Alexandre Grothendieck

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This is the first part of a two-part article about the life of Alexandre Grothendieck. The second part of the article will appear in the next issue of the *Notices*.

Et toute science, quand nous l'entendons non comme un instrument de pouvoir et de domination, mais comme aventure de connaissance de notre espèce à travers les âges, n'est autre chose que cette harmonie, plus ou moins vaste et plus ou moins riche d'une époque à l'autre, qui se déploie au cours des générations et des siècles, par le délicat contrepoint de tous les thèmes apparus tour à tour, comme appelés du néant.

And every science, when we understand it not as an instrument of power and domination but as an adventure in knowledge pursued by our species across the ages, is nothing but this harmony, more or less vast, more or less rich from one epoch to another, which unfurls over the course of generations and centuries, by the delicate counterpoint of all the themes appearing in turn, as if summoned from the void.

—*Récoltes et Semailles*, page P20

Alexandre Grothendieck is a mathematician of immense sensitivity to things mathematical, of profound perception of the intricate and elegant lines of their architecture. A couple of high points from his biography—he was a founding member of the Institut des Hautes Études Scientifiques (IHÉS) and received the Fields Medal in 1966—suffice to secure his place in the pantheon of twentieth century mathematics. But such details cannot capture the essence of his work, which is rooted in something far more organic and humble. As he wrote in his long memoir, *Récoltes et Semailles* (*Reapings and Sowings*, R&S), "What makes the quality of a researcher's inventiveness and imagination is the *quality of his attention* to hearing the voices of things" (emphasis in the original, page P27). Today Grothendieck's own voice, embodied in his written works, reaches us as if through a void: now seventy-six years old, he has for more than a decade lived in seclusion in a remote hamlet in the south of France.

Grothendieck changed the landscape of mathematics with a viewpoint that is "cosmically general", in the words of Hyman Bass of the University of Michigan. This viewpoint has been so thoroughly absorbed into mathematics that nowadays it is difficult for newcomers to imagine that the field was not always this way. Grothendieck left his deepest mark on algebraic geometry, where he placed emphasis on discovering relationships among mathematical objects as a way of understanding the objects themselves. He had an extremely powerful, almost other-worldly ability of abstraction that allowed him to see problems in a highly general context, and he used this ability with exquisite precision. Indeed, the trend toward increasing generality and abstraction, which can be seen across the whole field since the middle of the twentieth

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century, is due in no small part to Grothendieck's influence. At the same time, generality for its own sake, which can lead to sterile and uninteresting mathematics, is something he never engaged in.

Grothendieck's early life during World War II had a good deal of chaos and trauma, and his educational background was not the best. How he emerged from these deprived beginnings and forged a life for himself as one of the leading mathematicians in the world is a story of high drama—as is his decision in 1970 to abruptly leave the mathematical milieu in which his greatest achievements blossomed and which was so deeply influenced by his extraordinary personality.

Early Life

> Ce qui me satisfaisait le moins, dans nos livres de maths [au lycée], c'était l'absence de toute définition sérieuse de la notion de longueur (d'une courbe), d'aire (d'une surface), de volume (d'un solide). Je me suis promis de combler cette lacune, dès que j'en aurais le loisir.
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> What was least satisfying to me in our [high school] math books was the absence of any serious definition of the notion of length (of a curve), of area (of a surface), of volume (of a solid). I promised myself I would fill this gap when I had the chance.
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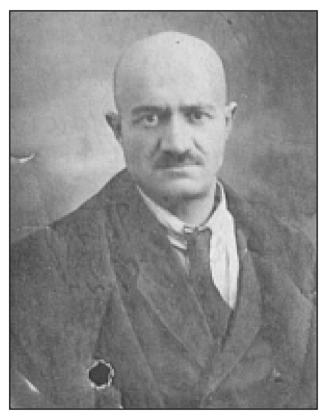
> —*Récoltes et Semailles*, page P3

Armand Borel of the Institute for Advanced Study in Princeton, who died in August 2003 at the age of 80, remembered the first time he met Grothendieck, at a Bourbaki seminar in Paris in November 1949. During a break between lectures, Borel, then in his mid-twenties, was chatting with Charles Ehresmann, who at forty-five years of age was a leading figure in French mathematics. As Borel recalled, a young man strode up to Ehresmann and, without any preamble, demanded, "Are you an expert on topological groups?" Ehresmann, not wanting to seem immodest, replied that yes, he knew something about topological groups. The young man insisted, "But I need a *real* expert!" This was Alexandre Grothendieck, age twenty-one—brash, intense, not exactly impolite but having little sense of social niceties. Borel remembered the question Grothendieck asked: Is every local topological group the germ of a global topological group? As it turned out, Borel knew a counterexample. It was a question that showed Grothendieck was already thinking in very general terms.

Grothendieck's time in Paris in the late 1940s was his first real contact with the world of mathematical research. Up to that time, his life story—at least what is known of it—contains few clues that he was destined to become a dominant figure in that world. Many of the details about Grothendieck's family background and early life are sketchy or unknown. Winfried Scharlau of the Universität Münster is writing a biography of Grothendieck and has studied carefully this part of his life. Much of the information in the following biographical sketch comes from an interview with Scharlau and from biographical materials he has assembled about Grothendieck [Scharlau].

Grothendieck's mother, Hanka, 1917.

Grothendieck's father, whose name may have been Alexander Shapiro, was born into a Jewish family in Novozybkov in Ukraine on October 11, 1889. Shapiro was an anarchist and took part in various uprisings in czarist Russia in the early twentieth century. Arrested at the age of seventeen, he managed to elude a death sentence, but, after escaping and being recaptured a few times, he spent a total of about ten years in prison. Grothendieck's father has sometimes been confused with another more famous activist also named Alexander Shapiro, who participated in some of the same political movements. This other Shapiro, who was portrayed in John Reed's book *Ten Days that Shook the World*, emigrated to New York and died there in 1946, by which time Grothendieck's father had already been dead for four years. Another distinguishing detail is that Grothendieck's father had only one arm. According to Justine Bumby, who lived with Grothendieck for a period in the 1970s and had a son by him, his father lost his arm in a suicide attempt while trying to avoid being captured by the police. Grothendieck himself may unwittingly have contributed to the confusion between the two Shapiros; for example, Pierre Cartier of the Institut des Hautes Études Scientifiques mentioned in [Cartier2] Grothendieck's



Grothendieck's father, Sascha, ca. 1922.

maintaining that one of the figures in Reed's book was his father.

In 1921 Shapiro left Russia and was stateless for the rest of his life. To hide his political past, he obtained identity papers with the name Alexander Tanaroff, and for the rest of his life he lived under this name. He spent time in France, Germany, and Belgium, where he associated with anarchist and other revolutionary groups. In the radical circles of Berlin in the mid-1920s, he met Grothendieck's mother, Johanna (Hanka) Grothendieck. She had been born on August 21, 1900, into a bourgeois family of Lutherans in Hamburg. Rebelling against her traditional upbringing, she was drawn to Berlin, which was then a hotbed of avant-garde culture and revolutionary social movements. Both she and Shapiro yearned to be writers. He never published anything, but she published some newspaper articles; in particular, between 1920 and 1922, she wrote for a leftist weekly newspaper called *Der Pranger*, which had taken up the cause of prostitutes living on the fringe of Hamburg society. Much later, in the late 1940s, she wrote an autobiographical novel called *Eine Frau*, which was never published.


A. Grothendieck as a child.

For most of his life, Tanaroff was a street photographer, an occupation that allowed him to earn an independent living without being in an employer-employee relationship that would have run counter to his anarchist principles. He and Hanka had each been married before, and each had a child from the previous marriage, she a daughter and he a son. Alexandre Grothendieck was born in Berlin on March 28, 1928, into a family consisting of Hanka, Tanaroff, and Hanka's daughter from her first marriage, Maidi, who was four years older than Alexandre. He was known in the family, and to his close friends later on, as Shurik; his father's nickname was Sascha. Although he never met his half-brother, Grothendieck dedicated to him the manuscript *A La Poursuite des Champs* (*Pursuing Stacks*), written in the 1980s.

In 1933, when the Nazis came to power, Shapiro fled Berlin for Paris. In December that year, Hanka decided to follow her husband, so she put her son in the care of a foster family in Blankenese, near Hamburg; Maidi was left in an institution for handicapped children in Berlin, although she was not handicapped (R&S, pages 472–473). The foster family was headed by Wilhelm Heydorn, whose remarkable life is outlined in his biography, *Nur Mensch Sein!* [Heydorn]; the book contains a photograph of Alexandre Grothendieck from 1934, and he is mentioned briefly. Heydorn had been a Lutheran priest and army officer, then left the church and worked as an elementary school teacher and a *Heilpraktiker* (which nowadays might be translated roughly as "practitioner of alternative medicine"). In 1930 he founded an idealistic political party called the "Menschheitspartei" ("Humanity Party"), which was outlawed by the Nazis. Heydorn had four children of his own, and he and his wife Dagmar, following their sense of Christian duty, took in several foster children who were separated from their families in the tumultuous period leading up to World War II.

Grothendieck remained with the Heydorn family for five years, between the ages of five and eleven, and attended school. A memoir by Dagmar Heydorn recalled the young Alexandre as being very free, completely honest, and lacking in inhibitions. During his time with the Heydorns, Grothendieck received only a few letters from his mother and no word at all from his father. Although Hanka still had relatives in Hamburg, no one ever came to visit her son. The sudden separation from his parents was highly traumatic for Grothendieck, as he indicated in *Récoltes et Semailles* (page 473). Scharlau speculated that the young Alexandre was probably not especially happy with the Heydorns. Having started life in a liberal home headed by a couple of anarchists, the stricter atmosphere of the Heydorn household probably chafed. He was actually closer to some other families who lived near the Heydorns, and as an adult he continued to write to them for many years. He also wrote to the Heydorns and visited Hamburg several times, the last time in the mid-1980s.

