Theoretical and Experimental Reflection Coefficients in Flexible Tubes as a Function of the Mach Number

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Abstract

The standard formulation of Wave Intensity Analysis (WIA) assumes that the flow velocity (U) in the conduit is << the velocity of propagation of waves (c) in the system, and Mach number, M=U/c, is negligible. However, in the large conduit arteries, U is relatively high due to ventricular contraction and c is relatively low due to the large compliance; thus M is > 0, and may not be ignored. Therefore, the aim of this study is to identify experimentally the relationship between M and the reflection coefficient in vitro.

Combinations of flexible tubes, of 2 m in length with isotropic and uniform circular cross sectional area along their longitudinal axes, were used to present mother and daughter tubes to produce a range of reflection coefficients. An approximately semi-sinusoidal pulse was generated at the inlet of the mother tube using a syringe pump, first in the condition of initial velocity, $U_0=0$, and when $U_0>0$ with steady flow to superimpose the pulse.

Pressure (P) and Velocity (U) were measured in the mother tube, wave speed was determined using the foot to foot and PU-loops methods. The theoretical reflection coefficient, R_t at M=0, has been compared to the experimental reflection coefficient, R at M>0, which was determined as dP-/dP+ as calculated using WIA.

The function R(M) changes significantly with the geometrical and mechanical features of the connected tubes. In our experiments, R increased significantly with small values of M. In the range of M=0-0.02, R increased by 4-36%. Therefore, we conclude that M significantly affects the magnitude of reflections.

1. Introduction

Arterial reflected waves play an important role in determining the features of the arterial pressure waveforms, including systolic pressure (1, 2). Also, arterial aging and pathologies (e.g. hypertension) affect both the magnitude and timing of arterial reflected waves and, therefore, the features of the pressure waveform (3, 4).

The magnitude of reflected waves can be quantified by means of the reflection coefficient; i.e. the ratio between the backward to forward components of the pressure waveform or of the wave intensity. This parameter is normally assumed to be independent of the mean value of blood velocity in the artery. In fact, to derive the standard WIA formulation the blood velocity is assumed to be << wave speed in arteries (M≈0), and, therefore, the convective component is neglected (5). While this assumption maybe reasonable in a considerable portion of the arterial tree, large conduit arteries, such as the aorta and pulmonary artery, present relatively low wall stiffness and relatively high blood velocity, neglecting the convective term may induce errors in the estimation of wave reflections. Therefore, the aim of the present work is to determine the effect of M on the reflection coefficient in flexible tubes *in vitro*.

2. Methods

2.1. Experimental setup

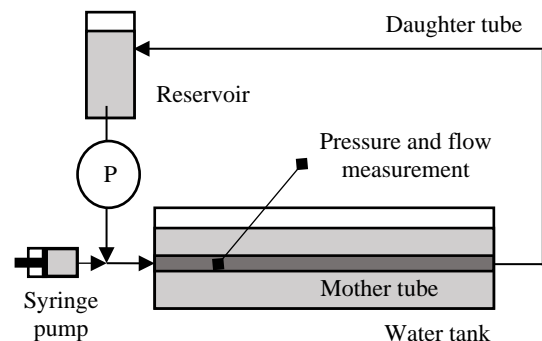The experimental setup is shown in Fig. 1.

Figure 1 - Schematic representation of the experimental setup. Pressure was recorded using a transducer tipped catheter, and flow using an ultrasound probe. All the components were located on a horizontal plane.

2.1.1. Tubes

We used five 2 meters long tubes as mother tubes and four > 9 meters long tubes of different size and material as daughter tubes. The length of the daughter tube was necessary to isolate the reflection generated at the connection between mother and daughter tubes from that generated at the connection between daughter tube and reservoir. Mechanical properties of the tubes are included in Table 1, and the geometrical and mechanical properties of each tube were uniform along its longitudinal axis. c has been determined by averaging the results obtained with the foot-to-foot and PU-loop techniques.

