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Oscillatory saturated flow of a second grade fluid in a channel

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ABSTRACT

An unsteady rotating flow of a second grade electrically conducting incompressible fluid through a channel with an oscillatory upper plate has been considered. The channel wall oscillates sinusoidally with a constant mean and the channel is packed with uniformly saturated porous medium and is subjected to a transverse magnetic field. The second grade fluid property through elasticity property of the fluid has been focused in the discussion. The method of solution is based upon a principle of superposebility of unsteady solution over basic steady solution. The important findings are: The greater corolis force and low permeability contribute to greater phase difference whereas elastic property of fluid reduces it. Fluid elasticity reduces the unsteady part of the primary velocity as well as secondary velocity. These are vital considerations for flow stability and resonance.

1. INTRODUCTION

An exact solution of Navier-Stokes equation arising out of the flow generated in a semi-infinite mass of an incompressible fluid by viscous action at an infinite flat plate oscillating in its own plane was first obtained by Stokes [1]. The flow of a viscous incompressible fluid near a porous oscillating infinite plate subject to suction or blowing has investigated by Biihler and Zierep [2]. Puri [3] analyzed viscoelastic fluid flow in a rotating system near an oscillating nonporous plate. It is of great interest to observe that by the earth's rotation, the small scale motion like the flow due to bathtub vortex and the large scale motion in the ocean as well as the atmosphere (Bachelor [4]) are strongly influenced. In the dynamics of thin sheets of fluid in the sense that their depth is very small relative to their horizontal extent, rotation plays an important role in fact.

Due to wider application of viscous incompressible fluid with the fluctuating flow past an infinite plate in the paper industry and other technological fields, several researchers have shown their interest towards this study. In a rotating system, Mazumdar [5] has studied an exact solution of oscillatory Couette flow and Ganapathy [6] has discussed on a note about oscillatory Couette flow. Singh [7] has also discussed an oscillatory hydromagnetic Couette flow in a rotating system. The heat and mass transfer in MHD flow of a viscous fluid past a vertical plate under oscillatory suction velocity have analyzed by Singh et al. [8]. The filtration of solids from liquids, drug permeation through human skin, the extraction of energy from geothermal region and the flow of the oil through porous rocks are associated with the flow of second grade fluid flow through porous media. That's why many scientists and engineers have been attracted to study the model. The flow through porous media also occurs in soil erosion and tile drainage, absorption and filtration processes of the ground water hydrology, irrigation and drainage problems, absorption and filtration processes in chemical engineering. The magnetohydrodynamics free convective flow with mass transfer of a viscous fluid through a porous medium bounded by an oscillating porous plate in slip flow regime has been discussed by Singh and Gupta [9]. Hayat et al. [10] have analyzed a non-Newtonian hydromagnetic oscillating flow in a rotating system. The analytical solution of a magnetohydrodynamics transient rotating flow of a second grade fluid in a porous space has been also studied by Hayat et al. [11]. They have considered flow of a conducting fluid in the presence of transverse magnetic field, thereby, they have accounted for the effect of an additional body force of electromagnetic origin. Das et al. [12] have discussed unsteady MHD Couette flow in a rotating system. Jana et al. [13] have analyzed unsteady flow of viscous fluid through a porous medium bounded by a porous plate in a rotating system. Sahoo et al. [14] have studied hydromagnetic oscillatory flow and heat transfer of a viscous liquid past a vertical porous plate in a rotating medium. Seth et al. [15] have studied unsteady hydromagnetic Couette flow within porous plates in a rotating system. Farhad et al. [16] have discussed the hydromagnetic rotating flow in a porous medium with slip condition and Hall current. Jana et al. [17] have studied the unsteady Couette flow through a porous medium in a rotating system. Saho [18] has discussed the effect of heat and mass transfer on MHD flow of a viscoelastic fluid through a porous medium bounded by an oscillating porous plate with slip flow regime. With the modified differential transform method, Rashidi et al. [19] have studied the heat transfer in a second grade fluid through a porous medium and arrived at some interesting results. Parida et al. [20] have investigated the MHD flow of a second grade fluid in a channel with porous wall.

