GRAVITY FLOW OF DUSTY VISCO-ELASTIC FLUID THROUGH AN INCLINED CHANNEL WITH THE EFFECT OF MAGNETIC FIELD

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Abstract

The gravity flow of dusty visco-elastic fluid through a inclined channel in the presence of transverse magnetic field is studied. It is found that as the values of time t increases then the velocity profiles for the liquid and the dust particles decreases. The mass concentration l increases the velocity of the liquid and the dust particles decreases and when the magnetic field term (Hartmann number M) increases the velocity of the liquid and dust particles decreases considerably. Also visco-elastic parameter λ increases and inclination angle θ increases the velocity of the liquid and the dust particles decreases .It is interesting to note that when visco-elastic parameter $\lambda \to 0$ and inclination angle θ or gravity parameter $g \to 0$

Key words: Visco-elastic fluid and fluid phase, Inclined hannel, Gravity, Hartmann Number.

Introduction

The subject of magnetohydrodynamics (MHD) has also developed in many directions and industry has exploited the use of magnetic fields in controlling a range of fluid and thermal processes. Many studies of the influence of magnetism
on electrically-conducting flows have been reported with a plethora of other physical phenomena. Poots [1] studied analytically the laminar natural convection
magnetohydrodynamic flows between parallel plane surfaces and also through
a horizontal circular tube incorporating viscous and Joule electrical dissipation
effects as well as internal energy generation. He showed that velocities andheat transfer rates were reduced by magnetic field. Soundalgekar and Takhar[2] investigated the MHD oscillatory flow past a flat plate, showing numerically that for flat plate flows magnetic field depresses heat transfer rates.

Visco-elastic fluid flows are important in petroleum industry and in the purification of crude oils. Other important Applications involving dust particles in boundary layers including oil salvation by natural winds, lunar surface erosion by the exhaust of a landing vehicle and dust entrainment in a cloud formed during a nuclear explosion. Singh et al has studied the unsteady flow of a conducting of fluid through a rectangular channel with time dependent pressure gradient [3]. The Magnetic field flow in a two dimensional diverging channel studied by Verma et al [4]. Hunt and Eleibouch have discussed the Magneto hydrodynamic flow in channel of variable cross section with strong transient magnetic field [5]. Soundalgekar have been discussed the oscillator MHD channel flow and heat transfer. Chakraborty and Borkakati have been obtained the solution of MHD flow and heat transfer of a dusty visco-elastic fluid down an inclined channel in porous medium [6]. Singh et al has discussed the free convection and mass transfer flow of Rivlin-Erickson fluid in the presence constant heat flux and uniform magnetic field [7-8].

The aim of this chapter is to study the gravity flow of a dusty visco-elastic liquid through an inclined channel with the effect of transverse magnetic field. The change in velocity profiles for dust and liquid particles for different values of time and different parameters has been depicted graphically.

Formulation and solution of the problem

The X-axis is taken along the plate and the Y-axis normal to it. The usual Prandtl boundary layer assumptions along with Maxwell's equations, together with Ohm's law and the law of electromagnetic conservation, are written in the case of zero-displacement and hall currents leads to the following equations:

$$\frac{\partial u'_1}{\partial t'} = -\frac{1}{\rho}\frac{\partial P'}{\partial x'} + \left(\gamma' + \beta\frac{\partial}{\partial t'}\right)\frac{\partial^2 u'_1}{\partial y^2} + \frac{KN_0}{\rho}(u'_2 - u'_1) + g\sin\theta - \frac{\sigma_1 B_0^2}{\rho'}u_1 \quad \ldots(1)$$

$$m\,\frac{\partial u'_2}{\partial t'} = K(u'_1 - u'_2) \quad \ldots(2)$$

where the u_1', u_2' denotes the velocity vector of fluid and dust particles respectively: p $'$ the pressure : ρ' the density of the fluid: γ' kinematic coefficient of viscosity: t $'$ the time: m, the mass of the dust particles : N_0, the number density of dust particles: K, the stokes resistance coefficient which for spherical particles of radius a is $6\pi\mu'$ a: μ , the coefficient of viscosity of fluid particles.

Which are to be solved subject to the boundary conditions.

$$t' = 0 \;,\quad u'_1 = u'_2 = 0$$

$$t' > 0 \; -\frac{1}{\rho}\frac{\partial P'}{\partial x'} = C \text{ (constant)} \quad \ldots(3)$$

$$y' = \mp h \;,\; u'_1 = 0, u'_2 \;,$$

Changing it into non dimensional form by putting

$$y = \frac{y'}{h} \;, t = \frac{\gamma' t'}{h^2}, u = \frac{u'_1 h}{\gamma'}, v = \frac{u'_2}{\gamma'}, p = \frac{p' h^2}{\rho' \gamma'^2}, x = \frac{x'}{h}$$

We have

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \left(1 - \lambda\frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial y^2} + \frac{l}{\sigma}(v-u) - M^2 u + R \quad \ldots(4)$$

$$\sigma\frac{\partial v}{\partial t} = u - v \quad \ldots(5)$$

Where

$\sigma = \frac{m\gamma'}{Kh^2}$ Relaxation time parameter.