By 1939, with war imminent, political pressure increased on the Heydorns, and they could no longer keep the foster children. Grothendieck was an especially difficult case, because he looked Jewish. The exact whereabouts of his parents were unknown, but Dagmar Heydorn wrote to the French consulate in Hamburg and managed to get a message to Shapiro in Paris and to Hanka in Nîmes. Once contact with his parents was made, Grothendieck, then 11 years old, was put on a train from Hamburg to Paris. He was reunited with his parents in May 1939, and they spent a brief time together before the war began.

It is not clear exactly what Grothendieck's parents were doing while he was in Hamburg, but they remained politically active. They went to Spain to fight in the Spanish Civil War and were among the many who fled to France when Franco triumphed. Because of their political activities, Hanka and her husband were viewed in France as dangerous foreigners. Some time after Grothendieck joined them there, Shapiro was put into the internment camp Le Vernet, the worst of all the French camps. It is probable that he never again saw his wife and son.

In August 1942 he was deported by the French authorities to Auschwitz, where he was killed. What happened to Maidi at this time is unclear, but eventually she married an American soldier and emigrated to the United States; she passed away a couple of years ago.

In 1940 Hanka and her son were put into an internment camp in Rieucros, near Mende. As internment camps went, the one at Rieucros was one of the better ones, and Grothendieck was permitted to go to the *lycée* (high school) in Mende. Nevertheless, it was a life of deprivation and uncertainty. He told Bumby that he and his mother were sometimes shunned by French people who did not know of Hanka's opposition to the Nazis. Once he ran away from the camp with the intention of assassinating Hitler, but he was quickly caught and returned. "This could easily have cost him his life", Bumby noted. He had always been strong and a good boxer, attributes that were useful at this time, as he was sometimes the target of bullying.

After two years, mother and son were separated; Hanka was sent to another internment camp, and her son ended up in the town of Chambon-sur-Lignon. André Trocmé, a Protestant pastor, had transformed the mountain resort town of Chambon into a stronghold of resistance against the Nazis and a haven for protecting Jews and others endangered during the war [Hallie]. There Grothendieck was taken into a children's home supported by a Swiss organization. He attended the Collège Cévenol, set up in Chambon to provide an education for the young people, and earned a *baccalauréat*. The heroic efforts of the Chambonnais kept the refugees safe, but life was nevertheless precarious. In *Récoltes et Semailles* Grothendieck mentioned the periodic roundups of Jews that would send him and his fellow students scattering to hide in the woods for a few days (page P2).

He also related some of his memories of his schooling in Mende and Chambon. It is clear that, despite the difficulties and dislocation of his youth, he had a strong internal compass from an early age. In his mathematics classes, he did not depend on his teachers to distinguish what was deep from what was inconsequential, what was right from what was wrong. He found the mathematics problems in the texts to be repetitive and presented in isolation from anything that would give them meaning. "These were the book's problems, and not *my* problems," he wrote. When a problem did seize him, he lost himself in it completely, without regard to how much time he spent on it (page P3).

From Montpellier to Paris to Nancy

> Monsieur Soula [mon professeur de calcul] m'assurait…que les derniers problèmes qui s'étaient encore posés en maths avaient été résolus, il y avait vingt ou trente ans, par un dénommé Lebesgue. Il aurait développé justement (drôle de coïncidence, décidement!) une théorie de la mesure et de l'intégration, laquelle mettait un point final à la mathématique.
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> Mr. Soula [my calculus teacher] assured me that the final problems posed in mathematics had been resolved, twenty or thirty years before, by a certain Lebesgue. He had exactly developed (an amusing coincidence, certainly!) a theory of measure and integration, which was the endpoint of mathematics.
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> —*Récoltes et Semailles*, page P4

By the time the war ended in Europe, in May 1945, Alexandre Grothendieck was seventeen years old. He and his mother went to live in Maisargues, a village in a wine-growing region outside of Montpellier. He enrolled at the Université de Montpellier, and the two survived on his student scholarship and by doing seasonal work in the grape harvest; his mother also worked at housecleaning. Over time he attended the university courses less and less, as he found that the teachers were mostly repeating what was in the textbooks. At the time, Montpellier "was among the most backward of French universities in the teaching of mathematics," wrote Jean Dieudonné [D1].

In this uninspiring environment, Grothendieck devoted most of his three years at Montpellier to filling the gap that he had felt in his high school textbooks about how to provide a satisfactory definition of length, area, and volume. On his own, he essentially rediscovered measure theory and the notion of the Lebesgue integral. This episode is one of several parallels between the life of Grothendieck and that of Albert Einstein; as a young man Einstein developed on his own ideas in statistical physics that he later found out had already been discovered by Josiah Willard Gibbs.

In 1948, having finished his Licenceès Sciences at Montpellier, Grothendieck went to Paris, the main center for mathematics in France. In an article about Grothendieck that appeared in a French magazine in 1995 [Ikonicoff], a French education official, André Magnier, recalled Grothendieck's application for a scholarship to go to Paris. Magnier asked him to describe the project he had been working on at Montpellier. "I was astounded," the article quoted Magnier as saying. "Instead of a meeting of twenty minutes, he went on for two hours explaining to me how he had reconstructed, 'with the tools available', theories that had taken decades to construct. He showed an extraordinary sagacity." Magnier also added: "Grothendieck gave the impression of being an extraordinary young man, but imbalanced by suffering and deprivation." Magnier immediately recommended Grothendieck for the scholarship.

Grothendieck's calculus teacher at Montpellier, Monsieur Soula, recommended he go to Paris and make contact with Cartan, who had been Soula's teacher. Whether the name Cartan referred to the father, Élie Cartan, who was then close to eighty years old, or his son, Henri Cartan, then in his mid-forties, Grothendieck did not know (R&S, page 19). When he arrived in Paris, in the autumn of 1948, he showed to mathematicians there the work he had done in Montpellier. Just as Soula had told him, the results were already known. But Grothendieck was not disappointed. In fact, this early solitary effort was probably critical to his development as a mathematician. In *Récoltes et Semailles,* he said of this time: "Without knowing it, I learned in solitude what is essential to the metier of a mathematician—something that no master can truly teach. Without having been told, I nevertheless knew 'in my gut' that I was a mathematician: someone who 'does' math, in the fullest sense of the word—like one 'makes' love" (page P5).

He began attending the legendary seminar run by Henri Cartan at the École Normale Supérieure. This seminar followed a pattern that Grothendieck was to take up with great vigor later in his career, in which a theme is investigated in lectures over the course of the year and the lectures are systematically written up and published. The theme for the Cartan seminar for 1948–1949 was simplicial algebraic topology and sheaf theory—then cutting-edge topics that were not being taught anywhere else in France [D1]. Indeed, this was not long after the notion of sheaves had been formulated by Jean Leray. In the Cartan seminar, Grothendieck encountered for the first time many of the outstanding mathematicians of the day, including Claude Chevalley, Jean Delsarte, Jean Dieudonné, Roger Godement, Laurent Schwartz, and André Weil. Among Cartan's students at this time was Jean-Pierre Serre. In addition to attending the Cartan seminar, Grothendieck went to a course on the then-new notion of locally convex spaces, given by Leray at the Collège de France.

As the son of the geometer Élie Cartan, as an outstanding mathematician in his own right, and as a professor at the École Normale Supérieure, Henri Cartan was in many ways the center of the Parisian mathematical elite. Also, he was one of the few French mathematicians who made efforts to reach out to German colleagues after the war. This was despite his intimate knowledge of the war's horrors: his brother, who had joined the Résistance, had been captured by the Germans and beheaded. Cartan and many of the top mathematicians of the time—such as Ehresmann, Leray, Chevalley, Delsarte, Dieudonné, and Weil—shared the common background of having been *normaliens*, meaning that they were graduates of the École Normale Supérieure, the most prestigious institution of higher education in France.

When Grothendieck joined Cartan's seminar, he was an outsider: not only was he a German speaker living in postwar France, but his meager educational background contrasted sharply with that of the group he found himself in. And yet in *Récoltes et Semailles*, Grothendieck said he did not feel like a stranger in this milieu and related warm memories of the "benevolent welcome" he received (pages 19–20). His outspokenness drew notice: in a tribute to Cartan for his 100th birthday, Jean Cerf recalled seeing in the Cartan seminar around this time "a stranger (it was Grothendieck) who took the liberty of speaking to Cartan, as if to his equal, from the back of the room" [Cerf]. Grothendieck felt free to ask questions, and yet, he wrote, he also found himself struggling to learn things that those around him seemed to grasp instantly and play with "like they had known them from the cradle." (R&S, page P6). This may have been one reason why, in October 1949, on the advice of Cartan and Weil, he left the rarefied atmosphere of Paris for the slower-paced Nancy. Also, as Dieudonné wrote [D1], Grothendieck was at this time showing more interest in topological vector spaces than in algebraic geometry, so Nancy was the natural place for him to go.

