MEE 10:99

On The Performance Of Two-Way Amplify-And-Forward Relay Networks

Hoang Le Nam

This thesis is presented as part of Degree of Master of Sciences in Electrical Engineering with emphasis on Radio Communications

Blekinge Institute of Technology
October 2010

School of Engineering
Department of Electrical Engineering
Blekinge Institute of Technology, Sweden
Supervisors: Trung Q. Duong and Prof. Hans-Jürgen Zepernick
Examiner: Prof. Hans-Jürgen Zepernick

Contact Information:

Author:

Hoang Le Nam
email: lnho08@student.bth.se

Supervisors:

Trung Q. Duong
Radio Communications Group (RCG)
School of Engineering, BTH
Blekinge Institute of Technology, Sweden
email: quang.trung.duong@bth.se

Prof. Hans-Jürgen Zepernick
Radio Communications Group (RCG)
School of Engineering, BTH
Blekinge Institute of Technology, Sweden
email: hans-jurgen.zepernick@bth.se

Examiner:

Prof. Hans-Jürgen Zepernick
Radio Communications Group (RCG)
School of Engineering, BTH
Blekinge Institute of Technology, Sweden
email: hans-jurgen.zepernick@bth.se

Abstract

The new paradigm of cooperative communications is a promising resolution to carry out MIMO technique. In this thesis work, we study the performance of cooperative relay networks in which the transmission from a source to a destination is assisted by one or several relaying nodes employing amplify-and-forward (AF) and decode-and-forward (DF) protocols. The performance of two-way (or bi-directional) AF relay networks, which are proposed to avoid the pre-log factor $\frac{1}{2}$ of spectral efficiency, is latter investigated. Specifically, the exact closed-form expressions for symbol error rate (SER), outage probability, and average sum-rate of bi-directional AF relay systems in independent but not identically distributed (i.n.i.d.) Rayleigh fading channels are derived. Our analyses are verified by comparing with the results from Monte-Carlo simulations.

Acknowledgements

First of all, I would like to express my deep gratitude to Prof. Hans Jürgen Zepernick for providing me such opportunity to work with talented people in Radio Communications Group.

I would especially like to thank my supervisor Duong Quang Trung for his enthusiastic guidance and support. His profound knowledge and experience in the research fields of wireless communications as well as his encouragement makes him a fantastic mentor.

I am most thankful to my parents and my sister for always loving and believing in me. Their endless loves for me are the most precious treasure during the course of my life.

Hoang Le Nam
2010, Sweden

Contents

Chapter 1

Introduction

Over the last two decades, the proliferation in the use of internet as well as wireless services has conducted an unprecedent technological evolution in the communications industry. Nowadays cell phones, pocket PCs, and laptops become more essential in modern life. However, such services as wireless broadband internet, mobile multimedia, gaming, and many other applications have tremendous demands on higher data rates, increased battery life, and more efficient transmission links but guaranteed quality of service (QoS).

Unlike the established wired networks which are easy to generate duplicated signals by deploying several wired links between any two nodes, the scarce resource of power and spectrum have been considering as the major challenges in wireless communications. The ground for the limited spectrum is that once it is exploited it cannot be overlapped. In addition, the portable wireless devices cannot always be adhered to power outlets as in wired system, which in turn causes the limitation on power utilization.

Furthermore, due to the broadcasting characteristic of wireless system and the physical phenomena such as diffraction, reflection, and attenuation, the signals transmitted between transceivers also experience enormous random fluctuations and thus make it challenging to design reliable networks. Recently, the excogitation of MIMO (Multi-Input Multi-Output) technology has contributed the most important resolution to these confrontations.

The theoretical idea of MIMO systems, i.e., deploying multiple antennas at the transceivers, offers a higher capacity and increases the quality and coverage without raising requirements on bandwidth and energy [1, 2]. By shaping and combining multiple paths of transmitted signals, it has been shown the magnificent advantage of MIMO, e.g., as comparing the spectral efficiency of up to 42 bps/Hz using VBLAST (Vertical Bell Laboratories Layered Space-time) with 3 bps/Hz in contemporary WLANs (Wireless Local Area Networks) [3]. Nevertheless, the employment of MIMO is only remarkable in high scattering environment [4] as well as the mobile portable devices themselves cannot attach many antennas to form MIMO channels.

In order to overcome such drawbacks of MIMO technology, cooperative communications have recently been proposed [5–9]. The key implication of cooperative prototype is the view of wireless nodes as a set of distributed antennas. Accordingly, the transmission and processing at terminals are executed in a distributed fashion, i.e., the communications between any source and destination nodes are assisted by several relay nodes. Theoretically, a set of relays is deployed to support the signal transmission in two separate phases. Firstly, the source broadcasts to the destination and the relays and during the second phase, signals after being processed will be relayed to the destination. Whereas the relays are far from the source terminal, the relaying and direct links can be considered as fading independently. In other words, the transmission between the source and destination nodes is through a full-rank MIMO channel. Moreover, by appropriately processing at the terminals, the information after being relayed can be combined at the destination to achieve a spatial diversity gain. Consequently, cooperative communication is a promising technique that can offer improved capacity, reliable transmission with saving power as well as expanded coverage areas.

The idea of using relay to support information transmission has been proposed early in the 1970's [5] and the capacity for the general relay network over Gaussian channel has been investigated later on [6]. Recently, it has drawn a great attention in research literature on cooperative techniques for fading environment [7–11]. The performance on multi-hop transmission via relays has been extensively considered in [12]. Furthermore, the works in [13,14] have provided not only the performance analysis but also the comparison on different cooperative protocols for the case of single and multiple relays.

The deployment of relays on the one hand promises much advance but on the other hand yields system more complicated as well as more interference between users. One of the resolutions for generating orthogonal channels is to employ time-division duplex or half-duplex protocol [9–11]. However, conventional half-duplex relay systems suffer a half of spectral efficiency as compared to the full-duplex protocol especially in high SNR regimes since the relay terminals can not transmit and receive at the same time.

A new class of wireless relay networks that increases the low capacity is two-way (or bi-directional) relay networking [15–19]. Dissimilar from the former half-duplex relaying protocols, in bi-directional relay networks both source and destination terminals transmit to the relay simultaneously in the first phase then the received signal will be forwarded to the source and the destination in the second phase. Although this modified protocol still has the pre-log factor of $\frac{1}{2}$, it considerably provides an improved total sum-rate since the communications between the two terminals take place in the same physical channel.

This thesis work is outlined as follows. Chapter 2 proposes the background as well as the commonly used protocols in cooperative communications. Chapter 3 represents the performance as well as comparison on cooperative gains between different protocols in three-terminal network. Moreover, Chapter 4 extends the analysis in Chapter 3 to the network including multiple relays. In Chapter 5, the idea of two-way amplify-and-forward networks and also the analyses on several important performance metrics such as average symbol error rate (SER), ergodic capacity and outage probability are introduced. Part of this chapter is published as [20]. Finally, Chapter 6 concludes the thesis works.

Chapter 2

Background on Cooperative Communications

2.1 A Prelude to Relaying

A typical MIMO communications from a transmitter T_x to a receiver R_x can be modeled as shown in Fig. 2.1 by forming physical arrays of antennas at the transceivers. Assuming the transmitter and receiver employs M_T and M_R antennas respectively, the transmit messages over M_T antennas and T_s time slots can be expressed as

$$\mathbf{X} = \begin{bmatrix} x_1^1 & x_2^1 & \cdots & x_{T_s}^1 \\ x_1^2 & x_2^2 & \cdots & x_{T_s}^2 \\ \vdots & \vdots & \ddots & \vdots \\ x_1^{M_T} & x_2^{M_T} & \cdots & x_{T_s}^{M_T} \end{bmatrix} \equiv \{x_t^n \ : \ n = 1, 2, \ldots, M_T\} \tag{2.1}$$

where x_t^n denotes the transmit message at discrete time $t,\ t = 1, 2, \ldots, T_s$ by antenna $n,\ n = 1, 2, \ldots, M_T$.

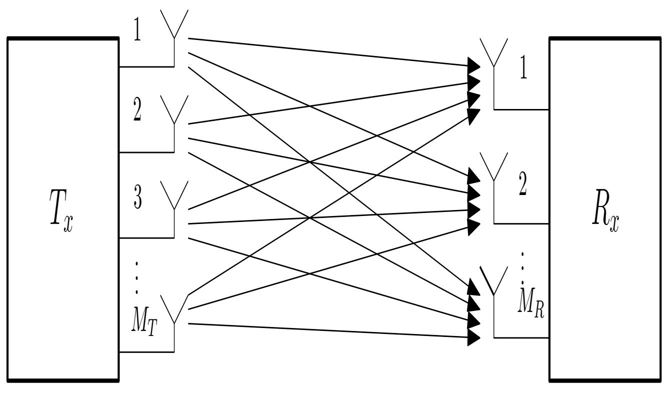

Figure 2.1: Typical MIMO physical arrays system.

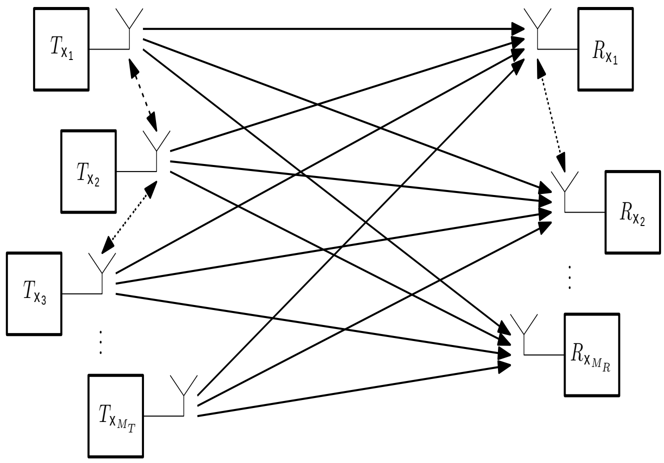

Figure 2.2: Equivalent multi-user virtual arrays system.

At the receiver, antenna m gets the respective message at time slot t

$$y_t^m = \sum_{n=1}^{M_T} h_{mn} x_t^n + z_t^m, \quad t = 1, 2, \ldots, T_s \tag{2.2}$$

where h_{mn} is the channel coefficient between receive antenna m and transmit antenna n and z_t^m the complex additive Gaussian noise at the receive antenna n at time t. It can be conveniently expressed in a more compact form as

$$\mathbf{Y} = \mathbf{HX} + \mathbf{Z} \tag{2.3}$$

where $\mathbf{Y} = \{y_t^m \,:\, 1 \leq t \leq T_s,\, 1 \leq m \leq M_R\}$ is the received symbols matrix of size $M_R \times T_s$, $\mathbf{H} = \{h_{mn} \,:\, 1 \leq m \leq M_R,\, 1 \leq n \leq M_T\}$ the channel amplitude gain matrix of size $M_R \times M_T$, and $\mathbf{Z} = \{z_t^m \,:\, 1 \leq t \leq T_s,\, 1 \leq m \leq M_R\}$ the additive white Gaussian noise (AWGN) matrix of size $M_R \times T_s$.

However, the terminals do have a limited capability for integrating a large number of antennas. Hence, by cooperating and efficiently sharing multiple users' antennas, an equivalent virtual array can be organized as in Fig. 2.2. In general, the issues in multi-antenna array system are arisen and handled at the physical layer, meanwhile there are multifaceted problems in virtual array systems

The molecule of a virtual array system is the three-terminal network as shown in Fig. 2.3, in which the information messages originated at the source node S travel to the destination D via the help of the relay R. Basically, the end-to-end transmission occurs in two disjointed phases: firstly, the broadcast phase, i.e., the source S transmits while both R and D receive; and secondly, multiple-access phase, i.e., (S, R) transmit while D receives. Conceptually, there exists four different types of relaying regimes deduced from the above two phases:

1. (1) $S \rightarrow (R, D)$ and (2) $(S, R) \rightarrow D$: the most general form;

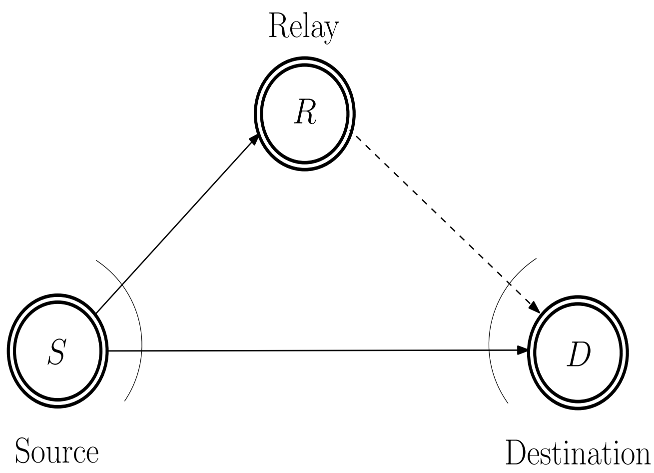

Figure 2.3: Canonical three-terminal cooperative system.

2. (1) $S \rightarrow R$ and (2) $(S, R) \rightarrow D$: D ignores the first phase signal;
3. (1) $S \rightarrow (R, D)$ and (2) $R \rightarrow D$: S does not transmit in the second phase;
4. (1) $S \rightarrow R$ and (2) $R \rightarrow D$: multi-hop transmission.

However, only the third and fourth approaches are usually employed since those regimes are easy to create orthogonal channels. Although multi-hop regime is widely used today, it does not produce diversity gain and it is mainly exploited to deal with the signal attenuation in long-range transmission.

2.2 Receiver Diversity

After receiving deformed copies of the original message, it is required to implement combining techniques at the destination so that the fading effects are extenuated. It can be done by various ways which are different in complexity and performance.

Selection Combining (SC): The receiver selects the path which has the best signal-to-noise ratio (SNR) among all distorted receipts.

Maximal Ratio Combining (MRC): The received fading paths are linearly weighted so that the outcome signal attains the maximum SNR.

Equal Gain Combining (EGC): Each branch is combined with the same weighting factor $w_m = e^{-j\Phi_m}$.

In addition, the performance of a specific receiver diversity technique can be evaluated by

Array gain: The average combined SNR to average path SNR ratio

$$G_a = \frac{\bar{\gamma}_{\text{comb}}}{\bar{\gamma}}. \tag{2.4}$$

Diversity gain: The decrease rate of the average error probability slope

$$G_d = -\frac{\log \bar{P}}{\log \bar{\gamma}}. \tag{2.5}$$

Moreover, in some diversity scheme, the average error probability can be written in the form $\bar{P} = A\bar{\gamma}^{-M}$ in which A is a constant depending on the specific modulation and coding, and M is called the diversity order. An M-antenna system which has the diversity order of M is defined to obtain full diversity order.

2.2.1 Diversity through SC

To demonstrate the implementation of SC, let us assume that M_R received channels are independent identically distributed (i.i.d.) Rayleigh fading with unit energy and the AWGN at each antenna is equal, i.e., each channel has the same signal-to-noise ratio (SNR) of γ. The algorithm is now simplified to compare the amplitude channel gain A_m for $m = 1, 2, \ldots, M_R$ and choose the largest $A_{\max} = \max\{A_m\}$.

The probability distribution function (PDF) of each amplitude channel coefficient is given by

$$p_{A_m}(a_m) = 2a_m\, e^{-a_m^2} \tag{2.6}$$

and the respective cumulative distribution function (CDF) of A_m is then

$$F_{A_m}(a_m) = P\{A_m < a_m\} = 1 - e^{-a_m^2} \tag{2.7}$$

The CDF of $A_{\max}$ is defined by the probability that $A_{\max}$ is below a certain threshold a as

$$F_{A_{\max}}(a) = P\{A_{\max} < a\} = P\{A_1, A_2, \ldots, A_{M_R} < a\} = \left[1 - e^{-a^2}\right]^{M_R} \tag{2.8}$$

Consequently, resulting in the corresponding PDF of $A_{\max}$ is deduced by employing derivation of $F_{A_{\max}}(a)$

$$p_{A_{\max}}(a) = 2M_R\, a\, e^{-a^2}\left[1 - e^{-a^2}\right]^{M_R-1} \tag{2.9}$$

For SC, the attained average SNR is thus

$$\gamma_{\mathrm{SC}} = \int_0^{\infty} \gamma a^2 p_{A_{\max}}(a)da = \gamma \sum_{m=1}^{M_R} \frac{1}{m} \tag{2.10}$$

As observed in (2.10), the technique of SC obtains an array gain of $\sum_{m=1}^{M_R} \frac{1}{m}$.