$l = \frac{mN_0}{\rho}$ Mass Concentration.

$\lambda = \frac{-\beta}{h^2}$ Visco-elastic parameter.

$M = \sqrt{\frac{\sigma_1}{\mu'}} B_0 h$ Hartmann number.

R= f $\sin\theta$. Where f = $\frac{gh^3}{\gamma'^2}$ Gravity parameter.

The boundary conditions are

$$\begin{aligned} &at\ t = 0,\ \ \bar{u} = 0, \bar{v} = 0 \\ &at\ t > 0,\ \ \bar{u} = 0,\ \bar{v} = 0\ \ at\ \ y = -1 \\ &\qquad\quad \bar{u} = 0, \bar{v} = 0\ \ at\ \ y = 1 \end{aligned} \qquad \ldots(6)$$

take $-\dfrac{\partial p}{\partial x} = C$ (constant) for t > 0.

Then the equation (4) becomes

$$\frac{\partial u}{\partial t} = C + \left(1 - \lambda \frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial y^2} + \frac{l}{\sigma}(v - u) - M^2 u + R \qquad \ldots(7)$$

Appling the Laplace Transform, we have from (5) and (7)

$$S\,\bar{u} = \frac{C}{S} + (1 - \lambda S)\frac{\partial^2 \bar{u}}{\partial y^2} + \frac{l}{\sigma}(\bar{v} - \bar{u}) - M^2 \bar{u} + \frac{R}{S} \qquad \ldots(8)$$

$$\sigma\, S\, \bar{v} = \bar{u} - \bar{v} \qquad \ldots(9)$$

where

$$\bar{u} = \int_0^\infty u\ e^{-st} dt, \qquad \bar{v} = \int_0^\infty v\ e^{-st} dt$$

The boundary conditions (6) are transformed to

$$\bar{u} = 0\ ,\ \bar{v} = 0\ \ at\ y = \pm 1 \qquad \ldots(10)$$

Solving equations (8) and (9) subject to the boundary conditions (10)

we have

$$\frac{d^2 \bar{u}}{dy^2} - \alpha_2^2 \bar{u} = \frac{-1}{(1 - \lambda S)}\left[\frac{C}{S} + \frac{R}{S}\right] \qquad \ldots(11)$$

Where

$$\alpha_2^2 = \frac{(S + M^2)(1 + \sigma S) + ls}{(1 - \lambda S)(1 + \sigma\, S)} \qquad \ldots(12)$$

Finally

$$\bar{u} = \frac{(C + R)}{\alpha_2^2\, S(1 - \lambda S)}\left\{1 - \frac{\cosh \alpha_2 y}{\cosh \alpha_2}\right\} \qquad \ldots(13)$$

$$\bar{v} = \frac{(C + R)}{\alpha_2^2 S(1 - \lambda S)(1 + \sigma S)}\left\{1 - \frac{\cosh \alpha_2 y}{\cosh \alpha_2}\right\} \qquad \ldots(14)$$

Applying Laplace Inversion formula

$$u=\frac{1}{2\pi i}\int_{\delta-i\infty}^{\delta+i\infty}\bar{u}\,e^{St}\,dt \qquad \ldots(15)$$

Here δ is greatest then the real part of all the Singularities of $\bar{u}$

$$u=\frac{1}{2\pi i}\int_{\delta-i\infty}^{\delta+i\infty}\frac{(C+R)}{\alpha_2^2\,S(1-\lambda S)}\left\{1-\frac{\cosh\alpha_2 y}{\cosh\alpha_2}\right\}e^{St}dt \qquad \ldots(16)$$

Taking Inversion Laplace Transform and with the help of calculus of residues the

equations (13) and (14) yields.