The objective of the present study is to account for

(i) Permeability of the uniformly saturated porous medium with the help of Darcian linear model.

(ii) The generated coriolis force due to rotation of the fluid with the frame of reference as a rigid body.

(iii) The second grade effect to account for the elasticity of the fluid contributing to non-Newtonian property.

2. FORMULATION OF THE PROBLEM

The unsteady flow of an electrically conducting incompressible second grade fluid through a porous medium bounded by infinite parallel plates at a distance d apart has been considered. Here, both the fluid and plates rotate about the z-axis normal to the plates with a constant angular velocity Ω. It is assumed that the plates are electrically non-conducting. The lower plate is at rest and upper plate oscillates with a velocity $U_1(t)=U_0(1+\varepsilon\cos\omega t)$ in its own plane about a non-zero constant mean velocity U_0. Here, ω is the frequency of oscillation. The x-axis is parallel to the direction of motion of the upper plate and origin is taken on the lower plate. Since the plates are infinite in extent, so all the physical variables, except the pressure, depends only on the variables z and t.

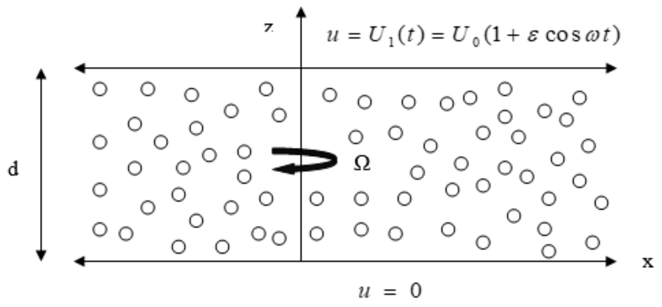

Figure 1. Flow geometry

The modified governing boundary layer equations account for the permeability of the medium are given by

$$\frac{\partial u}{\partial t}=-\frac{1}{\rho}\frac{\partial p^*}{\partial x}+\left(\nu+\alpha\frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial z^2}+2\Omega u-\frac{\nu u}{K_p} \tag{1}$$

$$\frac{\partial v}{\partial t}=-\frac{1}{\rho}\frac{\partial p^*}{\partial y}+\left(\nu+\alpha\frac{\partial}{\partial t}\right)\frac{\partial^2 v}{\partial z^2}-2\Omega v-\frac{\nu v}{K_p} \tag{2}$$

$$0=-\frac{1}{\rho}\frac{\partial p^*}{\partial z} \tag{3}$$

in which $\nu(=\mu/\rho)$ is the kinematic viscosity, $\alpha=\alpha_1/\rho$ and the modified pressure p^* is defined by

$$p^*=p_1-\frac{1}{2}\rho r^2\Omega^2-(2\alpha_1+\alpha_2)\left[\left(\frac{\partial u}{\partial z}\right)^2+\left(\frac{\partial v}{\partial z}\right)^2\right] \tag{4}$$

where p_1 being the fluid pressure and $r^2=x^2+y^2$

Equation (3) indicates that p^* is not a function of z and hence $p^*=p^*(x,y,t)$

The appropriate boundary conditions of the problem are

$$\left.\begin{aligned} z=0 &: u=v=0\\ z=d &: u=U_1(t)=U_0(1+\varepsilon\cos\omega t),\, v=0\end{aligned}\right\} \tag{5}$$

where ε is a constant.