2.2.2 Diversity through MRC

For simplicity, it is supposed that each fading channel at the receiver is independent with unit energy and can be performed as $h_m = |h_m| e^{j\Phi_m}$, and also the AWGN at each antenna $n_m \sim \mathcal{CN}(0, \sigma^2)$. The output of a maximal ratio combiner is the linear combination of the M_R received paths

$$\begin{aligned} y_{\text{MRC}} &= \sum_{m=1}^{M_R} w_m y_m \\ &= \sum_{m=1}^{M_R} w_m h_m x + n'_m \end{aligned} \tag{2.11}$$

in which x is the transmit signal with power P, w_m the weighting factors to the receive channel m, and

$$n'_m = \sum_{m=1}^{M_R} w_m n_m \tag{2.12}$$

the AWGN with variance $\sigma^2 \sum\limits_{m=1}^{M_R} |w_m|^2$. Therefore, the output instantaneous SNR of the combiner is given by

$$\gamma_{\text{MRC}} = \frac{P \left| \sum\limits_{m=1}^{M_R} w_m h_m \right|^2}{\sigma^2 \sum\limits_{m=1}^{M_R} |w_m|^2} \tag{2.13}$$

By using the Cauchy-Schwarz inequality, we have

$$\left| \sum_{m=1}^{M_R} w_m h_m \right|^2 \leq \left(\sum_{m=1}^{M_R} |w_m|^2 \right) \left(\sum_{m=1}^{M_R} |h_m|^2 \right) \tag{2.14}$$

The instantaneous SNR γ_{MRC} is maximum when setting $w_m = h_m^* / \sigma$ with h_m^* is the conjugate of h_m. Substituting to (2.13) we get

$$\gamma_{\text{MRC}} = \gamma \sum_{m=1}^{M_R} |h_m|^2 \tag{2.15}$$

The average output SNR is thereby determined by

$$\bar{\gamma}_{\text{MRC}} = M_R \gamma \tag{2.16}$$

The result in (2.16) shows that the average output SNR of the maximal ratio combiner is equivalent to the sum of all separate paths' average SNR.

2.2.3 Equal Gain Combining

To employ MRC technique, knowledge on the instantaneous SNR is required which is hard to quantify. In the simpler technique of EGC, the weighting gain is fixed as $w_m = e^{-j\Phi_m}$ so that the average SNR of combining output is given by

$$\begin{aligned}\bar{\gamma}_{\text{EGC}} &= \frac{\gamma}{M_R}\mathbb{E}\left\{\left(\sum_{m=1}^{M_R}|h_m|\right)^2\right\}\\ &= \frac{\gamma}{M_R}\left(\mathbb{E}\left\{\sum_{m=1}^{M_R}|h_m|^2\right\}+\sum_{m_1=1}^{M_R}\sum_{\substack{m_2=1\\m_2\neq m_1}}^{M_R}\mathbb{E}\left\{|h_{m_1}|\right\}\mathbb{E}\left\{|h_{m_2|}\right\}\right)\end{aligned} \tag{2.17}$$

Assuming each channel is Rayleigh fading with unit energy, we have

$$\mathbb{E}\left\{|h_{m_i}|\right\} = \frac{1}{2}\sqrt{\frac{\pi}{2}} \tag{2.18}$$

with $i = 1, 2$. Therefore, $\bar{\gamma}_{\text{EGC}}$ can be determined by

$$\bar{\gamma}_{\text{EGC}} = \frac{\gamma}{M_R}\left(M_R + M_R(M_R-1)\frac{\pi}{4}\right) = \gamma\left(1+(M_R-1)\frac{\pi}{4}\right) \tag{2.19}$$

2.3 Half-Duplex and Full-Duplex Relaying

Since each transmitter knows its own transmitted messages, it is theoretically able to operate in full-duplex protocol, i.e., terminals simultaneously transmit and receive in the same frequency band. However, the difficulty of precise interference removal in current radio manipulation prevents such operation. Despite the early researches on theoretic full-duplex relaying [5, 6], most recent practical relaying protocols are based on half-duplex operation. Such scheme forbids any node to transmit and receive at the same time in the same frequency channel. In other words, transmissions and receptions at the terminals occur in orthogonal channels. Fig. 2.4 shows a typical example of a relay network operating in time-division (TD) manner. The frequency-division (FD) approach is quite similar.

2.4 Relaying Strategies

Since the capacity region of general interference channel is not known even today, relay channel's capacity is still an open issue. Consequently, there is no relaying strategy judged as the best. In general, cooperative protocols are sorted by fixed and adaptive relaying.

In fixed relaying, the classifying criterion is the processing at the relays. When an information symbol received at a relay is processed through amplify-and-forward (AF)

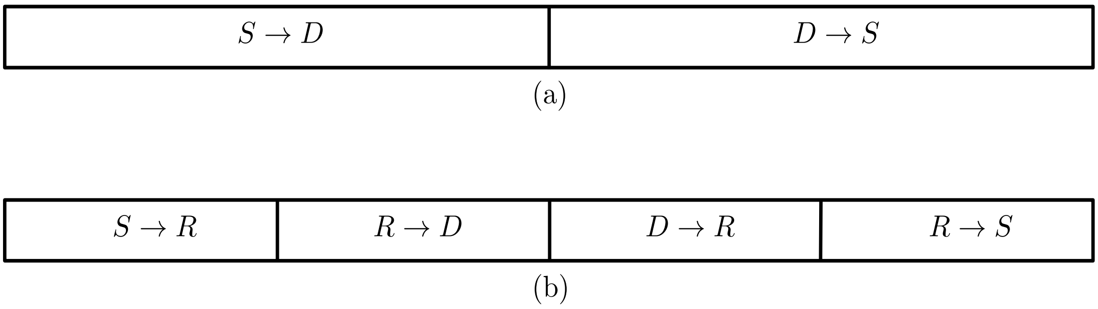

Figure 2.4: Example of TD half-duplex for (a) direct communications, (b) multi-hop communications.

scheme, it is simply amplified depending on available average transmit power of the relay before forwarded to the destination node in the next phase. On the other hand, the employment of fixed decode-and-forward (DF) strategy requires the relay firstly attempt to decode its received signal, then re-encode it and forward the decoded one to the destination.

Despite the advantage of easy execution, fixed relaying protocols suffer a loss of spectral efficiency; especially in the situation of reliable direct source-destination channel, the usage of relays would be wasted. With the aim of overcoming such drawback, adaptive DF relaying schemes have been proposed. The key idea behind that protocol is that when the relay cannot decode its received signal, it will not forward to the destination.

In order to form the orthogonal channels as well as diversity gain, the system employs the third scheme of cooperation in Section 2.1, i.e., the source broadcasts to both the relay and destination in the first phase and after processing the relay forwards the message to the destination.

2.4.1 AF Protocol

In the beginning phase, the signal received at both the relay and the destination can be represented respectively by

$$y_{\mathrm{R}} = h_{\mathrm{SR}} x + n_{\mathrm{R}}, \tag{2.20}$$

$$y_{\mathrm{SD}} = h_{\mathrm{SD}} x + n_{\mathrm{D}}, \tag{2.21}$$

in which x is the transmitted symbol with power constraint $\mathbb{E}\{x\} \leq P_{\mathrm{S}}$ and $n_{\mathrm{R}} \sim \mathcal{CN}(0, \sigma_{\mathrm{R}}^2)$, $n_{\mathrm{D}} \sim \mathcal{CN}$ (0, σ_{D}^2) the AWGN at the relay and destination, respectively. Moreover, in (2.20) and (2.21), h_{SR} and h_{SD} are the respective $S \to R$ and $S \to D$ channel fading coefficients. To satisfy the constraint on relay power P_{R}, the relay gains its received message by the scaling factor

$$\beta = \sqrt{\frac{P_{\mathrm{R}}}{P_{\mathrm{S}} |h_{\mathrm{SR}}|^2 + \sigma_{\mathrm{R}}^2}} \tag{2.22}$$

Hence the symbol received at the destination in the second phase is determined by

$$\begin{aligned} y_{\mathrm{RD}} &= h_{\mathrm{RD}}\beta y_R + n_{\mathrm{D}} \\ &= \sqrt{\frac{P_{\mathrm{R}}}{P_{\mathrm{S}}|h_{\mathrm{SR}}|^2 + \sigma_{\mathrm{D}}^2}} h_{\mathrm{SR}} h_{\mathrm{RD}} x + n'_{\mathrm{D}} \end{aligned} \tag{2.23}$$

where h_{RD} is the channel coefficient between the relay and the destination, and

$$n'_{\mathrm{D}} = \sqrt{\frac{P_{\mathrm{R}}}{P_{\mathrm{S}}|h_{\mathrm{SR}}|^2 + \sigma_{\mathrm{D}}^2}} h_{\mathrm{RD}} n_{\mathrm{R}} + n_{\mathrm{D}} \tag{2.24}$$

is also an AWGN with variance $\frac{P_{\mathrm{R}}|h_{\mathrm{RD}}|^2\sigma_{\mathrm{R}}^2}{P_{\mathrm{S}}|h_{\mathrm{SR}|^2}+\sigma_{\mathrm{D}}^2} + \sigma_{\mathrm{D}}^2$.

It is observed that the destination receives two copies of the original message hence the diversity techniques as discussed in Section 2.2 can be exploited . By MRC, the output of the combiner is given by

$$y_D = w_{\mathrm{SD}} y_{\mathrm{SD}} + w_{\mathrm{RD}} y_{\mathrm{RD}} \tag{2.25}$$

With perfect knowledge of h_{SR} and h_{SD}, the weighting factors w_{SD} and w_{RD} are chosen to maximize the resulting SNR as

$$w_{\mathrm{SD}} = \frac{h_{\mathrm{SD}}^*}{\sigma_{\mathrm{D}}^2} \quad \text{and} \quad w_{\mathrm{RD}} = \frac{\sqrt{\frac{P_{\mathrm{R}}}{P_{\mathrm{S}}|h_{\mathrm{SR}}|^2+\sigma_{\mathrm{D}}^2}} h_{\mathrm{SR}}^* h_{\mathrm{RD}}^*}{\frac{P_{\mathrm{R}}|h_{\mathrm{RD}}|^2\sigma_{\mathrm{R}}^2}{P_{\mathrm{S}}|h_{\mathrm{SR}}|^2+\sigma_{\mathrm{D}}^2} + \sigma_{\mathrm{D}}^2} \tag{2.26}$$

As in information theoretic view, the AF scheme performs a typical multiple-access channel, hence the instantaneous mutual information of the system is determined by

$$\mathcal{I}_{\mathrm{AF}} = \frac{1}{2}\log(1 + \gamma_{\mathrm{SD}} + \gamma_{\mathrm{RD}}) \tag{2.27}$$

where γ_{SD} and γ_{RD} are the instantaneous SNR of the $S \to D$ and $R \to D$ links, respectively, and are given by

$$\gamma_{\mathrm{SD}}^{\mathrm{AF}} = \frac{P_{\mathrm{S}}\left|h_{\mathrm{SD}}\right|^2}{\sigma_{\mathrm{D}}^2} \tag{2.28}$$

$$\gamma_{\mathrm{RD}}^{\mathrm{AF}} = \frac{P_{\mathrm{S}}\left|h_{\mathrm{RD}}\beta h_{\mathrm{SR}}\right|^2}{\left|h_{\mathrm{RD}}\beta\right|^2\sigma_{\mathrm{R}}^2 + \sigma_{\mathrm{D}}^2} = q\left(\frac{P_{\mathrm{S}}\left|h_{\mathrm{SR}}\right|^2}{\sigma_{\mathrm{R}}^2}, \frac{P_{\mathrm{R}}\left|h_{\mathrm{RD}}\right|^2}{\sigma_{\mathrm{D}}^2}\right) \tag{2.29}$$

with

$$q(x,y) = \frac{xy}{x+y+1}. \tag{2.30}$$

Since the relay operation does not contain decoding and encoding, the AF strategy has lower demand on processing hardware than the upcoming fixed decode-and-forward (FDF) and adaptive decode-and-forward (ADF) schemes.

2.4.2 FDF Protocol

In this scheme, the messages received at the relay and destination in the first phase is the same as (2.20) and (2.21). However, the second phase transmit signal is the re-encoded $\hat{x}$ of the original x. The received message at the destination in phase two is now

$$y_{\mathrm{RD}} = h_{\mathrm{RD}}\hat{x} + n_{\mathrm{D}} \tag{2.31}$$

The instantaneous mutual information of such protocol is therefore [6] and [9]

$$\mathcal{I}_{\mathrm{FDF}} = \frac{1}{2}\min\left\{\log\left(1+\gamma_{\mathrm{SR}}^{\mathrm{DF}}\right), \log\left(1+\gamma_{\mathrm{SR}}^{\mathrm{DF}}+\gamma_{\mathrm{SR}}^{\mathrm{DF}}\right)\right\} \tag{2.32}$$

where $\gamma_{\mathrm{XY}}^{\mathrm{DF}} = \frac{P_{\mathrm{X}}|h_{\mathrm{XY}}|^2}{\sigma_{\mathrm{Y}}^2}$.

From (2.32) it is observed that if the decoding at the relay is incorrect, the transmission via relay becomes meaningless. Therefore, such protocol is optimal when the link between the source and the relay is asymptotically perfect.

2.4.3 ADF protocol

In this protocol, the relay decide to forward only if it correctly decodes, hence the mutual information can be shown as

$$\mathcal{I}_{\mathrm{ADF}} = \begin{cases} \frac{1}{2}\log(1+\gamma_{\mathrm{SD}}^{\mathrm{DF}}) & : \gamma_{\mathrm{SR}}^{\mathrm{DF}} < \gamma_{th} \\ \frac{1}{2}\log(1+\gamma_{\mathrm{SD}}^{\mathrm{DF}}+\gamma_{\mathrm{RD}}^{\mathrm{DF}}) & : \gamma_{\mathrm{SR}}^{\mathrm{DF}} \geq \gamma_{th} \end{cases} \tag{2.33}$$

in which γ_{th} is the threshold of correct decoding.

2.5 Simulation on Average Achievable Rate

Considering the mutual information in (2.27), (2.32) and (2.33) as random variables, the average rates for AF, FDF and ADF schemes can be determined, respectively, as

$$R^{\mathrm{AF}} = \mathbb{E}\left\{\mathcal{I}_{\mathrm{AF}}\right\} = \frac{1}{2}\mathbb{E}\left\{\log(1+\gamma_{\mathrm{SD}}^{\mathrm{AF}}+\gamma_{\mathrm{RD}}^{\mathrm{AF}})\right\} \tag{2.34}$$

$$R^{\mathrm{FDF}} = \mathbb{E}\left\{\mathcal{I}_{\mathrm{FDF}}\right\} = \frac{1}{2}\mathbb{E}\left\{\log\left(1+\min\left\{\gamma_{\mathrm{SR}}^{\mathrm{DF}}, \gamma_{\mathrm{SD}}^{\mathrm{DF}}+\gamma_{\mathrm{RD}}^{\mathrm{DF}}\right\}\right)\right\} \tag{2.35}$$

$$R^{\mathrm{ADF}} = \mathbb{E}\left\{\mathcal{I}_{\mathrm{ADF}}\right\} = \frac{1}{2}\mathbb{E}\left\{\log(1+\gamma_{\mathrm{SD}}^{\mathrm{DF}})|\gamma_{\mathrm{SR}}^{\mathrm{DF}} < \gamma_{\mathrm{th}}\right\} + \frac{1}{2}\mathbb{E}\left\{\log(1+\gamma_{\mathrm{SD}}^{\mathrm{DF}}+\gamma_{\mathrm{RD}}^{\mathrm{DF}})|\gamma_{\mathrm{SR}}^{\mathrm{DF}} \geq \gamma_{\mathrm{th}}\right\} \tag{2.36}$$

where $\mathbb{E}\{.\}$ denotes the expectation operator. Assuming the same SNR of P/σ^2 at all the receiving nodes and $\gamma_{\mathrm{th}} = 3$, the plots in Fig. 2.5 represent the average achievable rates for AF, FDF as well as ADF protocols in symmetric networks with factor 1, i.e., $\Omega_{\mathrm{SD}} = \Omega_{\mathrm{SR}} = \Omega_{\mathrm{RD}} = 1$, and in asymmetric networks $\Omega_{\mathrm{SD}} = 1$, $\Omega_{\mathrm{SR}} = 2$ and $\Omega_{\mathrm{RD}} = 5$.

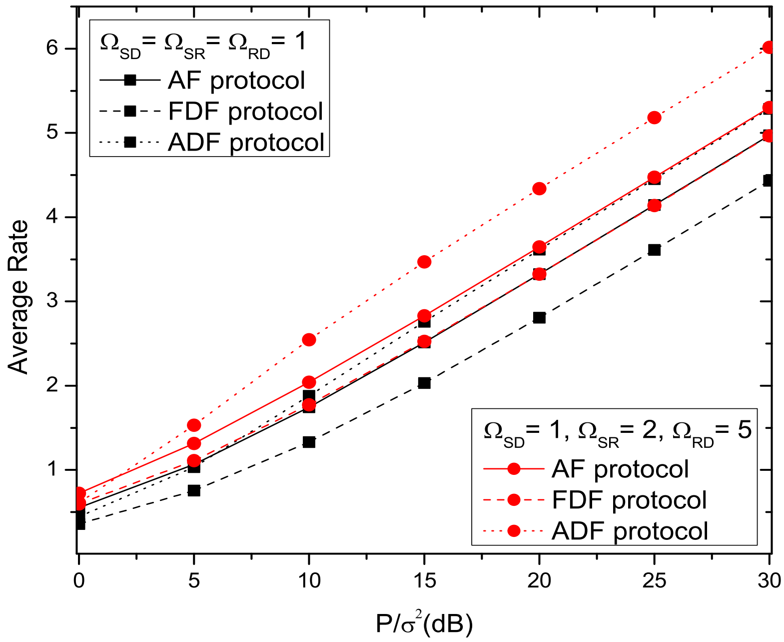

Figure 2.5: Simulations on average achievable rates.