The connection between mother and daughter tubes was realized overlapping them of approx. 1-2 cm. If the difference in diameter was too high, a third tube was inserted between mother and daughter to allow the connection.

Table 1 - Properties of the tubes used in the experiment, D_{in}: internal diameter, h: wall thickness, c: waves speed.

D_{in} [mm]	h [mm]	Material	c [m/s]
8	2	Silicone	25.02
9.525	0.8	Latex	12.70
10	1	Silicone	21.52
17	1.5	Rubber	20.76
19.05	2.4	Latex	14.17
20.6	1.7	Rubber	20.38

2.1.2. Pumps

Two pumps were used for this experiment: a piston pump, producing an approximately single semi-sinusoidal pulse, and a centrifugal pump providing a continuous flow. The speed of the centrifugal pump was adjusted to vary the flow; needed to provide a variation of M.

2.1.3. Measurements

Simultaneous pressure and flow were measured at the same axial location (52 cm from the inlet of the mother tube). Pressure was measured using a 6F pressure transducer tipped catheter (Millar, Inc, Huston, Texas, USA), and flow was measured using ultrasound flow probes (Transonic System, NY, USA). Data were sampled at 500 Hz.

2.2.   Theoretical reflection coefficient

When M≈0, the theoretical reflection coefficient, R_t, characterizing the connection between the mother and daughter tubes is calculated as follows:

$$R_t = \frac{\frac{A_1}{c_1} - \frac{A_2}{c_2}}{\frac{A_1}{c_1} + \frac{A_2}{c_2}} \qquad (1)$$

where A and c are the internal sectional areas and wave speeds of the tubes, respectively, and the subscripts (1, 2) indicate the mother and daughter tubes, respectively.

2.3.   Experimental reflection coefficient

The standard formulation of WIA was used to separate the pressure and velocity waveforms in their forward and backward components (6). The experimental reflection coefficient (R) was calculated as the ratio of the peak of the backward to forward component of the separated pressure waveform:

$$R = \frac{dP_-}{dP_+} \qquad (2)$$

2.3.   Data analysis

For each connection configuration, pressure and velocity waveforms were sampled and the data used in the calculations are the average of 7 recordings. The rotational speed of the centrifugal pump was varied between 0 and the maximum flow (which depended on the resistance provided by the mother and daughter tubes connected downstream). The whole range of flows was divided into 6-8 steps and P and U recordings were performed for each step.

Collected data were analyzed with a Matlab code (The Mathworks, Natick, MA, USA). The value of the reflection coefficient for each M was calculated averaging the results obtained in the 7 repeated measurements.

3.   Results

3.1.   Reflection coefficient at M=0

Table 2 shows a comparison between R_t calculated with (1) and R_0 (R when M=0) determined experimentally for each connection configuration used in the present work.

3.2.   R as a function of M

With all the connection configurations, an increase of M resulted in an increase of R (Figure 2). This was found for both negative and positive R_t, with a strong correlation between R and M. A fitting of R(M) provided an $R^2>0.92$ (linear fitting). Interestingly, the percentage of increase of R at a given M was different in different connection configurations as indicated by the slope (Table 2). For example, at M≈0.02, R increased by 50% in connection D, by 30% in H and by 10% in connection F.

4.   Discussion

The results showed a good agreement between the theoretical and the experimental reflection coefficients at

Connection	Mother $\varnothing$ [mm]	Daughter $\varnothing$ [mm]	R_t	R_0	Slope (95% C.I.)
A	20.6	8	0.78	0.696 ±0.013	8.7 – 25.9
B	17	8	0.697	0.694 ±0.010	12.3 – 14.0
C	19.05	8	0.817	0.761 ±0.011	10.9 – 16.3
D	10	8	0.290	0.285 ±0.005	5.6 – 8.8
E	17	10	0.499	0.505 ±0.004	6.3 – 8.2
F	19.05	19.6	0.105	0.076 ±0.011	2.8 – 4.5
G	9.525	10	0.212	0.206 ±0.006	2.1 – 2.7
H	9.525	8	0.473	0.531 ±0.008	2.2 – 3.4
I	10	19.6	-0.635	-0.605 ±0.009	2.1 – 3.4
J	10	17	-0.499	-0.465 ±0.008	2.1 – 3.7