$$u=\frac{(C+R)}{Q_2^2}\left\{1-\frac{\cosh Q_2 y}{\cosh Q_2}\right\}+\frac{4}{\pi}\sum_{r=0}^{\infty}\frac{(-1)^r\cos\left(\frac{2r+1}{2}\right)\pi y\,(1-\sigma S_5)^2(1+\lambda S_5)(C+R)e^{-S_5 t}}{(2r+1)\,A_5}$$

$$+\frac{4}{\pi}\sum_{r=0}^{\infty}\frac{(-1)^r\cos\left(\frac{2r+1}{2}\right)\pi y\,(1-\sigma S_6)^2(1+\lambda S_6)(C+R)e^{-S_6 t}}{(2r+1)\,A_6} \qquad \ldots(17)$$

and

$$v=\frac{(C+R)}{Q_2^2}\left\{1-\frac{\cosh Q_2 y}{\cosh Q_2}\right\}+\frac{4}{\pi}\sum_{r=0}^{\infty}\frac{(-1)^r\cos\left(\frac{2r+1}{2}\right)\pi y\,(1-\sigma S_5)(1+\lambda S_5)(C+R)e^{-S_5 t}}{(2r+1)\,A_5}$$

$$+\frac{4}{\pi}\sum_{r=0}^{\infty}\frac{(-1)^r\cos\left(\frac{2r+1}{2}\right)\pi y\,(1-\sigma S_6)(1+\lambda S_6)(C+R)e^{-S_6 t}}{(2r+1)\,A_6} \qquad \ldots(18)$$

Where

$Q_2^2=\alpha_2^2 \;\; at \;\; S\to 0$

$$A_5=\frac{1+l-2S_5\sigma+\sigma^2S_5^2+\lambda\left[\sigma^2S_5^2\,l+2lS_5^2\sigma-M^2\left(\sigma^2S_5^2+2\sigma S_5-1\right)\right]}{\left(1-S_5\sigma+\lambda S_5-\lambda S_5^2\sigma\right)}$$

$$A_6=\frac{1+l-2S_6\sigma+\sigma^2S_6^2+\lambda\left[\sigma^2S_6^2\,l+2lS_6^2\sigma-M^2\left(\sigma^2S_6^2+2\sigma S_6-1\right)\right]}{\left(1-S_6\sigma+\lambda S_6-\lambda S_6^2\sigma\right)}$$

$$S_5 = \frac{-X_5 + X_6}{2\sigma\left(1-\pi^2\lambda\left(\frac{2r+1}{2}\right)^2\right)}, \qquad S_6 = \frac{-X_5 - X_6}{2\sigma\left(1-\pi^2\lambda\left(\frac{2r+1}{2}\right)^2\right)}$$

Where

$$X_5 = 1 + l + M^2\sigma + \pi^2\sigma\left(\frac{2r+1}{2}\right)^2 - \pi^2\lambda\left(\frac{2r+1}{2}\right)^2$$

$$X_6 = \sqrt{X_5^2 - 4\sigma\left[1-\pi^2\lambda\left(\frac{2r+1}{2}\right)^2\right]\times\left[M^2 + \pi^2\left(\frac{2r+1}{2}\right)^2\right]}$$

Results and Discussion

The gravity flow of dusty visco-elastic fluid through a inclined channel in the presence of transverse magnetic field is studied. As different values of time t increases then the velocity profiles for the liquid and the dust particles decreases as shown in figures 1. From figure 2 it can be observed that as mass concentration l increases the velocity of the liquid and the dust particles decreases. When the magnetic field term (Hartmann number M) increases the velocity of the liquid and dust particles decreases considerably as seen from figures 3 and fig 4 it shows that as visco-elastic parameter λ increases the velocity of the liquid and the dust particles decreases. As the inclination angle θ increases the velocity of the liquid and the dust particles decreases as shown in figure 5. When the gravity parameter f increases then the velocity of the liquid and dust particles decreases as shown in figure 6. When the relaxation time parameter σ increases the velocity of the liquid and dust particles decreases as shown in figure 7. It is interesting to note that when visco-elastic parameter $\lambda \rightarrow 0$ and inclination angle θ or gravity parameter $g \rightarrow 0$ and then the present model becomes that of Singh in reference (7).