2.6 Advantages and Disadvantages

2.6.1 Advantages of Cooperative Communications

Performance gains: As discussed in [12–14], the employment of relaying cooperation can obtain significant performance gains, which in turn require lower transmission power as well as better cell coverage.

Quality of service: Through the help of relays, cooperative communications can assure a balance Quality-of-Service (QoS) to all users either at the cell edge or in shadowed environment.

Minimal infrastructure: Due to its distributed behavior, the deployment of relays consumes the least demand on infrastructure to facilitate the communications. It is therefore quite suitable for disaster communication.

2.6.2 Disadvantages of Cooperative Communications

More complex system functioning: Since the system is needed to schedule not only the direct but also the relaying traffic, the scheduler for cooperative communications becomes much more sophisticated. Furthermore, the requirements on handovers, security and relays selection are higher than the conventional non-cooperative system.

Interference: The distributed manipulation on the other hand generate extra interference between neighbor nodes. Therefore, a consideration on optimum cell coverage in terms of interference is needed.

Delay: The deployment of relays implies the repetition of messages during the data traffic, hence to convey delay-sensitive services, the selected relay protocol should be simple and transparent, which in turn has lower performance gains.

Chapter 3

Performance Analysis on Single Relay Networks

In this chapter, we consider the performance analysis on single relay cooperative networks. The discussion is concentrated on SER as well as the outage probability of the proposed AF, FDF and ADF protocols. From that we represent the comparison on the cooperative gains between these protocols and verify our analysis by Monte-Carlo simulation.

3.1 Exact Closed Form Expressions of SER

3.1.1 SER of AF Protocol

In AF scheme, the relay amplifies both the information message and noise, thus the equivalent noise, the output signal after MRC and the weighting factors at the destination in the second phase are given, respectively, by (2.24), (2.25) and (2.26). The instantaneous SNR is therefore

$$\gamma = \gamma_{\mathrm{SD}} + \gamma_{\mathrm{RD}} \tag{3.1}$$

The SER of AF protocol with M-PSK, M-AM and M-QAM modulations can be written, respectively, as [21]

$$P_s^{\mathrm{PSK}} = \frac{1}{\pi} \int_0^{\pi - \frac{\pi}{M}} e^{\frac{g_{\mathrm{PSK}}\gamma}{\sin^2\theta}} \mathrm{d}\theta \tag{3.2}$$

$$P_s^{\mathrm{AM}} = \frac{2\,(M-1)}{M\pi} \int_0^{\pi/2} e^{\frac{g_{\mathrm{AM}}\gamma}{\sin^2\theta}} \mathrm{d}\theta \tag{3.3}$$

$$P_s^{\mathrm{QAM}} = \frac{4}{\pi}\left(1 - \frac{1}{\sqrt{M}}\right) \int_0^{\pi/2} e^{\frac{g_{\mathrm{QAM}}\gamma}{\sin^2\theta}} \mathrm{d}\theta \tag{3.4}$$

where $g_{\mathrm{PSK}} = \sin^2(\pi/M)$, $g_{\mathrm{AM}} = 3/(M^2-1)$ and $g_{\mathrm{QAM}} = 3/[2(M-1)]$.

Defining the moment generating function (MGF) of a random variable X as

$$\Phi_X(s) = \int_{-\infty}^{\infty} p_X(x)\, e^{-sx} dx \tag{3.5}$$

the expressions of SER in (3.2), (3.3) and (3.4), respectively, can be reformulated as

$$P_s^{\mathrm{PSK}} = \frac{1}{\pi} \int_0^{\pi-\frac{\pi}{M}} \Phi_{\gamma_{\mathrm{SD}}}\left(\frac{g_{\mathrm{PSK}}}{\sin^2\theta}\right) \Phi_{\gamma_{\mathrm{RD}}}\left(\frac{g_{\mathrm{PSK}}}{\sin^2\theta}\right) \mathrm{d}\theta \tag{3.6}$$

$$P_s^{\mathrm{AM}} = \frac{2(M-1)}{M\pi} \int_0^{\pi/2} \Phi_{\gamma_{\mathrm{SD}}}\left(\frac{g_{\mathrm{AM}}}{\sin^2\theta}\right) \Phi_{\gamma_{\mathrm{RD}}}\left(\frac{g_{\mathrm{AM}}}{\sin^2\theta}\right) \mathrm{d}\theta \tag{3.7}$$

$$P_s^{\mathrm{QAM}} = \frac{4}{\pi}\left(1 - \frac{1}{\sqrt{M}}\right) \int_0^{\pi/2} \Phi_{\gamma_{\mathrm{SD}}}\left(\frac{g_{\mathrm{QAM}}}{\sin^2\theta}\right) \Phi_{\gamma_{\mathrm{RD}}}\left(\frac{g_{\mathrm{QAM}}}{\sin^2\theta}\right) \mathrm{d}\theta \tag{3.8}$$

Assuming $h_{\mathrm{XY}} \sim \mathcal{CN}(0, \Omega_{\mathrm{XY}})$ in which XY $\in$ {SD,SR,RD} and applying Jacobian transformation, the random variable $P_X h_{\mathrm{XY}}^2/\sigma_{\mathrm{Y}}^2$ has an exponential distribution with hazard rates $\sigma_Y^2/(P_X\Omega_{\mathrm{XY}})$. Applying this observation to (3.5), we get the MGF of γ_{SD} as follows:

$$\Phi_{\gamma_{\mathrm{SD}}}(s) = \frac{1}{1 + \frac{sP_{\mathrm{S}}\Omega_{\mathrm{SD}}}{\sigma_{\mathrm{D}}^2}} \tag{3.9}$$

Nevertheless, it is not straightforward to reach the MGF of γ_{RD} in AF protocol. To begin the calculation, let us first derive the PDF of γ_{RD}. Here, we have

$$\gamma_{\mathrm{RD}} = \frac{U_1U_2}{U_1+U_2+1} \approx \frac{U_1U_2}{U_1+U_2} \tag{3.10}$$

$$\gamma_{\mathrm{RD}} \approx \left(\frac{1}{U_1} + \frac{1}{U_2}\right)^{-1} \tag{3.11}$$

where $U_1 = P_{\mathrm{S}}\left|h_{\mathrm{SR}}\right|^2/\sigma_{\mathrm{R}}^2$ and $U_2 = P_{\mathrm{R}}\left|h_{\mathrm{RD}}\right|^2/\sigma_{\mathrm{D}}^2$.

Using the fact that the PDF of the random variable $V_i = U_i^{-1}$ is

$$p_{V_i}(v_i) = \frac{1}{v_i^2} p_{U_i}\left(\frac{1}{v_i}\right) \tag{3.12}$$

the PDF of $Z = V_1 + V_2$ is followed by

$$\begin{aligned} p_Z(z) &= \int_0^z p_{V_1}(z-v)\,p_{V_2}(v)\,\mathrm{d}v \\ &= \int_0^z \frac{1}{v^2(z-v)^2} p_{U_1}\left(\frac{1}{z-v}\right) p_{U_2}\left(\frac{1}{v}\right)\mathrm{d}v \end{aligned} \tag{3.13}$$

By noting that $\gamma_{\mathrm{RD}} \approx Z^{-1}$, the PDF of γ_{RD} is thereby

$$\begin{aligned} p_{\gamma_{\mathrm{RD}}}(\gamma^*) &= \frac{1}{\gamma^{*2}} \int_0^{1/\gamma^*} \frac{1}{v^2\left(\frac{1}{\gamma^*}-v\right)^2} p_{U_1}\left(\frac{1}{\frac{1}{\gamma^*}-v}\right) p_{U_2}\left(\frac{1}{v}\right)\mathrm{d}v \\ &= \int_0^1 \frac{\gamma^*}{v^2(1-v)^2} p_{U_1}\left(\frac{\gamma^*}{1-v}\right) p_{U_2}\left(\frac{\gamma^*}{v}\right)\mathrm{d}v \end{aligned} \tag{3.14}$$

On the assumption of Rayleigh fading, U_1 and U_2 are exponentially distributed with hazard rates $\alpha_{\mathrm{SR}} = \sigma_{\mathrm{R}}^2/(P_S\Omega_{\mathrm{SR}})$ and $\alpha_{\mathrm{RD}} = \sigma_{\mathrm{D}}^2/(P_R\Omega_{\mathrm{RD}})$, respectively. The PDF of γ_{RD} is therefore determined as

$$p_{\gamma_{\mathrm{RD}}}(\gamma^*) = \int_0^1 \frac{\gamma^*\alpha_{\mathrm{SR}}\alpha_{\mathrm{RD}}}{v^2(1-v)^2} e^{-\left(\frac{\alpha_{\mathrm{SR}}}{1-v}+\frac{\alpha_{\mathrm{RD}}}{v}\right)\gamma^*}\mathrm{d}v \tag{3.15}$$

Therefore, the MGF of γ_{RD} can be deduced as

$$\begin{aligned} \Phi_{\gamma_{\mathrm{RD}}}(s) &= \int_0^\infty e^{-s\gamma^*} \int_0^1 \frac{\gamma^*\alpha_{\mathrm{SR}}\alpha_{\mathrm{RD}}}{v^2(1-v)^2} e^{-\left(\frac{\alpha_{\mathrm{SR}}}{1-v}+\frac{\alpha_{\mathrm{RD}}}{v}\right)\gamma^*}\mathrm{d}v\mathrm{d}\gamma^* \\ &= \int_0^1 \frac{\alpha_{\mathrm{SR}}\alpha_{\mathrm{RD}}}{v^2(1-v)^2}\left(\int_0^\infty \gamma^* e^{-\left(\frac{\alpha_{\mathrm{SR}}}{1-v}+\frac{\alpha_{\mathrm{RD}}}{v}+s\right)\gamma^*}\mathrm{d}\gamma^*\right)\mathrm{d}v \end{aligned} \tag{3.16}$$

Due to the fact that

$$\int_0^\infty \gamma^* e^{-\left(\frac{\alpha_{\mathrm{SR}}}{1-v}+\frac{\alpha_{\mathrm{RD}}}{v}+s\right)\gamma^*}\mathrm{d}\gamma^* = \left(\frac{\alpha_{\mathrm{SR}}}{1-v}+\frac{\alpha_{\mathrm{RD}}}{v}+s\right)^{-2} \tag{3.17}$$

the expression in (3.16) can be rewritten as

$$\Phi_{\gamma_{\mathrm{RD}}}(s) = \int_0^1 \frac{\alpha_{\mathrm{SR}}\alpha_{\mathrm{RD}}}{\left(a_1+b_1v+c_1v^2\right)^2}\mathrm{d}v \tag{3.18}$$

where

$$a_1 = -\alpha_{\text{RD}} \tag{3.19}$$
$$b_1 = -s - \alpha_{\text{SR}} + \alpha_{\text{RD}} \tag{3.20}$$
$$c_1 = s \tag{3.21}$$

With the help of [22, eq. (2.172)], the closed form expression of (3.18) is followed by

$$\Phi_{\gamma_{\text{RD}}}(s) = \alpha_{\text{SR}}\alpha_{\text{RD}}\left[\frac{b_1}{\Delta a_1} + \frac{b_1 + 2c_1}{\Delta \alpha_{\text{SR}}} + \frac{4c_1}{\Delta^{3/2}}\left(\operatorname{arctanh}\frac{b_1 + 2c_1}{\sqrt{\Delta}} - \operatorname{arctanh}\frac{b_1}{\sqrt{\Delta}}\right)\right] \tag{3.22}$$

in which $\Delta = b_1^2 - 4a_1c_1$.

3.1.2 SER of FDF Protocol

In FDF scheme, the message pass through two states of decoding in cascade, hence the total probability of error is determined by [12]

$$P_s^{\text{FDF}} = P_s(\gamma_{\text{SR}}) + P_s(\gamma) - 2P_s(\gamma_{\text{SR}})P_s(\gamma) \tag{3.23}$$

It is observed from the previous section that the analyses on SER for M-PSK, M-AM and M-QAM modulations are similar except the parameter $g_{\text{PSK}}, g_{\text{AM}}$ and g_{QAM}. Thus from now on, we focus only on the specific PSK modulation with $g = g_{\text{PSK}}$. The SER expression in (3.23) is followed by

$$\begin{aligned} P_s^{\text{FDF}} =& \frac{1}{\pi}\int_0^{\pi-\frac{\pi}{M}} \Phi_{\gamma_{\text{SR}}}\left(\frac{g}{\sin^2\theta}\right)\mathrm{d}\theta + \frac{1}{\pi}\int_0^{\pi-\frac{\pi}{M}} \Phi_{\gamma}\left(\frac{g}{\sin^2\theta}\right)\mathrm{d}\theta \\ &- 2\left(\frac{1}{\pi}\right)^2 \int_0^{\pi-\frac{\pi}{M}} \Phi_{\gamma_{\text{SR}}}\left(\frac{g}{\sin^2\theta}\right)\mathrm{d}\theta \int_0^{\pi-\frac{\pi}{M}} \Phi_{\gamma}\left(\frac{g}{\sin^2\theta}\right)\mathrm{d}\theta \\ =& \frac{1}{\pi}\int_0^{\pi-\frac{\pi}{M}} \Phi_{\gamma_{\text{SR}}}\left(\frac{g}{\sin^2\theta}\right)\mathrm{d}\theta + \frac{1}{\pi}\int_0^{\pi-\frac{\pi}{M}} \Phi_{\gamma_{\text{SD}}}\left(\frac{g}{\sin^2\theta}\right)\Phi_{\gamma_{\text{RD}}}\left(\frac{g}{\sin^2\theta}\right)\mathrm{d}\theta \\ &- 2\left(\frac{1}{\pi}\right)^2 \int_0^{\pi-\frac{\pi}{M}} \Phi_{\gamma_{\text{SR}}}\left(\frac{g}{\sin^2\theta}\right)\mathrm{d}\theta \int_0^{\pi-\frac{\pi}{M}} \Phi_{\gamma_{\text{SD}}}\left(\frac{g}{\sin^2\theta}\right)\Phi_{\gamma_{\text{RD}}}\left(\frac{g}{\sin^2\theta}\right)\mathrm{d}\theta \end{aligned} \tag{3.24}$$

Assuming the channel coefficients $h_{\text{SR}}, h_{\text{SD}}$ and h_{RD} are i.n.i.d. Rayleigh fading, the MGF of a $X \to Y$ channel is

$$\Phi_{\gamma_{\text{XY}}}(s) = \frac{1}{1 + \frac{sP_{\text{X}}\Omega_{\text{XY}}}{\sigma_{\text{Y}}^2}} \tag{3.25}$$

Therefore, SER of FDF protocol can now be deduced as

$$\begin{aligned}P_s^{\text{FDF}} =& f\left(1+\frac{gP_{\text{S}}\Omega_{\text{SR}}}{\sin^2\theta\sigma_{\text{D}}^2}\right)+f\left(\left(1+\frac{gP_{\text{S}}\Omega_{\text{SD}}}{\sin^2\theta\sigma_{\text{R}}^2}\right)\left(1+\frac{gP_{\text{S}}\Omega_{\text{RD}}}{\sin^2\theta\sigma_{\text{R}}^2}\right)\right)\\ &-2f\left(1+\frac{gP_{\text{S}}\Omega_{\text{SR}}}{\sin^2\theta\sigma_{\text{D}}^2}\right)f\left(\left(1+\frac{gP_{\text{S}}\Omega_{\text{SD}}}{\sin^2\theta\sigma_{\text{R}}^2}\right)\left(1+\frac{gP_{\text{S}}\Omega_{\text{RD}}}{\sin^2\theta\sigma_{\text{R}}^2}\right)\right)\end{aligned} \tag{3.26}$$

where

$$f(y(\theta))=\frac{1}{\pi}\int_0^{\pi-\frac{\pi}{M}}\frac{1}{y(\theta)}\mathrm{d}\theta \tag{3.27}$$

3.1.3 SER of ADF Protocol

In ADF scheme, it is assumed that in phase two the relay transmits whenever it correctly decodes its received information, i.e., the relay transmit power is

$$\bar{P}_{\text{R}}=\begin{cases}P_{\text{R}}: & \text{correct decoding}\\ 0: & \text{otherwise}\end{cases} \tag{3.28}$$

The instantaneous SNR of the two-phase transmission now can be performed as

$$\gamma=\begin{cases}\gamma_{\text{SD}}: & \text{incorrect decoding } (\bar{P}_{\text{R}}=0)\\ \gamma_{\text{SD}}+\gamma_{\text{RD}}: & \text{otherwise}\end{cases} \tag{3.29}$$

With the relay power constraint above, the probability that the relay can not decode is $P_s(\gamma_{\text{SR}})$ and correct decoding $1-P_s(\gamma_{\text{SR}})$ in which $\gamma_{\text{SR}}=P_{\text{S}}h_{\text{SR}}^2/\sigma_{\text{R}}^2$. The total SER after two transmitting phases is then

$$\begin{aligned}P_s^{\text{ADF}} =& P_s(\gamma=\gamma_{\text{SD}})P_s(\gamma_{\text{SR}})+P_s(\gamma=\gamma_{\text{SD}}+\gamma_{\text{RD}})\left[1-P_s(\gamma_{\text{SR}})\right]\\ =&\left(\frac{1}{\pi}\right)^2\int_0^{\pi-\frac{\pi}{M}}\Phi_{\gamma_{\text{SD}}}\left(\frac{g}{\sin^2\theta}\right)\mathrm{d}\theta\int_0^{\pi-\frac{\pi}{M}}\Phi_{\gamma_{\text{SR}}}\left(\frac{g}{\sin^2\theta}\right)\mathrm{d}\theta\\ &+\frac{1}{\pi}\int_0^{\pi-\frac{\pi}{M}}\Phi_{\gamma_{\text{SD}}}\left(\frac{g}{\sin^2\theta}\right)\Phi_{\gamma_{\text{RD}}}\left(\frac{g}{\sin^2\theta}\right)\mathrm{d}\theta\left[1-\frac{1}{\pi}\int_0^{\pi-\frac{\pi}{M}}\Phi_{\gamma_{\text{SR}}}\left(\frac{g}{\sin^2\theta}\right)\right]\mathrm{d}\theta\\ =& f\left(1+\frac{gP_{\text{S}}\Omega_{\text{SD}}}{\sin^2\theta\sigma_{\text{D}}^2}\right)f\left(1+\frac{gP_{\text{S}}\Omega_{\text{SR}}}{\sin^2\theta\sigma_{\text{R}}^2}\right)\\ &+f\left(\left(1+\frac{gP_{\text{S}}\Omega_{\text{SD}}}{\sin^2\theta\sigma_{\text{D}}^2}\right)\left(1+\frac{gP_{\text{R}}\Omega_{\text{RD}}}{\sin^2\theta\sigma_{\text{D}}^2}\right)\right)\left[1-f\left(1+\frac{gP_{\text{S}}\Omega_{\text{SR}}}{\sin^2\theta\sigma_{\text{R}}^2}\right)\right]\end{aligned} \tag{3.30}$$

in which $f(.)$ is defined in (3.27).