Apprenticeship in Nancy

> …l'affection circulait…depuis ce premier moment où j'ai été reçu avec affection à Nancy, en 1949, dans la maison de Laurent et Hélène Schwartz (où je faisais un peu partie de la famille), celle de Dieudonné, celle de Godement (qu'en un temps je hantais également régulièrement). Cette chaleur affectueuse qui a entouré mes premiers pas dans le monde mathématique, et que j'ai eu tendance un peu à oublier, a été importante pour toute ma vie de mathématicien.
>
> …the affection circulated…from that first moment when I was received with affection in Nancy in 1949, in the house of Laurent and Hélène Schwartz (where I was somewhat a member of the family), in that of Dieudonné, in that of Godement (which at that time also became one of my regular haunts). This affectionate warmth that surrounded my first steps in the mathematical

> world, and that I have had some tendency to forget, was important in my entire life as a mathematician.
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> —*Récoltes et Semailles*, page 42

In the late 1940s, Nancy was one of the strongest mathematical centers in France; indeed, the fictitious Nicolas Bourbaki was said to have come from the "University of Nancago", a name that makes reference to Weil's time at the University of Chicago as well as to his fellow Bourbakists in Nancy. The Nancy faculty included Delsarte, Godement, Dieudonné, and Schwartz. Among Grothendieck's fellow students at Nancy were Jacques-Louis Lions and Bernard Malgrange, who like Grothendieck were students of Schwartz, as well as Paulo Ribenboim, a Brazilian who at twenty-two years of age arrived in Nancy about the same time as Grothendieck.

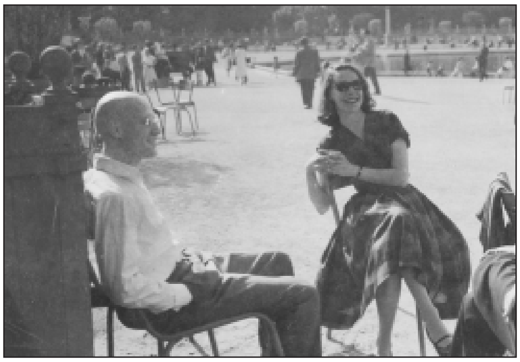
According to Ribenboim, who is today a professor emeritus at Queen's University in Ontario, the pace in Nancy was less hectic than in Paris, and professors had more time for the students. Ribenboim said he had the impression that Grothendieck had come to Nancy because his lack of background had made it hard for him to follow Cartan's high-powered seminar. Not that Grothendieck came out and said this: "He was not the guy who would admit he didn't understand!" Ribenboim remarked. Nevertheless, Grothendieck's extraordinary talents were apparent, and Ribenboim remembered looking up to him as an ideal. Grothendieck could be extremely intense, sometimes expressing himself in a brazen way, Ribenboim recalled: "He was not mean, but very demanding of himself and everyone else." Grothendieck had very few books; rather than learning things by reading, he would try to reconstruct them on his own. And he worked very hard. Ribenboim remembered Schwartz telling him: You seem to be a nice, well-balanced young man; you should make friends with Grothendieck and do something so that he is not only working.

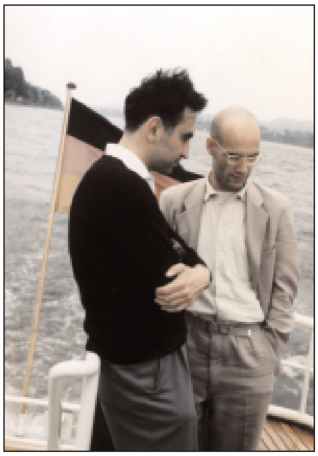
Dieudonné and Schwartz were running a seminar in Nancy on topological vector spaces. As Dieudonné explained in [D1], by this time Banach spaces and their duality were well understood, but locally convex spaces had only recently been introduced, and a general theory for their duality had not yet been worked out. In working in this area, he and Schwartz had run into a series of problems, which they decided to turn over to Grothendieck. They were astonished when, some months later, he had solved every one of them and gone on to work on other questions in functional analysis. "When, in 1953, it was time to grant him a doctor's degree, it was necessary to choose from among six papers he had written, any one of which was at the level of a good dissertation," Dieudonné wrote. The paper chosen for his thesis was "Produits tensoriels topologiques et espaces nucléaires," which shows the first signs of the generality of thinking that would come to characterize Grothendieck's entire oeuvre. The notion of nuclear spaces, which has had wide applications, was first proposed in this paper. Schwartz popularized Grothendieck's results in a Paris seminar, "Les produits tensoriels d'après Grothendieck," published in 1954 [Schwartz]. In addition, Grothendieck's thesis appeared as a monograph in 1955 in the *Memoirs of the AMS* series; it was reprinted for the seventh time in 1990 [Gthesis].

Grothendieck's work in functional analysis "was quite remarkable," commented Edward G. Effros of the University of California at Los Angeles. "He was arguably the first to realize that the algebraic/categorical methods that flourished after the Second World War could be used in this highly analytic branch of functional analysis." In some ways, Grothendieck was ahead of his time. Effros noted that it took at least fifteen years before Grothendieck's work was fully incorporated into mainstream Banach space theory, partly because of a reluctance to adopt his more algebraic perspective. The influence of his work has grown in recent years, Effros said, with the "quantization" of Banach space theory, for which Grothendieck's categorical approach is especially well suited.

Top: Party at Hirzebruch home, 1961 Arbeitstagung (left to right) Dorothea von Viereck, Raoul Bott, Grothendieck.
Center, with Michael Atiyah.
Bottom: Bonn, 1961, excursion during Arbeitstagung, Ioan James, Michael Atiyah, Grothendieck.

Although Grothendieck's mathematical work had gotten off to a promising start, his personal life was unsettled. He lived in Nancy with his mother, who as Ribenboim recalled was occasionally bedridden because of tuberculosis. She had contracted the disease in the internment camps. It was around this time that she was writing her autobiographical novel *Eine Frau*. A liaison between Grothendieck and an older woman who ran the boarding house where





Top: Paris, with Karin Tate, 1964. Bottom: with E. Luft, an excursion on the Rhine, 1961.

he and his mother rented rooms resulted in the birth of his first child, a son named Serge; Serge was raised mostly by his mother. After he finished his Ph.D., Grothendieck's prospects for permanent employment were bleak: he was stateless, and at that time it was difficult for noncitizens to get permanent jobs in France. Becoming a French citizen would have entailed military service, which Grothendieck refused to do. Since 1950 he had had a position through the Centre National de la Recherche Scientifique (CNRS), but this was more like a fellowship than a permanent job. At some point he considered learning carpentry as a way to earn money (R&S, page 1246(*)).

Laurent Schwartz visited Brazil in 1952 and told people there about his brilliant young student who was having trouble finding a job in France. As a result Grothendieck received an offer of a visiting professor position at the Universidade de São Paulo, which he held during 1953 and 1954. According to José Barros-Neto, who was then a student in São Paulo and is now a professor emeritus at Rutgers University, Grothendieck made a special arrangement so that he would be able to return to Paris to attend seminars that took place in the fall. The second language for the Brazilian mathematical community was French, so it was easy for Grothendieck to teach and converse with his colleagues. In going to São Paulo, Grothendieck was carrying on a tradition of scientific exchange between Brazil and France: in addition to Schwartz, Weil, Dieudonné, and Delsarte had all visited Brazil in the 1940s and 1950s. Weil came to São Paulo in January 1945 and stayed until the fall of 1947, when he went to the University of Chicago. The mathematical ties between France and Brazil continue to this day. The Instituto de Matemática Pura e Aplicada in Rio de Janeiro has a Brazil-France cooperative agreement that brings many French mathematicians to IMPA.

In *Récoltes et Semailles*, Grothendieck referred to 1954 as "the wearisome year" (*"l'année pénible"*) (page 163). For the whole year he tried without success to make headway on the problem of approximation in topological vector spaces, a problem that was resolved only some twenty years later by methods different from those Grothendieck was attempting to use. This was "the only time in my life when doing mathematics became burdensome for me!" he wrote. This frustration taught him a lesson: always have several mathematical "irons in the fire," so that if one problem proves too stubborn, there is something else to work on.

Chaim Honig, a professor at the Universidade de São Paulo, was an assistant in the mathematics department when Grothendieck was there, and they became good friends. Honig said Grothendieck led a somewhat spartan and lonely existence, living off of milk and bananas and completely immersing himself in mathematics. Honig once asked Grothendieck why he had gone into mathematics. Grothendieck replied that he had two special passions, mathematics and piano, but he chose mathematics because he thought it would be easier to earn a living that way. His gift for mathematics was so abundantly clear, said Honig, "I was astonished that at any moment he could hesitate between mathematics and music."

Grothendieck planned to write a book on topological vector spaces with Leopoldo Nachbin, who was in Rio de Janeiro, but the book never materialized. However, Grothendieck taught a course in São Paulo on topological vector spaces and wrote up the notes, which were subsequently published by the university. Barros-Neto was a student in the course and wrote an introductory chapter for the notes, giving some basic prerequisites. Barros-Neto recalled that at the time he was in Brazil Grothendieck was talking about changing fields. He was "very, very ambitious," Barros-Neto said. "You could sense that drive—he had to do something fundamental, important, basic."