Elimination of p^* from equations (1), (2) and (3) by differentiating with respect to z we get

$$\frac{\partial^2 u}{\partial z\,\partial t}=\left(\nu+\alpha\frac{\partial}{\partial t}\right)\frac{\partial^3 u}{\partial z^3}+2\Omega\frac{\partial v}{\partial z}-\frac{\nu}{K}\frac{\partial u}{\partial z} \tag{6}$$

$$\frac{\partial^2 v}{\partial z\,\partial t}=\left(\nu+\alpha\frac{\partial}{\partial t}\right)\frac{\partial^3 v}{\partial z^3}-2\Omega\frac{\partial u}{\partial z}-\frac{\nu}{K}\frac{\partial v}{\partial z} \tag{7}$$

Integration of above equations with respect to z resulted in the followings

$$\frac{\partial u}{\partial t}=\left(\nu+\alpha\frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial z^2}+2\Omega v-\frac{\nu}{K_p}u+A(t), \tag{8}$$

$$\frac{\partial v}{\partial t}=\left(\nu+\alpha\frac{\partial}{\partial t}\right)\frac{\partial^2 v}{\partial z^2}-2\Omega u-\frac{\nu}{K_p}v+B(t), \tag{9}$$

where A and B are constants of integration and are functions of t. The resulting boundary layer equations (8) and (9) can be combined into following partial differential equation

$$\frac{\partial q}{\partial t}=\left(\nu+\alpha\frac{\partial}{\partial t}\right)\frac{\partial^2 q}{\partial z^2}+\frac{\partial U_1}{\partial t}-\left(2i\Omega+\frac{\nu}{K}\right)(q-U_1) \tag{10}$$

The boundary conditions (5) are reduce to

$$\left.\begin{aligned} z=0 &: q=0\\ z=h &: q=U_1(t)\end{aligned}\right\} \tag{11}$$

where

$$q=u+iv, \tag{12}$$

is the fluid velocity in the complex form. It should be noted that equation (10) includes the Newtonian fluid as a special case when non-Newtonian viscoelastic parameter $\alpha=0$. If $\Omega=0$, the equation reduces to that of second grade fluid in an inertial frame without rotating frame of reference.

3. SOLUTION OF THE PROBLEM

In order to solve equation (10) subject to the boundary conditions (11), we look for the solution based upon the principle superpossebility i.e. the unsteady transient solution is super imposed on the basic steady solution given by Schlitzing and Gersten [21].

$$q(\eta,t)=U_0\left[q_0(\eta)+\frac{\varepsilon}{2}\left\{q_1(\eta)e^{i\omega t}+q_2(\eta)e^{-i\omega t}\right\}\right] \tag{13}$$

$$\left.\begin{aligned}&\text{where}\quad \eta=\frac{z}{d},\, q_0(\eta)=u_0(\eta)+iv_0(\eta)\\ &\text{and}\ q_1(\eta)e^{i\omega t}+q_2(\eta)e^{-i\omega t}=u_1(\eta,t)+iv_1(\eta,t)\end{aligned}\right\} \tag{14}$$

Using (13) and (14) into equations (10) and (11) and then collecting harmonic and non-harmonic terms, we obtain

$$\frac{d^2q_0}{d\eta^2}-\left(2iR+\frac{1}{K_p}\right)q_0=-\left(2iR+\frac{1}{K_p}\right) \tag{15}$$

$$\frac{d^2q_1}{d\eta^2}-(1+i\lambda\beta)^{-1}\left(2iR+\frac{1}{K_p}+i\lambda\right)q_1=-(1+i\lambda\beta)^{-1}\left(2iR+\frac{1}{K_p}+i\lambda\right) \tag{16}$$

$$\frac{d^2q_2}{d\eta^2}-(1-i\lambda\beta)^{-1}\left(2iR+\frac{1}{K_p}-i\lambda\right)q_2=-(1-i\lambda\beta)^{-1}\left(2iR+\frac{1}{K_p}-i\lambda\right) \tag{17}$$

$$\left.\begin{aligned}&\eta=0: q_0=q_1=q_2=0\\ &\eta=1: q_0=q_1=q_2=1\end{aligned}\right\} \tag{18}$$

In above equations, $R=\Omega^2d^2/\nu$ is the dimensionless rotation parameter, $\lambda=\omega d^2/\nu$ is dimensionless oscillating parameters, $\beta=\alpha/d^2$ is the dimensionless second grade parameter and $K_p=K/d^2$ is the permeability of the medium.