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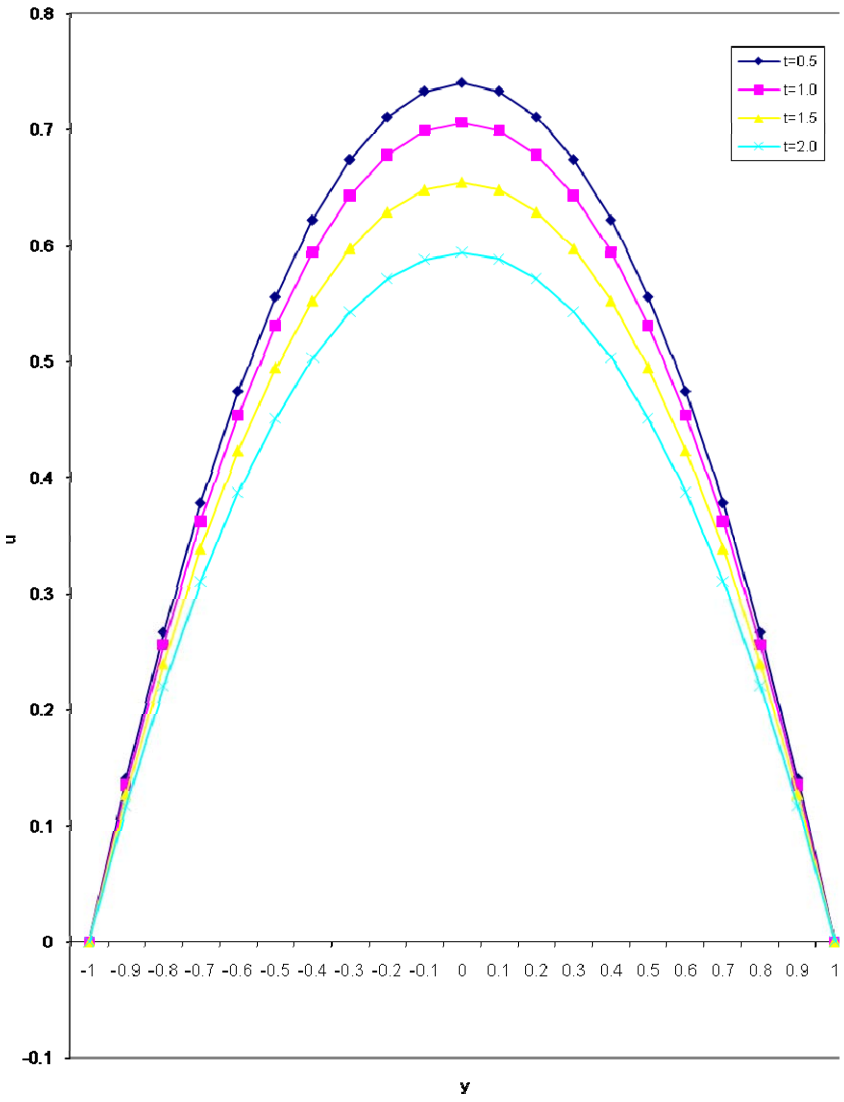

Figure 1 shows varation of velocity profile of liquid for different values of time and at fixed (σ=0.8, λ=0.5, t=0.5, f=0.5, $\theta=30^0$, M=0.5)