A Rising Star

> La chose essentielle, c'était que Serre à chaque fois sentait fortement la riche substance derrière un énoncé qui, de but en blanc, ne m'aurait sans doute fait ni chaud ni froid—et qu'il arrivait à "faire passer" cette perception d'une substance riche, tangible, mystérieuse—cette perception qui est en même temps *désir* de connaître cette substance, d'y pénétrer.
>
> The essential thing was that Serre each time strongly sensed the rich meaning behind a statement that, on the page, would doubtless have left me neither hot nor cold—and that he could "transmit" this perception of a rich, tangible, and mysterious substance—this perception that is at the same time the

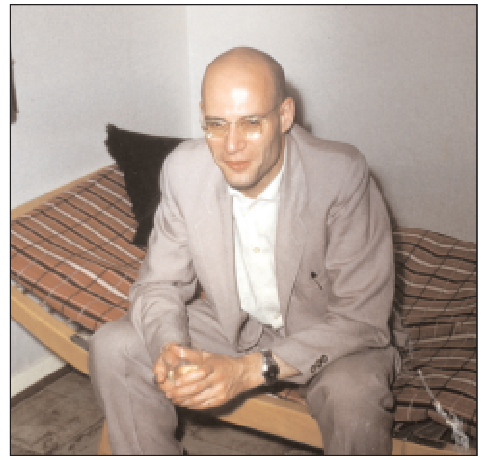
desire to understand this substance, to penetrate it.

—*Récoltes et Semailles*, page 556

Bernard Malgrange of the Université de Grenoble recalled that after Grothendieck wrote his thesis he asserted that he was no longer interested in topological vector spaces. "He told me, 'There is nothing more to do, the subject is dead'," Malgrange recalled. At that time, students were required to prepare a "second thesis", which did not contain original work but which was intended to demonstrate depth of understanding of another area of mathematics far removed from the thesis topic. Grothendieck's second thesis was on sheaf theory, and this work may have planted the seeds for his interest in algebraic geometry, where he was to do his greatest work. After Grothendieck's thesis defense, which took place in Paris, Malgrange recalled that he, Grothendieck, and Henri Cartan piled into a taxicab to go to lunch at the home of Laurent Schwartz. They took a cab because Malgrange had broken his leg skiing. "In the taxi Cartan explained to Grothendieck some wrong things Grothendieck had said about sheaf theory," Malgrange recalled.

After leaving Brazil Grothendieck spent the year of 1955 at the University of Kansas, probably at the invitation of N. Aronszajn [Corr]. There Grothendieck began to immerse himself in homological algebra. It was while he was at Kansas that he wrote "Sur quelques points d'algèbre homologique," which came to be known informally among specialists as the "Tôhoku paper" after the name of the journal in which it appeared, the *Tôhoku Mathematical Journal* [To]. This paper, which became a classic in homological algebra, extended the work of Cartan and Eilenberg on modules. Also while he was in Kansas, Grothendieck wrote "A general theory of fiber spaces with structure sheaf," which appeared as a report of the National Science Foundation. This report developed his initial ideas on nonabelian cohomology, a subject to which he later returned in the context of algebraic geometry.

Around this time, Grothendieck began corresponding with Jean-Pierre Serre of the Collège de France, whom he had met in Paris and later encountered in Nancy; a selection of their letters was published in the original French in 2001 and in a dual French-English version in 2003 [Corr]. This was the beginning of a long and fruitful interaction. The letters display a deep and vibrant mathematical bond between two very different mathematicians. Grothendieck shows a high-flying imagination that is frequently brought back to earth by Serre's incisive understanding and wider knowledge. Sometimes in the letters Grothendieck displays a surprising level of ignorance: for example, at one point, he asks Serre if the Riemann zeta function has infinitely many zeros ([Corr], page 204). "His knowledge of classical algebraic geometry was practically zero," recalled Serre. "My own knowledge of classical algebraic geometry was a little bit better, but not very much, but I tried to help him with that. But…there were so many open questions that it didn't matter." Grothendieck was not one for keeping up on the latest literature, and to a large degree he depended on Serre to tell him what was going on. In *Récoltes et Semailles* Grothendieck wrote that most of what he learned in geometry, apart from what he taught himself, he learned from Serre (pages 555–556). But Serre did not simply teach Grothendieck things; he was able to digest ideas and to discuss them in a way that Grothendieck found especially compelling. Grothendieck called Serre a "detonator," one who provided a spark that set the fuse burning for an explosion of ideas.



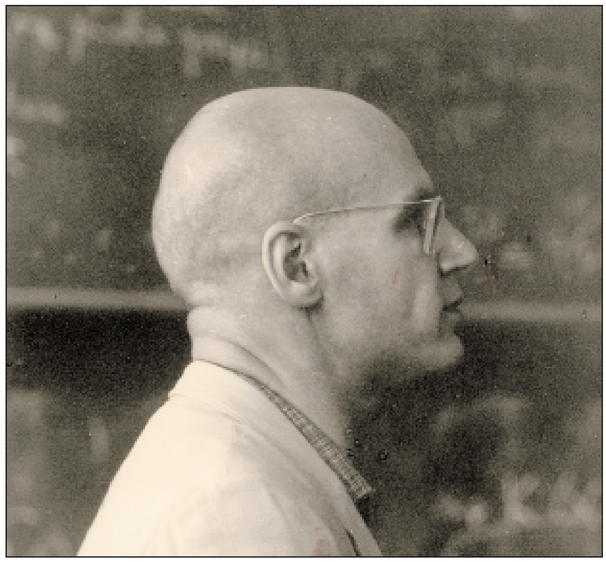
During an Arbeitstagung in 1961, an evening at the Hirzebruch home in Bonn.

Indeed, Grothendieck traced many of the central themes of his work back to Serre. For example, it was Serre who around 1955 described the Weil conjectures to Grothendieck in a cohomological context—a context that was not made explicit in Weil's original formulation of the conjectures and was the one that could hook Grothendieck (R&S, page 840). Through his idea of a "Kählerian" analogue of the Weil conjectures, Serre also inspired Grothendieck's conception of the so-called "standard conjectures," which are more general and would imply the Weil conjectures as a corollary (R&S, page 210).

When Grothendieck returned to France in 1956 after his year in Kansas, he held a CNRS position and spent most of his time in Paris. He and Serre continued to correspond by letter and to talk regularly by telephone. This was when Grothendieck began to work more deeply in topology and algebraic geometry. He "was bubbling with ideas," recalled Armand Borel. "I was sure something first-rate would come out of him. But then what came out was even much higher than I had expected. It was his version of Riemann-Roch, and that's a fantastic theorem. This is really a masterpiece of mathematics."

The Riemann-Roch theorem was proved in its classical form in the mid-nineteenth century. The question it addresses is, What is the dimension of the space of meromorphic functions on a compact Riemann surface having poles of at most given orders at a specified finite set of points? The answer is the Riemann-Roch formula, which expresses the dimension in terms of invariants of the surface—thereby providing a profound link between the analytic and topological properties of the surface. Friedrich Hirzebruch made a big advance in 1953, when he generalized the Riemann-Roch theorem to apply not just to Riemann surfaces but to projective nonsingular varieties over the complex numbers. The mathematical world cheered at this tour de force, which might have seemed to be the final word on the matter.

"Grothendieck came along and said, 'No, the Riemann-Roch theorem is *not* a theorem about varieties, it's a theorem about morphisms between varieties'," said Nicholas Katz of Princeton University. "This was a fundamentally new point of view…the very statement of the theorem completely changed." The basic philosophy of category theory, that one should pay more attention to the arrows between objects than to the objects themselves, was just then beginning to have an influence. "What [Grothendieck] did is he applied this philosophy on a very hard piece of mathematics," Borel said. "This was really in the spirit of categories and functors, but no one had ever thought about doing this in such a hard topic…. If people had been given that statement and had understood it, there might have been others who would have been able to prove it. But the statement itself was ten years ahead of anybody else."


Bonn, around 1965.

This theorem, which was also proved independently by Gerard Washnitzer in 1959 [Washnitzer], applies not just to a complex algebraic variety—the case where the ground field has characteristic zero—but to any proper smooth algebraic variety regardless of the ground field. The Hirzebruch-Riemann-Roch theorem then follows as a special case. A far-reaching generalization of the Riemann-Roch theorem came in 1963, with the proof by Michael Atiyah and Isadore Singer of the Atiyah-Singer Index Theorem. In the course of his proof, Grothendieck introduced what are now called Grothendieck groups, which essentially provide a new kind of topological invariant. Grothendieck himself called them K-groups, and they provided the starting point for the development of topological K-theory by Atiyah and Hirzebruch. Topological K-theory then provided the inspiration for algebraic K-theory, and both have been active fields of research ever since.

The *Arbeitstagung*, which means literally "working meeting," was begun by Hirzebruch at the Universität Bonn and has been a forum for cutting-edge mathematics research for more than forty years. It was at the very first *Arbeitstagung* in July 1957 that Grothendieck spoke about his work on Riemann-Roch. But in a curious twist, the result was not published under his name; it appears in a paper by Borel and Serre [BS] (the proof also appeared later as an *exposé* in volume 6 of *Séminaire de Géometrie Algébrique du Bois Marie* from 1966-67). While visiting the IAS in the fall of 1957, Serre received a letter from Grothendieck containing an outline of the proof (November 1, 1957, letter in [Corr]). He and Borel organized a seminar to try to understand it. As Grothendieck was busy with many other things, he suggested to his colleagues that they write up and publish their seminar notes. But Borel speculated that there may have been another reason Grothendieck was not interested in writing up the result himself. "The main philosophy of Grothendieck was that mathematics should be reduced to a succession of small, natural steps," Borel said. "As long as you have not been able to do this, you have not understood what is going on…. And his proof of Riemann-Roch used a trick, *une astuce*. So he didn't like it, so he didn't want to publish it…. He had many other things to do, and he was not interested in writing up this trick."

This was not the last time Grothendieck would revolutionize the viewpoint on a subject. "This just kept happening over and over again, where he would come upon some problem that people had thought about for, in some cases, a hundred years…and just completely transformed what people thought the subject was about," Katz remarked. Grothendieck was not only solving outstanding problems but reworking the very questions they posed.

