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Pixel Fusion Based Curvelets and Wavelets Denoise Algorithm

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***Abstract*—Curvelets denoise approach has been widely used in many fields for its ability to obtain high quality result images. However artifacts those appear in the result images of curvelets approach prevent its application in some fields such as medical image. A denoise algorithm is proposed to perform pixel fusion to result images of curvelets and wavelets approaches. Firstly curvelets denoise result image and hidden Markov tree based wavelets denoise result image are calculated. Then image regions are analyzed with quadtree decomposition. Finally result images are obtained with weighted pixel fusion method. Experimental results demonstrate that the algorithm improves the visual quality of result images efficiently and suppresses the artifacts in result images evidently. And the results of the algorithm applied to ultrasonic medical images also indicate that the algorithm can be used efficiently in medical image fields.**

***Index Terms*—image denoise, curvelets, wavelets, pixel fusion.**

I. Introduction

Wavelets transform has been widely employed to solve image-denoising problem by representing signals in different frequency bands [1]. A noise signal generally has greatly different frequency properties from those of source image signal. So wavelet-based denoising methods can apply a suitable threshold to the noisy image wavelet transform coefficients and get the estimate of source image. Recently hidden Markov tree based wavelets transform denoise methods have been extensively studied. These methods use Gaussian mixture distributions to model the wavelet coefficients and a Markov tree to capture the correlation of the wavelet coefficients [2]. These studies show that these methods are efficient for image denoise.

Wavelets are successful in representing point discontinuities in one dimension, but less successful in two dimensions [3][4]. As a new multiscale representation suited for edges and other singularities along curves, the curvelets transform was introduced in [3]. Curvelets transform was successfully used in image denoise in [4], and obtained higher quality result images than common wavelets transform methods. Another combined curvelets and wavelets denoise approach in the frequency domain was proposed in [5]. The curvelets transform denoise method has been used in some fields, such as remote sense and SAR image processing [6].

However artifacts usually appear in result images when curvelets transform are used for image denoise in [4] and [5]. Sometimes artifacts cannot be permitted in result images. For example in medical image processing, artifacts can bring mistake in diagnosis. One of the main contributions of this paper is a pixel fusion based denoise algorithm that suppresses efficiently the artifacts in the result images and improves the visual quality of the result images evidently.

II. Curvelet Transform Denoise

Assume a smooth univariate function $\psi : R \to R$ satisfies

$$\int \frac{|\hat{\psi}(\omega)|^2}{|\omega|^2} d\omega < \infty , \quad (1)$$

for each $a > 0, b \in R, \theta \in [0, 2\pi)$, the bivariate ridgelet $\psi_{a,b,\theta} : R^2 \to R^2$ can be defined as

$$\psi_{a,b,\theta}(x) = a^{-1/2} \psi((x_1 \cos\theta + x_2 \sin\theta - b)/a) . \quad (2)$$

And ridgelet coefficients of an integrable bivariate function $f(x)$ are defined as

$$R_f(a,b,\theta) = \int \psi_{a,b,\theta}(x) f(x) dx . \quad (3)$$

The reconstruction formula can be given as

$$f(x) = \int_0^{2\pi} \int_{-\infty}^{\infty} \int_0^{\infty} R_f(a,b,\theta) \psi_{a,b,\theta}(x) \frac{da}{a^3} db \frac{d\theta}{4\pi} . \quad (4)$$

More details about the ridgelet transform can be found in [7]. The curvelet decomposition can be performed as follows [4].

1) Subband decomposition. The image f is decomposed into subbands $f \mapsto (P_0 f, \Delta_1 f, \Delta_2 f, \ldots)$.

2) Smooth partitioning. Each subband is smoothly windowed into squares.

3) Renormalization. Each resulting square is renormalized to unit scale.

4) Ridgelet analysis. Each square is analyzed via the ridgelet transform.

Donoho *et al* have solved the implementation of curvelet discrete transform in 2000. Recently they proposed fast