Solving the ordinary differential equations (15)-(17) subject to the boundary conditions (18), we get

$$q_0(\eta)=1-\frac{\sinh l(1-\eta)}{\sinh l}, \tag{19}$$

$$q_1(\eta)=1-\frac{\sinh m(1-\eta)}{\sinh m} \tag{20}$$

$$q_2(\eta)=1-\frac{\sinh n(1-\eta)}{\sinh n} \tag{21}$$

We note that the result of Singh [7] can be recovered when the dimensionless material parameter of the second grade fluid is zero. The solution (19) corresponds to the steady part which gives u_0 and v_0 as the primary and secondary velocity components respectively. From equation (19) with the large value of *R*, we have

$$u_0=\mathrm{Re}(q_0)\approx 1-e^{-P\eta}\cos Q\eta \tag{22}$$

$$v_0=\mathrm{Im}(q_0)\approx 1-e^{-P\eta}\sin Q\eta \tag{23}$$

where *P* and *Q* are the real and imaginary parts of *l*.

The solutions (20) and (21) together give the unsteady part of the flow. These solutions depend on β. For large *R*, the primary and secondary velocity components u_1 and v_1 respectively for the fluctuating flow are given by

$$u_1(\eta,t)=\mathrm{Re}(\hat{q}_0(\eta,t))\approx 2\cos\,\omega t-e^{-S\eta}\cos(T\eta-\omega t)-e^{-E\eta}\cos(F\eta+\omega t) \tag{24}$$

$$v_1(\eta,t)=\mathrm{Im}(\hat{q}_0(\eta,t))\approx e^{-S\eta}\sin(T\eta-\omega t)+e^{-E\eta}\sin(F\eta+\omega t) \tag{25}$$

where *S* and *T* are the real and imaginary parts of *m* and *E* and *F* are the real and imaginary parts of *n*.

The amplitudes and phase differences are:

$$R_0=\sqrt{u_0^2+v_0^2},\ \theta_0=\tan^{-1}\left(v_0/u_0\right), \tag{26}$$

$$R_1=\sqrt{u_1^2+v_1^2},\theta_1=\tan^{-1}\left(v_1/u_1\right) \tag{27}$$

4. RESULTS AND DISCUSSION

The governing equations (8) and (9) with boundary conditions (11) are solved basing upon a principle of superpossebility. The transient solution has been superimposed on the basic solution i.e. steady solution. This method has a bearing in the present problem due to oscillatory motion of the upper plate which sets in an oscillatory motion in the flow domain under consideration. Following Schlichting and Gersten [21] we have solved the governing equations. The effects of pertinent parameters such as porosity parameter (K_p) and rotation parameter (R) are discussed on velocity distribution and surface criterion i.e. skin friction.

Fig.2 enunciates primary velocity distribution exhibiting the effects of rotation (R) and permeability parameter (Kp). The distribution is parabolic in nature. From the figure, it is observed that an increase in coriolis force *R* has an accelerating effect. Now, comparing the cases of Kp=0.5 and Kp=0.1, it is concluded that an increase in permeability parameter reduces the velocity since the increasing value of Kp lessens its impact on fluid motion as $\mathrm{Kp}\to\infty$ represents the absence of porous matrix, a clear flow. One striking feature of the distribution is that higher value of R i.e. greater rotation leads to greater non-linearity of the distribution contributing to hike in velocity near the plate. This is due to greater coriolis force. Thus it is concluded that the value of the parameter R is to be assigned suitably to control the growth of boundary layer.