A New World Opens

> [J'ai fini] par me rendre compte que cette idéologie du "nous, les grands et nobles esprits…", sous une forme particulièrement extrême et virulente, avait sévi en ma mère depuis son enfance, et dominé sa relation aux autres, qu'elle se plaisait à regarder du haut de sa grandeur avec une commisération souvent dédaigneuse, voire méprisante.
>
> [I eventually] realized that this ideology of "we, the grand and noble spirits…", in a particularly extreme and virulent form, raged in my mother since her childhood and dominated her relations to others, whom she liked to view from the height of her grandeur with a pity that was frequently disdainful, even contemptuous.
>
> —*Récoltes et Semailles*, page 30

According to Honig, Grothendieck's mother was with him at least part of the time that he was in Brazil, though Honig says he never met her. Whether she was with him in Kansas is not clear. When Grothendieck returned to France in 1956, they may not have continued living together. In a letter to Serre written in Paris in November 1957, Grothendieck asked whether he might be able to rent a Paris apartment that Serre was planning to vacate. "I am interested in it for my mother, who is not doing so well in Bois-Colombes, and is terribly isolated," Grothendieck explained [Corr]. In fact, his mother died before the year's end.

Friends and colleagues say that Grothendieck spoke with great admiration, almost adulation, of both of his parents. And in *Récoltes et Semailles* Grothendieck expressed a deep and elemental love for them. For years he had in his office a striking portrait of his father, painted by a fellow detainee in the Le Vernet camp. As Pierre Cartier described it, the portrait showed a man with his head shaved and a "fiery expression" in the eyes [Cartier1]; for many years Grothendieck also shaved his head. According to Ribenboim, Hanka Grothendieck was very proud of her brilliant son, and he in turn had an extremely deep attachment to his mother.

After her death, Grothendieck went through a period of soul-searching, during which he stopped all mathematical activity and thought about becoming a writer. After several months, he decided to return to mathematics, to finish work on some of the ideas he had begun developing. This was 1958, the year that, as Grothendieck put it, was "probably the most fecund of all my mathematical life." (R&S, page P24) By this time he was living with a woman named Mireille, whom he was to marry a few years later and with whom he had three children, Johanna, Mathieu, and Alexandre. Mireille had been close to Grothendieck's mother and, according to several people who knew them, was quite a bit older than he was.

With Mireille and baby Mathieu, Paris, May 1965.

John Tate of the University of Texas at Austin and his wife at the time, Karin Tate, spent the academic year 1957–58 in Paris, where they met Grothendieck for the first time. Grothendieck displayed none of the arrogance he attributed to his mother. "He was just friendly, and at the same time rather naive and childlike," John Tate recalled. "Many mathematicians are rather childlike, unworldly in some sense, but Grothendieck more than most. He just seemed like an innocent—not very sophisticated, no pretense, no sham. He thought very clearly and explained things very patiently, without any sense of superiority. He wasn't contaminated by civilization or power or one-upmanship." Karin Tate recalled that Grothendieck had a great capacity for enjoyment, he was charming, and he loved to laugh. But he could also be extremely intense, seeing things in black-and-white with no shades of gray. And he was honest: "You always knew where you stood with him," she said. "He didn't pretend anything. He was direct." Both she and her brother, Michael Artin of the Massachusetts Institute of Technology, saw similarities between Grothendieck's personality and that of their father, Emil Artin.

Grothendieck had "an incredible idealistic streak," Karin Tate remembered. For example, he refused to have any rugs in his house because he believed that rugs were merely a decorative luxury. She also remembered him wearing sandals made out of tires. "He thought these were fantastic," she said. "They were a symbol of the kind of thing he respected—you take what you have, and you make do." In his idealism, he could also be wildly impractical. Before Grothendieck and Mireille visited Harvard for the first time in 1958, he gave her one of his favorite novels so that she could improve her rather weak knowledge of English. The novel was *Moby Dick*.

The Birth of the New Geometry

> Avec un recul de près de trente ans, je peux dire maintenant que c'est l'année [1958] vraiment où est née la vision de la géometrie nouvelle, dans le sillage

des deux maître-outils de cette géometrie: les schémas (qui représentent une métamorphose de l'ancienne notion de "variété algébrique"), et les topos (qui représentent une métamorphose, plus profonde encore, de la notion d'espace).

With hindsight of thirty years, I can now say that [1958] is the year where the vision of the new geometry was really born, in the wake of two master-tools of this geometry: schemes (which represent a metamorphosis of the old notion of "algebraic variety"), and toposes (which represent a metamorphosis, yet more profound, of the notion of space).

—*Récoltes et Semailles*, page P23

In August 1958, Grothendieck gave a plenary lecture at the International Congress of Mathematicians in Edinburgh [Edin]. The talk outlined, with a remarkable prescience, many of the main themes that he was to work on for the next dozen years. It was clear by this time that he was aiming to prove the famous conjectures of André Weil, which hinted at a profound unity between the discrete world of algebraic varieties and the continuous world of topology.

At this time, algebraic geometry was evolving rapidly, with many open questions that did not require a great deal of background. Originally the main objects of study were varieties over the complex numbers. During the early part of the twentieth century, this area was a specialty of Italian mathematicians, such as Guido Castelnuovo, Federigo Enriques, and Francesco Severi. Although they developed many ingenious ideas, not all of their results were proved rigorously. In the 1930s and 1940s, other mathematicians, among them B. L. van der Waerden, André Weil, and Oscar Zariski, wanted to work with varieties over arbitrary fields, particularly varieties over fields of characteristic p, which are important in number theory. But, because of the lack of rigor of the Italian school of algebraic geometry, it was necessary to build new foundations for the field. This is what Weil did in his 1946 book *Foundations of Algebraic Geometry* [Weil1].

Weil's conjectures appeared in his 1949 paper [Weil2]. Motivated by problems in number theory, Weil studied a certain zeta function that had been introduced in special cases by Emil Artin; it is called a zeta function because it was defined in analogy to the Riemann zeta function. Given an algebraic variety V defined over a finite field of characteristic p, one can count the number of points of V that are rational over this field, as well as the corresponding numbers for each finite extension field. These numbers are then incorporated into a generating function, which is the zeta function of V. Weil proved for both curves and abelian varieties three facts about this zeta function: it is rational, it satisfies a functional equation, and its zeros and poles have a certain specific form. This form, once a change of variables is made, corresponds exactly to the Riemann hypothesis. Moreover, Weil observed that, if V arose from reduction modulo p of a variety W in characteristic zero, then the Betti numbers of W can be read off the zeta function of V, when the zeta function is expressed as a rational function. The Weil conjectures ask whether these same facts hold true if one defines such a zeta function for a projective nonsingular algebraic variety. In particular, would topological data such as the Betti numbers emerge in the zeta function? This conjectured link between algebraic geometry and topology hinted that some of the new tools, such as cohomology theory, that were then being developed for topological spaces, could be adapted for use with algebraic varieties. Because of its similarity to the classical Riemann hypothesis, the third of the Weil conjectures is sometimes called the "congruence Riemann hypothesis"; this one turned out to be the most difficult of the three to prove.

"As soon as [the Weil] conjectures were made, it was clear that they were somehow going to play a central role," Katz said, "both because they were fabulous just as 'black-box' statements, but also because it seemed obvious that solving them required developing incredible new tools that would somehow have to be incredibly valuable in their own way—which turned out to be completely correct." Pierre Deligne of the Institute for Advanced Study said that it was the conjectured link between algebraic geometry and topology that attracted Grothendieck. He liked the idea of "turning this dream of Weil into powerful machinery," Deligne remarked.

Grothendieck was not interested in the Weil conjectures because they were famous or because other people considered them to be difficult. Indeed, he was not motivated by the challenge of hard problems. What interested him were problems that seemed to point to larger, hidden structures. "He would aim at finding and creating the home which was the problem's natural habitat," Deligne noted. "That was the part that interested him, more than solving the problem." This approach contrasts with that of another great mathematician of the time, John Nash. In his mathematical prime, Nash searched out specific problems considered by his colleagues to be the most important and challenging [Nasar]. "Nash was like an Olympian athlete," remarked Hyman Bass of the University of Michigan. "He was interested in enormous personal challenges." If Nash is an ideal example of a problem-solver, then Grothendieck is an ideal example of a theory-builder. Grothendieck, said Bass, "had a sweeping vision of what mathematics could be."

In the fall of 1958, Grothendieck made the first of his several visits to the mathematics department at Harvard University. Tate was a professor there, and the chairman was Oscar Zariski. By this time Grothendieck had reproved, by recently developed cohomological methods, the connectedness theorem that was one of Zariski's biggest results, proved in the 1940s. According to David Mumford of Brown University, who was Zariski's student at the time, Zariski never took up the new methods himself, but he understood their power and wanted his students to be exposed to them, and this was why he invited Grothendieck to Harvard.

Zariski and Grothendieck got along well, Mumford noted, though as mathematicians they were very different. It was said that Zariski, when he got stuck, would go to the blackboard and draw a picture of a self-intersecting curve, which would allow him to refresh his understanding of various ideas. "The rumor was that he would draw this in the corner of the blackboard, and then he would erase it and then he would do his algebra," explained Mumford. "He had to clear his mind by creating a geometric picture and reconstructing the link from the geometry to the algebra." According to Mumford, this is something Grothendieck would never do; he seemed never to work from examples, except for ones that were extremely simple, almost trivial. He also rarely drew pictures, apart from homological diagrams.

