Mathematical Modeling of Real Estate Financial Analysis of Mortgage Loan

Mohammad Tanzil Hasan[1,*], M. Abul Hossion[2]

[1]Department of Mathematics, Bangabandhu Sheikh Mujibur Rahman Maritime University, Bangladesh.
[2]Department of Physics, Bangabandhu Sheikh Mujibur Rahman Maritime University, Bangladesh.

*Corresponding author: Mohammad Tanzil Hasan, Department of Mathematics, Bangabandhu Sheikh Mujibur Rahman Maritime University, Bangladesh.
Email: tanzil.math@bsmrmu.edu.bd

Abstract

Mathematics of finance for mathematical modeling of real estate financial analysis of mortgage loan is considered in this article. Financial mathematics describes the application of mathematical methods and mathematical modeling to solve financial problems. It is at times referred to as financial engineering, quantitative finance, and computational finance. Inflation and its effect on standard mortgage loan instrument causes problem for both borrowers and lenders which are analyzed. Adjustable interest rate which is called adjustable rate mortgage or variable payment provisions that change with economic conditions is also examined here. In addition, graphical comparisons have been shown between constant payment mortgage and price level adjusted mortgage loan. For the analysis, mathematical models are described with examples. Mathematical computations have been done by using Mathematica. This paper may be beneficial for the persons who exactly want to better understand the real estate for their personal investment and for making financing decisions.

Keywords

Real Estate, Interest Rate, Inflation, Mortgage, PLAM

1. Introduction

The term real in real estate comes from the term realty [1], which has, for centuries, meant land and things permanently attached (the latter would include immovable things such as buildings and other structures). All other items not considered realty have been designated as personality, which include all movable things (e.g. automobiles, shares of stock, bank accounts, and patents). The term estate has evolved to mean "all that a person owns," including both realty and personality. Hence, the portion of a person's estate that consists of realty has come to be known as real estate. However, in current business practice, although the term realty is sometimes used, we generally use the term real estate to mean land and all things permanently attached [2].

Real estate, the real asset, provides the space humans need and costs money to physically produce. If exists because humans put their money and effort into transforming vacant land into useful space [3]. The wealth of real estate represents all previous contributions humans made to improve land and the prevailing demand supply conditions in the asset market that affect current wealth levels [4]. During each period, new contributions, investments, add to the level of wealth.

Consequently, the accumulated stock of wealth and the flow of investment in real estate compared to other assets indicate the relative importance of human and space relationships in the economy. Now we concentrate on the real estate asset. Leases allow for the use of real estate for limited periods of time. Mortgages allow the owners of real estate to borrow money using real estate as collateral. Both leases and mortgages grant rights to document holders to receive the in-

comes from real estate, but they don't allow holders of the documents to indefinitely control the physical space for human use. Creation of leases and mortgages results in creation of real estate financial assets. Patterns in the creation of real estate financial assets represent another indicator of the importance of real estate in the economy.

Real estate not only provides physical shelter but also have physical significant impact on the lives of dwellers in terms of skill enhancement, income generation, increased security, health, self-confidence and human dignity. Considerable research interest is attached to the question of how the acquisition of residential real estate has been attached. Bangladesh, like many other developing countries face an acute shortage of affordable housing both in the urban and rural areas [5]. The main constraints in housing production are the unavailability and high cost of housing finance in relation to house hold incomes. Though the high cost of housing finance in comparison to house hold income, Parvez and Rana [6] tried to investigate the causes of increasing consumer demand for real estate housing. Uddin Md. Kutub et. al. [7] presented residential real estate for financing and investment for the fixed rate mortgage loans.

The benefits of real estate investments may be classified as follows: cash flow, tax shelter, and proceeds of sale [8]. Cash flow is the difference between the sum of cash received and the sum of cash disbursed during a period, usually a year. It is not income. It is not earning. It is not profit. It is just the difference between the cash that comes in and the cash goes out. The tax shelter benefits are the savings derived from preferential income tax treatment of the earnings from the property [9]. The proceeds of sale are the moneys that the investor receives when the property is sold and settled and income taxes are paid.