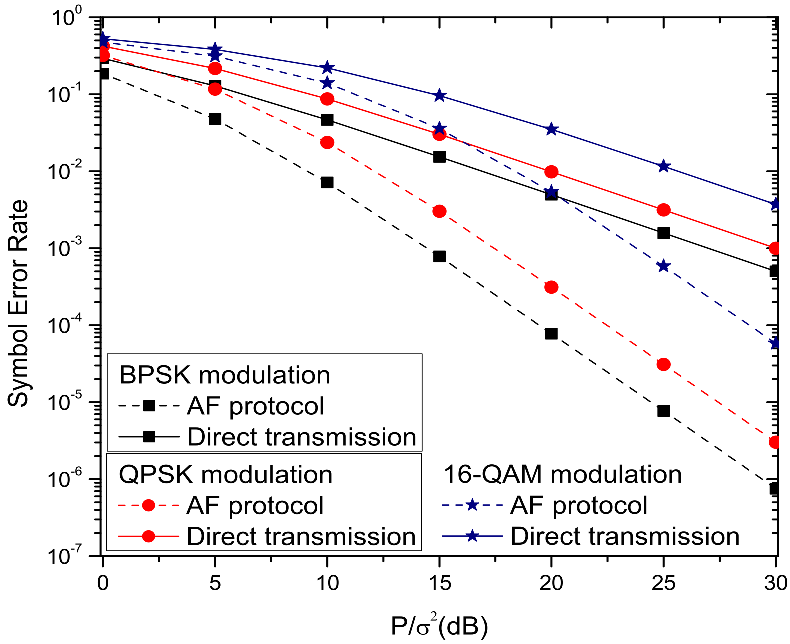

Figure 3.1: Comparisons on SER between AF relay networks and direct transmission employing some basic modulation techniques.

3.2 Numerical Results

3.2.1 Simulations on SER with Different Modulation Schemes

Assuming $\Omega_{\text{SD}} = \Omega_{\text{SR}} = \Omega_{\text{RD}} = 1$ and the same SNR of P/σ^2 at all nodes, it can be seen from Fig. 3.1, Fig. 3.2 and Fig. 3.3 that by using the same modulation technique the deployment of a single relay performs better SER than direct transmission.

3.2.2 Validation on the Analysis

With the aim of verifying the derivations above we represent the Monte-Carlo simulation as comparing with our analyses with the assumption of the same transmit SNR of P/σ^2. Fig. 3.4 demonstrates the symmetric case $(\Omega_{\text{SR}} = \Omega_{\text{RD}} = \Omega_{\text{SD}} = 1)$ and Fig. 3.5 the asymmetric case $(\Omega_{\text{SD}} = 1, \Omega_{\text{SR}} = 2, \Omega_{\text{RD}} = 5)$. From these figures it has been shown that the analyses give fitted results to the simulations.

3.3 Comparison on the Cooperative Gains

Although the exact SER formulations can be numerically calculated, they are excessively complicated and it is difficult to get perception on the performance from these expressions. For simplification purpose, the approximations of SER for AF and ADF protocols are introduced in the following.

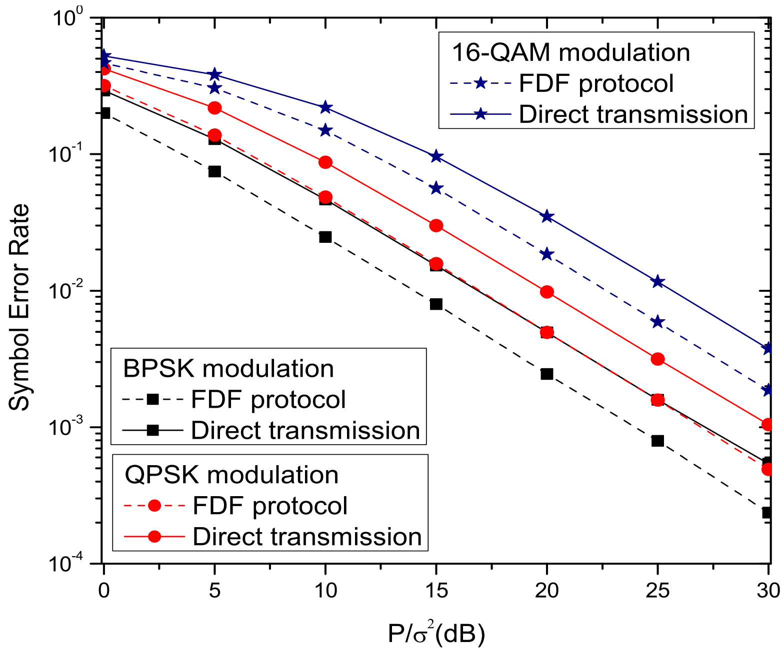

Figure 3.2: Comparisons on SER between FDF relay networks and direct transmission employing some basic modulation techniques.

3.3.1 Asymptotic Approximation of SER for ADF Protocol

The MGF in (3.25) can be rewritten in the form

$$\Phi_{\gamma_{\mathrm{XY}}}(s) = \left(1 + \frac{P_{\mathrm{X}}\Omega_{\mathrm{XY}}}{\sigma_{\mathrm{Y}}^2} s\right)^{-1} \tag{3.31}$$

At high SNR, i.e., $\frac{P_X}{\sigma_Y^2} \gg 1$, it can be approximated by

$$\Phi_{\gamma_{\mathrm{XY}}}(s) \approx \left(P_{\mathrm{X}}\frac{\Omega_{\mathrm{XY}}}{\sigma_{\mathrm{Y}}^2} s\right)^{-1} \tag{3.32}$$

Consequently, the asymptotic approximation of the closed form in (3.30) is achieved as follows

$$\begin{aligned} P_s^{\mathrm{ADF}} &\approx \frac{\sigma_{\mathrm{R}}^2\sigma_{\mathrm{D}}^2}{P_{\mathrm{S}}^2}\frac{a^2}{g^2}\frac{1}{\Omega_{\mathrm{SD}}\Omega_{\mathrm{SR}}} + \frac{\sigma_{\mathrm{D}}^4}{P_{\mathrm{S}}P_{\mathrm{R}}}\frac{b}{g^2}\frac{1}{\Omega_{\mathrm{SD}}\Omega_{\mathrm{RD}}} \\ &\approx \frac{\sigma_{\mathrm{D}}^2}{P_{\mathrm{S}}\Omega_{\mathrm{SD}}g^2}\left(\frac{a^2\sigma_{\mathrm{R}}^2}{P_{\mathrm{S}}\Omega_{\mathrm{SR}}} + \frac{b\sigma_{\mathrm{D}}^2}{P_{\mathrm{R}}\Omega_{\mathrm{RD}}}\right) \end{aligned} \tag{3.33}$$

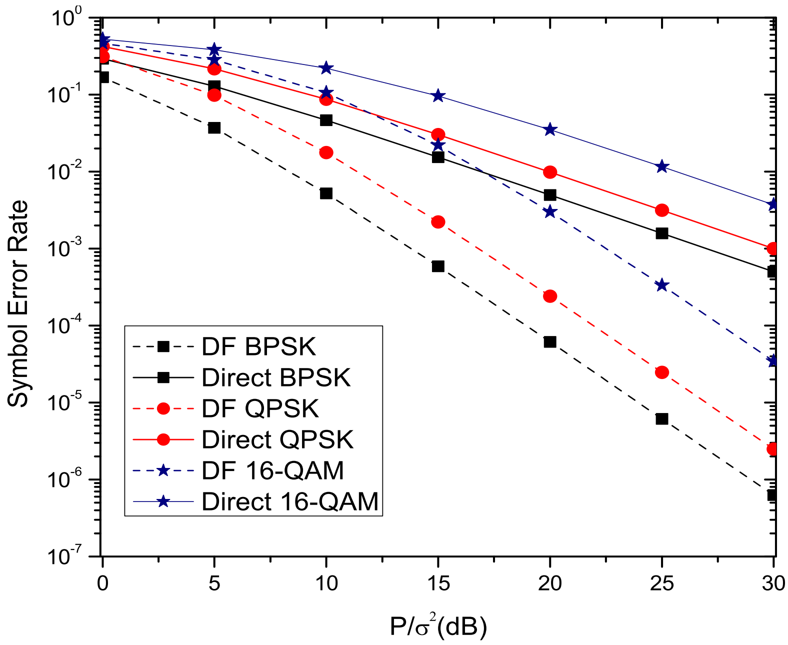

Figure 3.3: Comparisons on SER between ADF relay networks and direct transmission employing some basic modulation techniques.

in which

$$a = \frac{1}{\pi}\int_0^{\pi-\frac{\pi}{M}} \sin^2\theta \mathrm{d}\theta \qquad = \frac{M-1}{2M} + \frac{\sin\left(\frac{2\pi}{M}\right)}{4\pi} \tag{3.34}$$

$$b = \frac{1}{\pi}\int_0^{\pi-\frac{\pi}{M}} \sin^4\theta \mathrm{d}\theta \qquad = \frac{3(M-1)}{8M} + \frac{\sin\left(\frac{2\pi}{M}\right)}{4\pi} - \frac{\sin\left(\frac{4\pi}{M}\right)}{32\pi} \tag{3.35}$$

3.3.2 Asymptotic Approximation of SER for AF Protocol

Through the analysis in [13] at high SNR, the parameters in (3.22) can be approximated as

$$b_1 \approx -s \tag{3.36}$$

$$\Delta \approx s^2 \tag{3.37}$$

$$\frac{4c_1}{\Delta^{3/2}}\left(\operatorname{arctanh}\frac{b_1+2c_1}{\sqrt{\Delta}} - \operatorname{arctanh}\frac{b_1}{\sqrt{\Delta}}\right) \approx 0 \tag{3.38}$$

The MGF in (3.22) is therefore tightly approximated at high SNR as

$$\Phi_{\gamma_{\mathrm{RD}}}(s) \approx \frac{\alpha_{\mathrm{SR}}}{s} + \frac{\alpha_{\mathrm{RD}}}{s} = \frac{\sigma_{\mathrm{R}}^2}{sP_{\mathrm{S}}\Omega_{\mathrm{SR}}} + \frac{\sigma_{\mathrm{D}}^2}{sP_{\mathrm{R}}\Omega_{\mathrm{RD}}} \tag{3.39}$$

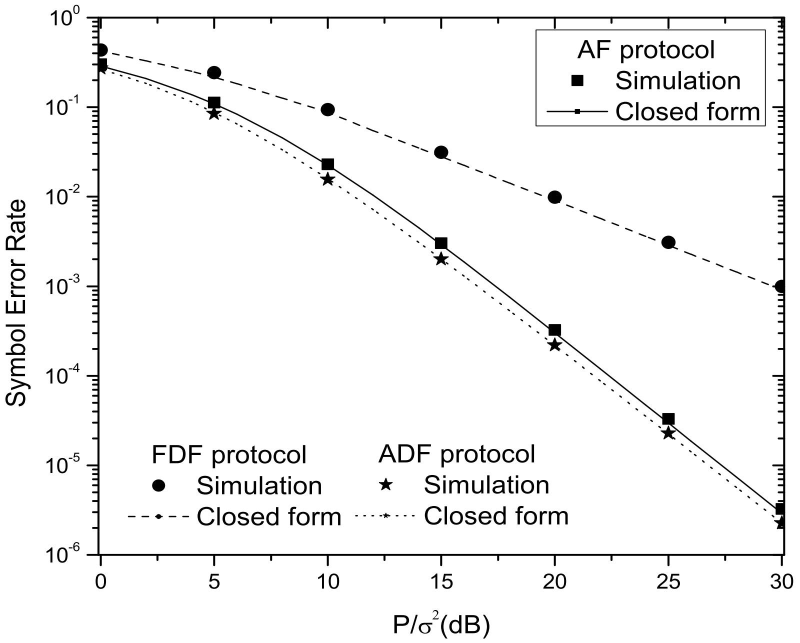

Figure 3.4: SER in symmetric system using QPSK, $\Omega_{\text{SR}} = \Omega_{\text{RD}} = \Omega_{\text{SD}} = 1$.

Consequently, the approximation of SER for AF scheme is determined by

$$P_s^{\text{AF}} \approx \frac{b\sigma_{\text{D}}^2}{g^2 P_{\text{S}}\Omega_{\text{SD}}}\left(\frac{\sigma_{\text{R}}^2}{P_{\text{S}}\Omega_{\text{SR}}} + \frac{\sigma_{\text{D}}^2}{P_{\text{R}}\Omega_{\text{RD}}}\right) \tag{3.40}$$

3.3.3 Cooperative Gains of AF and ADF scheme

To provide the comparison on the two protocols, let us define

$$\Delta = \frac{P_s^{\text{AF}}}{P_s^{\text{ADF}}} = \frac{b\left(\frac{\sigma_{\text{R}}^2}{P_{\text{S}}\Omega_{\text{SR}}} + \frac{\sigma_{\text{D}}^2}{P_{\text{R}}\Omega_{\text{RD}}}\right)}{\frac{a^2\sigma_{\text{R}}^2}{P_{\text{S}}\Omega_{\text{SR}}} + \frac{b\sigma_{\text{D}}^2}{P_{\text{R}}\Omega_{\text{RD}}}} \tag{3.41}$$

In view of (3.33), (3.40) and (3.41), some statements on the cooperative gains at high SNR are revealed as follows:

- Both AF and ADF schemes can attain diversity order of two.
- For a fixed M, i.e., inflexible modulation regime, Δ only depends on the specific source-to-relay and relay-to-destination links despite of the direct link channel
- As $\Omega_{\text{SR}} \ll \Omega_{\text{RD}}$, i.e., the quality of the channel between source and relay is much worse than between relay and destination.

$$\Delta \approx \frac{b}{a^2} > 1 \tag{3.42}$$

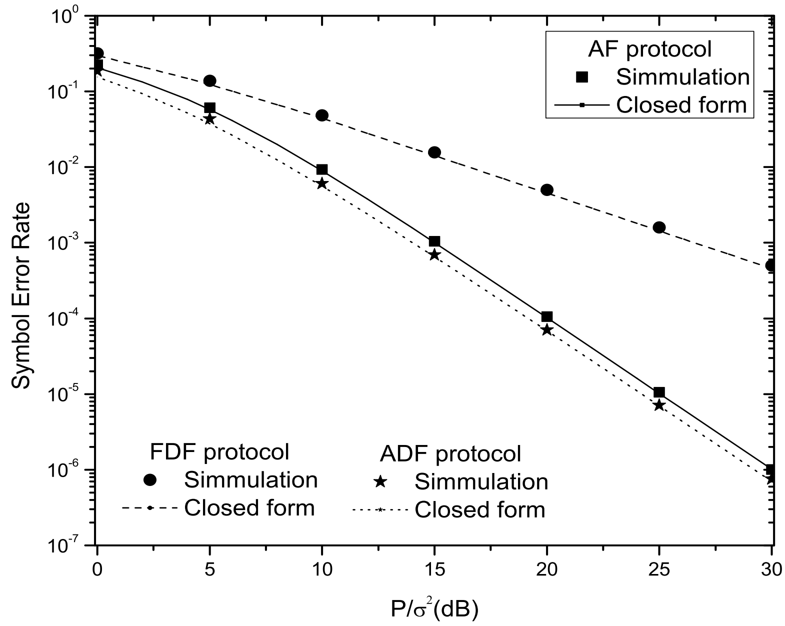

Figure 3.5: SER in asymmetric system using QPSK, $\Omega_{\text{SD}} = 1$, $\Omega_{\text{SR}} = 2$ and $\Omega_{\text{RD}} = 5$.