Table 2 - Comparison between the theoretical reflection coefficient R_t and the experimentally determined R_0 at M=0. R_0 is expressed as mean ± SD.
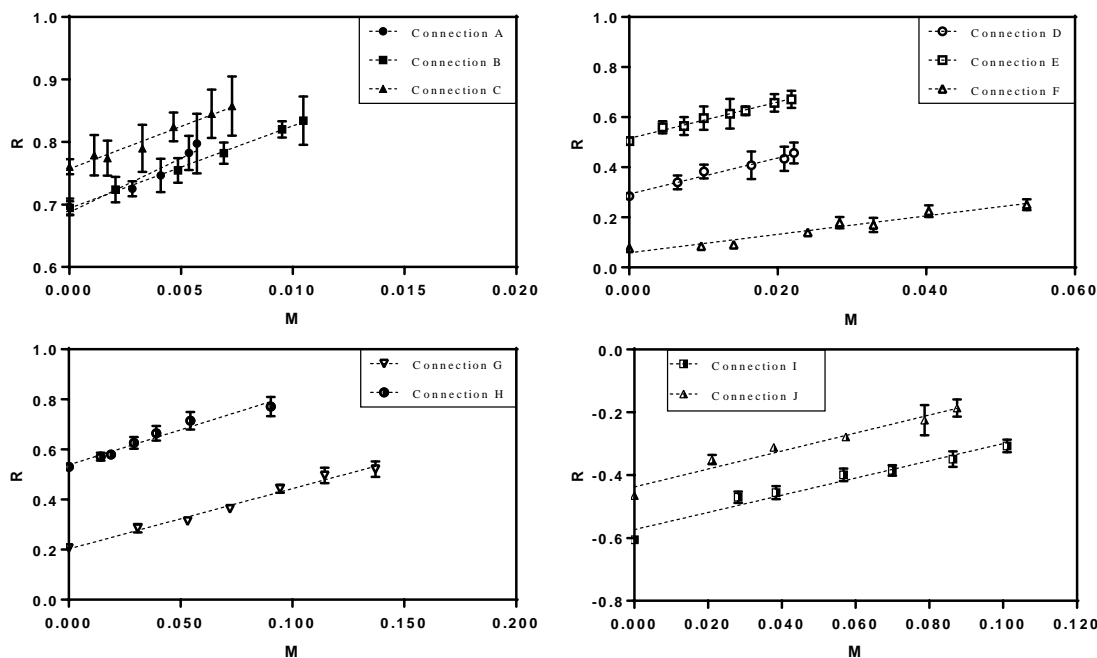

Figure 2 - R as a function of M for all the connection described in table 2. Data are presented as mean and error bars indicate SD.

initial U=0. The largest error was found for connection A (R_t=0.78 and R_0=0.70). The theoretical reflection coefficient assumes an ideal connection between tubes with no overlapping. On the other hand, the real connection between tubes may have introduced a deviation of R_0 from R_t.

All the connections used in this study showed an increase in R with increasing M. However, this finding has different implications for positive and negative reflections. In the case of positive reflections, the superimposed steady state flow enhances the discontinuity given by the connection between mother and daughter tubes. On the other hand, the amplitude of negative reflections is decreased by increasing M, with R approaching 0. Therefore, positive and negative reflections present an opposite behavior with increasing M.