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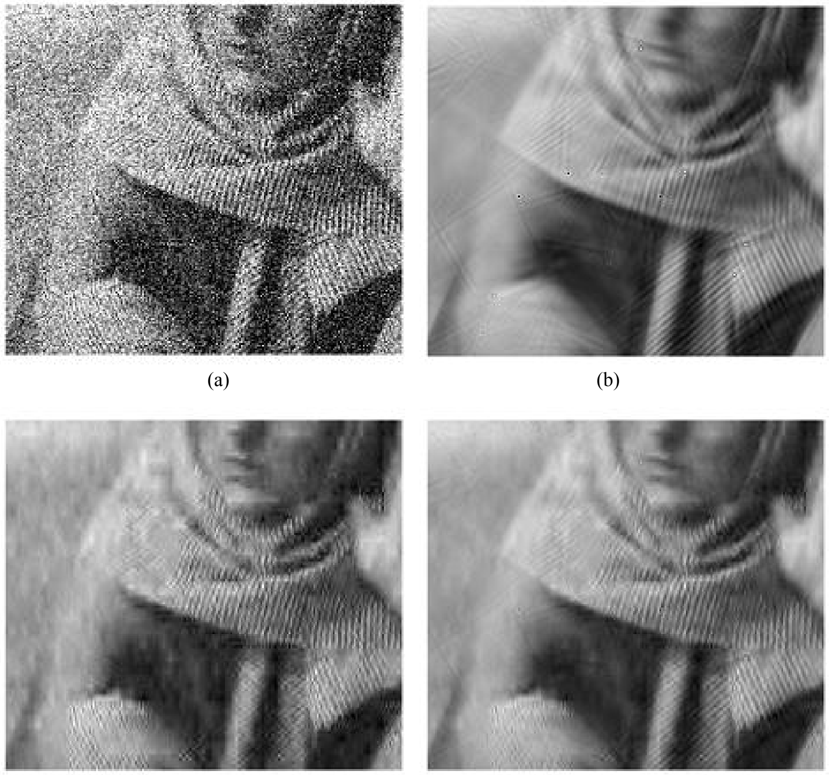


(a) (b) (c) (d)

Figure 1 Result Images of Different Denoise Approaches
(a) Noisy image (b) CTD method (c) HWTD method (d) Pixel fusion method

curvelet discrete transform algorithm [8]. In this paper USFFT algorithm is used to calculate curvelet transform.

Curvelet denoise is performed in this paper just as in [4]. Assume that white noise with a certain deviation is superposed to the source image, and hard threshold approach is used to calculate the unknown curvelet coefficients.

III. PIXEL FUSION BASED ALGORITHM

A. *Characteristics of Result Images*

For the various properties of curvelets transform denoise (CTD) approach and hidden Markov tree model based wavelets transform denoise (HWTD) approach, there is evident difference in the result images. The noise image of Barbara with the noise deviation 50 is illustrated in Fig. 1a. The result images of CTD and HWTD are showed in Fig. 1b and Fig. 1c respectively. More properties of the two methods can be observed from these images.

An evident smooth region that is the background of the woman lies in the left side in images. This region is smoother in Fig. 1b than in Fig. 1c, it is to say that CTD approach can obtain better quality in smooth region than HWTD method. It is also obvious that some segments those look like scratch in the glass appear in the result image of CTD method. These segments do not exist in the source image. And they are artifacts of CTD method. It is also clear that there is no artifact in the result image of HWTD method.

There is a detail region that includes various directional texture of the scarf in the middle of the images. This detail region is restored elaborately in Fig. 1c by HWTD approach. And in Fig. 1b this detail region is partially restored by CTD approach. Another scarf texture region with just one direction appears in the image bottom. This region is exactly restored by CTD approach in Fig. 1b, but not by HWTD approach.

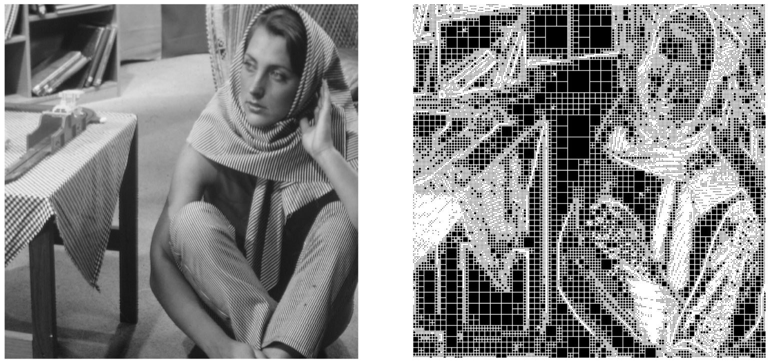
So we can expect higher visual quality result images by employing CTD approach and HWTD approach in different regions. But new artifacts may be introduced in the region boundary between different regions. In this paper a pixel fusion based algorithm is introduced to obtain better result images.

B. *Quadtree Decomposition*

Quadtree decomposition is an efficient tool for image structure analysis. The quadtree decomposition is described as follows.

Denote the noisy image with size $m \times m$ as I , the threshold of quadtree decomposition is denoted as T , and the suggested value of T is 10.