A loan used to purchase or maintain a home, land, or the other types of real estate, secured by the property itself is usually known as mortgage loan [10]. The mortgage serves as a means of promoting the best use of society's finite resources: people and land. It provides for the ready transferability of land and for the improvement or working of that land by those unable to buy the property with their current resources. Mortgages play an even more important role in maintaining the market in residential housing.

The study of finance and investment attracted a lot of attention due to various business applications including real estate and housing sector [11-13]. Mathematics of finance is the most important topic in the economic world and real estate is one of the most influential subjects in finance [14]. It is the branch of applied mathematics concerned with the financial market [15].

The aim of this study is to investigate the inflation and mortgage pricing problem, and the price label adjusted mortgage loan. In this article, mortgage interest rates are determined in the following section. We present in section 3, how the inflation effects on the lender and borrower in the long term for mortgage pricing. In section 4, the price label adjusted mortgage loan is analyzed and compared with the fixed rate mortgage loan. The financial mathematical formula and programming language Mathematica have been used to solve the problems. Finally concluding remarks are given in section 5.

2. Determinants of Mortgage Interest Rates

Changing economy conditions have forced the real state finance industry to go through an important evolution. These changing conditions now require lenders and borrowers to have a better understanding of the sources of funds used for lending and the nature of how risk, economic growth, and inflation affect the availability and the cost of mortgage funds [16]. When considering the determinants of interest rates on mortgage loans used to finance single-family residences, we must consider the demand and supply of mortgage funds. The market rate of interest on mortgage loans is established by what borrowers are willing to pay for the use of funds over a specified period of time and what lenders are willing to accept in the way of compensation for the use of such funds [17].

2.1 The Real Rate of Interest: Underlying Considerations

When discussing market interest rates on mortgages, we should keep in mind these interest rates are based on a number of considerations. One fundamental relationship that is common to investments requiring use of funds in the economy is that they earn at least the real rate of interest. This is the minimum rate of interest that must be earned by savers to induce them to divert the use of resources (funds) from present consumption to future consumption.

2.2 Interest Rates Risk and Inflation Expectations

Besides the real rate of interest, a concern that all investors have when making investment decisions is how inflation will affect investment returns. The rate of inflation is of particular importance to investors and lenders making or purchasing loans made at fixed rates of interest over long periods of time. The uncertainty about what interest rate to change, when a loan is made can be referred to as interest rate risk [18]. For example, anticipated inflation may have been 6 percent at the time our Tk.100000 loan was made, but if actual inflation turns out to be 8 percent, this means the interest rate

that should have been changed. We say that the anticipated rate of inflation at the time loan was made was 6 percent.

2.3 Prepayment Risk

The risk that the loan will be prepaid when the interest rate falls below the loan contract rate is referred to as prepayment risk.

2.4 Other Risks

There are additional risks that lenders and investors consider that may vary by type of loan or investment. For example, the liquidity or marketability of loans and investments will also affect the size of the premium that must be earned. Securities that can be easily sold and resold in well-established markets will require lower premiums that those that are more difficult to sell. This is called liquidity risk.

2.5 A Summary of Factors Important in Mortgage Pricing

The interest rate charged on a particular mortgage loan will depend on the real interest rate, anticipated inflation, interest rate risk, prepayment risk and other risk. These relationships can be summarized in general as follows:

$$i = r + p + f$$

In other words, when pricing or setting the rate of interest i on a mortgage loan, the lender must charge a premium p sufficiently high to compensate for default and other risks and a premium f that reflects anticipated inflation to earn a real rate of interest r, which is competitive with real returns available on other investment opportunities in the economy.

**Fig 1. Various Types of Real Estate Pictures.**

3. Inflation and Mortgage Pricing Problem

The inflation relates to mortgage lending and cause difficulty for lenders and borrowers desiring to make constant payment loans with fixed interest rates. Let's assume initially that a Tk.1, 000,000 loans is made at a time when no inflation is expected. The loan is expected to be outstanding for a 12 years period. Because there is no inflation, an inflation premium (f) is not required; hence, the lender will earn a return equivalent to the risk less interest rate (r), plus a premium for risk (p) over the period of the loan. We assume that the interest rate charged under such assumptions would be 4 percent, representing a 3 percent real rate of interest and a risk premium of 1 percent over the period of the loan. Assuming a constant payment, fixed interest rate loan made in an inflation less environment, the lender would collect constant payments of approximately Tk.8,753.34 per month, base on the loan constant for 4 percent and 12 years. This amount is Figure 2 as a straight line (RP) over the life of the loan.