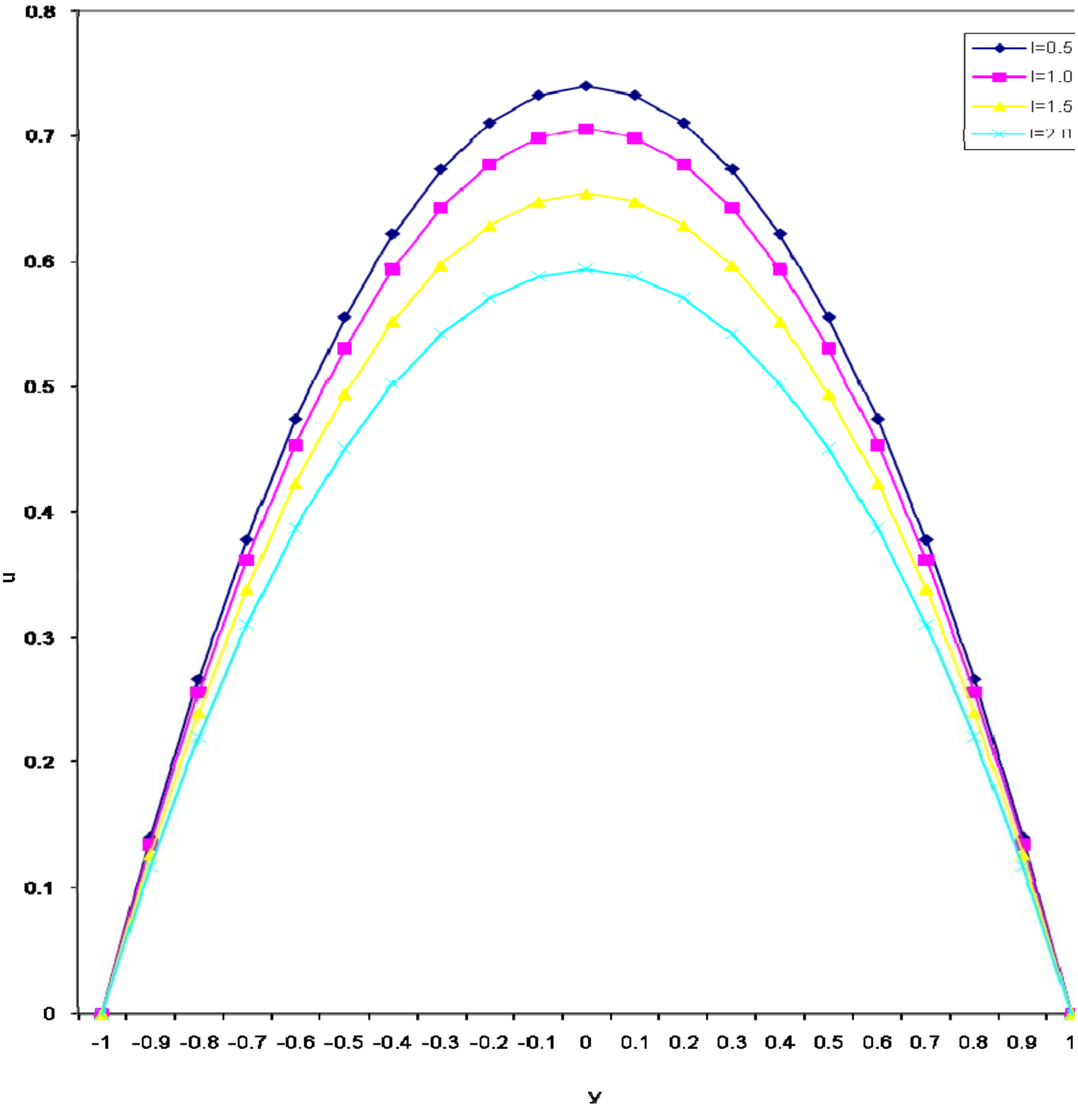

Figure 2 shows varation of velocity profile of liquid for different values of l mass concentration and at fixed (σ=0.8, λ=0.5, t=0.5, f=0.5, $\theta=30^0$, M=0.5)

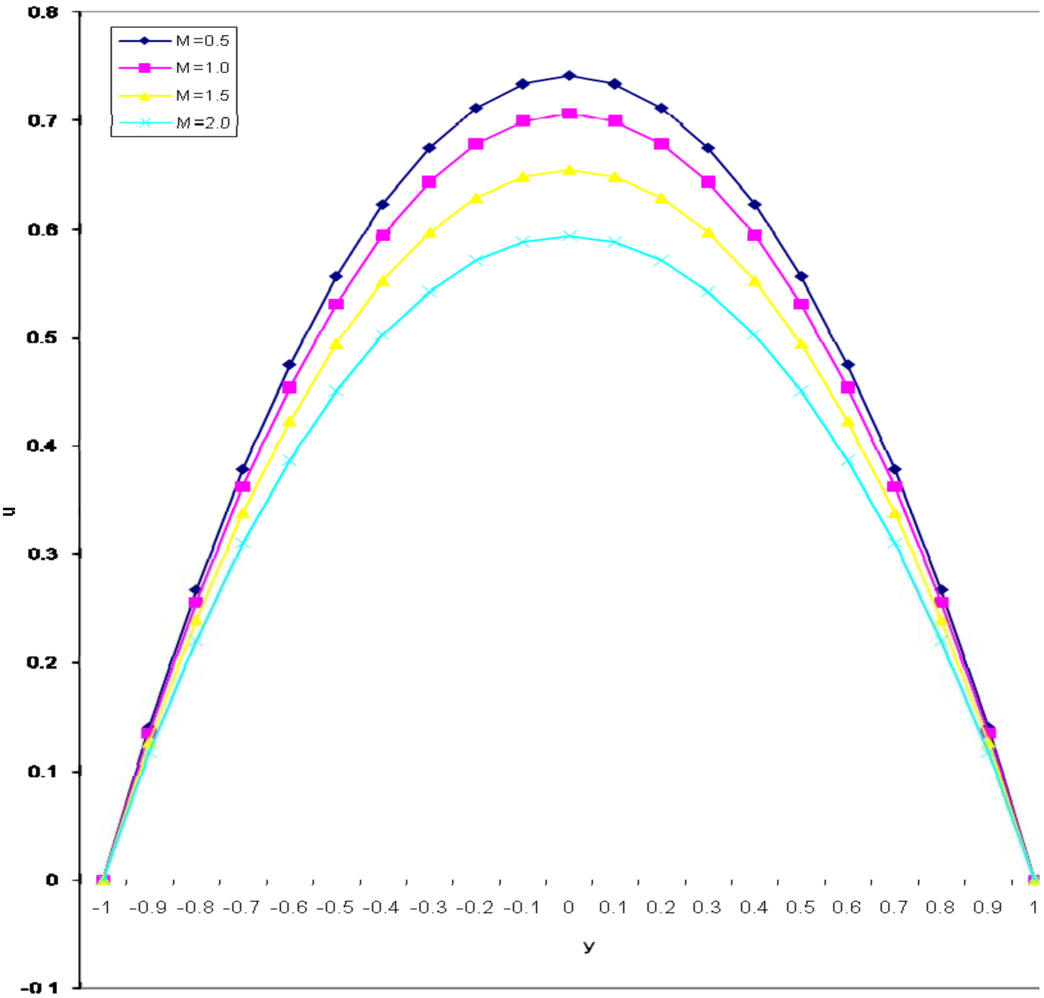

Figure 3 shows varation of velocity profile of liquid for different values of M Hartmann and at fixed (σ=0.8, λ=0.6, t=0.6, f=0.6, $\theta=30^0$, M=0.6)