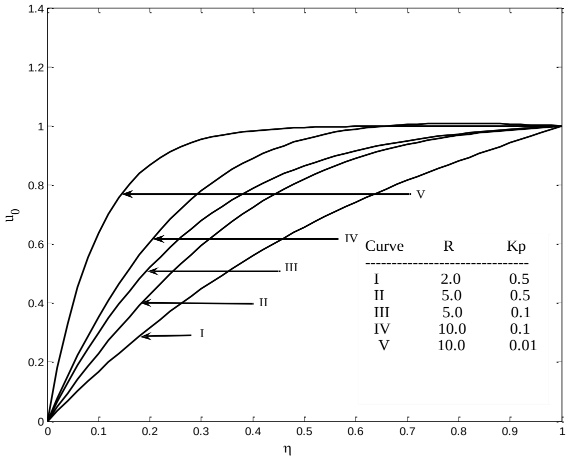

Figure 2. Variation of primary velocity u_0 with η for various values of R and K_p

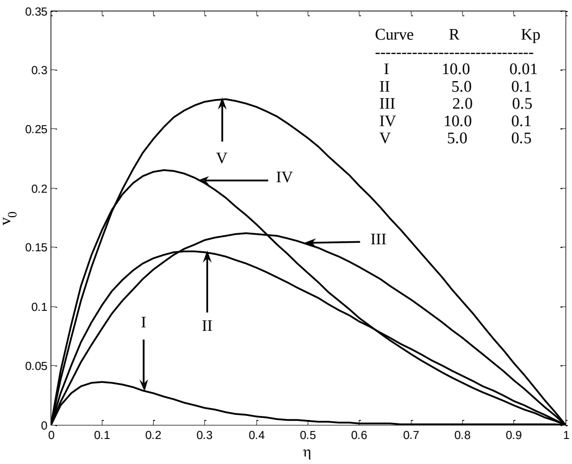

Figure 3. Variation of secondary velocity v_0 with η for various values of R and K_p

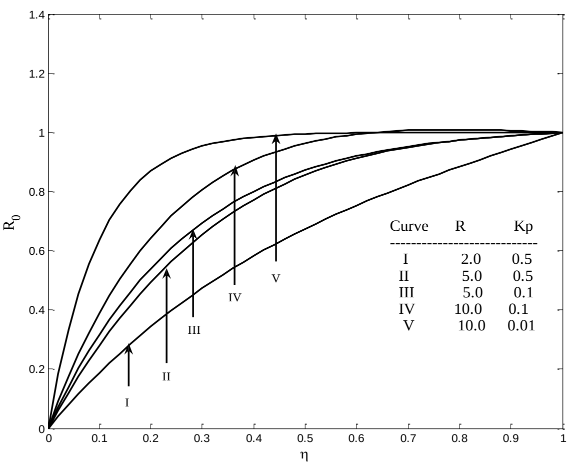

Figure 4. Variation of resultant velocity R_0 with η for various values of R and K_p

Fig.3 reveals that secondary velocity gets enhanced by higher value of R and Kp and is more symmetrical in nature except the case of higher value of R(R=10) associated with low permeability (Kp=0.01). Further the secondary velocity decreases for low value of Kp. The coriolis force has no significant effect on secondary velocity.

Fig.4 presents the resultant of the steady part of the velocity. It is seen that higher rotation accelerates the steady part of velocity whereas porous parameter decelerates it.

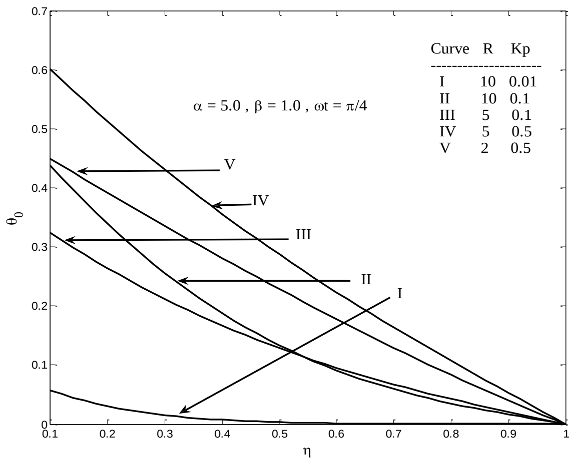

Figure 5. Variation of phase angle θ_0 with η for various values of R and K_p

From fig.5 it is observed that both rotation parameter and porosity parameter increase the phase angle throughout the flow domain. On careful observation it further reveals that low value of K_p ($K_p = 0.01$) and higher value of R ($R=10$) significantly reduce the phase angle. Therefore, it is suggested that for obtaining greater phase difference, greater coriolis force and low permeability of the medium is essential (curves I and IV). It is remarked that the phase difference is large at the lower plate which is at rest. On the other hand, oscillatory plate has no phase difference because the phase difference sets in due to increasing viscous resistance offered by the static plate. It is maximum at the lower static plate where the resistance is maximum.