- On the other hand, when $\Omega_{\text{SR}} \gg \Omega_{\text{RD}}$,

$$\Delta \approx 1 \tag{3.43}$$

  From the expressions in (3.43) and (3.42) it can be concluded that if the source-to-relay link is much better, the selection between AF and ADF protocol almost does not affect the SER. Therefore, it is suggested to employ AF scheme in this case due to its simplicity.

- The SER performance of ADF protocol is always better than AF since Δ is always larger than 1.

3.3.4 Simulations on Cooperative Gains

This section provides the verification on the comparison in the previous section with the assumptions that all the receiving nodes have the same SNR of P/σ^2 and the channel between the source and destination is characterized by factor 1,i. e. $\Omega_{\text{SD}} = 1$. Simulations as well as approximations on SER of both AF and ADF protocols in the symmetric case ($\Omega_{\text{SR}} = \Omega_{\text{RD}} = 1$) are presented through the plot in Fig. 3.6. To illustrate the asymmetric case, Fig. 3.7 and Fig. 3.8 give examples of $\Omega_{\text{SR}} \gg \Omega_{\text{RD}}$ and $\Omega_{\text{SR}} \ll \Omega_{\text{RD}}$ by the specific channels ($\Omega_{\text{SR}} = 5$ and $\Omega_{\text{RD}} = 0.5$), and ($\Omega_{\text{SR}} = 0.5$ and $\Omega_{\text{RD}} = 5$). For the sake of convenience, only the QPSK modulation is investigated; however, the approaches to other modulation techniques are quite similar. It can be seen from the figures that the approximations are tightly at high SNR.

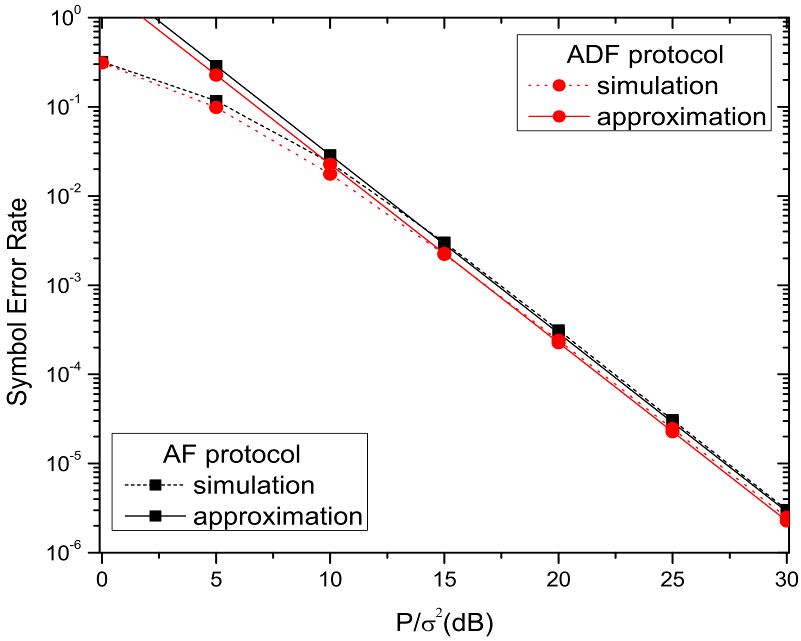

Figure 3.6: Simulations and approximations on SER of AF and ADF protocols in symmetric system using QPSK, $\Omega_{\text{SD}} = \Omega_{\text{SR}} = \Omega_{\text{RD}} = 1$.

3.4 Outage Probability

In the context of the channel experiencing very slow fading, instead of the usage of averages the concept of outage is introduced. An outage event is defined as the circumstance when the signal performance is worse than an acceptable level of communication.

Assuming all the nodes have the same SNR $\frac{P_{\text{S}}}{\sigma_{\text{R}}} = \frac{P_{\text{S}}}{\sigma_{\text{D}}} = \frac{P_{\text{R}}}{\sigma_{\text{D}}} = \gamma_0$, the approximations on outage probability in high SNR regimes for AF, FDF and ADF protocols are determined by [9]

$$\begin{aligned} P_{\text{out}}^{\text{AF}}(\gamma_0, R) =& P\{\mathcal{I}_{\text{AF}} < R\} = P\left\{\gamma^{\text{AF}} < \left(\frac{2^{2R}-1}{\gamma_0}\right)\right\} \\ &\approx \left(\frac{1}{2\Omega_{\text{SD}}}\frac{\Omega_{\text{SR}}+\Omega_{\text{RD}}}{\Omega_{\text{SR}}\Omega_{\text{RD}}}\right)\left(\frac{2^{2R}-1}{\gamma_0}\right)^2 \end{aligned} \tag{3.44}$$

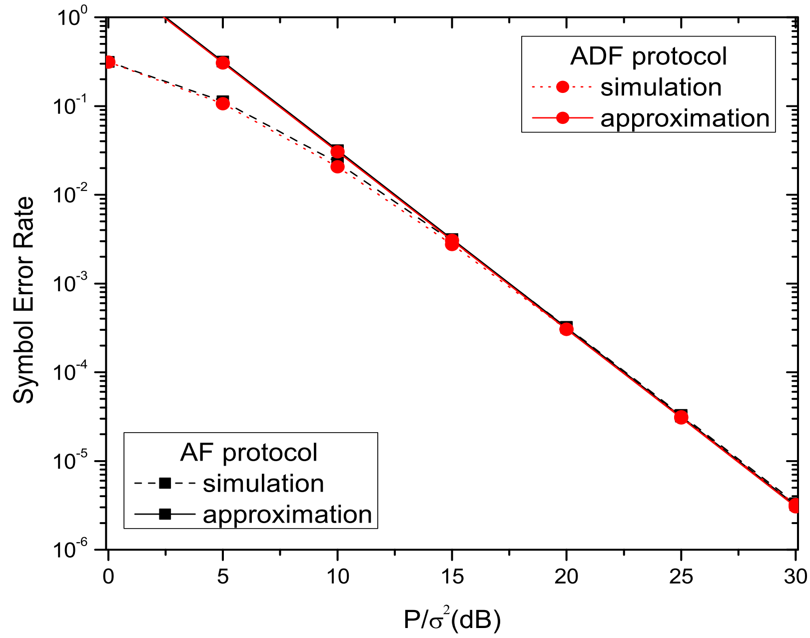

Figure 3.7: Simulations and approximations on SER of AF and ADF protocols in asymmetric system using QPSK, $\Omega_{\mathrm{SD}} = 1, \Omega_{\mathrm{SR}} = 5$ and $\Omega_{\mathrm{RD}} = 0.5$.

$$\begin{aligned} P_{\mathrm{out}}^{\mathrm{FDF}}(\gamma_0, R) =& P\{\mathcal{I}_{\mathrm{FDF}} < R\} \\ =& P\left\{|h_{\mathrm{SR}}|^2 < \left(\frac{2^{2R}-1}{\gamma_0}\right)\right\} \\ &+ P\left\{|h_{\mathrm{SR}}|^2 \geq \left(\frac{2^{2R}-1}{\gamma_0}\right)\right\} P\left\{|h_{\mathrm{SD}}|^2 + |h_{\mathrm{RD}}|^2 < \left(\frac{2^{2R}-1}{\gamma_0}\right)\right\} \\ \approx& \frac{1}{\Omega_{\mathrm{SR}}} \frac{2^{2R}-1}{\gamma_0} \end{aligned} \tag{3.45}$$

$$\begin{aligned} P_{\mathrm{out}}^{\mathrm{ADF}}(\gamma_0, R) =& P\{\mathcal{I}_{\mathrm{ADF}} < R\} \\ =& P\left\{|h_{\mathrm{SR}}|^2 < \left(\frac{2^{2R}-1}{\gamma_0}\right)\right\} P\left\{|h_{\mathrm{SD}}|^2 < \left(\frac{2^{2R}-1}{\gamma_0}\right)\right\} \\ &+ P\left\{|h_{\mathrm{SR}}|^2 \geq \left(\frac{2^{2R}-1}{\gamma_0}\right)\right\} P\left\{|h_{\mathrm{SD}}|^2 + |h_{\mathrm{RD}}|^2 < \left(\frac{2^{2R}-1}{\gamma_0}\right)\right\} \\ \approx& \left(\frac{1}{2\Omega_{\mathrm{SD}}} \frac{\Omega_{\mathrm{SR}} + 2\Omega_{\mathrm{RD}}}{\Omega_{\mathrm{SR}}\Omega_{\mathrm{RD}}}\right) \left(\frac{2^{2R}-1}{\gamma_0}\right)^2 \end{aligned} \tag{3.46}$$

in which R is the spectral efficiency.

Fig. 3.9 demonstrates the simulations as well as approximations on outage probability of all the proposed protocols with the assumption that $R = 1$ and $\Omega_{\mathrm{SD}} = \Omega_{\mathrm{SR}} = \Omega_{\mathrm{RD}} = 1$.

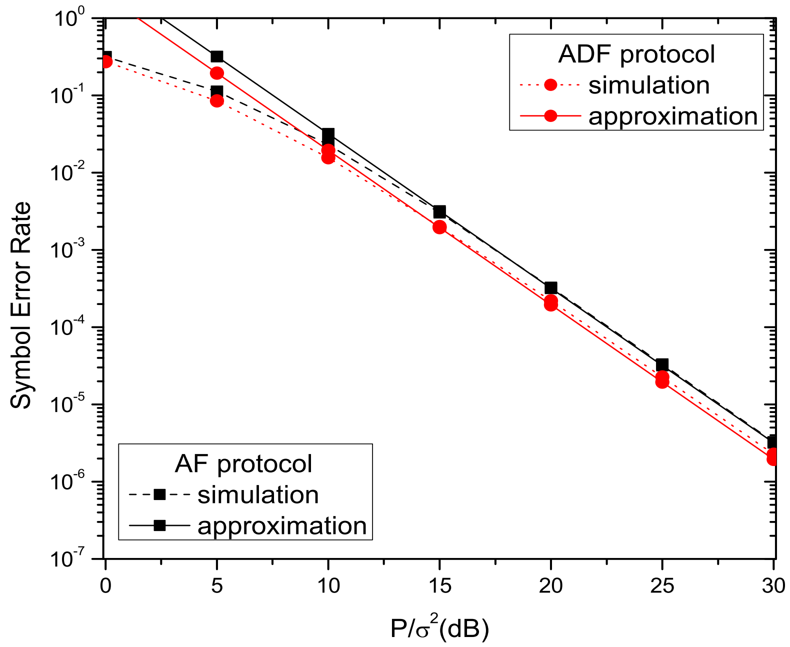

Figure 3.8: Simulations and approximations on SER of AF and ADF protocols in asymmetric system using QPSK, $\Omega_{\mathrm{SD}} = 1, \Omega_{\mathrm{SR}} = 0.5$ and $\Omega_{\mathrm{RD}} = 5$.

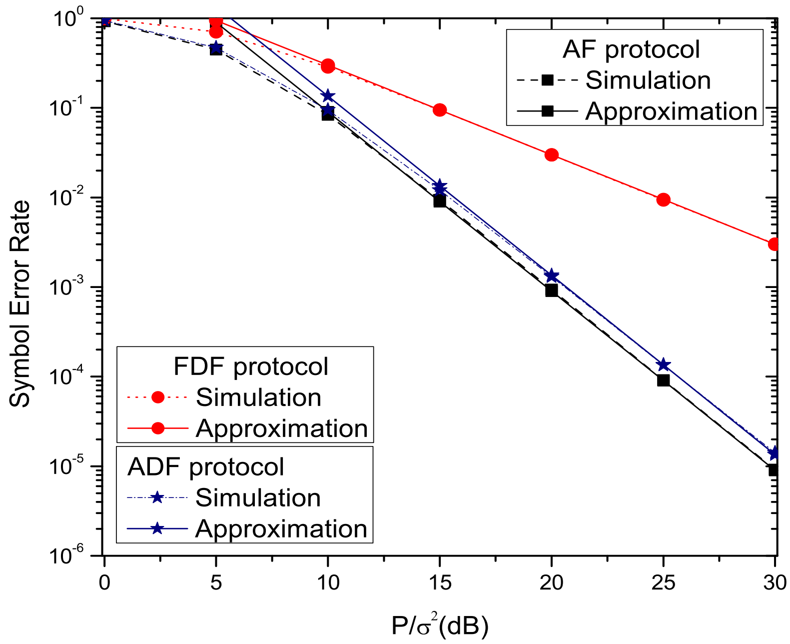

Figure 3.9: Simulations and approximations on outage probability, $\Omega_{\mathrm{SD}} = \Omega_{\mathrm{SR}} = \Omega_{\mathrm{RD}} = 1$, $R = 1$.

Chapter 4

SER Performance of Cooperative Multi-Relay Networks

The former chapter has provided the analysis on SER performance of single-relay cooperative networks for both AF and DF protocols. This chapter performs an extension to the analysis on the generalized multi-relay scenario.

Consider a typical cooperative N-relay network consisting of one source S, one destination D and N relay nodes $R_1, R_2, \ldots, R_N$ as depicted in Fig. 4.1. In the first phase, the source broadcasts to all the other nodes. After processing, i.e., decoding or amplifying, signal from each relay is forwarded to the destination via orthogonal channel in the next phase. Finally the destination exploit MRC technique to combine the received signals from the two-phase transmission.

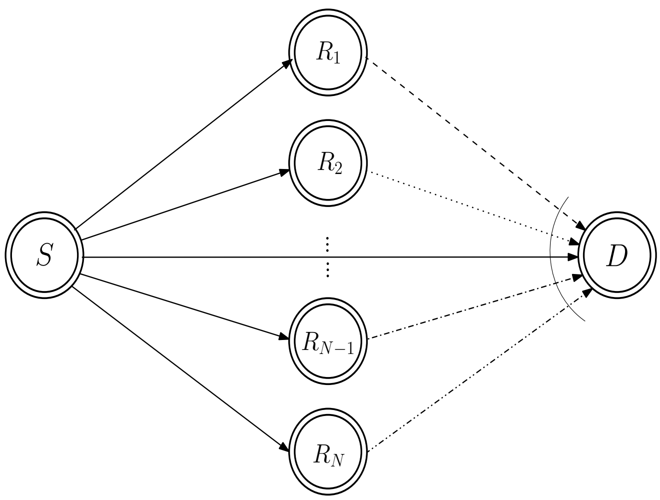

Figure 4.1: Typical multiple relays system.

4.1 Multi-Relay AF Scheme

4.1.1 System Model

During the broadcast phase the received message at the destination and the relay k is, respectively

$$y_{\mathrm{D}_0} = h_{\mathrm{SD}} x + n_{\mathrm{D}_0} \tag{4.1}$$

$$y_{\mathrm{R}_k} = h_{\mathrm{SR}_k} x + n_{\mathrm{R}_k} \tag{4.2}$$

In the second phase, the destination receives from the k-th relay the signal

$$y_{\mathrm{D}_k} = h_{\mathrm{RD}_k} \beta_k y_{\mathrm{R}_k} + n_{\mathrm{D}_k} \tag{4.3}$$

The notation $h_{\mathrm{XY}} \sim \mathcal{CN}(0, \Omega_{\mathrm{XY}})$ with XY $\in \{\mathrm{SD}, \mathrm{SR}_k, \mathrm{RD}_k\}$ in (4.1), (4.2) and (4.3) performs the channel fading coefficients and $n_{\mathrm{Z}_k} \sim \mathcal{CN}(0, \sigma^2_{\mathrm{Z}_k})$ with Z $\in$ {D,R} the AWGN in which $k = 0$ represents the channel between the source and the destination. The amplifying gain β_k at the k-th relay is similar to (2.22) given by

$$\beta_k = \sqrt{\frac{P_{\mathrm{R}_k}}{P_{\mathrm{S}} \left|h_{\mathrm{SR}_k}\right|^2 + \sigma^2_{\mathrm{R}_k}}} \tag{4.4}$$

By the use of MRC, the instantaneous SNR of multiple relays AF protocol is

$$\gamma = \gamma_{\mathrm{SD}} + \sum_{k=1}^{N} \gamma_{\mathrm{RD}_k} \tag{4.5}$$

in which

$$\gamma_{\mathrm{SD}} = \frac{P_{\mathrm{S}} \left|h_{\mathrm{SD}}\right|^2}{\sigma^2_{\mathrm{D}_0}} \tag{4.6}$$

$$\gamma_{\mathrm{RD}_k} = q\left(\frac{P_{\mathrm{S}} \left|h_{\mathrm{SR}_k}\right|^2}{\sigma^2_{\mathrm{R}_k}}, \frac{P_{\mathrm{R}_k} \left|h_{\mathrm{RD}_k}\right|^2}{\sigma^2_{\mathrm{D}_k}}\right) \tag{4.7}$$

and $q(x, y)$ is defined in (2.30).