An expected inflation rate of 5 percent caused a 30 percent rise in the monthly mortgage payment from Tk.8, 753.34 to Tk.11, 380.3, or Tk.2, 627. Significant increase in these monthly payments can be easily seen by examining curve NPD in Figure 2. This curve represents the real value of the monthly payments that the lender will receive over the 12- year loan period. The NPD curve is important because the lender, realizing that inflation is going to occur, expects that the constant stream of Tk.11, 380.3 payments to be received over time will be worth less and less because of lost purchasing power. In order to earn the same real interest rate, the real value of the payment stream (NPD) must be greater than RP in the early years, since it will fall below RP in the later years. This relationship is referred to as tilting the real payment

stream in the early years to make up for the loss in purchasing power in the later years.

One final observation about the tilt effect is that, as the rate of the inflation increases, the tilt effect increases. In Figure 3, we show the effect of an increase in inflation from 5% to 8% per year.
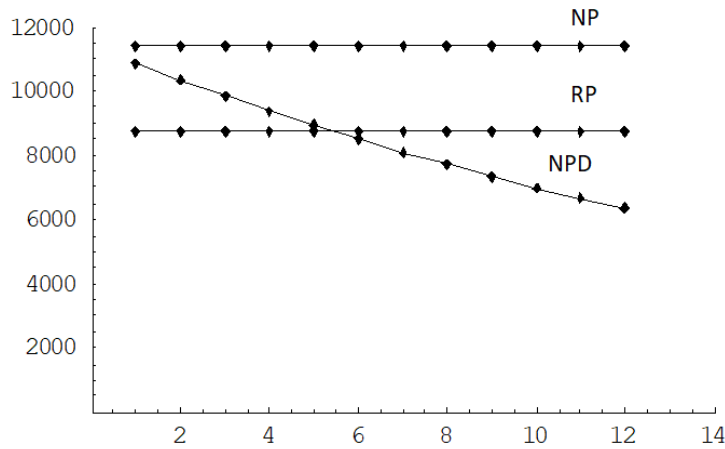
NP=Nominal payments; RP=Real payments; NPD=Nominal payments deflated.

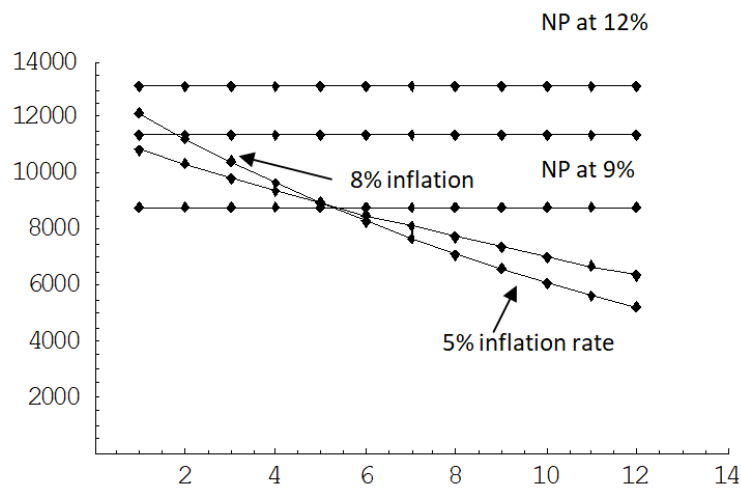**Fig 2. Real and Nominal Values of Mortgage Payments.**

Fig 3. Relationship between Real and Nominal Payments at Various Rate of Inflation.

The curve corresponding to monthly payments deflated at 8 percent indicates that the real value of monthly payments on 12 percent mortgage exceeds the real value of payments on the 9 percent mortgage for about the first 5 years of the loan term. The tilt effect makes it even more difficult for borrowers to qualify for loans based on their current income and make payments from the current income.

4. Adjustable Rate and Variable Payment Mortgages

One concept that has been discussed as a remedy to the imbalance problem for savings institutions is the price level adjusted mortgage (PLAM). The Mortgage rate i-an expected real rate of interest (r), a risk premium (p), and an expected inflation (f)-we displayed the following equation

$$i = r + p + f$$

After estimating initial values for r and p, the PLAM loan balance would be adjusted up or down by a price index.