When Grothendieck was first invited to Harvard, Mumford recalled, he had some correspondence with Zariski before the visit. This was not long after the era of the House Un-American Activities Committee, and one requirement for getting a visa was swearing that one would not work to overthrow the government of the United States. Grothendieck told Zariski he would refuse to take such a pledge. When told he might end up in jail, Grothendieck said jail would be acceptable as long as students could visit and he could have as many books as he wanted.

In Grothendieck's lectures at Harvard, Mumford found the leaps into abstraction to be breathtaking. Once he asked Grothendieck how to prove a certain lemma and got in reply a highly abstract argument. Mumford did not at first believe that such an abstract argument could prove so concrete a lemma. "Then I went away and thought about it for a couple of days, and I realized it was exactly right," Mumford recalled. "He had more than anybody else I've ever met this ability to make an absolutely startling leap into something an order of magnitude more abstract…. He would always look for some way of formulating a problem, stripping apparently everything away from it, so you don't think anything is left. And yet something *is* left, and he could find real structure in this seeming vacuum."

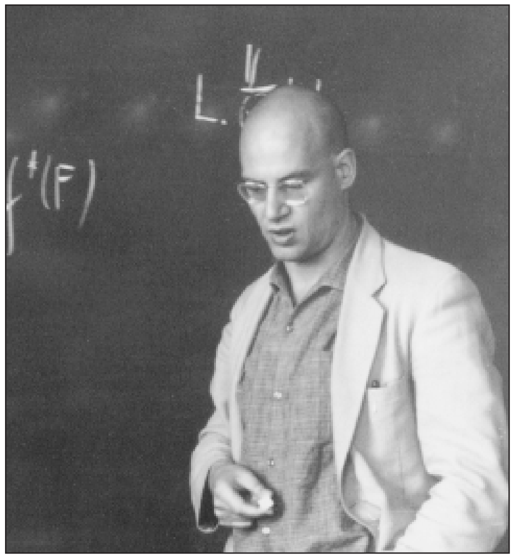
The Heroic Years

> Pendant les années héroiques de l'IHÉS, Dieudonné et moi en avons été les seuls membres, et les seuls aussi à lui donner crédibilité et audience dans le monde scientifique. …Je me sentais un peu comme un cofondateur "scientifique", avec Dieudonné, de mon institution d'attache, et je comptais bien y finir mes jours! J'avais fini par m'identifier fortement à l'IHÉS….
>
> During the heroic years of the IHÉS, Dieudonné and I were the only members, and the only ones also giving it credibility and an audience in the scientific world. …I felt myself a bit like a "scientific" co-founder, with Dieudonné, of the institution where I was on the faculty, and I counted on ending my days there! I ended up strongly identifying with the IHÉS….
>
> —*Récoltes et Semailles*, page 169

In June 1958, the Institut des Hautes Études Scientifiques (IHÉS) was formally established in a meeting of its sponsors at the Sorbonne in Paris. The founder, Léon Motchane, a businessman with a doctoral degree in physics, had a vision of establishing in France an independent research institution akin to the Institute for Advanced Study in Princeton. The original plan for the IHÉS was to focus on fundamental research in three areas: mathematics, theoretical physics, and the methodology of human sciences. While the third area never gained a foothold, within a decade the IHÉS had established itself as one of the world's top centers for mathematics and theoretical physics, with a small but stellar faculty and an active visitor program.

According to the doctoral thesis of historian of science David Aubin [Aubin], it was at the Edinburgh Congress in 1958, or possibly before, that Motchane persuaded Dieudonné and Grothendieck to accept professorships at the newly established IHÉS. Cartier wrote in [Cartier2] that Motchane originally wanted to hire Dieudonné, who made it a condition of his taking the position that an offer also be made to Grothendieck. Because the IHÉS has been from the start independent of the state, there was no problem in hiring Grothendieck despite his being stateless. The two professors formally took up their positions in March 1959, and Grothendieck started his seminar on algebraic geometry in May of that year. René Thom, who had received a Fields Medal at the 1958 Congress, joined the faculty in October 1963, and the IHÉS section on theoretical physics was launched with the appointments of Louis Michel in 1962 and of David Ruelle in 1964. Thus by the mid-1960s, Motchane had assembled an outstanding group of researchers for his new institute.

Up to 1962, the IHÉS had no permanent quarters. Office space was rented from the Fondation Thiers, and seminars were given there or at universities in Paris. Aubin reported that an early visitor to the IHÉS, Arthur Wightman, was expected to work from his hotel room. It is said that, when a visitor noted the inadequate library, Grothendieck replied, "We don't read books, we write them!" Indeed, in the early years, much of the institute's activity centered on the "Publications mathématiques de l'IHÉS," which began with the initial volumes of the foundational work *Éléments de Géométrie Algébrique*, universally known by its acronym EGA. In fact, the writing of EGA had begun half a year before Dieudonné and Grothendieck formally took their positions at the IHÉS; a reference in [Corr] dates the beginning of the writing to the autumn of 1958.



Around 1965.

The authorship of EGA is attributed to Grothendieck, "with the collaboration of Jean Dieudonné." Grothendieck wrote notes and drafts, which were fleshed out and polished by Dieudonné. As Armand Borel explained it, Grothendieck was the one who had the global vision for EGA, whereas Dieudonné had only a line-by-line understanding. "Dieudonné put this in a rather heavy style," Borel remarked. At the same time, "Dieudonné was of course fantastically efficient. No one else could have done it without ruining his own work." For some wanting to enter the field at that time, learning from EGA could be a daunting challenge. Nowadays it is seldom used as an introduction to the field, as there are many other, more approachable texts to choose from. But those texts do not do what EGA aims to do, which is to explain fully and systematically some of the tools needed to investigate schemes. When he was at Princeton University, Gerd Faltings, now at the Max-Planck-Institut für Mathematik in Bonn, encouraged his doctoral students to read EGA. And for many mathematicians today, EGA remains a useful and comprehensive reference. The current IHÉS director, Jean-Pierre Bourguignon, says that the institute still sells over 100 copies of EGA every year.

Grothendieck's plans for what EGA would cover were enormous. In a letter to Serre from August 1959, he gave a brief outline, which included the fundamental group, category theory, residues, duality, intersections, Weil cohomology, and "God willing, a little homotopy." "Unless there are unexpected difficulties or I get bogged down, the multiplodocus should be ready in 3 years' time, or 4 at the outside," Grothendieck optimistically wrote, using his and Serre's joking term "multiplodocus," meaning a very long paper. "We will be able to start doing algebraic geometry!" he crowed. As it turned out, EGA ran out of steam after almost exponential growth: chapters one and two are each one volume, chapter three is two volumes, and the last, chapter four, runs four volumes. Altogether, they comprise 1,800 pages. Despite its falling short of Grothendieck's plans, EGA is a monumental work.

It is no coincidence that the title of EGA echoes the title of the series by Nicolas Bourbaki, *Éléments de Mathématique*, which in turn echoes Euclid's *Elements*: Grothendieck was a member of Bourbaki for several years, starting in the late 1950s and was close to many of the other members. Bourbaki was the pseudonym for a group of mathematicians, most of them French, who collaborated on writing a series of foundational treatises on mathematics. Dieudonné was a founder of the Bourbaki group, together with Henri Cartan, Claude Chevalley, Jean Delsarte, and André Weil. Usually there were about ten members, and the group's composition evolved over the years. The first Bourbaki book appeared in 1939, and the group's influence was at its height during the 1950s and 1960s. The purpose of the books was to provide axiomatic treatments of central areas of mathematics at a level of generality that would make the books useful to the largest number of mathematicians. The books were born in a crucible of animated and sometimes heated discussions among the group's members, many of whom had strong personalities and highly individual points of view. Borel, who was a member of Bourbaki for 25 years, wrote that this collaboration may have been "a unique occurrence in the history of mathematics" [Borel]. Bourbaki pooled the efforts of some of the top mathematicians of the day, who unselfishly and anonymously devoted a good deal of time and energy to writing texts that would make a wide swath of the field accessible. The texts had a large impact, and by the 1970s and 1980s, there were grumblings that Bourbaki had too much influence. Also, some criticized the style of the books as being excessively abstract and general.

The work of Bourbaki and that of Grothendieck bear some similarities in the level of generality and abstraction and also in the aim of being foundational, thorough, and systematic. The main difference is that Bourbaki covered a range of mathematical areas, while Grothendieck focused on developing new ideas in algebraic geometry, with the Weil conjectures as a primary goal. In addition, Grothendieck's work was very much centered on his own internal vision, whereas Bourbaki was a collaborative effort that forged a synthesis of the viewpoints of its members.

Borel described in [Borel] the March 1957 meeting of Bourbaki, dubbed the "Congress of the inflexible functor" because of Grothendieck's proposal that a Bourbaki draft on sheaf theory be redone from a more categorical viewpoint. Bourbaki abandoned this idea, believing it could lead to an endless cycle of foundation-building. Grothendieck "could not really collaborate with Bourbaki because he had his big machine, and Bourbaki was not general enough for him," Serre recalled. In addition, Serre remarked, "I don't think he liked very much the system of Bourbaki, where we would really discuss drafts in detail and criticize them. …That was not his way of doing mathematics. He wanted to do it himself." Grothendieck left Bourbaki in 1960, though he remained close to many of its members.