1) At first the entire image is divided into four equally rectangular blocks. The blocks are symbolized as $\{b_j \mid j = 1,2,3,4\}$. Predetermined threshold T is applied to these blocks. A block is marked as a homogeneous block if



(a) (b)

Figure 2 Quadtree Decomposition Result
(a) Source Barbara image (b) Quadtree Decomposition result

$$\max_{1\le j\le 4}\{b_j\}-\min_{1\le j\le 4}\{b_j\}\le T\ . \tag{5}$$

If a block is not a homogeneous block, further splitting is applied to it.

2) The division step for each block is repeated independently and recursively until all the blocks are either homogeneous blocks or become small blocks with size 1×1.

3) A quadtree matrix Q with exactly the same size of $m\times m$ as the entire image is established by assigning every element $Q(x,y)$, where $1\le x\le m$, $1\le y\le m$, with a number that equates to the size of the block where the image pixel $I(x,y)$ lies. For example elements in a block with size 4×4 are assigned 4.

After decomposed with quadtree method, $Q(x,y)$ can be used to represent the image activity. In an edge or detail region the gray value changes greatly, the total region cannot be judged as a homogeneous block and is divided into small blocks. In very activity regions the size of blocks will be 1×1. So small block size implies small $Q(x,y)$ value, that is a region with small block size is edge or detail region.

Let V be a value set of Q,

$$V=\{v_i \mid v_i=Q(x,y), 1\le x\le m, 1\le y\le m\} \tag{6}$$

The total element number of V is denoted as V_{num}. Let $S(v_i)$ denote the element number of Q whose block size equals to v_i, where $1\le i\le V_{num}$. For Barbara image with size 512×512, the size v_i of quadtree decomposition blocks is 1, 2, 4, 8, 16 and 32. Decomposition blocks of image Barbara are displayed in Fig 2, where $T=10$. Tests show that the quadtree decomposition results of original images are similar with that of the result images of CTD approach and HWTD approach. So in our proposed algorithm the quadtree decomposition of result image of HWTD is employed for image region structure analysis.

C. Proposed Algorithm

Denote result image of our proposed algorithm as *R,* this pixel fusion based algorithm is described as follows.

1) Applying HWTD approach in [2] to obtain result image *W*.

2) Applying CTD approach in [4] to obtain result image *C*.

3) Get quadtree matrix *Q* with applying quadtree decomposition to *C*.

4) $R(x,y)$ is calculated as

$$R(x,y)=cW(x,y)+dC(x,y) \tag{7}$$

where

$$c=\begin{cases}0.5, & 3\le v_i\le 8\\ 0.5-0.5\times\dfrac{1}{m\times m}\sum S(v_i), & others\end{cases}\ , \tag{8}$$

$$d=1-c\ . \tag{9}$$

IV. Experiments and Discussion

A. Experiments of Standard Images

We compared the result images after applying HWTD, CTD and pixel fusion approach to some test images those are widely used in other literature, such as Lena, Boat, Barbara, *et al*. The result image of pixel fusion approach applied to noisy image Barbara in Fig 1a is illustrated in Fig. 1d. It can be observed that artifacts are efficiently suppressed in the left smooth region of the result image in Fig. 1d. It is also clear that this region in the result image of pixel fusion approach is restored smoother than HWTD approach. The visual quality of various directional texture region of scarf with proposed approach is also as clear as HWTD approach. And the texture region of scarf in the image bottom is evident as in result image of CTD approach. These observation results show that the proposed pixel fusion approach obtains better visual quality result image than HWTD approach and CTD approach.