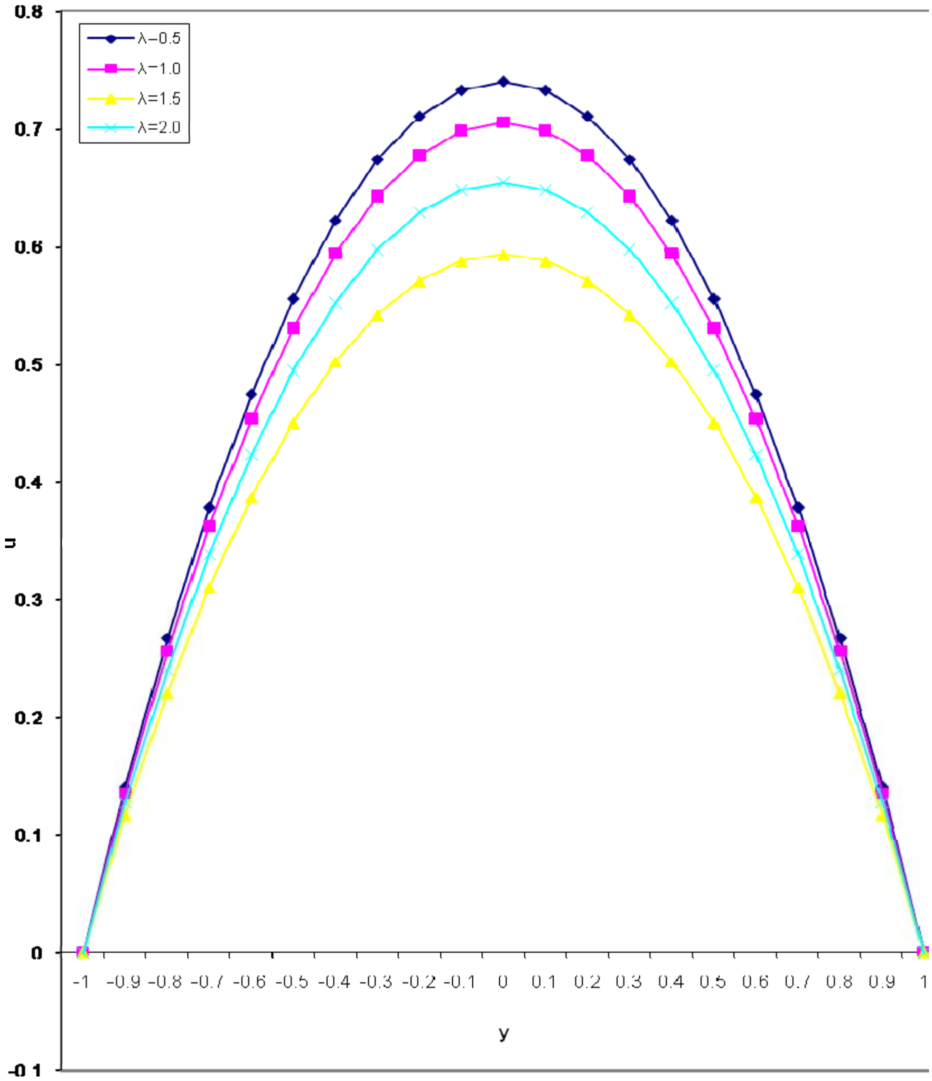

Figure 4 shows varation of velocity profile of liquid for different values of λ visco-elesto parameter and at fixed (σ=0.8, λ=0.6, t=0.6, f=0.6, $\theta=30^0$, M=0.6)