Interestingly, the slope of the linear fitting was not equal for all the connections. It seems the slopes depends on the relative geometrical and mechanical properties of the connected tubes. These results indicate that higher values R_0 produce higher slopes of R as a function of M. However, connection H shows a significantly lower slope with respect to D and E but has similar R_0, and comparable to connection F and G (R≤0.2 for M=0). Therefore, other elements seem to play a role in determining the trend of R as a function of M. For example, connection A, B, C, D, and E were made between silicone or rubber tubes having similar values of wall stiffness (c=20-25 m/s), while connection F, G, and H were between latex and silicone/rubber, with latex being significantly more compliant (c=12-15 m/s). This could possibly explain the difference between H and D-E. The slopes of the normalized reflection coefficient (R/R_0) have also been compared. A significant difference was found between all the connections. Figure 3 (top), a different slope can be identified between connection A and E even for very low values of M ($\approx 10^{-2}$).
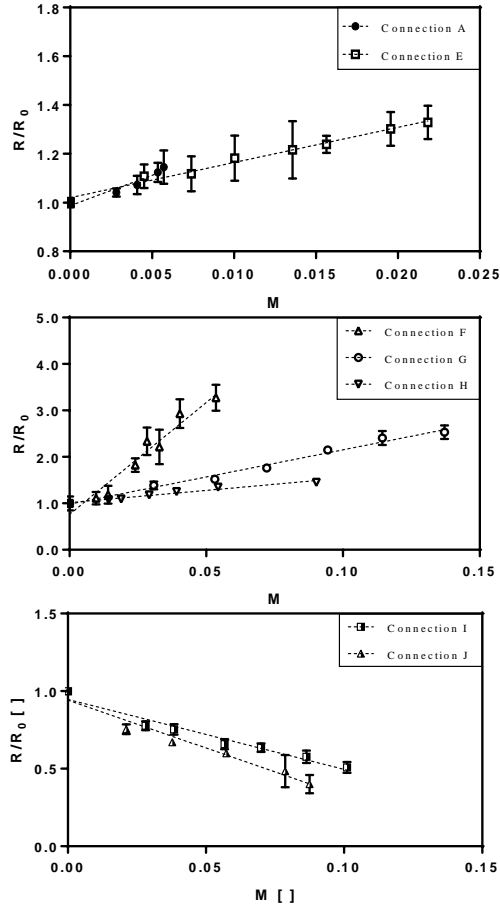
Figure 3 - Normalized reflection coefficient (R/R_0) as a function of M. Data are presented as mean (SD).

It is worth considering that R is bounded between -1 and 1 by definition, since $P_i = P_t - P_r$, where P_i is the incident pressure wave, P_t is the transmitted pressure wave and P_r is the reflected pressure wave. Therefore R(M) is expected to reach a plateau at high values of M. However, this trend was not shown in the range of M used in our results, except for negative reflections. When fitting the results with 2nd order polynomial functions, the sign of the quadratic term was negative only in 4 of the connection configurations with positive R.

The dependence of R on M becomes relevant when studying wave propagation in large conduit arteries. In the main pulmonary artery, M is significantly higher in healthy subjects than in hypertensive (0.071 ± 0.045 and 0.023 ± 0.019, respectively) due to the pathological arterial stiffening (using data collected in (7)). Therefore, when comparing the two groups, the experimental R may not completely describe the remodelling process caused by hypertension and the effect of M has also to be taken into account.

One of the limitations of this work is the lack of high values of M. This might have been due to the needed lengthy daughter tube, which introduced a high hydraulic resistance, limiting the flow rate produced by the continuous flow pump.

5.   Conclusion

In our experiments, R is dependent on M, and a significant alteration of R can be obtained for M <0.05. Moreover, the function R(M) seems to be dependent on the relative geometrical and mechanical properties of the connected mother and daughter tubes. This could be relevant when considering waves reflections in large conduit arteries and the alteration of R with arterial pathologies.

More work is needed to define a mathematical formulation to describe the dependency of R from M and to explain the different behavior of different types of connections.

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