Development of Production Function for a Solar Water Heating System

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Abstract: ***Inspite of the present high worldwide interest in substituting solar produced energy for conventional non-renewable energy resources, very little research work could be found on development of production functions for solar systems. The production function for a solar system can help to predict its energy output in different conditions and also helps to obtain the economically optimum design for an application. This paper shows the development of production functions for a forced flow solar water heating system for residential buildings, hotels and hospitals in Mumbai. They give the annual solar energy output as a function of collector area, tank volume and daily average hot water requirement of the load.***

Keywords: Solar water heating system, Production function, Multiple polynomial regression

1. Introduction

A production function is a technical relationship specifying the maximum amount of output capable of being produced by each and every set of specified inputs [1]. It is defined for a given state of technical knowledge. It is mostly used in management, economics and shop-floor production. A production function can be expressed in the following form: -

$Q = f(X_1, X_2, X_3, X_4 \dots X_n)$

Where Q is the quantity of output and $X_1, X_2, X_3, X_4 \dots\dots X_n$ are quantities of various inputs.

2. Forms of Production Function

2.1 Cobb-Douglas production function [1]

The Cobb-Douglas type of production function has been used most frequently in attempts to express mathematically the input-output relationship of firms or industries. This function is of the form:

$$Q = A x_1^{a1} x_2^{a2}$$

Where Q is the output and x_1 & x_2 are variable inputs. A, a1, a2 are constants. If this function is transformed into logarithms it reduces to a simple linear equation and can be solved by the familiar means of least squares. Example, it is used for finding output at a given time for the inputs as labor and capital [2].

2.2 Polynomial production function [3]

A particularly simple production function is a multiple-variable polynomial with coefficients based on regression analysis of system performance in which data is taken either from computer models or actual equipment. It is expressed in the form:

$$Q = \sum_{i=1}^{n} p_i(x_i)$$

Where, p_i is the polynomial in the parameter x_i.

Polynomial production function can be selected to have any number of terms required to ensure the best accuracy as adjudged by an F-test. The coefficients and exponents do not provide any additional information as is the case in other production functions below. The principal advantage of polynomial is its simplicity.

2.3 Logarithmic and exponential production function [3]

A production function can be formed from logarithms or their inverse exponents as follows:

$$Q = \ln \prod_{i=1}^{N} a_i x_i^{ni}$$

Where n_i are powers (integral or non-integral) and a_i are coefficients. This form is equivalent to

$$Q = A + n_1 \ln x_1 + n_2 \ln x_2 + \dots$$

The coefficients n_i can be evaluated by regression methods using data relating Q to x_i. The logarithmic form has a limited number of coefficients and possibly lower regression accuracy than polynomial form.

2.4 Power-Law production functions [3]

A production function can be formed from the product of powers of physical inputs. It can be represented as follows:

$$Q = v_0 \prod_{i=1}^{N} x_i^{v_i}$$

As in the previous cases the v_i's are determined by regression analysis of performance data. The exponents in the power-law production function are similar to those in the logarithmic expression and represent the percentage change in output for a percentage change in input. It is easy to show that

$$\frac{dQ}{Q} = \sum_{i=1} v_i \frac{d x_i}{x_i}$$

In addition, the sums of the v_i's represent the return-to-scale. Returns-to-scale are the change in output Q resulting from a uniform unit change in all inputs x_i. If it is greater than one, returns-to-scale are increasing; if less than one, decreasing; if equal to one, proportionate.

2.5 Numerical production functions [3]

In this type of production function performance data itself can be taken as production function instead of evaluating it by regression methods. However, no information regarding returns-to-scale can be determined from a numerical production function. Also, this production function requires digital computers to perform optimization calculations.