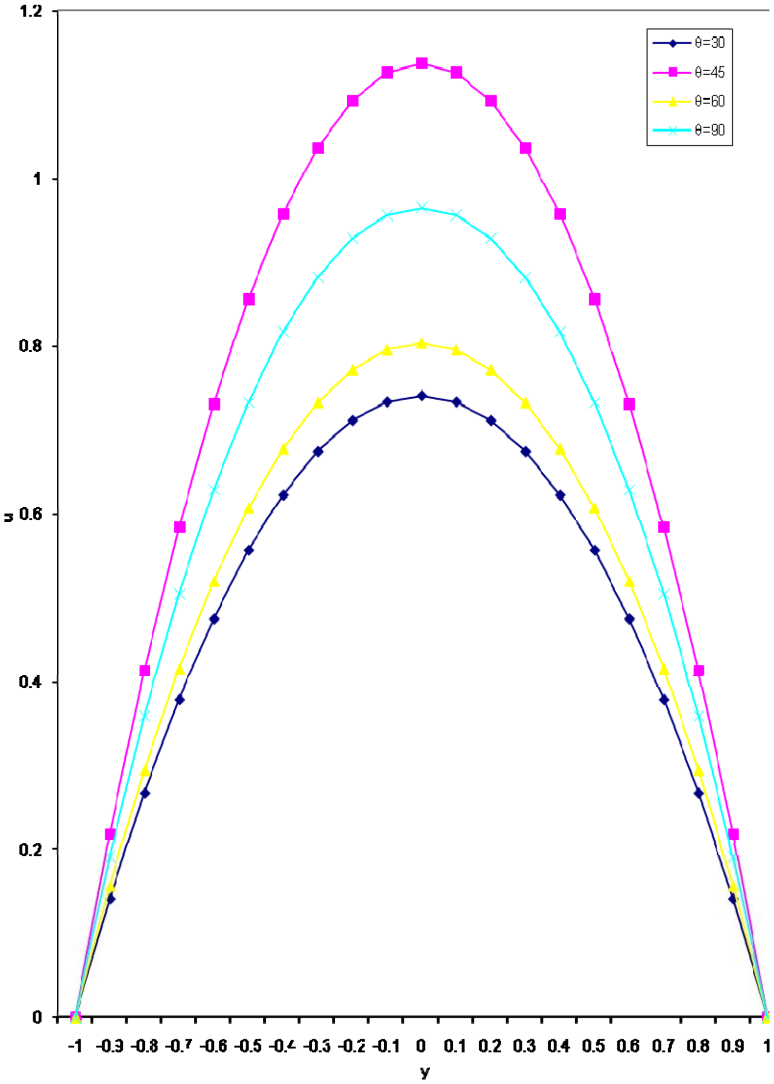

Figure 5 shows varation of velocity profile of liquid for different values of θ inclination angale and at fixed (σ=0.8, λ=0.5, t=0.5, f=0.5, $\theta=30^0$, M=0.5)