Stories have circulated that Grothendieck left Bourbaki because of clashes with Weil, but in fact the two had only a brief overlap: following the edict that members must retire at age 50, Weil left the group in 1956. Nevertheless, it is true that Grothendieck and Weil were very different as mathematicians. As Deligne put it, "Weil felt somewhat that Grothendieck was too ignorant of what the Italian geometers had done and what all the classical literature was, and Weil did not like the style of building a big machine. …Their styles were quite different."

Apart from EGA, the other major part of Grothendieck's oeuvre in algebraic geometry is *Séminaire de Géométrie Algébrique du Bois Marie*, known as SGA, which contains written versions of lectures presented in his IHÉS seminar. They were originally distributed by the IHÉS. SGA 2 was copublished by North-Holland and Masson, while the remaining volumes were published by Springer-Verlag. SGA 1 dates from the seminars of 1960–1961, and the last in the series, SGA 7, dates from 1967–1969. In contrast to EGA, which is intended to set foundations, SGA describes ongoing research as it unfolded in Grothendieck's seminar. He presented many of his results in the Bourbaki Seminar in Paris, and they were collected in FGA, *Fondements de la Géométrie Algébrique*, which appeared in 1962. Together, EGA, SGA, and FGA total around 7,500 pages.

The Magic Fan

> [S]'il y a une chose en mathématique qui (depuis toujours sans doute) me fascine plus que toute autre, ce n'est ni "le nombre", ni "la grandeur", mais toujours *la forme*. Et parmi les mille-et-un visages que choisit la forme pour se révéler à nous, celui qui m'a fasciné plus que tout autre et continue à me fasciner, c'est *la structure* cachée dans les choses mathématiques.
>
> [I]f there is one thing in mathematics that fascinates me more than anything else (and doubtless always has), it is neither "number" nor "size", but always *form*. And among the thousand-and-one faces whereby form chooses to reveal itself to us, the one that fascinates me more than any other and continues to fascinate me, is *the structure* hidden in mathematical things.
>
> —*Récoltes et Semailles*, page P27

In the first volume of *Récoltes et Semailles*, Grothendieck presents an expository overview of his work intended to be accessible to nonmathematicians (pages P25–48). There he writes that, at its most fundamental level, this work seeks a unification of two worlds: "the *arithmetic world*, in which live the (so-called) 'spaces' having no notion of continuity, and the *world of continuous size*, in which live the 'spaces' in the proper sense of the term, accessible to the methods of the analyst". The reason the Weil conjectures were so tantalizing is exactly that they provided clues about this unity. Rather than trying to solve the Weil conjectures directly, Grothendieck greatly generalized their entire context. Doing so allowed him to perceive the larger structures in which the conjectures lived and of which they provided only a fleeting glimpse. In this section of *Récoltes et Semailles*, Grothendieck explained some of the key ideas in his work, including *scheme*, *sheaf*, and *topos*.

Basically, a scheme is a generalization of the notion of an algebraic variety. Given the array of finite fields of prime characteristic, a scheme produces in turn an array of varieties, each with its distinct geometry. "The array of these different varieties of different characteristics can be visualized as a sort of 'infinite fan of varieties' (one for each characteristic)," Grothendieck wrote. "The 'scheme' *is* this magic fan, which links, like so many different 'branches', the 'avatars' or 'incarnations' of all the possible characteristics." The generalization to a scheme allows one to study in a unified way all the different "incarnations" of a variety. Before Grothendieck, "I don't think people really believed you could do that," commented Michael Artin. "It was too radical. No one had had the courage to even think this may be the way to work, to work in complete generality. That was very remarkable."

Starting with the insight of the nineteenth-century Italian mathematician Enrico Betti, homology and its dual cohomology were developed as tools for studying topological spaces. Basically, cohomology theories provide invariants, which can be thought of as "yardsticks" for measuring this or that aspect of a space. The great hope, sparked by the insight implicit in the Weil conjectures, was that cohomological methods for topological spaces could be adapted for use with varieties and schemes. This hope was realized to a great extent in the work of Grothendieck and his collaborators. "It was like night and day to [bring] these cohomological techniques" into algebraic geometry, Mumford noted. "It completely turned the field upside down. It's like analysis before and after Fourier analysis. Once you get Fourier techniques, suddenly you have this whole deep insight into a way of looking at a function. It was similar with cohomology."

The notion of a sheaf was conceived by Jean Leray and further developed by Henri Cartan and Jean-Pierre Serre. In his groundbreaking paper known as FAC ("Faisceaux algébriques cohérents", [FAC]), Serre showed how sheaves could be used in algebraic geometry. Without saying exactly what a sheaf is, Grothendieck described in *Récoltes et Semailles* how this notion changed the landscape: When the idea of a sheaf came along, it was as if the good old standard cohomology "yardstick" suddenly multiplied into an infinite array of new "yardsticks", in all sizes and forms, each perfectly suited to its own unique measuring task. What is more, the category of all sheaves over a space carries so much information that one can essentially "forget" what the space is. All the information is in the sheaf—what Grothendieck called the "silent and sure guide" that led him on the path to his discoveries.

The notion of topos, Grothendieck wrote, is "a metamorphosis of the notion of a space." The concept of a sheaf provides a way of translating from the topological setting, where the space lives, to the categorical setting, where the category of sheaves lives. A topos, then, can be described as a category that, without necessarily arising from an ordinary space, nevertheless has all of the "nice" properties of a category of sheaves. The notion of topos, Grothendieck wrote, highlights the fact that "what really counts in a topological space is not at all its 'points' or its subsets of points and their proximity relations and so forth, but rather the *sheaves* on the space and the *category* that they form."

To come up with the idea of topos, Grothendieck "thought very deeply about the notion of space," Deligne commented. "The theory he created to understand those conjectures of Weil was first to create the concept of topos, a generalization of the notion of space, then to define a topos adapted to the problem," he explained. Grothendieck also showed that "one can really work with it, that the intuition we have about ordinary space works [on a topos] also. …This was a very deep idea."

In *Récoltes et Semailles* Grothendieck commented that from a technical point of view much of his work in mathematics consisted in developing the cohomology theories that were lacking. Étale cohomology was one such theory, developed by Grothendieck, Michael Artin, and others, specifically to apply to the Weil conjectures, and indeed it was one of the key ingredients in their proof. But Grothendieck went yet further, developing the concept of a *motive*, which he described as the "ultimate cohomological invariant" of which all others are different realizations or incarnations. A full theory of motives has remained out of grasp, but the concept has generated a good deal of mathematics. For example, in the 1970s Deligne and Robert Langlands of the IAS conjectured precise relationships between motives and automorphic representations. This conjecture, now part of the so-called Langlands Program, first appeared in print in [Langlands]. James Arthur of the University of Toronto said that proving this conjecture in full generality is decades away. But, he pointed out, what Andrew Wiles did in the proof of Fermat's Last Theorem was essentially to prove this conjecture in the case of two-dimensional motives that come from elliptic curves. Another example is the work of Vladimir Voevodsky of the IAS on motivic cohomology, for which he received the Fields Medal in 2002. This work builds on some of Grothendieck's original ideas about motives.

In looking back on this brief retrospective of his mathematical work, Grothendieck wrote that what makes up its essence and power is not results or big theorems, but "ideas, even dreams" (page P51).

The Grothendieck School

> Jusqu'au moment du premier "réveil,"
> en 1970, les relations à mes élèves, tout

> comme ma relation à mon propre travail, était une source de satisfaction et de joie, un des fondements tangibles, irrécusables, d'un sentiment d'harmonie dans ma vie, qui continuait à lui donner un sens….
>
> Until the moment of the first "awakening", in 1970, the relations with my students, just like my relation to my own work, was a source of satisfaction and joy, one of the tangible, unimpeachable bases of a sense of harmony in my life, which continued to give it meaning….
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> —*Récoltes et Semailles*, page 63

During a visit to Harvard in the fall of 1961, Grothendieck wrote to Serre: "The mathematical atmosphere at Harvard is ravishing, a real breath of fresh air compared with Paris, which is getting more gloomy every year. Here, there are a fair number of intelligent students who are beginning to be familiar with the language of schemes and ask for nothing more than to work on interesting problems, which obviously are not in short supply" [Corr]. Michael Artin was at Harvard at that time as a Benjamin Peirce instructor, after having finished his thesis with Zariski in 1960. Immediately after his thesis, Artin set about learning the new language of schemes, and he also became interested in the idea of étale cohomology. When Grothendieck came to Harvard in 1961, "I asked him to tell me the definition of étale cohomology," Artin recalled with a laugh. The definition had not yet been formulated precisely. Said Artin, "Actually we argued about the definition for the whole fall."

After moving to the Massachusetts Institute of Technology in 1962, Artin gave a seminar on étale cohomology. He spent much of the following two years at the IHÉS working with Grothendieck. Once the definition of étale cohomology was in hand, there was still a lot of work to be done to tame the theory and make it into a tool that could really be used. "The definition looked marvelous, but it came with no guarantees that anything was finite, or could ever be computed, or anything," Mumford commented. This was the work that Artin and Grothendieck plunged into; one product was the Artin representability theorem. Together with Jean-Louis Verdier, they directed the 1963–64 seminar, which focused on étale cohomology. That seminar was written up in the three volumes of SGA 4, which total nearly 1,600 pages.