Fig.6 depicts the variation of unsteady primary velocity. A close observation reveals that an increase in second grade parameter β , decreases the velocity at all the layers. The magnitude of decrease in u_1, commensurate with higher value of β . This is in conformity with the fact that elastic property exhibiting the non-Newtonian characteristics resists the unsteadiness and hence decreases the unsteady part of primary velocity.

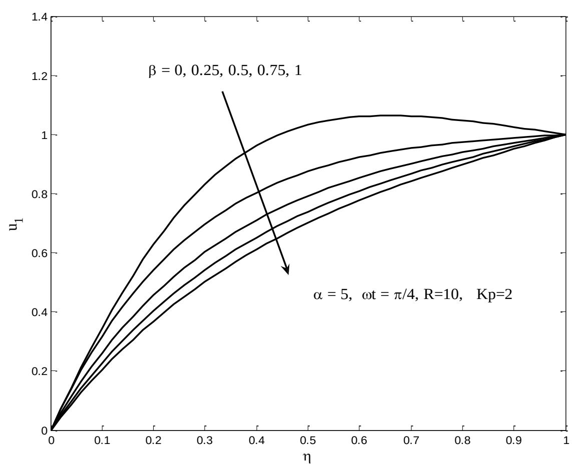

Figure 6. Variation of unsteady primary velocity u_1 with η for various values of β

Fig.7 shows the variation of unsteady secondary velocity in a channel with oscillating upper plate. It is seen that an increase in β , non-Newtonian parameter exhibiting the second grade fluid property(elasticity), opposes the secondary velocity, same as the primary velocity (Fig.6). The profile structure is symmetrical and attains maximum in case of $\beta = 0$ i.e. for Newtonian fluid.

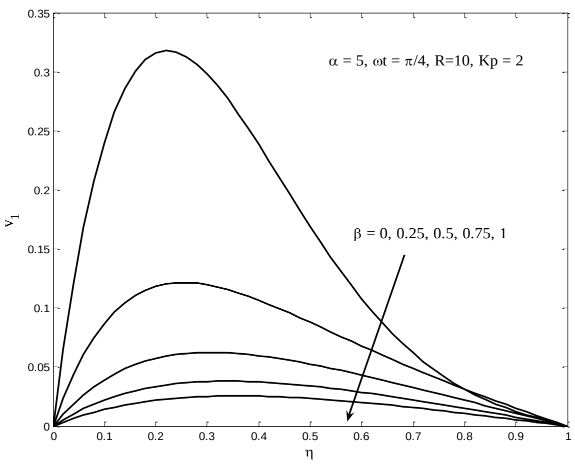

Figure 7. Variation of unsteady secondary velocity v_1 with η for various values of β

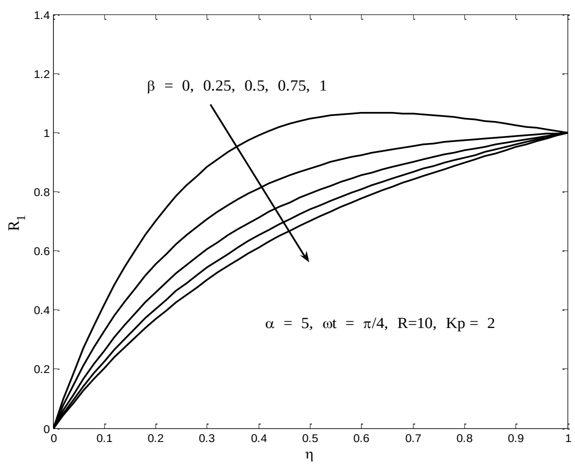

Figure 8. Variation of resultant velocity R_1 with η for various values of β

Fig.8 shows the contribution of non-Newtonian parameter β on resultant of unsteady part. The effect remains same as that of primary velocity (Fig.6).