Payments would then be based, on a new loan balance, adjusted for inflation.

Example:

▪ Assume that a mortgage is made for Tk.1, 000,000 for 20 years at an interest rate of 4 percent $(r + p)$. The lender and borrower may agree that the loan balance will be indexed to the customer price index & adjusted annually. Assume inflation continued at an annual rate 5 percent for the remaining loan term.

Input

ir = 0.09   **Ir1** = .04   **A** = 1000000   **n** = 20

Output

| Year	Beg Loan Bal.	Yearly pay.	interest	principle Rep	Ending Loan Bal.
1	1000000	73581.8	40000.	33581.8	966418.
2	1.01474×10^6	77260.8	40589.6	36671.3	978068.
3	1.02697×10^6	81123.9	41078.9	40045.	986926.
4	1.03627×10^6	85180.1	41450.9	43729.2	992543.
5	1.04217×10^6	89439.1	41686.8	47752.3	994418.
6	1.04414×10^6	93911.	41765.6	52145.5	991994.
7	1.04159×10^6	98606.6	41663.7	56942.8	984651.
8	1.03388×10^6	103537.	41355.3	62181.6	971702.
9	1.02029×10^6	108714.	40811.5	67902.3	952384.
10	$1. \times 10^6$	114149.	40000.1	74149.3	925854.
11	972147.	119857.	38885.9	80971.	891176.
12	935735.	125850.	37429.4	88420.4	847314.
13	889680.	132142.	35587.2	96555.	793125.
14	832781.	138749.	33311.3	105438.	727343.
15	763710.	145687.	30548.4	115138.	648572.
16	681000.	152971.	27240.	125731.	555269.
17	583033.	160620.	23321.3	137298.	445734.
18	468021.	168651.	18720.8	149930.	318091.
19	333996.	177083.	13359.8	163723.	170272.
20	178786.	185937.	7151.44	178786.	0
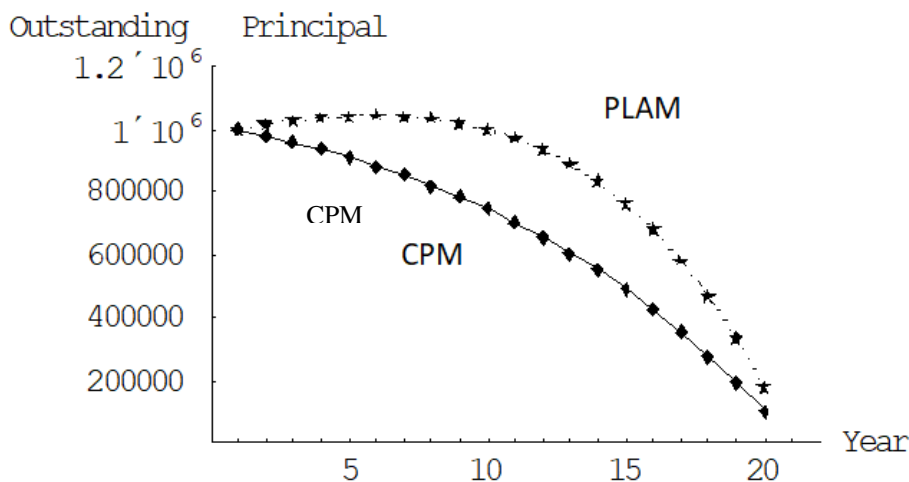
Fig 4. Outstanding Loan Balances.
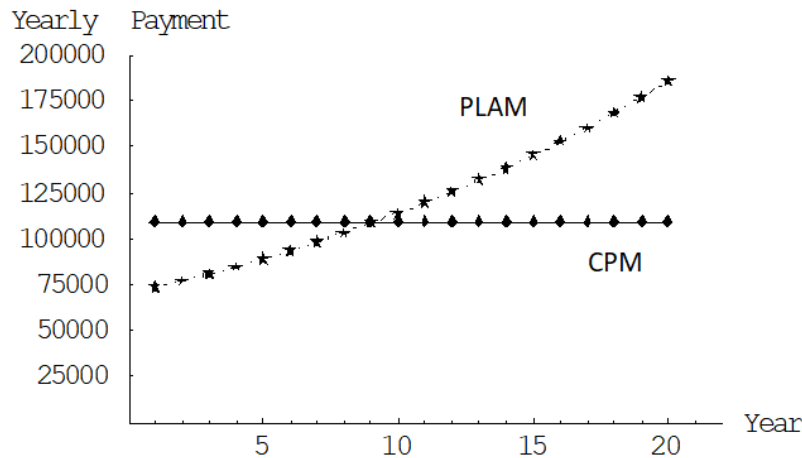
Note: The above figures (Fig 4 and Fig 5) represent payments and loan balance patterns, Tk.1, 000,000 PLAM, 4%, inflation = 5% per year, versus Tk.1, 000,000 CPM, 9% interest, 20 years