4.1.2 SER for AF Multiple Relays

With the instantaneous SNR in (4.5), the respective MGF can be given by

$$\Phi_{\gamma}(s) = \Phi_{\gamma_{\mathrm{SD}}}(s) \prod_{k=1}^{N} \Phi_{\gamma_{\mathrm{RD}_k}}(s) \tag{4.8}$$

By employing the calculation in Section 3.3.2, the approximation of each element $\Phi_{\gamma_{\text{RD}_k}}(s)$ is determined by

$$\Phi_{\gamma_{\text{RD}_k}}(s) \approx \frac{\sigma^2_{\text{R}_k}}{sP_S\Omega_{\text{SR}_k}} + \frac{\sigma^2_{\text{D}_k}}{sP_{R_k}\Omega_{\text{RD}_k}} \tag{4.9}$$

In the interest of simplicity, let us assume that $\Omega_{\text{SR}_k} = \Omega_{\text{SR}}$, $\Omega_{\text{RD}_k} = \Omega_{\text{RD}}$ and $\frac{\sigma^2_{\text{R}_k}}{P_\text{S}} = \frac{\sigma^2_{\text{D}_k}}{P_{\text{R}_k}} = \frac{\sigma^2_{\text{D}_0}}{P_\text{S}} = \alpha_0$, then $\Phi_{\gamma_{\text{RD}_k}}(s)$ is simply

$$\Phi_{\gamma_{\text{RD}_k}}(s) \approx \frac{\alpha_0}{s}\left(\frac{1}{\Omega_{\text{SR}}} + \frac{1}{\Omega_{\text{RD}}}\right) \tag{4.10}$$

Therefore SER for multi-relay AF scheme is deduced as

$$P_s^{\text{AF}} = \frac{\eta(N+1)}{\Omega_{\text{SD}}}\left(\frac{\alpha_0}{g}\right)^{N+1}\left(\frac{1}{\Omega_{\text{SR}}} + \frac{1}{\Omega_{\text{RD}}}\right)^N \tag{4.11}$$

in which

$$g = \begin{cases} \sin^2(\pi/M) & : M-\text{PSK} \\ 3/(M^2-1) & : M-\text{AM} \\ 3/[2(M-1)] & : M-\text{QAM} \end{cases} \tag{4.12}$$

$$\eta(m) = \begin{cases} \frac{1}{\pi}\int\limits_0^{\pi-\frac{\pi}{M}} (\sin\theta)^{2m}\,\mathrm{d}\theta & : M-\text{PSK} \\ \frac{2(M-1)}{M\pi}\int\limits_0^{\pi/2} (\sin\theta)^{2m}\,\mathrm{d}\theta & : M-\text{AM} \\ \frac{4}{\pi}\left(1-\frac{1}{\sqrt{M}}\right)\int\limits_0^{\pi/2} (\sin\theta)^{2m}\,\mathrm{d}\theta & : M-\text{QAM} \end{cases} \tag{4.13}$$

For the sake of simplicity, let us consider only the M-PSK modulation and the approaches to SER of the others are precisely similar.

4.2 Multi-Relay DF Scheme

4.2.1 System Model

During the first phase, the destination and relays receive the similar message as in (4.1) and (4.2). However, on the assumption that each relay k only encodes and forwards whenever it can decode its receive message precisely, the receive message at the destination in the next phase is thereby

$$y_{\text{D}_k} = \begin{cases} n_{\text{D}_k} & : \text{relay } k \text{ can not decode} \\ h_{\text{RD}_k}x + n_{\text{D}_k} & : \text{correct decoding} \end{cases} \tag{4.14}$$

By employing MRC the instantaneous SNR of multi-relay DF protocol is determined by

$$\gamma = \begin{cases} \gamma_{\text{SD}} & : \text{ all of relays are incorrect decoding} \\ \gamma_{\text{SD}} + \sum\limits_{k=1}^{j} \gamma_{\text{RD}_k} & : \ j \text{ of } N \text{ relays decode correctly} \end{cases} \tag{4.15}$$

4.2.2 SER for DF Multiple Relays

The probability of correct decoding at the k-th relay is

$$P_k^d = 1 - \frac{1}{\pi} \int\limits_0^{\pi-\frac{\pi}{M}} \Phi_{\gamma_{\text{SR}_k}} \left(\frac{g}{\sin^2\theta} \right) \mathrm{d}\theta \tag{4.16}$$

Through the help of the approximation in (3.32), P_k^d can be approximated by

$$P_k^d \approx 1 - \frac{\eta(1)\alpha_0}{g\Omega_{\text{SR}}} \tag{4.17}$$

As a consequence, the probability that exactly j in the set N of relays decode properly is determined by

$$P_{\{j\}}^d = \prod_{k=1}^{j} P_k^d \prod_{l=j+1}^{N} \left(1 - P_l^d\right) = \left(1 - \frac{\eta(1)\alpha_0}{g\Omega_{\text{SR}}}\right)^j \left(\frac{\eta(1)\alpha_0}{g\Omega_{\text{SR}}}\right)^{N-j} \tag{4.18}$$

and the probability that no relay can properly decode the broadcast signal from the source is followed by

$$P_{\{0\}}^d = \prod_{k=1}^{N} \left(1 - P_k^d\right) = \left(\frac{\eta(1)\alpha_0}{g\Omega_{\text{SR}}}\right)^N \tag{4.19}$$

The average SER of multi-relay DF scheme is then determined as

$$\begin{aligned} P_s^{\text{DF}} &= \sum_{j=1}^{N} \binom{N}{j} P_{\{j\}}^d P_s \left(\gamma = \gamma_{\text{SD}} + \sum_{k=1}^{j} \gamma_{\text{RD}_k} \right) + P_{\{0\}}^d P_s \left(\gamma = \gamma_{\text{SD}} \right) \\ &= \sum_{j=1}^{N} \binom{N}{j} P_{\{j\}}^d \frac{1}{\pi} \int\limits_0^{\pi-\frac{\pi}{M}} \Phi_{\gamma_{\text{SD}}} \left(\frac{g}{\sin^2\theta} \right) \prod_{k=1}^{j} \Phi_{\gamma_{\text{RD}_k}} \left(\frac{g}{\sin^2\theta} \right) \mathrm{d}\theta \\ &\quad + \frac{P_{\{0\}}^d}{\pi} \int\limits_0^{\pi-\frac{\pi}{M}} \Phi_{\gamma_{\text{SD}}} \left(\frac{g}{\sin^2\theta} \right) \mathrm{d}\theta \end{aligned} \tag{4.20}$$

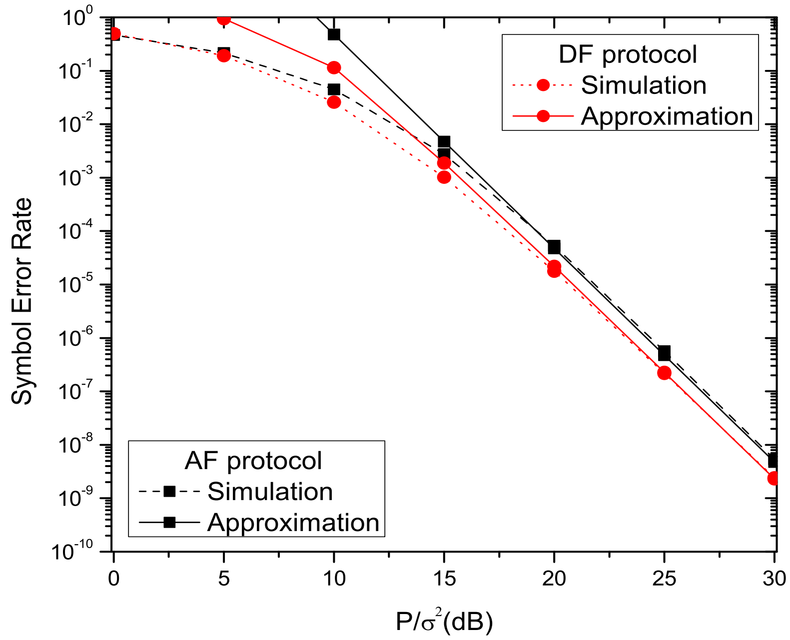

Figure 4.2: Simulations and approximations on SER in 3 relays system using 8-PSK, $\Omega_{\mathrm{SD}} = \Omega_{\mathrm{SR}} = \Omega_{\mathrm{RD}} = 1$.

Since $\Phi_{\gamma_{\mathrm{XY}}}(s) \approx \frac{\alpha_0}{\Omega_{\mathrm{XY}} s}$

$$\begin{aligned} P_s^{\mathrm{DF}} \approx & \sum_{j=1}^{N} \binom{N}{j} \left(1 - \frac{\eta(1)\alpha_0}{g\Omega_{\mathrm{SR}}}\right)^j \left(\frac{\eta(1)\alpha_0}{g\Omega_{\mathrm{SR}}}\right)^{N-j} \frac{\eta(j+1)\alpha_0^{j+1}}{\Omega_{\mathrm{SD}}\Omega_{\mathrm{RD}}^j g^{j+1}} \\ & + \left(\frac{\eta(1)\alpha_0}{g\Omega_{\mathrm{SR}}}\right)^N \frac{\eta(1)\alpha_0}{g\Omega_{\mathrm{SD}}} \\ \approx & \frac{1}{\Omega_{\mathrm{SD}}} \left(\frac{\alpha_0}{g}\right)^{N+1} \left(\sum_{j=1}^{N} \binom{N}{j} \left(\frac{\eta(1)}{\Omega_{\mathrm{SR}}}\right)^{N-j} \frac{\eta(j+1)}{\Omega_{\mathrm{RD}}^j} + \frac{1}{\Omega_{\mathrm{SR}}^N}\eta^{N+1}(1)\right) \end{aligned} \tag{4.21}$$

4.3 Numerical Results

In this section, to illustrate the SER performance of multi-relay networks, we consider a network consisting of three relays deploying 8-PSK and the direct link channel is characterized by $\Omega_{\mathrm{SD}} = 1$. Fig. 4.2 represents the SER performance of a symmetric system with $\Omega_{\mathrm{SR}} = \Omega_{\mathrm{RD}} = 1$. The plots in Fig. 4.3 and Fig. 4.4 demonstrate specific asymmetric systems with the constraint ($\Omega_{\mathrm{SR}} = 0.5$ and $\Omega_{\mathrm{RD}} = 5$) and ($\Omega_{\mathrm{SR}} = 5$ and $\Omega_{\mathrm{RD}} = 0.5$), respectively.

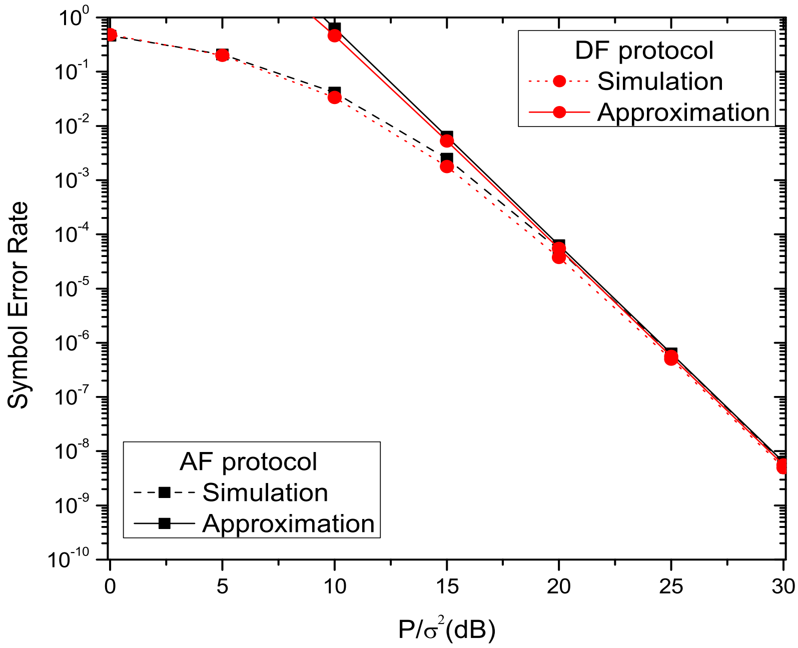

Figure 4.3: Simulations and approximations on SER in 3 relays system using 8-PSK, $\Omega_{\text{SD}} = 1, \Omega_{\text{SR}} = 5$ and $\Omega_{\text{RD}} = 0.5$.

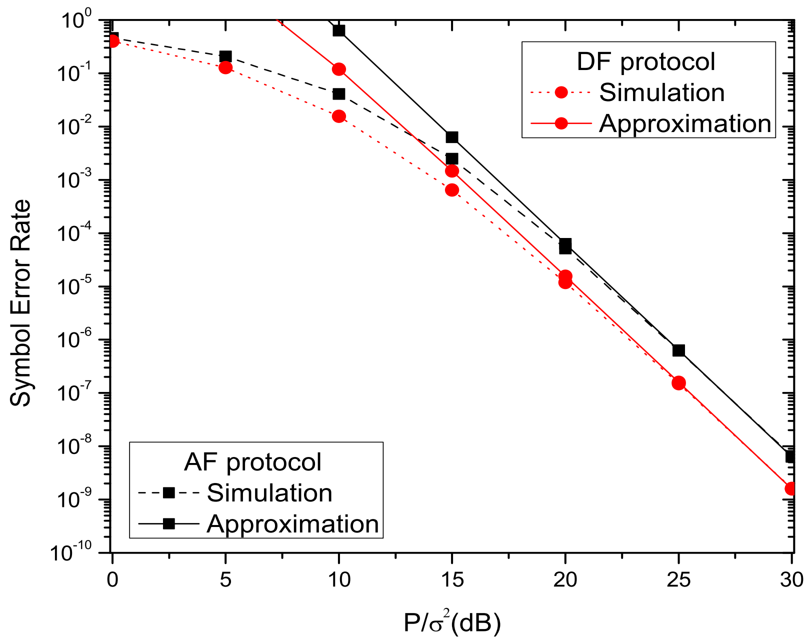

Figure 4.4: Simulations and approximations on SER in 3 relays system using 8-PSK, $\Omega_{\text{SD}} = 1, \Omega_{\text{SR}} = 0.5$ and $\Omega_{\text{RD}} = 5$.

Chapter 5

Two-Way AF Relay Networks

5.1 Multi-hop Half-Duplex AF Relay Networks

A specific dual-hop cooperative half-duplex relay system is modeled as shown in Fig. 5.1 where the two terminals, namely T_1 and T_2, communicate to each other through the help of the relay R. It is assumed that there is no direct connection between nodes T_1 and T_2 and the system operates in half-duplex protocol, i.e., all the nodes cannot transmit and receive at the same time on the same frequency channel. The communicating process takes place in two separate phases. Firstly, the source node transmits only to the relay and after being amplified at the relay, the scaled signal is forwarded to the destination node. The reverse path from T_2 to T_1 proceeds in the similar manner by transiting at the relay.

As considering the transmission from node T_1 to T_2, the first phase received symbol at the relay R is given by

$$y_R = h_{T_1} x_{T_1} + n_R \tag{5.1}$$

where h_{T_1} is the amplitude channel gain of $T_1 \rightarrow R$ link, $x_{T_1} \sim \mathcal{CN}$ (0, P_{T_1}) the transmitted signal from node T_1 and $n_R \sim \mathcal{CN}$ (0, σ_R^2) the relay's AWGN. At the relay, the received signal y_R is scaled to

$$\beta = \sqrt{\frac{P_R}{P_{T_1} \left|h_{T_1}\right|^2 + \sigma_R^2}} \tag{5.2}$$

where P_R is the relay's average transmit power. Hence, the signal received at the destination in the next phase is followed by

$$y_{T_2} = h_{T_2} \beta h_{T_1} x_{T_1} + h_{T_2} \beta n_R + n_{T_2} \tag{5.3}$$

where h_{T_2} is the $R \rightarrow T_2$ channel coefficient and $n_{T_2} \sim \mathcal{CN}$ (0, $\sigma_{T_2}^2$) the AWGN at node T_2. As a consequence the respective average achievable rate as using dual-hop

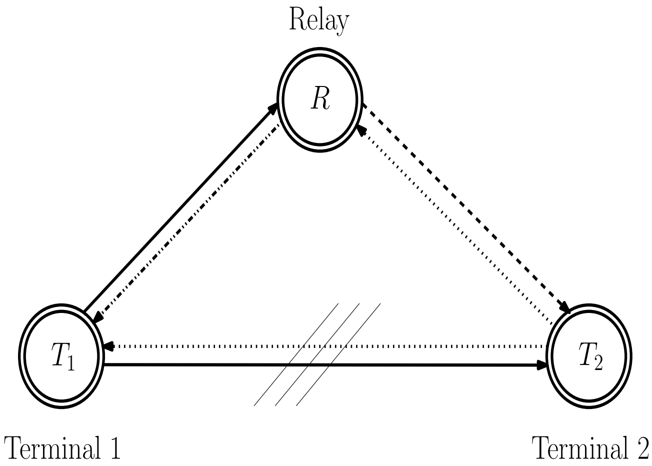

Figure 5.1: Conventional half-duplex relay network.