Fig.9 shows the variation of phase angle θ_1 due to unsteady contribution of velocity profile. It is seen that phase difference decreases with an increase in β (the second grade fluid property) and attains minimum for $\beta = 1.0$. The striking feature of the variation is that phase difference attains maximum in case of Newtonian fluid ($\beta = 0$). Thus, the second grade fluidity induces a phase difference in unsteady transient motion of the fluid flow. Moreover, second grade fluid with higher grade property gives rise to thinner boundary layer structure which is desirable.

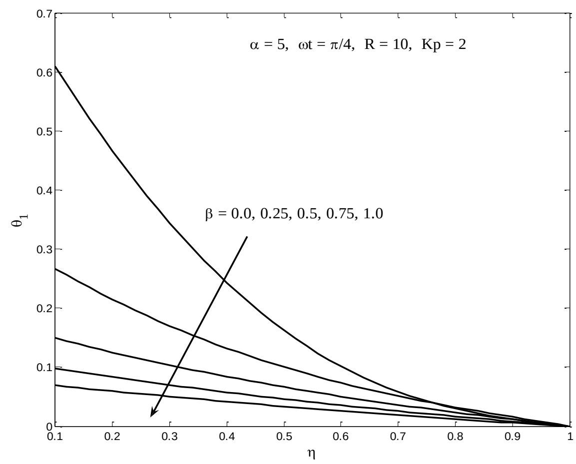

Figure 9. Variation of phase angle θ_1 with η for various values of β

5. CONCLUSION

I. Greater rotation leads to hike in velocity near the plate.
II. The coriolis force has no significant effect on secondary velocity.
III. The higher coriolis force and low permeability contribute to greater phase difference whereas elasticity property reduces it. This may be of design requirement for obtaining resonance (Fig. 9).
IV. Fluid elasticity reduces the unsteady part as well as resultant of the primary velocity and opposes the secondary velocity.

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APPENDIX

$$l=\sqrt{2iR+(1/K_p)},\ m=\left[\frac{2iR+(1/K_p)+i\lambda}{(1+i\lambda\beta)}\right]^{\frac{1}{2}},$$

$$n=\left[\frac{2iR+(1/K_p)-i\lambda}{(1-i\lambda\beta)}\right]^{\frac{1}{2}}$$

$$P=(1/\sqrt{2})\left[\sqrt{\left(1/K_p\right)^2+4R^2}+\left(1/K_p\right)^2\right]^{\frac{1}{2}},$$

$$Q=(1/\sqrt{2})\left[\sqrt{\left(1/K_p\right)^2+4R^2}-\left(1/K_p\right)^2\right]^{\frac{1}{2}}$$

$$S=(C_1)^{-1}\left[\sqrt{\sqrt{A_1^2+B_1^2}+A_1}\right],$$

$$T=(C_1)^{-1}\left[\sqrt{\sqrt{A_1^2+B_1^2}-A_1}\right]$$

$$E=(C_1)^{-1}\left[\sqrt{\sqrt{A_2^2+B_2^2}+A_2}\right],$$

$$F=(C_1)^{-1}\left[\sqrt{\sqrt{A_2^2+B_2^2}-A_2}\right]$$

$$A_1=(1/K_p)+(2K+\lambda)\lambda\beta,$$

$$B_1=(2K+\lambda)-(1/K_p)\lambda\beta,$$

$$A_2=(1/K_p)+(2K-\lambda)\lambda\beta$$

$$B_2=(2K-\lambda)-(1/K_p)\lambda\beta,\ C_1=\sqrt{2(1+\lambda^2\beta^2)}$$