Fig 5. Yearly Payments.

Comments:

- The outstanding loan balance exceeds the original loan amount. So it decreases very slowly.
- If the loan is refinanced before the end of its term, there is a built in a penalty for the borrower.
- Periodic payment is increasing.
- The PLAM loan riskier to the lender because more consideration must be given to future market values of real estate and future borrower income.
- The borrower and lender can change the rate of inflation.

5. Conclusion

We illustrated a number of problems concerning financing situations that borrowers and lenders might face. However, we have chosen examples that illustrate the main concepts and approaches to solving important problems. In fixed and adjustable payments mortgage loans, we discussed various approaches to pricing and structuring fixed loans with adjustable interest rates mortgage loans. Loans with adjustable interest rates become necessary from time to time, depending on the rate of economic expansion and expected rate of inflation. We saw the price of interest rate on the loan depends on a number of factors, including various types of risk that affected mortgage lenders. Although we have analyzed only real estate problems in this article but all of the concepts may apply equally to the analysis of income producing or investment in housing.

References

[1] Creedy, J. and Wall, N. F. (1979). Real Estate Investment by Objective, McGraw-Hill.

[2] William, B.B. and Jeffrey, D.F. (2005). Real Estate Finance and Investment, McGraw-Hill.

[3] Wu, T. (2019). Analysis of Financing Plan for Real Estate Development Project. Open Journal of Business and Management 7 (3): 1338–45.

[4] Wofford, L. E. (1983). Real Estate, John Wiley and Sons.

[5] Sarker, M., Siddiquee, M. and Rehan, S. (2008). Real Estate Financing in Bangladesh: Problems, Programs, and Prospects, SSRN Electronic Journal, Vol.7 No. 2.

[6] Parvez, M., Rana, S. (2021). Investing the Causes of Increasing Consumer Demand for Real Estate Housing in Bangladesh, International Journal of Housing Markets and Analysis, Vol 14, Issue 5.

[7]   Uddin Md. Kutub et al. (2011), Residential Real Estate for Financing and Investment, Vol 3, No. 4, European Journal of Business and Management.

[8]   Hwang, Sungjoo, Park, M., and Hyun-Soo Lee. (2013). Dynamic Analysis of the Effects of Mortgage-Lending Policies in a Real Estate Market. Mathematical and Computer Modeling, 57 (9): 2106–20.

[9]   Boaz, G. and Suzanne, G. (2006). Real Estate Millionaire, McGraw-Hill.

[10] https://www.investopedia.com/terms/m/mortgage.asp.

[11] Seldin and Maury (1980), Real Estate Investment for Profit through Appreciation, Prentice Hall.

[12] Robert, S. (2004). Successful Real Estate Investing, John Wiley and Sons.

[13] Edward A. Glickman. (2014). An Introduction to Real Estate Finance, Academic Press.

[14] Zima, P. and Robert, L.B. (2005). Mathematics of Finance, Schaum's Outline Series, McGraw-Hill.

[15] John B.C. (2001). Real Estate Perspective, McGraw-Hill.

[16] Li, Xiangrong. (2010). Mathematical Model of Housing Loans. Modern Economy 01 (03): 168–70.

[17] Richard F. B. (2003). The Basic of Financial Mathematics, Spring.

[18] Ko, Po Sheng, Wu, C. C., and Chang, C. F. (2012). Mathematical Modeling Analysis of Credit Risk in Home Loan. Advanced Materials Research 433–440: 2342–48.