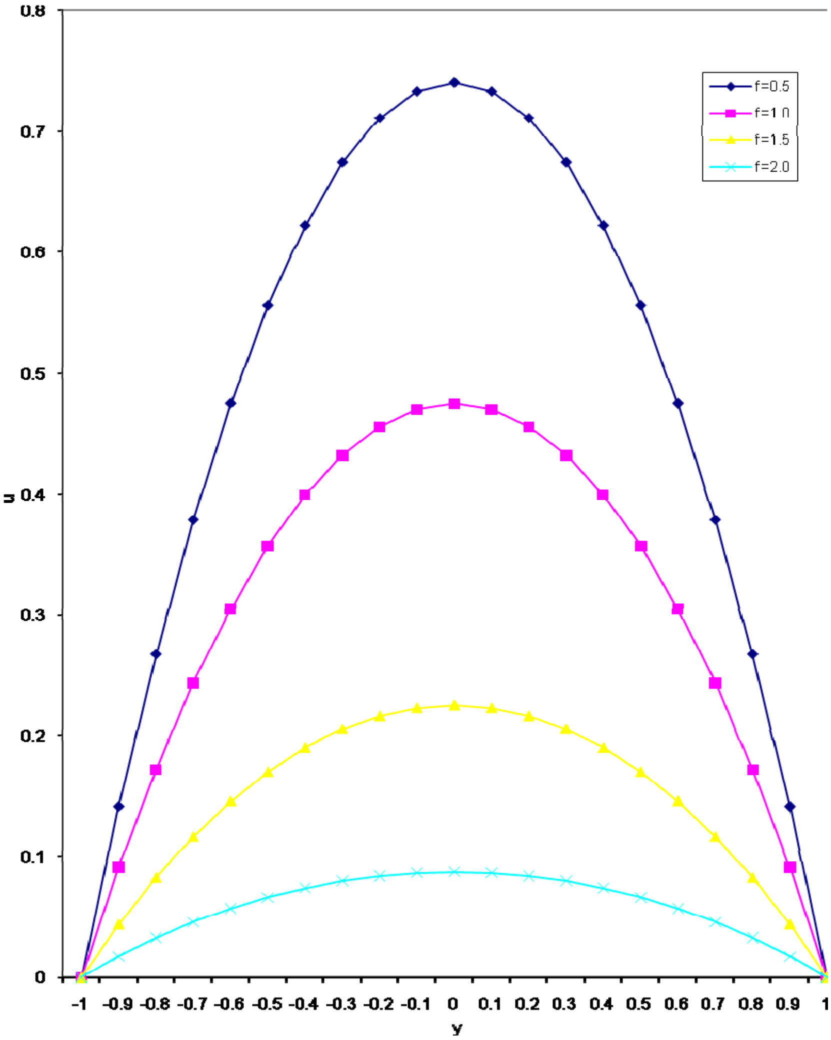

Figure 6 shows varation of velocity profile of liquid for different values of f gravity parameter and at fixed (σ=0.8, λ=0.5, t=0.5, f=0.5, $\theta=30^0$, M=0.5)

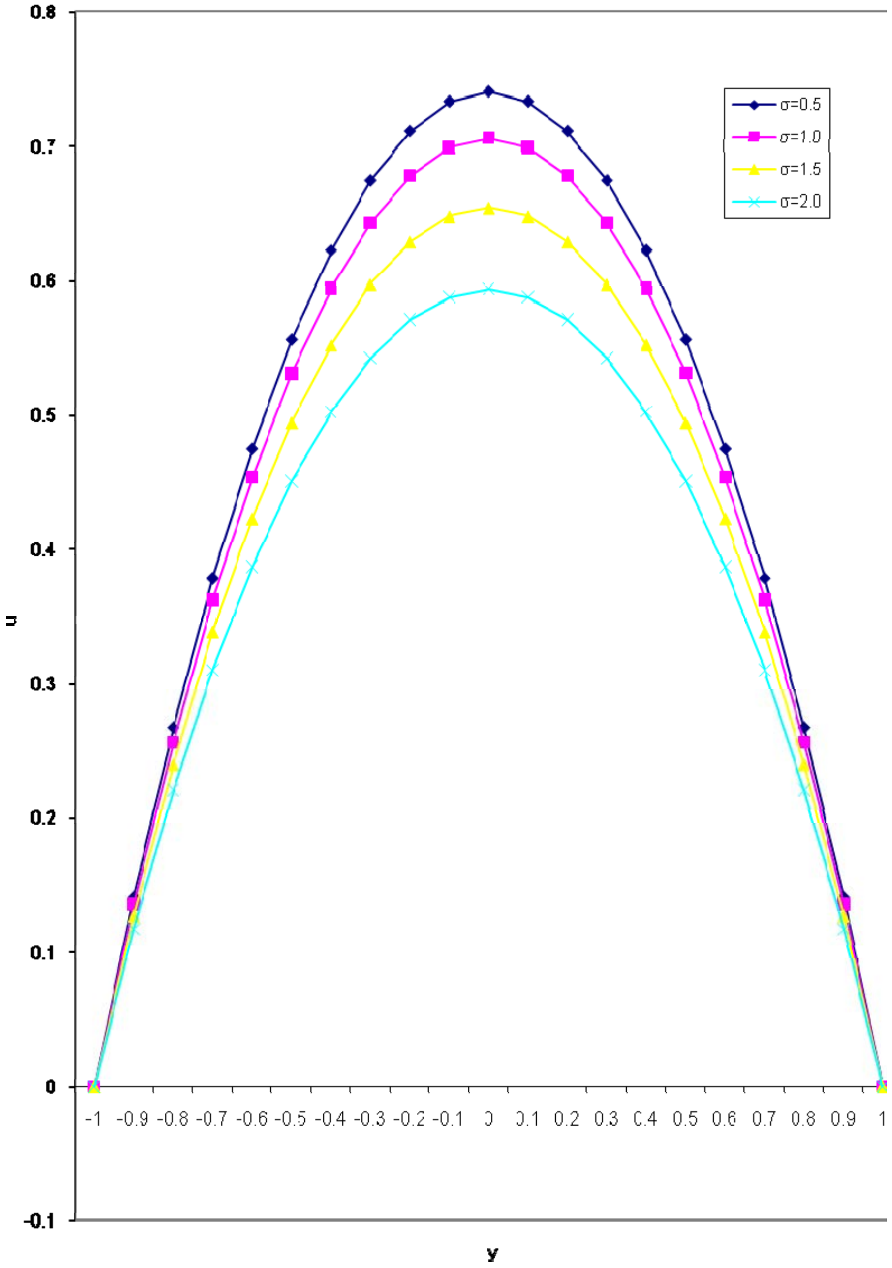

Figure 7 shows varation of velocity profile of liquid for different values of σ relaxiation time and at fixed (σ=0.8, λ=0.5, t=0.5, f=0.5, $\theta=30^0$, M=0.5)