Peak signal noise ratio (PSNR) is often employed to compare

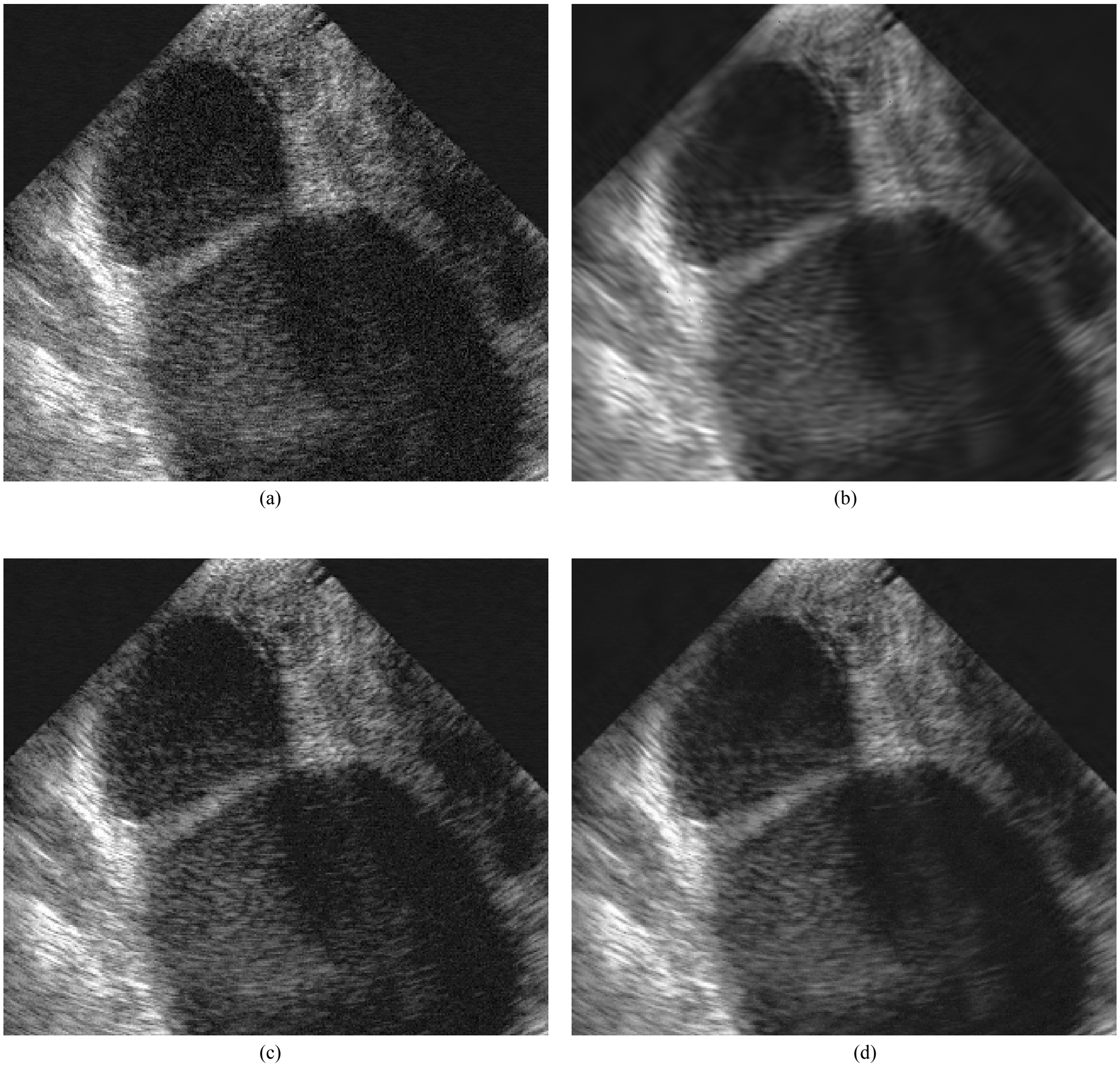


Figure 3 Result Images of Ultrasound Image Denoise
(a) Source image (b) CTD method (c) HWTD method (d) Pixel fusion method

gray value difference between result image and original image. Some PSNR results of different approaches are listed in Table I. It can be observed that all the HWTD, CTD and pixel fusion approaches obtained better PSNR value than median filter approach. It can also be observed that the pixel fusionobtains the highest PSNR value. So the pixel fusionalgorithm also improves the PSNR value while obtaining high visual quality result images.

Table I. PSNR of Different Denoise Approach

Image (deviation)	Noise Image	Median Filter	HWTD	CTD	Pixel Fusion
Lena (20)	22.12	28.51	30.91	31.18	31.85
Lena (30)	18.58	25.62	29.01	29.49	29.97
Lena (50)	14.15	21.55	26.78	27.26	27.67
Barbara (20)	22.10	23.83	27.54	25.46	27.70
Barbara (30)	18.58	22.58	25.65	24.45	25.88
Barbara (50)	14.15	20.12	23.65	23.37	24.04
Boat (20)	22.10	26.93	28.96	28.23	29.41
Boat (30)	18.58	24.68	27.17	26.91	27.70
Boat (50)	14.14	21.15	25.12	25.15	25.65

B. Applied to Ultrasound Image

Artifacts can be efficiently suppressed when the pixel fusion algorithm is used for image denoise. And the pixel fusion algorithm can be employed for medical image denoise. Some ultrasound images acquired from medical ultrasound machines are tested with HWTD, CTD and pixel fusion algorithms. Some result images are showed in Fig. 3. It can be observed that the artifacts those appeared in the result images of CTD approach are efficiently suppressed in the result images of pixel fusion approach. And it is showed that the pixel fusion algorithm obtains the best denoise visual quality, and this has also been confirmed by experienced doctors.

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