AF scheme is deduced by

$$R^{\mathrm{AF}} = \frac{1}{2}\,\mathbb{E}\left\{\log\left(1+\frac{P_{T_1}\left|\beta h_{T_1}h_{T_2}\right|^2}{\sigma_R^2\left|\beta h_{T_2}\right|^2+\sigma_{T_2}^2}\right)\right\}$$
$$= \frac{1}{2}\,\mathbb{E}\left\{1+q\left(\frac{P_{T_1}\left|h_{T_1}\right|^2}{\sigma_R^2},\frac{P_R\left|h_{T_2}\right|^2}{\sigma_D^2}\right)\right\} \tag{5.4}$$

where $\mathbb{E}\{.\}$ denotes the expectation function and $q\{.\}$ is defined in (2.30).

5.2 Two-Way AF Relay Networks

In order to overwhelm the loss of spectral efficiency due to the pre-log factor of $\frac{1}{2}$, the two-way AF relaying scheme has been proposed. A specific bi-directional relay system is shown in Fig. 5.2. Each of the two nodes T_1 and T_2 desires to transmit a signal to the other due to the assumption of no direct link all traffics need to transit at the relay R. In the suggested two-way AF relay network, during the first phase both terminals' messages are sent to the relay which in turn are amplified and broadcasted to both destinations in the next phase.

In this protocol, the message received in phase one at the relay is now given by

$$y_R = h_{T_1}x_{T_1} + h_{T_2}x_{T_2} + n_R \tag{5.5}$$

where $x_{T_2} \sim \mathcal{CN}\,(0, P_{T_2})$ is the symbol transmitted from node T_2. Consequently, the scale factor of the relay is followed by

$$\beta = \sqrt{\frac{P_R}{P_{T_1}\left|h_{T_1}\right|^2 + P_{T_2}\left|h_{T_2}\right|^2 + \sigma_R^2}} \tag{5.6}$$

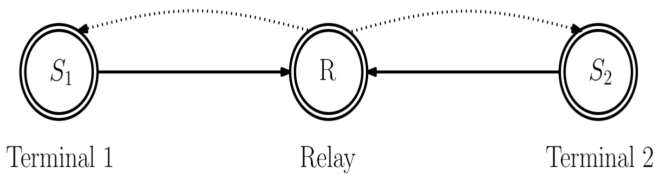

Figure 5.2: Two-way relay network.

According to the assumption on channel reciprocity, in the next phase the terminals T_1 and T_2 receive, respectively, the signal

$$y_{T_1} = h_{T_1}\beta h_{T_1}x_{T_1} + h_{T_1}\beta h_{T_2}x_{T_2} + h_{T_1}\beta n_R + n_{T_1} \tag{5.7}$$

$$y_{T_2} = h_{T_2}\beta h_{T_1}x_{T_1} + h_{T_2}\beta h_{T_2}x_{T_2} + h_{T_2}\beta n_R + n_{T_2} \tag{5.8}$$

Presuming the terminals T_1 and T_2 have perfect knowledge on their channel coefficients, the self-interference in (5.7) and (5.8) can be fully eliminated before decoding. As a result, the total sum-rate of bi-directional AF protocol is determined by

$$R_{\text{AF}}^{\text{sum}} = \frac{1}{2}\,\mathbb{E}\left\{\log\left(1+\frac{P_{T_1}\left|\beta h_{T_1}h_{T_2}\right|^2}{\sigma_R^2\left|\beta h_{T_2}\right|^2+\sigma_{T_2}^2}\right)\right\} + \frac{1}{2}\,\mathbb{E}\left\{\log\left(1+\frac{P_{T_2}\left|\beta h_{T_1}h_{T_2}\right|^2}{\sigma_R^2\left|\beta h_{T_1}\right|^2+\sigma_{T_1}^2}\right)\right\} \tag{5.9}$$

5.3 Performance Comparison between Two-Way AF Relay and Dual-hop AF Relay Networks

In this section, to evaluate the performance of bidirectional AF relay networks, we compare SER, average sum-rate and outage probability of such networks with the conventional dual-hop AF relay networks. Assuming $h_{T_i} \sim \mathcal{CN}(0,\Omega_i)$ with $i = 1, 2$, the same SNR of P/σ^2 at all receiving nodes and $\gamma_{\text{th}} = 3$, by using QPSK modulation the comparisons in Fig. 5.3, Fig. 5.4 and Fig. 5.5 are represented in the i.i.d. case $\Omega_1 = \Omega_2 = 1$ and the i.n.i.d case $\Omega_1 = 3, \Omega_2 = 5$.

5.4 Statistic Metrics of Two-Way AF Relay Networks

For the sake of simplicity, we assume an equal transmit power P at T_1, T_2, and R, and equal variance σ^2 for the AWGN at the three nodes. Denoting $h_{T_i} \sim \mathcal{CN}(0,\Omega_i)$ with $i = 1, 2$ as the independent channel amplitude gain for the link from T_i to the relay R. Consequently, the instantaneous SNR γ_{T_i} of terminal T_i, deduced from (5.9) is given by

$$\gamma_{T_i} = \frac{P\left|h_{T_i}h_{T_j}\beta\right|^2}{\sigma^2\left(\left|h_{T_j}\beta\right|^2+1\right)} \tag{5.10}$$

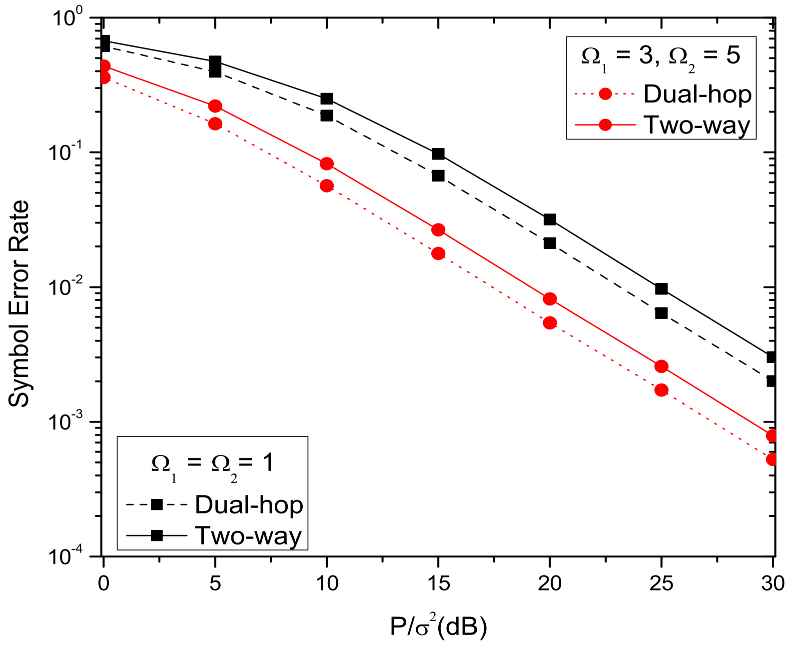

Figure 5.3: Comparisons on SER between dual-hop and two-way AF relay networks using QPSK.

where

$$\beta^2 = \frac{1}{|h_{T_i}|^2 + |h_{T_j}|^2 + \frac{\sigma^2}{P}} \tag{5.11}$$

By denoting $\gamma_i = \frac{P|h_{T_i}|^2}{\sigma^2}$ and $\gamma_j = \frac{P|h_{T_j}|^2}{\sigma^2}$ $(j = 1, 2 \text{ and } i \neq j)$ the respective instantaneous SNR for $T_i \to R$ and $T_j \to R$ links, γ_{T_i} can be rewritten as

$$\gamma_{T_i} = \frac{\gamma_i \gamma_j}{2\gamma_i + \gamma_j + 1} \approx \frac{\gamma_i \gamma_j}{2\gamma_i + \gamma_j} \tag{5.12}$$

and γ_i and γ_j are independently exponentially distributed random variables with hazard rates $1/(\gamma_0\Omega_i)$ and $1/(\gamma_0\Omega_j)$ respectively, where $\gamma_0 = P/\sigma^2$ is the average transmit SNR. It is observed that the performance of bi-directional relay networks strictly depends on the statistical behavior of the SNR given in (5.12). Due to the asymmetry of γ_{T_i}, i.e., occurring the factor 2 in the denominator of (5.12)), the statistic metrics of γ_{T_i}, e.g., CDF, PDF, and MGF, cannot be deduced directly by applying the approach in [12]. From this point of view, we start our derivation by rewriting γ_{T_i} in a more tractable form

$$\gamma_{T_i} = (X_i + X_j)^{-1} \tag{5.13}$$

in which $X_i = 1/\gamma_i$ and $X_j = 2/\gamma_j$. By deploying Jacobian transformation, we can

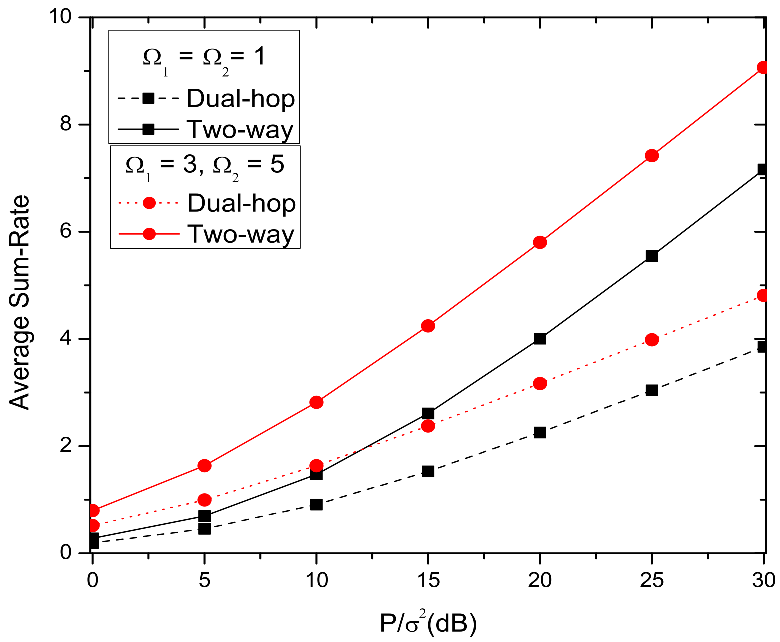

Figure 5.4: Comparisons on average sum-rate between dual-hop and two-way AF relay networks.

obtain the PDF of X_i

$$p_{X_i}(x_i) = \frac{1}{\Omega_i \gamma_0 x_i^2} e^{-1/(\Omega_i \gamma_0 x_i^2)} \tag{5.14}$$

as well as the PDF of X_j

$$p_{X_j}(x_j) = \frac{2}{\Omega_j \gamma_0 x_j^2} e^{-2/(\Omega_j \gamma_0 x_j^2)} \tag{5.15}$$

Hence, the MGF of X_i and X_j can be respectively given by

$$\Phi_{X_i}(s) = \mathbb{E}_{X_i}\left\{e^{-sx_i}\right\} = \int_0^\infty \frac{1}{\Omega_i \gamma_0 x_i^2} e^{-sx_i - 1/(\Omega_i \gamma_0 x_i^2)} dx_i \tag{5.16}$$

and

$$\Phi_{X_j}(s) = \mathbb{E}_{X_j}\left\{e^{-sx_j}\right\} = \int_0^\infty \frac{2}{\Omega_j \gamma_0 x_j^2} e^{-sx_j - 2/(\Omega_j \gamma_0 x_j^2)} dx_j \tag{5.17}$$

Through the help of [22, eq. (3.471.9)], (5.16) and (5.17) can be rewritten in closed form as

$$\Phi_{X_i}(s) = 2\sqrt{\frac{s}{\bar{\gamma}_i}} \mathcal{K}_1\left(2\sqrt{\frac{s}{\bar{\gamma}_i}}\right), \tag{5.18}$$

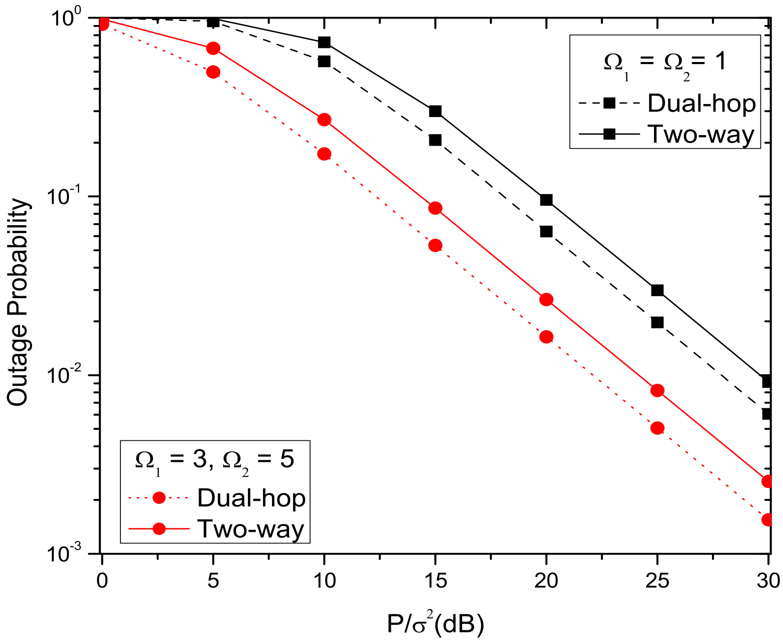

Figure 5.5: Comparisons on outage probability between dual-hop and two-way AF relay networks.

$$\Phi_{X_j}(s) = 2\sqrt{\frac{2s}{\bar{\gamma}_j}}\mathcal{K}_1\left(2\sqrt{\frac{2s}{\bar{\gamma}_j}}\right) \tag{5.19}$$

where $\bar{\gamma}_i = \gamma_0\Omega_i$, $\bar{\gamma}_j = \gamma_0\Omega_j$, and $\mathcal{K}_v(z)$ is defined as the v-th order modified Bessel function of the second kind [22, eq. (8.432.6)].

Since X_i and X_j are statistically independent, the MGF of a random variable $Y_i = X_i + X_j$ can be expressed as

$$\Phi_{Y_i}(s) = 4\sqrt{\frac{2}{\bar{\gamma}_i\bar{\gamma}_j}}\, s\, \mathcal{K}_1\left(2\sqrt{\frac{s}{\bar{\gamma}_i}}\right)\mathcal{K}_1\left(2\sqrt{\frac{2s}{\bar{\gamma}_j}}\right) \tag{5.20}$$

Noting that $\gamma_{T_i} = 1/Y_i$, the CDF of γ_{T_i} is then deduced as

$$F_{\gamma_{T_i}}(\gamma) = 1 - F_{Y_i}(1/\gamma) \tag{5.21}$$

By the inverse Laplace transformation $\mathcal{L}^{-1}\{.\}$, it can be shown that

$$\begin{aligned} F_{\gamma_{T_i}}(\gamma) &= 1 - \mathcal{L}^{-1}\{\Phi_{Y_i}(s)/s\}\big|_{y_i=1/\gamma} \\ &= 1 - 4\sqrt{\frac{2}{\bar{\gamma}_i\bar{\gamma}_j}}\int_0^\infty \mathcal{K}_1\left(2\sqrt{\frac{s}{\bar{\gamma}_i}}\right)\mathcal{K}_1\left(2\sqrt{\frac{2s}{\bar{\gamma}_j}}\right)e^{sy_i}ds\Big|_{y_i=1/\gamma} \end{aligned} \tag{5.22}$$

Substituting [23, eq. (13.2.20)] to (5.20) and (5.21), the closed-form expression of $F_{\gamma_{T_i}}(\gamma)$ is obtained as follows

$$F_{\gamma_{T_i}}(\gamma) = 1 - a\gamma \exp(-b_i\gamma)\,\mathcal{K}_1(a\gamma) \tag{5.23}$$

where $a = 2\sqrt{\frac{2}{\bar{\gamma}_1\bar{\gamma}_2}}$ and $b_i = 1/\bar{\gamma}_i + 2/\bar{\gamma}_j$. From [22, eq. (8.486.12)], it is observed that

$$\frac{d\mathcal{K}_v(z)}{dz} = -\mathcal{K}_{v-1}(z) - \frac{v}{z}\mathcal{K}_v(z) \tag{5.24}$$

The PDF of γ_{T_i}, defined by $p_{\gamma_{T_i}}(\gamma)$ is then achieved from differentiating (5.23) with respect to γ

$$p_{\gamma_{T_i}}(\gamma) = a\gamma\, e^{-b_i\gamma}\left[a\mathcal{K}_0(a\gamma) + b_i\mathcal{K}_1(a\gamma)\right] \tag{5.25}$$

The derivation to the compact form of $p_{\gamma_{T_i}}(\gamma)$ readily allows us to obtain closed-form expressions for several important performance metrics as follows in the next section.