3. Production Functions Applied to Solar Systems [3]

The calculation of a technical production function is not an exact science; an empirical method is used to relate energy output to solar system parameters such as collector size, efficiency and orientation, storage size, control strategy, and many other factors. Nearly all inputs to a solar system production function are subject to a law of diminishing returns. This basic technological law states that extra output from a system diminishes relatively when successive equal units of an input are added to fixed amounts of other inputs. The production function can be applied to the solar systems like solar space heating, solar service water heating, solar space cooling, solar dehumidification, low temperature process heat, crop drying, distillation, swimming pool heating etc. The use of a production function is explained through an example problem as follows [3]:

A production function for a particular solar space-heating system in Chicago is of the form:

$$Q_s = L\left\{0.8 + \ln\left[\left(\frac{Ac}{L}\right)^{1/3}\left(\frac{S}{L}\right)^{1/20}\right]\right\}$$

Where, Ac is the collector area, L is the annual energy demand of 100 GJ, and S is the storage size. We have to determine the expected increase in annual energy delivery Qs for a 1 percent increase in both storage size and collector size if the load remains unchanged.

Expanding the above equation;

$$Qs = L\left(0.8 + \frac{1}{3}\ln Ac + \frac{1}{20}\ln S - \frac{23}{60}\ln L\right)$$

For small changes in Ac or S, the differential approximation can be used. Differentiating we have

$$dQs = \frac{L}{3}\frac{dAc}{Ac} + \frac{L}{20}\frac{dS}{S}$$

Therefore, for a 1 percent increase in collector area Ac (dAc/Ac=0.01),

$$dQs = \frac{100{,}000MJ}{3} \times 0.01 = 333MJ$$

For a 1 percent increase in storage S,

$$dQs = \frac{100{,}000MJ}{20} \times 0.01 = 50MJ$$

From this example, a one percent increase in collector area Ac delivers more than six times more energy than the same percentage increase in storage delivers.

In this paper we will explain the procedure for the development of production function for a forced flow solar water heating system done by us.

4. Production Function Development for a Forced Flow Solar Water Heating System

4.1 Performance data generation by simulation:

When a large amount of hot water is required for supplying heat in residential buildings or in commercial establishment, large arrays of flat plate collectors are used and forced circulation is maintained with a water pump. The most common system is shown below.

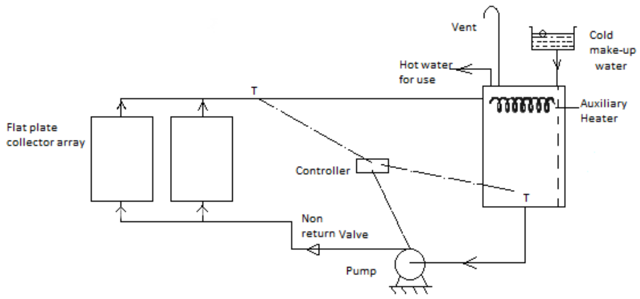

Figure: Closed loop forced circulation solar water heating system [4]

Water from the storage tank is pumped through a collector array, where it is heated and then flows back into the storage tank. Whenever hot water is withdrawn for use, cold make-up water takes its place because of the ball-float control shown in make-up water tank. The pump for maintaining the forced circulation is operated by an on-off controller which senses the difference between the temperature of the water at exit of the collectors and bottom of the storage tank. The pump is switched on whenever this difference exceeds a certain value and off when it falls below a certain value. Provision is also made for an auxiliary heater which is switched on if the hot water delivery temperature is not high enough [4]. In this system the dependent variable i.e. output is the annual energy delivery by the solar collectors, known as useful energy gain of a system. This depends on its independent variables i.e. various inputs to the system, like physical configuration, operating temperature, solar radiation level, ambient temperature, wind speed, fluid flow rate, collector orientation, tank volume, insulation to tank, load profile etc. In the following study we will find the relationship between the output i.e. solar energy gain and the inputs i.e. collector area, tank volume and daily hot water requirement.

The production function is developed for large forced flow solar water heating systems in or near Mumbai for applications such as hotels, residential buildings, hospitals. These sites have a daily load profile which is well represented by the Rand profile [5]. A load hot water temperature requirement of 60°C, with ambient temperature of 25°C is assumed. The performance data for this system is obtained by simulating a forced flow solar water heating system on TRNSYS17 software [6]. Weather Data for location of Mumbai, latitude 19.1° is taken from the TRNSYS17 weather database, which is based on monthly values that Meteonorm data source generates through its worldwide centers located in more than 1000 locations in more than 150 countries.