There might be disagreement with Grothendieck's "gloomy" assessment of the Parisian mathematical scene of the early 1960s, but there is no question that it got an enormous boost when he returned to the IHÉS in 1961 and restarted his seminar. The atmosphere was "fantastic", Artin recalled. The seminar was well populated by the leading lights of Parisian mathematics, as well as mathematicians visiting from other places. A group of brilliant and eager students began to collect around Grothendieck and to write their theses under his direction (the IHÉS does not give degrees, so formally they were students at universities in and around Paris). By 1962 the IHÉS had moved to its permanent home in the middle of a serene, tree-filled park called the Bois-Marie, in the Paris suburb of Bures-sur-Yvette. The gazebo-like building where the seminar was held, with its large picture windows and open, airy feel, provided an unusual and dramatic setting. Grothendieck was the dynamic center of the activities. "The seminars were highly interactive," recalled Hyman Bass, who visited the IHÉS in the 1960s, "but Grothendieck dominated whether he was the speaker or not." He was extremely rigorous and could be rather tough on people. "He was not unkind, but also not coddling," Bass said.

Grothendieck developed a certain pattern of working with students. A typical example is that of Luc Illusie of the Université de Paris-Sud, who became a student of Grothendieck's in 1964. Illusie had been participating in the Paris seminar of Henri Cartan and Laurent Schwartz, and it was Cartan who suggested that Illusie might do a thesis with Grothendieck. Illusie, who had until that time worked only in topology, was apprehensive about meeting this "god" of algebraic geometry. As it turned out, Grothendieck was quite kind and friendly and asked Illusie to explain what he had been working on. After Illusie had spoken for a short time, Grothendieck went to the board and launched into a discussion of sheaves, finiteness conditions, pseudo-coherence, and the like. "It was like a sea, like a continuous stream of mathematics on the board," Illusie recalled. At the end of it Grothendieck said that the next year he would devote his seminar to L-functions and l-adic cohomology and that Illusie should help to write up the notes. When Illusie protested that he knew nothing about algebraic geometry, Grothendieck said it didn't matter: "You will learn quickly."

And Illusie did. "His lectures were so clear, and he made so many efforts to recall what was necessary, all the prerequisites," Illusie remarked. Grothendieck was an excellent teacher, very patient and adept at explaining things clearly. "He took time to explain very simple examples showing how the machinery works," Illusie said. Grothendieck discussed formal properties that are often glossed over as being "trivial" and therefore too obvious to require explanation. Usually "you don't specify them, you don't spend time," Illusie said, but such things are pedagogically very useful. "Sometimes it was a bit lengthy, but it was very good for understanding."

Grothendieck gave Illusie the assignment of writing up notes for some *exposés* of the seminars—namely, *exposés* I, II, and III of SGA 5. The notes done, "I was shivering when I handed them to him," Illusie recalled. A few weeks later, Grothendieck asked Illusie to come to his home to discuss the notes; he often worked at home with colleagues and students. When Grothendieck took the notes out and set them on the table, Illusie saw that they were blackened with penciled comments. The two sat there for several hours as Grothendieck went over each comment. "He could criticize for a comma, for a period, he could criticize for an accent, he could criticize also on the substance of the thing very deeply and propose another organization—it was all kinds of comments," Illusie said. "But all his comments were very up to the point." This kind of line-by-line critique of written notes was typical of Grothendieck's way of working with students. Illusie recalled that a couple of students could not bear this kind of close criticism and ended up writing their theses with someone else. One was nearly reduced to tears after an encounter with Grothendieck. Illusie said, "Some people I remember didn't like it so much. You had to comply with that. …[But] they were not petty criticisms."

Nicholas Katz was also given an assignment when he visited the IHÉS as a postdoc in 1968. Grothendieck suggested that Katz could give a lecture in the seminar about Lefschetz pencils. "I had heard of Lefschetz pencils but really knew as little as is possible to know about them except for having heard of them," Katz recalled. "But by the end of the year I had given a few talks in the seminar, which now exist as part of SGA 7. I learned a tremendous amount from it, and it had a big effect on my future." Katz said that Grothendieck would come to the IHÉS perhaps one day a week to talk to the visitors. "What was completely amazing is he would then somehow get them interested in something, give them something to do," Katz explained. "But with, it seemed to me, a kind of amazing insight into what was a good problem to give to that particular person to think about. And he was somehow mathematically incredibly charismatic, so that it seemed like people felt it was almost a privilege to be asked to do something that was part of Grothendieck's long range vision of the future."

Barry Mazur of Harvard University still remembers today the question that Grothendieck posed to him in one of their first conversations at the IHÉS in the early 1960s, a question that Gerard Washnitzer had originally asked Grothendieck. The question: Can an algebraic variety defined over a field give topologically different manifolds by two different embeddings of the field into the complex numbers? Serre had given some early examples showing that the two manifolds could be different, and Mazur went on to do some work in homotopy theory with Artin that was inspired by this question. But at the time Grothendieck posed it, Mazur was a dedicated differential topologist, and such a question would not have occurred to him. "For [Grothendieck], it was a natural question," Mazur said. "But for me, it was precisely the right kind of motivation to get me to begin to think about algebra." Grothendieck had a real talent for "matching people with open problems. He would size you up and pose a problem that would be just the thing to illuminate the world for you. It's a mode of perceptiveness that's quite wonderful, and rare."

In addition to work with students and colleagues at the IHÉS, Grothendieck maintained correspondence with a large number of mathematicians outside Paris, some of whom were working on parts of his program in other places. For example, Robin Hartshorne of the University of California at Berkeley was at Harvard in 1961 and got the idea for his thesis topic, concerning Hilbert schemes, from Grothendieck's lectures there. Once the thesis was done, Hartshorne sent a copy to Grothendieck, who was by then back in Paris. In a reply dated September 17, 1962, Grothendieck made some brief positive remarks about the thesis. "The next three or four pages [of the letter] are all of his ideas about further theorems that I might be able to develop and other things that one might like to know about the subject," Hartshorne said. Some of the things the letter suggested are "impossibly difficult," he noted; others show a remarkable prescience. After this outpouring of ideas, Grothendieck returned to the thesis and offered three pages of detailed comments.

In his 1958 talk at the Edinburgh Congress, Grothendieck had outlined his ideas for a theory of duality, but because he was busy with other topics in his IHÉS seminar, it did not get treated there. So Hartshorne offered to give a seminar on duality at Harvard and write up the notes. Over the summer of 1963, Grothendieck fed Hartshorne about 250 pages of "pre-notes" that formed the basis for the seminar, which Hartshorne began in the fall of 1963. Questions from the audience helped Hartshorne to develop and refine the theory, which he began to write up in a systematic fashion. He would send each chapter to Grothendieck to critique. "It would come back covered with red ink all over," Hartshorne recalled. "So I'd fix everything he said, and then I would send him the new version. And it would come back again covered with more red ink." After realizing that this was a potentially endless process, Hartshorne decided one day to send the manuscript off to be published; it appeared in the Springer Lecture Notes series in 1966 [Hartshorne].

Grothendieck "had so many ideas that he kept basically all the serious people working in algebraic geometry in the world busy during that time," Hartshorne observed. How did he keep such an enterprise going? "I don't think there is a simple answer," Artin replied. But certainly Grothendieck's energy and breadth were factors. "He was very dynamic, and he did cover a lot of territory," Artin said. "One thing that was remarkable was that he had complete control of the field, which was not inhabited by slouches, for a period of 12 years or so."

During his IHÉS years, Grothendieck's devotion to mathematics was total. His tremendous energy and capacity for work, combined with a tenacious fidelity to his internal vision, produced a flood of ideas that swept many into its currents. He did not shrink from the daunting program he had set for himself, but plunged straight in, taking on tasks great and small. "His mathematical agenda was much more than any single human being could do," Bass commented. He parceled out much of the work to his students and collaborators, while also taking on a great deal himself. What motivated him, as he explained in *Récoltes et Semailles*, was simply the desire to understand, and indeed those who knew him then confirm that he was not propelled by any sense of competition. "At the time, there was never a thought of proving something before somebody else," Serre explained. And in any case, "he could not be in competition with anybody, in a sense, because he wanted to do things his own way, and essentially nobody else wanted to do the same. It was too much work."

The dominance of the Grothendieck school had some detrimental effects. Even Grothendieck's distinguished IHÉS colleague, René Thom, felt the pressure. In [Fields], Thom wrote that his relations with Grothendieck were "less agreeable" than with his other IHÉS colleagues. "His technical superiority was crushing," Thom wrote. "His seminar attracted the whole of Parisian mathematics, whereas I had nothing new to offer. That made me leave the strictly mathematical world and tackle more general notions, like morphogenesis, a subject which interested me more and led me towards a very general form of 'philosophical' biology."

In the historical remarks that appear at the end of his 1988 textbook *Undergraduate Algebraic Geometry*, Miles Reid wrote: "[T]he Grothendieck personality cult had serious side effects: many people who had devoted a large part of their lives to mastering Weil foundations suffered rejection and humiliation. …[A] whole generation of students (mainly French) got themselves brainwashed into the foolish belief that a problem that can't be dressed up in high-powered abstract formalism is unworthy of study." Such "brainwashing" was perhaps an inevitable by-product of the fashions of the times, though Grothendieck himself never pursued abstraction for abstraction's sake. Reid also noted that, apart from the small number of students of Grothendieck who could "take the pace and survive," the people who benefited most from his ideas were those influenced at a distance, particularly American, Japanese, and Russian mathematicians. Pierre Cartier sees Grothendieck's heritage in the work of such Russian mathematicians as Vladimir Drinfeld, Maxim Kontsevich, Yuri Manin, and Vladimir Voevodsky. Said Cartier, "They capture the true spirit of Grothendieck, but they are able to combine it with other things."

Photographs used in this article are courtesy of Friedrich Hirzebruch, Karin Tate, and the website of the Grothendieck Circle (`http://www.grothendieck-circle.org`).

The second part of this article will appear in the next issue of the Notices.

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