5.5 Performance Analysis

5.5.1 Average Symbol Error Rate

In order to obtain the closed-form expression of SER at the terminal T_i, it is required to derive the MGF of γ_{T_i}:

$$\begin{aligned}\Phi_{\gamma_{T_i}}(s) &= \mathbb{E}_{\gamma_{T_i}}\left\{e^{-s\gamma}\right\} \\ &= \int_0^\infty a\gamma\left[a\mathcal{K}_0(a\gamma) + b_i\mathcal{K}_1(a\gamma)\right]\, e^{(-b_i-s)\gamma} d\gamma \end{aligned} \tag{5.26}$$

With the help of [22, eq. (6.621.3)], $\Phi_{\gamma_{T_i}}(s)$ is then given by

$$\begin{aligned}\Phi_{\gamma_{T_i}}(s) = \frac{4a^2}{3(a+b_i+s)^2}&\left[{}_2F_1\left(2,\frac{1}{2};\frac{5}{2};\frac{b_i-a+s}{b_i+a+s}\right)\right. \\ &\left. + \frac{4b_i}{a+b_i+s}\,{}_2F_1\left(3,\frac{3}{2};\frac{5}{2};\frac{b_i-a+s}{b_i+a+s}\right)\right]\end{aligned} \tag{5.27}$$

where ${}_2F_1(a,b;c;z)$ is the Gauss hypergeometric function. Specifically, the SER of T_i for M-PSK can be expressed as

$$P_s^{\text{PSK}} = \frac{1}{\pi}\int_0^{\pi-\frac{\pi}{M}} \Phi_{\gamma_{T_i}}\left(\frac{g_{\text{PSK}}}{\sin^2\theta}\right) \mathrm{d}\theta \tag{5.28}$$

5.5.2 Average Sum-Rate

The average sum-rate of the bi-directional relay networks as defined in (5.9) can be shown in the form

$$R_{\text{sum}} = \sum_{i=1}^{2} \underbrace{\mathbb{E}_{\gamma_{T_i}} \left\{ \frac{1}{2} \log_2 \left(1 + \gamma_{T_i}\right) \right\}}_{I} \tag{5.29}$$

From (5.25), the summand of (5.29) can be rewritten as

$$I = \frac{a^2}{2\ln 2} \int_0^\infty \ln\left(1+\gamma\right) \gamma e^{(-b_i\gamma)} \mathcal{K}_0\left(a\gamma\right) \mathrm{d}\gamma + \frac{ab_i}{2\ln 2} \int_0^\infty \ln\left(1+\gamma\right) \gamma e^{(-b_i\gamma)} \mathcal{K}_1\left(a\gamma\right) \mathrm{d}\gamma \tag{5.30}$$

With the aim of evaluating I, it is needed to calculate each integral

$$J = \int_0^\infty \ln\left(1+x\right) x e^{(-b_i x)} \mathcal{K}_n\left(ax\right) \mathrm{d}x \tag{5.31}$$

in which $n = 0, 1$. Through the helps of [24, eq. (8.4.23.3)], [24, eq. (8.4.6.5)], [24, eq. (8.3.2.21)], J can be rewritten as

$$J = \sqrt{\pi} \int_0^\infty x e^{-(b_i - a)x} H_{2,2}^{1,2}\left[x \left| \begin{matrix} (1,1),(1,1) \\ (1,1),(0,1) \end{matrix} \right. \right] H_{1,2}^{2,0}\left[2ax \left| \begin{matrix} (\frac{1}{2},1) \\ (n,1),(-n,1) \end{matrix} \right. \right] \mathrm{d}x \tag{5.32}$$

where $H_{C,D}^{A,B}[\cdot]$ is the Fox's H-function [24, eq. (8.3.1.1)]. Therefore, the integral J can be evaluated with the help of [25, eq. (2.6.2)] as

$$J = \sqrt{\pi}\left(b_i - a\right)^{-2} H_{1,[2:1],0,[2:2]}^{1,2,0,1,2}\left[\begin{matrix} \frac{1}{b_i - a} \\ \\ \frac{2a}{b_i - a} \end{matrix} \left| \begin{matrix} (2,1) \\ (1,1),(1,1);(\frac{1}{2},1) \\ -- \\ (1,1),(0,1);(n,1),(-n,1) \end{matrix} \right. \right] \tag{5.33}$$

in which $H_{E,[A:C],F,[B:D]}^{K,N,N',M,M'}[\cdot]$ is the generalized Fox's H-function [25, eq. (2.2.1)]. By substituting (5.33) in (5.30), we obtain the closed-form expression of the summand I, which in turn helps us to obtain the average sum-rate as following

$$R_{\text{sum}} = \sum_{i=1}^{2} \frac{\sqrt{\pi} a \left(b_i - a\right)^{-2} \xi_i}{2\ln 2} \times H_{1,[2:1],0,[2:2]}^{1,2,0,1,2}\left[\begin{matrix} \frac{1}{b_i - a} \\ \\ \frac{2a}{b_i - a} \end{matrix} \left| \begin{matrix} (2,1) \\ (1,1),(1,1);(\frac{1}{2},1) \\ -- \\ (1,1),(0,1);(i-1,1),(1-i,1) \end{matrix} \right. \right] \tag{5.34}$$

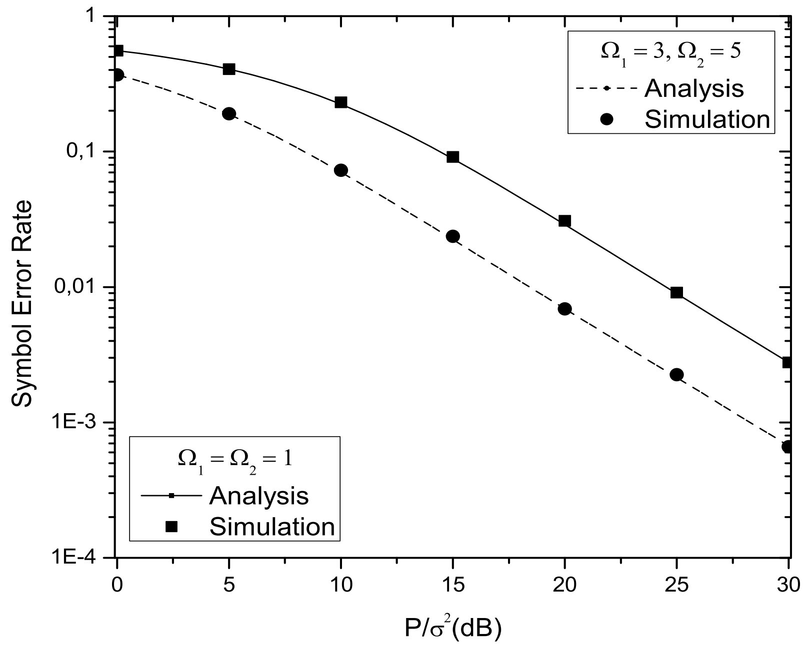

Figure 5.6: SER of bi-directional AF relay networks using QPSK.

where $\xi_1 = a$ and $\xi_2 = b_i$.

5.5.3 Outage Probability

Defining the outage probability P_{out} at T_i as the probability that the instantaneous SNR is lower than a given threshold γ_{th}, i.e., $P\{\gamma_{T_i} \leq \gamma_{\text{th}}\}$. From (5.23), P_{out} is easily obtained as follows:

$$P_{\text{out}} = 1 - a\gamma_{\text{th}} \exp\left(-b_i\gamma_{\text{th}}\right) \mathcal{K}_1\left(a\gamma_{\text{th}}\right) \tag{5.35}$$

5.6 Numerical Results and Discussions

In order to verify our analytical expressions, we represent Monte-Carlo simulation results and compare them with our analysis. We investigate the specific system employing QPSK modulation with $\gamma_{\text{th}} = 3$ for two specific cases: 1) i.i.d. case: (e.g., $\Omega_1 = \Omega_2 = 1$) and 2) i.n.i.d. case: (e.g., $\Omega_1 = 3, \Omega_2 = 5$). The performance metrics such as SER, average sum-rate, and outage probability for the bi-directional AF relay network are illustrated in Fig. 5.6, Fig. 5.7, and Fig. 5.8, respectively. From the three figures, it can be seen that the analyses give close results to the simulation.

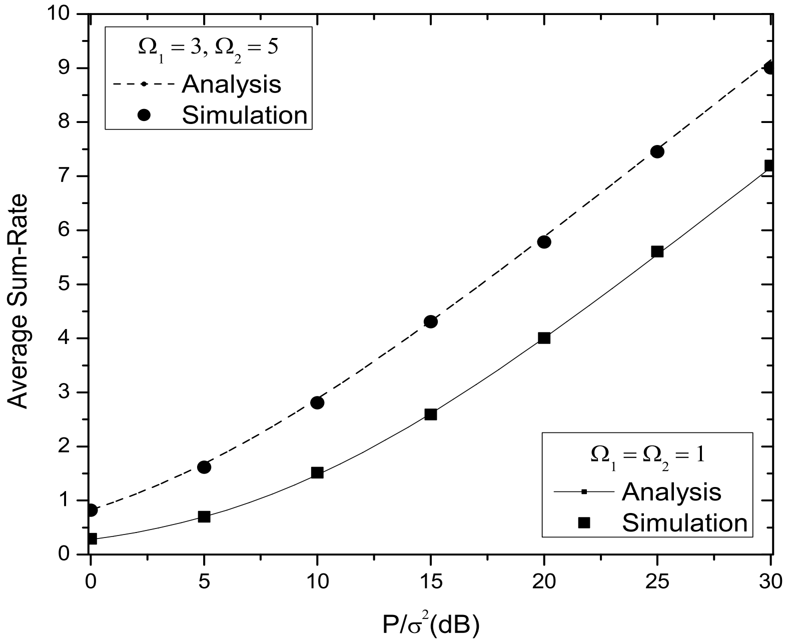

Figure 5.7: Average sum-rate of bi-directional AF relay networks.

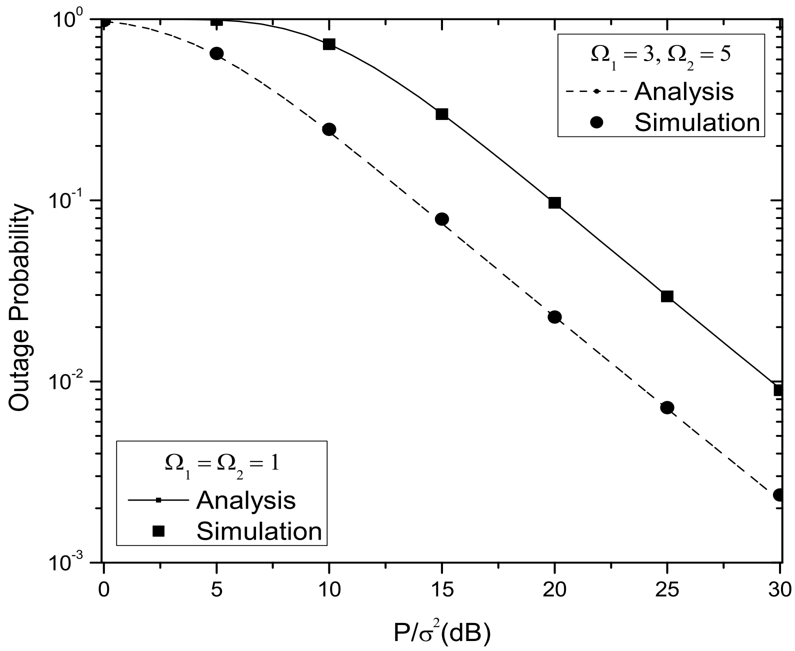

Figure 5.8: Outage probability of bi-directional AF relay networks.

Chapter 6

Conclusions

MIMO has been introduced as one of the most significant contributions in research literature. However it is impossible to attach many antennas in a mobile headset. Therefore, cooperative communications have been proposed in wireless relay networks by forming virtual arrays of antennas. A set of half-duplex relays is deployed to transmit the signal in two phases. In the first time-slot, the source broadcasts to the destination and also the relays. During the second phase, signal after being amplified or decoded will be relayed to the destination. The manner of two time-slots transmission yields the main disadvantage of conventional relaying networks as half capacity compared to MIMO system.

From these points of view, two-way (or bi-directional) relay networking has been introduced. At first, not only the source but the destination transmit to the relay simultaneously and in the next phase the received signal will be forwarded to both the source and the destination. Hence this altered protocol has extensively conducted an improved total sum-rate.

This thesis has provided the performance analysis on the conventional as well as two-way relay networks. In Chapter 3, the exact closed form expressions of SER in single-relay networks have been derived. Moreover, based on the approximations of SER at high SNR, it has provided the comparison on cooperative gains between the AF and ADF protocols. The outage probability of such networks is also investigated. Chapter 4 represents the extension to the SER analysis in the previous chapter on the generalized multi-relay networks. The bi-directional AF relay networks have been introduced and analyzed on SER, average sum-rate and outage probability in Chapter 5.

Bibliography

[1] İ. E. Telatar, "Capacity of multi-antenna Gaussian channels," *European Trans. Telecommun.*, vol. 10, no. 6, pp. 585–595, Nov./Dec. 1999.

[2] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 16, no. 8, pp. 1451–1458, Oct. 1998.

[3] G. J. Foschini, "Layered space-time architecture for wireless communication in a fading environment when using multi-element antennas," *Bell Labs Tech. J.*, pp. 41–59, Oct 1996.

[4] D.-S. Shiu, G. J. Foschini, M. J. Gans, and J. M. Kahn, "Fading correlation and its effect on the capacity of multielement antenna systems," *IEEE Trans. Commun.*, vol. 48, no. 3, pp. 502–513, Mar. 2000.

[5] E. C. van der Meulen, "Three-terminal communication channels," *Advances in Applied Probability*, vol. 3, pp. 120–154, 1971.

[6] T. M. Cover and A. A. El Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inf. Theory*, vol. 25, no. 5, pp. 572–584, Sep. 1979.

[7] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity—Part I: System description," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927–1938, Nov. 2003.

[8] ——, "User cooperation diversity—Part II: Implementation aspects and performance analysis," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1939–1948, Nov. 2003.

[9] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.

[10] A. Høst-Madsen and J. Zhang, "Capacity bounds and power allocation for wireless relay channels," *IEEE Trans. Inf. Theory*, vol. 51, no. 6, pp. 2020–2040, Jun. 2005.

[11] M. A. Khojastepour, A. Sabharwal, and B. Aazhang, "On capacity of gaussian cheap relay channel," in *Proc. IEEE Global Commununications Conf.*, Dec. 2003, pp. 1776–1780.

[12] M. O. Hasna and M.-S. Alouini, "Harmonic mean and end-to-end performance of transmission systems with relays," *IEEE Trans. Commun.*, vol. 52, no. 1, pp. 130–135, Jan. 2004.

[13] T. Q. Duong and H.-J. Zepernick, "On the performance gain of hybrid decode-amplify-forward cooperative communications," *EURASIP Journal on Wireless Communications and Networking*, 2009, article ID 479463, 10 pages, 2009. doi:10.1155/2009/479463.

[14] ——, "Hybrid decode-amplify-forward cooperative communications with multiple relays," in *Proc. IEEE Wireless Commun. and Networking Conf.*, Budapest, Hungary, Apr. 2009, pp. 1–6.

[15] B. Rankov and A. Wittneben, "Spectral efficient protocols for half-duplex fading relay channels," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 2, pp. 379–389, Feb. 2007.

[16] P. Popovski and H. Yomo, "Wireless network coding by amplify-and-forward for bi-directional traffic flows," *IEEE Commun. Lett.*, vol. 11, no. 1, pp. 16–18, Jan. 2007.

[17] T. Unger and A. Klein, "Duplex schemes in multiple antenna two-hop relaying," *EURASIP Journal on Advances in Signal Processing*, 2008, article ID 128592, 14 pages, 2008. doi:10.1155/2008/128592.

[18] Y. Han, S. H. Ting, C. K. Ho, and W. H. Chin, "High rate two-way amplify-and-forward half-duplex relaying with OSTBC," in *Proc. of the IEEE 67th Vehicular Technology Conference*, Marina Bay, Singapore, May 2008, pp. 2426–2430.

[19] I. Hammerstrom, M. Kuhn, C. Esli, J. Zhao, A. Wittneben, and G. Bauch, "MIMO two-way relaying with transmit CSI at the relay," in *Proc. of the IEEE 8th Workshop on Signal Processing Advances in Wireless Communications*, Helsinki, Finland, Jun. 2007, pp. 1–5.

[20] Q. T. Duong, L.-N. Hoang, and V. N. Q. Bao, "On the performance of two-way amplify-and-forward relay networks," *IEICE Trans. Commun.*, vol. E92-B, no. 12, pp. 3957–3959, Dec. 2009.

[21] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels: A Unified Approach to Performance Analysis*. New York: John Wiley & Sons, 2000.

[22] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 6th ed. San Diego, CA: Academic Press, 2000.

[23] G. E. Roberts and H. Kaufman, *Table of Laplace Transforms*. W. B. Saunders, 1966.

[24] A. P. Prudnikov, Y. A. Brychkov, and O. I. Marichev, *Integrals and Series*. New York: Gordon and Breach Science, 1990, vol. 3.

[25] A. M. Mathai and R. K. Saxena, *The H-function with Applications in Statistics and Other Disciplines*. New York: Wiley, 1978.