The vertical storage tank volume was varied in the range 2000 to 10000 liters and average daily hot water load in the range 2000 to 9000 liters per day. The TRNSYS17 program developed generated total solar energy gain over the year i.e. from 1st Jan to 31st December.

4.2 Developing Regression Equations

Multiple polynomial regression was applied to the data obtained for each collector area. The independent variables were taken as tank volume V_T, in liters; daily hot water load D_L, in liters/day and the dependent variable is Qs (kJ/year) i.e. solar energy gain.

Other system parameters assumed are:
- Collector fin efficiency: 0.9
- Number of covers: 1
- Collector tilt angle: 19.1°
- Controller upper dead band temperature: 10°C
- Controller lower dead band temperature: 1°C
- Pump flow rate is 0.01 kg/m^2 of collector area.
- Stratified vertical storage tank with 5 nodes is used.

The software used for finding multiple polynomial regressions was from the website www.xuru.com [7]. The following production functions were found for the collector array areas for which data was generated:

1. Collector area 40 m^2:

$$Qs = -1.13478\ V_T^2 + 1.24626\ V_T.\ D_L - 2.87576\ D_L^{\ 2} + 6402.636054\ V_T + 25416.58163\ D_L + 74881462.59 \quad (1)$$

Coefficient of determination was calculated [8] [9], and found to be = R^2 = 0.9940

Sample correlation coefficient, $|r|$ = 0.9969

2. Collector area 60 m^2:

$$Qs = -1.21506\ V_T^{\ 2} + 0.714853\ V_T.\ D_L - 1.70012\ D_L^{\ 2} + 12158.84765\ V_T + 24241.58415\ D_L + 98949971 \quad (2)$$

With R^2 = 0.9929, $|r|$ = 0.9964

3. Collector area 80 m^2:

$$Qs = -1.04903\ V_T^{\ 2} + 1.10977\ V_T.\ D_L - 1.0219\ D_L^{\ 2} + 9230.09519\ V_T + 27228.80474\ D_L + 136886003.7 \quad (3)$$

With R^2 = 0.9883, $|r|$ = 0.9991

4. Collector area 100 m^2:

$$Qs = -1.11447\ V_T^{\ 2} + 1.52074\ V_T.\ D_L - 2.46149\ D_L^{\ 2} + 8286.91918\ V_T + 37459.49044\ D_L + 139097620.4 \quad (4)$$

With R^2 = 0.9816, $|r|$ = 0.9907

In these equations, Qs is total annual solar energy output, including heat contained in hot water at temperatures above 60°C in the top node of the tank. This energy above 60°C is a waste as our load needs only 60°C heat. In monsoon months temperature often does not reach 60°C, due to which the auxiliary heater comes on anyway regardless of collector area. We therefore calculate a useful solar energy output which is the heat obtained from solar collectors, which is up to 60°C. It is calculated as annual load heat requirement [daily load * specific heat of water * temp rise of water (60-25) * 365 - Auxiliary energy consumption]. A correlation is found for Qs with Qus for each collector area.

5. Results and Conclusions

We can use these equations for sizing a solar water heating system, which is explained through the following example:

A residential building in Mumbai has a daily hot water load of 2000 liters per day at 60°C. For this system required total heat Q_T is calculated as (m.*Cp.*ΔT) = 106.9 MJ. Assuming that our solar water heating system will have a solar fraction of 90%, the annual useful solar heat Qus would be (0.9 * Q_T =) 96.21 MJ/yr.

From the relation Qs = 0.5134 Qus + (8 * 10^7), for a 40 m^2 collector area, we find a system with Qs of 129.4 MJ/yr. is needed.

Taking daily load = 2000 liters/day and tank volume 3000 liters, we get from equations (1) & (2) for different collector areas:

40 m^2: Qs = 130.6 MJ
60 m^2: Qs = 170.5 MJ

We find that our annual design requirement of Qs=129.4 MJ is met by a 40 m^2 collector area with a storage tank of 3000 liters, thus giving us a suitable solar water heater design.

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